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A fractional-order model of Lassa Hemorrhagic Fever and its Influence on the Pregnant Women

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Abstract: In this work, we aim to explains and analyzes the mathematical model of equations that depicts Lassa hemorrhagic fever (LHF) dynamics in the pregnant women population. We presented the mathematical formulation of LHF in the form of a fractional-order model (FOM) based on the Caputo fractional derivative. We considered the generalized mean value theorem to illustrate the positivity, existence and uniqueness of the obtained solutions. Moreover, we investigated the stability analysis at all equilibrium points and obtained the basic reproduction number R_0 and study its effect on the behavior of the pregnant women population. This FOM is solved by two methods, one of them is called the generalized Mittag-Leffler function method (GMLFM) described an analytic-approximate solution, another method give the numerical solution is called the predictor-corrector method (PCM). GMLFM is used to give appropriate results as a series of time variable to get the desired solution at any time and PCM is implemented to get the FOM solution over a long period of time. The efficiency of the given methods and the effect of the fractional operator was clarified by simulations of our results in some figures.

Keywords: Lassa Hemorrhagic Fever, Fractional Differential Equations, Stability Analysis, Generalized Mittag-Leffler Function Method, Predictor-Corrector Method.

1 Introduction

Lassa hemorrhagic fever (LHF) is a type of viral hemorrhagic fever happened by the Lassa vitus also transmitted by rodents [1]. Rodents are considered the main and an important ressrvoirs of rodent-borne zoonosis world wide and its transfer to humans by aerosol spread either from the *Mastomys natalensis* (multimammate rats) [2] or via the direct contact with infection persons, this illness belongs to the family arenaviridae virus which is enveloped, negative-sense and single-standard RNA genome. The genome divided into two types a small (S) segment and a large (L) segment [3]. LHF is often fatal disease endemic over most of West Africa [4,5]. The disease outbreaks discovered in many cities such as Central Africa Republic, Liberia, Sierra Leone and Nigeria [6,7,8]. In 1969, the Borno state in Nigeria was considered the first town described the LHF disease [6]. There are many cases each year estimated 300,000 to 500,000 cases of LHF and also 5,000 case of deaths in a year [9,10,11]. Many symptoms takes place during the infection include fever, headaches, muscle pains, vomiting and weakness, this symptoms are not observed in about 80 percent of infected peoples but 20 percent of those persons have acute multi-system disease whereat the virus survives in urine form 3 to 8 weeks after infection [12]. The incubation period of LHF is 6-21 days after exposure of infection and the outbreaks of disease become the height of the dry season compared to the rainy season [13,14,15]. Although, developments of studying and researches about LHF in the last decades but there is no vaccine or immunotherapy available for humans until 2019. To prevent or treat this disease outbreaks implies isolating humans whose are injured and not contact with the rats especially in the geographic regions.

No doubt, the controlling of rodents and the food protection from them are the essential reason for transition the infection to peoples which happens by the ingestion or inhalation [16]. Despite, progresses of studying there exist about 15-20 percent of LHF patients who hospitalized people shall die out of illness [12,17,18]. LHF is the essential reason for

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death the fetus and mother during pregnancy period (see [19,20,17]). LHF has a high risk of death to the fetus through pregnancy period and to the mother. At beginning of pregnancy the fetal losses increases and the mortality rate hight about 80 percent [21]. Mothers in LHF getting better quickly after uterine emptying throught miscarriage or natural childbirth. Recently, many scholars and authors have focused our attentions about LHF and its impact on maternity during pregnancy (see e.g., [22,23,24,25,26]).

In this paper, we investigate the FOM of pregnant women as follow:

$$\begin{aligned}
{}_0^C D_t^\alpha U &= (\eta^\alpha - \gamma^\alpha)N^\alpha - \xi^\alpha UV + (\phi^\alpha - \gamma^\alpha)U + \theta^\alpha W,\\
{}_0^C D_t^\alpha V &= \xi^\alpha UV - \phi^\alpha U - (\theta^\alpha + \phi^\alpha)V,\\
{}_0^C D_t^\alpha W &= \phi^\alpha V - \theta^\alpha W,\\
{}_0^C D_t^\alpha Z &= (\gamma^\alpha - \eta^\alpha)N^\alpha + \gamma^\alpha U + \theta^\alpha V,
\end{aligned} \tag{1}$$

where both U,V,W and Z are denotes the susceptible, recovered, infected and death population of pregnant women in that country, respectively. ξ is the rate of infectious women, η denotes the rate of adult pregnant women, N is all adult women in a given country, γ is named the rate of pregnant women die with natural death or other disease, θ is the of death in the dying population and ϕ express to the rate of recovered women.

The aim of this manuscript is to analyze the lassa hemorrhagic fever of the fractional-order pregnant women population by various methods as GMLFM (an approximate solution) and PCM (a numerical solution). Nevertheless, we deduce that the effect of change in the value of α on the pregnant women population.

The highlight in our manuscript is arranged as follows. In Section 2, we display many essential definitions in our studying. We explain the non-negative solution of model (1) in Section 3. Moreover, the stability of model (1) and its equilibrium fixed points are investigated in Section 4. In addition, Section 5 clears the analysis of proposed methods for solving the model (1). Numerical simulation results are supported to confirm the theoretical analysis in Section 6. Finally, we show our conclusion in Section 7.

