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Controllability of Stochastic Evolution Equations Driven by Poisson Random Measure

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Abstract: In this paper, we investigated controllability of a stochastic partial equation driven by Poisson random measure. The stochastic equation was presented as a stochastic evolution equation which is an abstract formulation for stochastic partial equations. Using semigroup theory, we consider the mild solution of stochastic equation. And then the successive approximations method is used to consider the controllability in this paper. An application to a Parabolic SPDE is given in the last section of the paper.

Keywords: Controllability,Stochastic Evolution Equations, Poisson Random Measure, Successive Approximations.

1. Introduction

It is well know, parabolic SPDEs [1] driven by Poisson random measure had been initially introduced and discussed by Walsh [21], where he mentioned the cable equation as example. Stochastic partial equations driven by Lévy processes have been widely studied when the equations are place in Hilbert spaces. There exists some literature on this subject, for instance, Albeverio, Wu and Zhang [2], Rockner and Zhang [20] and Loka, Osendal and Proske [15] (and references therein). In [10,11], the author investigated SPDEs which be in certain Banach space, the space is of M type p, driven by Poisson random measure. The obtained are existence, uniqueness and regularity of the solution of the equations.

On the other hand, control theory of stochastic systems has recently received a lot of attention. See, for example, Balasubramaniam and Dauer [3–5], Subalakshmi, Balasubramaniam and Park [19] and Mahmudov [16,17]. There are many literature presents in references therein. Almost all of the paper, the stochastic systems be studied are the stochastic systems driven by Brownian motion with finite trace nuclear covariance operator. The results of these papers were obtained using properties of operators, compact semi groups and the fixed-point theorem.

In recent years, the extensions of stochastic control systems with Gaussian white noise to the systems with Poisson white noise have been recently discussed only in a rather few number of publications. In this paper, we consider stochastic systems driven by Poisson white noise which can be present as Poisson random measure. The method that will be used to study the existence of solution for stochastic equations driven by Poisson white noise is used in this paper. The successive approximations method and semigroup theory are used in the paper.

The paper is organized as follows: section 2, we introduce the basic notations and assumptions which be necessary to formulate the results in next section. Then the main result be presented in the third section and the proof be given. In section 4, an example is presented to prove the Theorem.

2. Preliminaries

In this section, we present some definitions and preliminary results that will be used in next sections. Let $(\Omega, \mathcal{F}, P)$ be a probability space with filtration $\{\mathcal{F}_t : 0 \le t \le T\}$. H, Z and U are separable Hilbert spaces and the space H with norm $\|\cdot\|$. $\mathcal{L}(U; H)$ is the space of all linear bounded operators from U to H.

In this paper, our point of interest is the controllability

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of stochastic systems driven by Poisson random measures. So, in that follow, we give the definition of the Poisson random measure.

Definition 2.1 [11, Definition 2.2] Let $(Z, \mathcal{B}(Z))$ be a measurable space and $(\Omega, \mathcal{F}, P)$ be a probability space. A time homogeneous Poisson random measure $N(t,z)$ on $(\mathcal{B}(\mathbb{R}^+), \mathcal{B}(Z))$ is a collection of random variables $\{N(I,A); I \in \mathcal{B}(\mathbb{R}^+), A \in \mathcal{B}(Z)\}$ such that

i) $N(I, \varnothing) = 0$, $I \in \mathcal{B}(\mathbb{R}^+)$ and $N(\varnothing, A) = 0$ a.s. for $A \in \mathcal{B}(Z)$;
ii) N is a.s. σ-additive;
iii) N is independently scattered, i.e. for any family of disjoint sets $(I_1, A_1) \cdots, (I_n, A_n) \in (\mathcal{B}(\mathbb{R}^+), \mathcal{B}(Z))$, the random variables

$$N(I_1, A_1) \cdots, N(I_n, A_n) \text{ are independent;}$$

iv) For each $A \in \mathcal{B}(Z)$ and $I \in \mathcal{B}(\mathbb{R}^+)$ such that $EN(I,A)$ is finite, $N(I,A)$ is a Poisson random variable with parameter $EN(I,A)$.

The measure ν is called the characteristic measure of N which is defined by

$$\nu : A \in \mathcal{B}(Z) \longmapsto EN((0,1], A) \in [0, \infty].$$

The compensator $\gamma : \mathcal{B}(\mathbb{R}^+) \times \mathcal{B}(Z) \to \mathcal{B}(\mathbb{R}^+)$ of N is the unique predictable random measure and such that

$$N((0,t], A) - \gamma((0,t], A)$$

is a martingale for each $A \in \mathcal{B}(Z)$. Then we call a Poisson random measure $\widetilde{N}$ compensated, if $\widetilde{N} = N - \gamma$.

Remark 2.1 If $\gamma(dz, dt) = \nu(dz)dt$, then there is a Lévy process $\{Y(t)\}_{t\geq 0}$ and $N(dt, dz)$ is the Poisson random measure associated to the additive process $\{Y(t)\}_{t\geq 0}$. Where dt denotes the Lesbegues measure on $\mathcal{B}(\mathbb{R}^+)$, and $\nu(dz)$ is a σ-finite measure on $(Z\backslash\{0\}; \mathcal{B}(Z\backslash\{0\})$. The measure $\nu(dz)$ is called Lévy measure associated to $\{Y(t)\}_{t\geq 0}$.

Now, we consider a stochastic evolution equation driven by Poisson random measures of the form

$$\begin{cases} dX(t) = [AX(t) + (Bu)(t)]dt \\ \quad + \int_Z g(t, X(t), z) N(dt, dz) \\ X(0) = x_0 \in H \end{cases} \tag{1}$$

where $g : I \times H \times Z \to H$ is a given function and $x_0 \in H$. The control function $u(\cdot)$ be given in $L^2(I;U)$ which is a Banach space of admissible control function and the space U is a real separable Banach space with the norm $\|\cdot\|_u$. The operator B is a bounded linear operator from U into H that is $B \in \mathcal{L}(U;H)$ and the operator $A : D(A) \to H$ is the infinitesimal generator of a compact analytic semigroup $\{S(t)\}_{t\geq 0}$ which is a uniformly bounded linear operator.

In order to consider controllability of the system (1), we present some spaces and a Proposition that will be used in Section 3. Let the set

$$L_2(\nu) = \{h : \Omega \times \mathbb{R}^+ \to L(Z,H), h \text{ is predictable cáglád process such that } \int_0^t \int_Z E\|h(s,z)\|^2 \nu(dz)ds < \infty\}$$

and $\mathbb{D}_2([0,t]; L_2(\Omega, \mathcal{F}, P; H))$ is a Skorohod space equipped with following norm

$$\|X\|_d^2 := E \sup_{0\leq s\leq t} \|X(s)\|^2, \quad X(s) \in \mathbb{D}_2([0,t]; L_2(\Omega, \mathcal{F}, P; H)).$$

From the Skorohod space, we know that $\mathbb{D}_2([0,t]; L_2(\Omega, \mathcal{F}, P; H))$ is complete, so there exists a bounded linear operator

$$I(h) : L_2(\nu) \to \mathbb{D}_2([0,t]; L_2(\Omega, \mathcal{F}, P; H)),$$

where $I(h) := \int_0^t \int_Z h(s,z)N(ds,dz)$. The next proposition is the Proposition 3.3 in paper [10].

Proposition 2.1 Assume Z and H are separable Banach spaces, and where the space H is a Hilbert space. Let $\mathcal{B}(Z)$ and $\mathcal{B}(H)$ are Borel σ-algebras. Then there exists some constant $C_4 < \infty$ such that for all Poisson random measures N on $\mathcal{B}(Z) \times \mathcal{B}(\mathbb{R}^+)$ with characteristic measure ν which is symmetric Lévy measure, and all functions $h : \Omega \times \mathbb{R}^+ \times Z \to H$ belongs to $L_2(\nu)$ we have

$$\|\int_{0+}^s \int_Z h(\tau,z)N(d\tau,dz)\|_d^r \leq CE(\int_0^t \int_Z \|h(s,z)\|^2 N(d\tau,dz))^{\frac{r}{2}}, \quad 0 < r < \infty,$$

and

$$\|\int_{0+}^s \int_Z h(\tau,z)N(d\tau,dz)\|_d^2 \leq C_4(\int_0^t \int_Z E\|h(s,z)\|^2 \nu(dz)ds).$$

We give a definition of mild solution of the problem (1) and controllability of the stochastic evolution equation (1).

Definition 2.2 A function $X(t) \in \mathbb{D}_2([0,t]; L_2(\Omega, \mathcal{F}, P; H))$ is said to a mild solution of (1), if $X(0) = x_0$ and $X(t)$ satisfied the following integral equation

$$X(t) = S(t)x_0 + \int_0^t S(t-s)Bu(s)ds + \int_0^{t+} \int_Z S(t-s)g(s, X(s), z)N(ds,dz). \tag{2}$$

Definition 2.3 The stochastic evolution equation (1) is said to be controllable on $[0,T]$, if for every $X(0) = x_0 \in H$ and $x_1 \in H$, there exists a control $u \in L^2(I;U)$ such that the mild solution $X(\cdot)$ of (1) satisfied $X(T) = x_1$.

For the controllability of the system (1), we need to introduce the following hypotheses.

(H1) For semigroup $\{S(t)\}_{t\geq 0}$, there exists positive constants C_1 and w such that

$$\|S(t)\| \leq C_1 e^{-wt}, \quad t \geq 0;$$

(H2) Assume that for any $T > 0$, there exist some constants K_T and L_T such that

$$\begin{aligned}&\int_Z \|g(t,X(t),z)\|^2\nu(dz)\\&\quad\le K_T(1+\|X(t)\|^2)\\&\int_Z \|g(t,X_1(t),z)-g(t,X_2(t),z)\|^2\nu(dz)\\&\quad\le L_T\|X_1(t)-X_2(t)\|^2;\end{aligned}$$

(H3) The linear operator W from $L^2(I;U)$ into H defined by

$$Wu=\int_0^T S(T-s)Bu(s)ds$$

has an invertible operator W^{-1} defined on $H\backslash\ker W$ and there exist positive constants C_2, C_3 such that

$$\|B\|^2\le C_2 \quad \text{and} \quad \|W^{-1}\|^2\le C_3.$$

(H4) The characteristic measure ν of Poisson random measure $N(dt,dz)$ in equation (1) is a symmetric Lévy measure.

3. Main Result

Now, we present main result of our paper in this section and then give a proof of the result.

Theorem 3.1 Suppose hypotheses (H1)-(H4) are satisfied, then the system (1) is completely controllable on $[0,T]$.

We know that we want to prove the theorem is true just to prove that the system (1) exists a mild solution $X(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for all x_0 and T. In this section, we prove existence of the solution by successive approximations method and induction method. The proof is complete by following Lemmas.

At first, from the hypothesis, we define the control

$$\begin{aligned}u(t)=&-W^{-1}[S(t)x_0\\&+\int_0^T\int_Z S(T-(s-))g(s-,X(s-),z)N(ds,dz),\end{aligned}$$

using the control, the operator Φ defined by

$$\begin{aligned}(\Phi X)(t)=&S(t)x_0-\int_0^t S(t-\tau)BW^{-1}[S(T)x_0\\&+\int_0^T\int_Z S(T-(s-))g(s-,X(s-),z)N(ds,dz)]d\tau\\&+\int_0^t S(t-(s-))g(s-,X(s-),z)N(ds,dz).\end{aligned}\tag{3}$$

Therefore, we have that the system (1) exists a mild solution if and only if the operator $\Phi X(t)$ has a fixed point, that is there exists $X(t)$ satisfying $\Phi X(t)=X(t)$. In order to prove that the operator defined by equation (3) has a fixed point, we set

$$X_1(t)=S(t)x_0-\int_0^t S(t-s)BW^{-1}S(T)x_0ds,$$

and

$$\begin{aligned}X_{n+1}(t)=&X_1(t)-\int_0^t S(t-\tau)BW^{-1}\\&\times\int_0^T\int_Z S(T-(s-))g(s-,X_n(s-),z)N(ds,dz)]d\tau\\&+\int_0^t S(t-(s-))g(s-,X_n(s-),z)N(ds,dz)\\=&X_1(t)+I_1(t)+I_2(t).\end{aligned}\tag{4}$$

For $(t,\omega)\in I\times\Omega$ and $n\in\mathbb{N}$(where $I_1(t)$ and $I_2(t)$ denote the second and third term respectively on the right-hand side of the second equality). We have following result for $(X_n(t))_{n\in\mathbb{N}}$.

Lemma 3.2 $X_1(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ and if $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for some $n\in\mathbb{N}$, then $h(t,X_n(t)):=S(t-(s-))g(s-,X_n(s-),z)\in L_2(\nu)$ for any arbitrarily fixed $t\in[0,T]$.

Proof: For any fixed $t\in[0,T]$, we defined

$$\begin{aligned}X_n^m(s):=&X_n(0)\\&+\sum_{k=0}^{2^m+1}X_n(\tfrac{kt}{2^m})\mathbf{1}_{(kt/2^m,((k+1)t+1)/2^m]}(s),\quad m\in\mathbb{N}\end{aligned}$$

for $s\in[0,t]$. Clearly, $X_n^m(s)$ is $\mathcal{F}_s$-predictable. And we set

$$h^m(t,x(t)):=S(t-s)g(s,X_n^m(s),z), m\in\mathbb{N}\tag{5}$$

for $s\in[0,t]$, then $h^m(t,X_n(t))$ is $\mathcal{F}_t$-predictable. Let us show that for any fixed $m\in\mathbb{N}$ and $t\in[0,T]$ the function $h^m(t,X_n(t))$ defined by (5) belong to $L_2(\nu)$. In fact, by the assumption (H2) and Schwarz inequality, we have

$$\begin{aligned}&E\int_Z\|h^m(t,X_n(t))\|^2\nu(dz)\\&\quad=E\int_Z\|S(t-s)g(s,X_n^m(t),z)\|^2\nu(dz)\\&\quad\le E\int_Z\|S(t-s)\|^2\|g(s,X_n^m(t),z)\|^2\nu(dz)\\&\quad\le C_1e^{-w(t-s)}\cdot K_T(1+E\|X_n^m(t)\|^2)\\&\quad<\infty.\end{aligned}\tag{6}$$

Hence, $h^m(t,X_n(t))$ is well defined by (5). Moreover, by (6), we have

$$\begin{aligned}&\int_0^t\int_Z E\|h^m(t,X_n(t))\|^2\nu(dz)\\&\quad\le\int_0^t C_1e^{-w(t-s)}\cdot K_T(1+E\|X_n^m(t)\|^2)ds<\infty\end{aligned}$$

for $t\in[0,T]$. Thus $h^m(t,X_n(t))\in L_2(\nu)$.

On the other hand, by the second inequality of the assumption (H2), we have

$$\begin{aligned}&E\int_0^t\int_Z\|h^m(t,X_n(t))-h^l(t,X_n(t))\|^2\nu(dz)\\&\le\int_0^t\|S(t-s)\|^2ds\cdot\int_0^t\int_Z E\|g(s,X_n^m(t-),z)\\&\quad-g(s,X_n^l(t-),z)\|^2\nu(dz)ds\\&\le\int_0^t C_1e^{-w(t-s)}ds\cdot L_T\int_0^t E\|X_n^m(t-)-X_n^l(t-)\|^2)ds\\&\longrightarrow 0\ \text{ as }\ m,l\to\infty,\end{aligned}$$

where the last line follows from the existence of the left limit $X_n(s-)$ of $X_n(s)$ in $L^2(\Omega)$. Hence $h(t,X_n(t))\in L^2(\nu)$ for any fixed $t\in[0,T]$.

Lemma 3.3 If $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for some $n\in\mathbb{N}$, then the two integral terms $I_1(t)$ and $I_2(t)$ on the right-hand side of (4) are well defined and $I_1(t),\ I_2(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$.

Proof: First, we prove that $I_1(t)$ and $I_2(t)$ are well defined. For the second integral $I_2(t)=\int_0^t\int_Z S(t-s)g(s,X_n(t),z)N(ds,dz)$. Remarking that by Lemma 3.2, we have $h(t,X_n(t))\in L_2(\nu)$, thus

$I_2(t)$ is well defined (and in fact) as the $L^2(\Omega)$-limit of the Cauchy sequences $\{\int_0^{t+}\int_Z h^m(s,X(s))N(ds,dz)\}_{m\in\mathbb{N}}$. Then, the first integral

$$I_1(t)=\int_0^t S(t-\tau)BW^{-1} \times \int_0^T\int_Z S(T-(s-))g(s-,X(s-),z)N(ds,dz)]d\tau$$

is well defined as the $L^2(\Omega)$-limit of the Cauchy sequences

$$\{\int_0^t S(t-\tau)BW^{-1}\int_0^T\int_Z h^m(s,X(s))N(ds,dz)]d\tau\}_{m\in\mathbb{N}}.$$

Second, we prove that $I_2(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$, and induction that $I_1(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$. First of all, we observe that $I_2(t)$ is right continuous in variable t which is obvious from the fact that $I_2(t)$ is a well-defined integral, since the upper limit of $I_2(t)$ is given by the right limit of t. Now, for any fixed $t\in[0,T]$, we set

$$J_t(r):=\int_0^{r+}\int_Z h(s,X(s))N(ds,dz),\quad r\in[0,t]. \tag{7}$$

Then, by (5), $\{J_t(r)\}$ is square integrable $\mathcal{F}_t$-martingale with quadratic variational process given by the following (non-stochastic) integral

$$<J_t(r)>=\int_0^r\int_Z\|S(t-s)g(s,X_n(s),z)\|^2\nu(dz)ds\quad r\in[0,t]. \tag{8}$$

Since we have derived that $h(t,X_n(t))\in L_2(\nu)$ from the proof of Lemma 3.2, it is well know that (see e.g. Theorem I.6.9 of Ikeda and Watanabe 1981) $J_t(r)$ has a cádlág version in the variable $r\in[0,t]$. On the other hand, we have $I_2(t)=J_t(t)$. Hence $I_2(t)$ is $\mathcal{F}_t$-measurable. Moreover, since

$$\begin{aligned}&E[J_r(r)-J_t(r)]^2\\&=E\int_0^{r+}\int_Z\|[S(r-s)\\&\quad-S(t-s)]g(s,z,X_n(s))\|^2\nu(dz)ds\\&\le E\int_0^r\|[S(r-s)-S(t-s)]\|^2\\&\quad\times\int_0^{r+}\int_Z\|g(s,z,X_n(s))\|^2\nu(dz)ds,\end{aligned}$$

that is

$$L^2(\Omega)-\lim_{r\uparrow t}(J_r(r)-J_t(r))=0,$$

we have

$$\begin{aligned}L^2(\Omega)-\lim_{r\uparrow t}I_2(t)&=L^2(\Omega)-\lim_{r\uparrow t}J_r(r)\\&=L^2(\Omega)-\lim_{r\uparrow t}J_t(r)\\&=J_t(t-).\end{aligned}$$

Hence, the left $L^2(\Omega)$-limit of $I_2(t)$ exist for all $t\in[0,T]$. Combining with the right continuity of $I_2(t)$ in t, we get that $I_2(t)$ is modified cádlág in t.

By (8), we obtain

$$\begin{aligned}&E\|I_2(t)\|^2=E[J_t(t)]^2\\&=E\int_0^t\int_Z\|S(t-s)g(s,z,X_n(s))\|^2\nu(dz)ds\\&\le\int_0^t\|S(t-s)\|^2ds\cdot E\int_0^t\int_Z\|g(s,z,X_n(s))\|^2\nu(dz)ds\\&\le\int_0^t\|S(t-s)\|^2ds\cdot\int_0^t K_T(1+E\|X_n(s))\|^2)ds\\&<\infty,\end{aligned} \tag{9}$$

that is $I_2(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$.

Since $I_1(t)=\int_0^t S(t-\tau)BW^{-1}I_2(T)d\tau$ and the semigroup $S(t-\tau)$ is continuous in t. Hence, $I_1(t)$ is continuous in the variable t. By (9), we get

$$\begin{aligned}E\|I_1(t)\|^2&=E\int_0^t\|S(t-\tau)\|^2\|B\|^2\|W^{-1}\|^2\|I_2(T)\|^2d\tau\\&<\infty.\end{aligned}$$

So we have $I_1(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$.

Lemma 3.4 $\forall T>0$, $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for all $n\in\mathbb{N}$.

Proof: First, we prove that if $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for some $n\in\mathbb{N}$ then $X_{n+1}(t)\in\mathbb{D}_2$. By Lemma 3.2, we have $X_1(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$. Thus by (4) and Lemma 3.3, it suffices to show that $I_1(t),\ I_2(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$.

Since $\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ is closed under linear operations, and by (4), we have

$$\begin{aligned}E\|X_{n+1}\|^2&\le 9E\|X_1(t)\|^2+9E\|I_1(t)\|^2\\&\quad+9E\|I_2(t)\|^2<\infty.\end{aligned}$$

Therefore, we conclude that

$$X_{n+1}(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H)).$$

Now, combining Lemma 3.2 with the result that if $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for some $n\in\mathbb{N}$, then $X_{n+1}(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$, we obtain that $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for all $n\in\mathbb{N}$ by induction. Hence, the sequence $(X_n(t,\omega))_{n\in\mathbb{N}}$ is well defined by (4) and $X_n(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ for all $n\in\mathbb{N}$.

In what follows, we will show that $\{X_n(t,\omega)\}_{n\in\mathbb{N}}$ converges in $\mathbb{D}_2$ to a solution of (1), which complete the proof of Theorem 3.1.

Firstly, we give the existence proof of solution to (1). Set

$$F_n(t):=E\sup_{s\in[0,t]}\|X_{n+1}(s)-X_n(s)\|^2,n\in\mathbb{N},\ t\in[0,T].$$

Then by (4), Schwartz inequations and Fibini theorem, we have

$$\begin{aligned}&E\|X_{n+1}(t)-X_n(t)\|^2\\&\le 2E\|\int_0^t S(t-\tau)BW^{-1}\{S(T)x_0\\&\quad+E\int_0^T\int_Z S(T-(s-))[g(s-,X_n(s-),z)\\&\quad-g(s-,X_{n-1}(s-),z)]N(ds,dz)\}d\tau\|^2\\&\quad+2E\|\int_0^t S(t-(s-))[g(s-,X_n(s-),z)\\&\quad-g(s-,X_{n-1}(s-),z)]N(ds,dz)\|^2\\&:=J_1+J_2.\end{aligned} \tag{10}$$

For the term J_1, by the assumption (H4) and Proposition 2.1 we obtain that

$$\begin{aligned} J_1 &\le 2E\int_0^t \|S(t-\tau)\|^2\|B\|^2\|W^{-1}\|^2\|S(T)x_0 \\ &\quad \times \int_0^T\int_Z S(T-(s-))[g(s-,X_n(s-),z) \\ &\quad -g(s-,X_{n-1}(s-),z)]N(ds,dz)\|^2 d\tau \\ &\le 2C_1C_2C_3\int_0^t e^{-wt}dt\cdot\|S(T)\|^2\|x_0\|^2 \\ &\quad \times E\int_0^T\int_0^t\int_Z \|S(T-(s-))\|^2 \\ &\quad \times\|g(s-,X_n(s-),z) \\ &\quad -g(s-,X_{n-1}(s-),z)\|^2\nu(dz)dsd\tau \\ &\le 2tC_1C_2C_3\int_0^t e^{-ws}dsC_1C_4\int_0^T e^{-wt}dt \\ &\quad \times L_T\int_0^t E\|X_n(s)-X_{n-1}(s)\|^2 ds, \end{aligned} \tag{11}$$

and for the term J_2, we have

$$\begin{aligned} J_2 &\le 2E\int_0^t \|S(t-(s-))[g(s-,X_n(s-),z) \\ &\quad -g(s-,X_{n-1}(s-),z)]\|^2\nu(dz)ds \\ &\le 2C_1\int_0^t e^{-ws}dsC_4L_T\int_0^t E\|X_n(s)-X_{n-1}(s)\|^2 ds. \end{aligned} \tag{12}$$

Hence, from (10), (11) and (12), by induction we get

$$F_n(t)\le [C_TL_T]^{n-1}\int_0^t dt_1\int_0^{t_1}dt_2\cdots\int_0^{t_{n-2}}F_1(t_{n-1})dt_{n-1}, \tag{13}$$

where C_T is a constant about $C_1,\ C_2,\ C_3,\ C_4$ and $\int_0^t e^{-ws}ds$.

Thus, we obtain that

$$0\le F_n(t)\le \frac{const\cdot[C_TL_T]^{n-1}}{(n-1)!}\quad n\in\mathbb{N},$$

which implies that the series $\sum_{n\in\mathbb{N}} F_n(t)$ converges uniformly on $[0,T]$. Therefore, the sequence $X_n(t)$ converges uniformly for $t\in[0,T]$ and a.s. $\omega\in\Omega$. Let $X(t)$ be the limit of the sequence $X_n(t)$ as $n\to\infty$. It is easy to see that $X(t)$ is $\mathcal{F}_t$-adapted.

It remains to show $X(t)$ satisfies an equation of the form (4), namely, we need to prove

$$\begin{aligned} X(t) &= X_1(t)-\int_0^t S(t-\tau)BW^{-1} \\ &\quad\times\int_0^T\int_Z S(T-(s-))g(s-,X(s-),z)N(ds,dz)]d\tau \\ &\quad+\int_0^t S(t-(s-))g(s-,X(s-),z)N(ds,dz), \end{aligned} \tag{14}$$

where $X_1(t)=S(t)x_0-\int_0^t S(t-s)BW^{-1}S(T)x_0ds$.

In fact, by equations (4), (11) and (12), we have

$$\begin{aligned} &E\|\{X_n(t)-X_1(t)-\int_0^t S(t-\tau)BW^{-1} \\ &\quad\times\int_0^T\int_Z S(T-(s-))g(s-,X(s-),z)N(ds,dz)]d\tau \\ &-\int_0^t S(t-(s-))g(s-,X(s-),z)N(ds,dz)\}\|^2 \\ &\le C_TL_T\int_0^t E\|X_{n-1}(s-)-X(s-)\|^2 ds. \end{aligned} \tag{15}$$

Since $X_n(t)$ converges uniformly for $t\in[0,T]$ and a.s. $\omega\in\Omega$, we can take the limit as $n\to\infty$ through the integral of the variable s over $[0,t]$ on the right-hand side of the inequality (15), from which we obtain equation (14). Moreover, from (14) and by a similar argument as in the proofs of Lemma 3.2, 3.3 and 3.4, we conclude that $X(t)$ has a version which is modified cádlág in t. Thus $X(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$ is the solution of (3).

Then, we prove the uniqueness. Let $X^1(t)$ and $X^2(t)$ be two solutions of (3), then $X^1(t),X^2(t)\in\mathbb{D}_2$. Set

$$H(t):=E\|X^1(t)-X^2(t)\|^2.$$

From $X^1(t),X^2(t)\in\mathbb{D}_2([0,T];L_2(\Omega,\mathcal{F},P;H))$, we have that $H(t)$ is modified cálág in variable t and $\sup_{t\in[0,T]}H(t)\le E\sup_{t\in[0,T]}\|X^1(t)-X^2(t)\|^2<\infty$. By equation (14) and the same argument as in the proof of existence, we get

$$H(t)\le C_TL_T\int_0^t H(s)ds.$$

By Gronwall inequality, we obtain the uniqueness. Thus we prove Theorem 3.1.

4. Example

Consider the following Parabolic stochastic partial differential equation of the form

$$\begin{aligned} \frac{\partial X(t,x,\omega)}{\partial t} &= \frac{1}{2}\frac{\partial^2}{\partial x^2}X(t,x,\omega)+Bu(t) \\ &\quad+\int_Z g(t,z,X(t,x,\omega))\eta_t(dz,\omega) \\ X(0,x,\omega) &= u_0(x)\in H \end{aligned} \tag{16}$$

with the following assumption:

(1)Let $A=\frac{1}{2}\frac{\partial^2}{\partial x^2}$ and B is a bounded linear operator from the control space $\mathbb{D}_2$ to H;

(2)The semi group of the operator A is

$$S(t,x)=\begin{cases}\frac{1}{\sqrt[2]{2\pi t}}e^{-\frac{x^2}{2t}} & t>0,\ x\in\mathbb{R} \\ \delta_x & t=0\end{cases},$$

that is $S(t,x)$ be the fundamental solution of the operator $\frac{\partial}{\partial t}-\frac{1}{2}\frac{\partial^2}{\partial x^2}$;

(3)$\eta_t(dz,\omega)$ is Poisson white noise defined heuristically as the Radon-Nikodym derivative

$$\eta_t(dz,\omega)=\frac{\tilde{N}(dt,dz,\omega)}{dt}(t),\ \ t\in[0,\infty).$$

Then, the equation (16) has an abstract formulation of the following linear stochastic equation in a Hilbert space

$$\begin{aligned} \frac{dX(t)}{dt} &= AX(t)+Bu(t)+\int_Z g(t,X(t),z)\frac{\tilde{N}(dz,dt)}{dt}, \\ &\quad t\in[0,T], \\ X(0) &= u_0\in H, \end{aligned}$$

where $\tilde{N}(dz,dt)=N(dz,dt)-\nu(dz)dt$. One rigorous formulation of (16) can be given by the following integral equation

$$\begin{aligned} X(t,x) &= \int_{\mathbb{R}} S(t,x-y)u_0(y)dy \\ &+\int_0^t\int_{\mathbb{R}} S(t-s,x-y)Bu(t)dyds \\ &+\int_0^t\int_{\mathbb{R}}\int_Z S(t-s,x-y)g(s,X(s,y),z)dyN(dz,ds) \\ &-\int_0^t\int_{\mathbb{R}}\int_Z S(t-s,x-y)g(s,X(s,y),z)dy\nu(dz)ds. \end{aligned}$$

Hence by Theorem 3.1, for $S(t,x), x \in [0,L] \subset [0,\infty)$, the system (16) is completely controllable on $[0,T]$.

5. Conclusion

In this paper, we employ semigroup theory to consider the mild solution of a stochastic partial differential equation driven by a Poisson random measure and then the successive approximation is used to consider the controllability of the stochastic equation. The stochastic equation is presented as a stochastic evolution which is an abstract formulation for stochastic partial equations. We obtain the result for stochastic system with Poisson white noise. Our analysis of controllability for stochastic systems of both conceptual value and practical interest. One interesting future direction would be the stochastic system with more compiled noise such as Levy processes (c.f. [20]), since many random disturbances of system is an inherently more compiled then Brownian processes. To capture the infection process for controllability of systems, approximate controllability for stochastic systems in Hilbert space are preferable [19].

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References

[1] Applebaum, D. Lévy Processes and Stochastic Calculus. Cambridge University Press, Cambridge (2004).

[2] S. Albeverio, J.L. Wu, and T. S. Zhang: Parabolic SPDEs driven by Poisson white noise. Stochastic Processes and Their Applications, 74(1), 21-36, 1998.

[3] Balachandran, K. and Dauer, JP.Controllability of nonlinear systems via fixed-point theorems, Journal of optimization theory and applications, 53(3): 345–352, 1987.

[4] Balasubramaniam, P. and Dauer, JP. Controllability of semilinear stochastic evolution equations in Hilbert space, Journal of Applied Mathematics and Stochastic Analysis, 14(4): 329–339, 2001.

[5] Balasubramaniam, P. and Dauer, JP.Controllability of semilinear stochastic evolution equations with time delays, Publicationes Mathematicae Debrecen, 63(3): 279–291, 2003.

[6] Bashirov, A.E. and Kerimov, K.R. On controllability conception for stochastic systems, SIAM journal on control and optimization, 35: 384, 1997.

[7] Curtain, R.F. and Ichikawa, A. The separation principle for stochastic evolution equations, SIAM Journal on Control and Optimization, 15: 367, 1977.

[8] Da Prato, G.and Zabczyk, J. Stochastic Equations in Infinite Dimensions, Cambridge Univ. Press, Cambridge, UK, 1992.

[9] Gorniewicz, L. and Ntouyas, SK and O'Regan, D. Controllability of semilinear differential equations and inclusions via semigroup theory in Banach spaces, Reports on Mathematical Physics, 56(3): 437–470, 2005.

[10] Hausenblas, E. Existence, Uniqueness and Regularity of Parabolic SPDEs driven by Poisson random measure. Electron. J. Probab. 10, 1496C1546, (electronic) (2005).

[11] Hausenblas, E. SPDEs driven by Poisson random measure with non Lipschitz coefficients: existence results.Probability Theory and Related Fields, 137(1),161-200, 2007.

[12] Knoche, C.SPDEs in infinite dimension with Poisson noise, Comptes Rendus Mathematique, 339(9): 647–652, 2004.

[13] Krylov, N. V. and RozovskiiB. L. Stochastic evolution equations, J. Soviet Mathematics 16, 1233C1277,(1981).

[14] Lindquist, A. On feedback control of linear stochastic systems, SIAM J. Control,11(2):323–343,1973.

[15] Loka, A. Osendal, B. and Proske, F. Stochastic partial differential equations driven by Lévy space-time white noise. Annals of Applied Probability,14(3)1506-1528,2004.

[16] Mahmudov, N.I. Controllability of semilinear stochastic systems in Hilbert spaces, Journal of Mathematical Analysis and Applications, 288(1) 197–211, 2003.

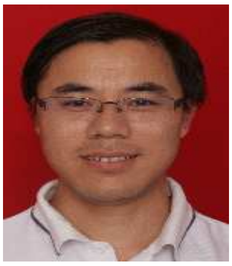
[17] Mahmudov, N.I.Approximate controllability of semilinear deterministic and stochastic evolution equations in abstract spaces, SIAM journal on control and optimization, 42:1604, 2003.

[18] Pazy,A. Semigroups of Linear Operators and Applications to Partial Differential Equations. Springer-Verlag, Berlin, 1983.

[19] Subalakshmi, R. and Balachandran, K. and Park, JY.Controllability of semilinear stochastic functional integrodifferential systems in Hilbert spaces, Nonlinear Analysis: Hybrid Systems,3(1):39–50, 2009.

[20] Röckner, M. and Zhang, T.Stochastic evolution equations of jump type: existence, uniqueness and large deviation principles,Potential Analysis, 26(3):255–279,2007.

[21] Walsh,J.B. An introduction to stochastic partial differential equations. In: Ecole d'Été de Probabilités de St. Flour XIV, Lecture Notes in Mathematics, vol. 1180. Springer, Berlin, pp. 266-439,1986.



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