2 Preliminaries

Here, we present an assess definitions belong to fractional calculus (see [27]).

Definition 1. *The Riemann-Liouville fractional integral of order $\alpha > 0$ is given by*

$$\begin{aligned}
{}_aI_t^\alpha f(t) &= \frac{1}{\Gamma(\alpha)}\int_a^t (t-\xi)^{\alpha-1} f(\xi)d\xi, \qquad \alpha > 0, \qquad t > a,\\
{}_aI_t^0 f(t) &= f(t),
\end{aligned}$$

where $\Gamma(.)$ is given by

$$\Gamma(z) = \int_0^\infty x^{z-1}e^{-x}dx, \qquad \Re(z) > 0.$$

Definition 2. *The Riemann-Liouville fractional derivatives take the form*

$${}_aD_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(\alpha-n)}\frac{d^n}{dt^n}\int_a^t \frac{f(\xi)}{(t-\xi)^{\alpha-n+1}}d\xi, & n-1 < \alpha < n, n \in \mathbb{N},\\ \frac{d^n f(t)}{dt^n}, & \alpha = n, n \in \mathbb{N}. \end{cases}$$

Definition 3. *The Caputo fractional derivative of order $\alpha > 0$ is defined as follows*

$${}_a^C D_t^\alpha f(t) = \begin{cases} \frac{1}{\Gamma(n-\alpha)}\int_a^t (t-\xi)^{n-\alpha-1} f^{(n)}(\xi)d\xi, & n-1 < \alpha < n, n \in \mathbb{N},\\ \frac{d^n f(x)}{dt^n}, & \alpha = n, n \in \mathbb{N}. \end{cases}$$

In addition to, we introduce some properties of fractional calculus (see references [28,29])

$$ {}_a^C D_x^\alpha \, {}_a I_x^\alpha f(x) = f(x), $$

$$ {}_a I_x^\alpha \, {}_a^C D_x^\alpha f(x) = f(x) - \sum_{k=0}^{n-1} f^{(k)}(a) \frac{(x-a)^k}{\Gamma(k+1)}. $$

Definition 4.*In (1902-1905), the Mittag-Leffler functions E_α and $E_{\alpha,\beta}$ defined by the power series as follows:*

$$ E_\alpha(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(n\alpha+1)}, \qquad E_{\alpha,\beta}(x) = \sum_{n=0}^{\infty} \frac{x^n}{\Gamma(n\alpha+\beta)}, \qquad \alpha,\beta > 0. \tag{2} $$

So, the Caputo fractional derivative of the GMLF (see e.g.,[30,31,32,33]) is given by

$$ {}_0^C D_t^\alpha E_\alpha(ax^\alpha) = \sum_{n=1}^{\infty} a^n \frac{x^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)}. \tag{3} $$

3 Non-negative solution

Assume that $R_+^4 = \{X \in R^4 : X \geq 0\}$, where $X(t) = (U,V,W,Z)^T$. To prove this solution, we present some essential relations.

Theorem 1.*[34] Suppose that $f(y) \in C(0,a]$, $D^\alpha f(y) \in C(0,a]$, for $\alpha \in (0,1]$. Then we have*

$$ f(y) = f(0) + \frac{1}{\Gamma(\alpha)} (D^\alpha f)(\zeta)(y)^\alpha, \ \zeta \in (0,y], \forall x \in (0,a]. $$

Corollary 1.*[35] Let $f(y) \in C[0,a]$, $D^\alpha f(y) \in C(0,a]$. From theorem 1, we deduce that if $D^\alpha f(y) \geq 0$, then $f(y)$ is not diminishing, when $D^\alpha f(y) \leq 0$, then $f(y)$ is non-increasing.*

Theorem 2.*There exist a unique solution $X(t) = (U,V,W,Z)^T$ for (1) at $t \geq 0$ and the solution will remain in R_+^4.*

*Proof.*From [36], we can acquires the existence and uniqueness of the model (1) in $(0,\infty)$. Now, we shall clear that R_+^4 is positively invariant domain.
Then

$$ \begin{aligned} {}_0^C D_t^\alpha U \mid_{U=0} &= (\eta^\alpha - \gamma^\alpha) N^\alpha + \theta^\alpha W \geq 0, \\ {}_0^C D_t^\alpha V \mid_{V=0} &= -\phi^\alpha U \leq 0, \\ {}_0^C D_t^\alpha W \mid_{W=0} &= \phi^\alpha V \geq 0, \\ {}_0^C D_t^\alpha Z \mid_{Z=0} &= (\gamma^\alpha - \eta^\alpha) N^\alpha + \gamma^\alpha U + \theta^\alpha V \geq 0. \end{aligned} $$

According to corollary 1, we will conclude that the solution remain in R_+^4.

4 Stability and Equilibrium points

In this section, we determine the equilibrium points and their stability properties of the model (1). To get these points of the fractional-order pregnant women population by putting the right hand side of model (1) equal to zero to acquire the following system of equations as

$$ \begin{aligned} {}_0^C D_t^\alpha U &= 0, \\ {}_0^C D_t^\alpha V &= 0, \\ {}_0^C D_t^\alpha W &= 0. \end{aligned} $$

By solving the above equations, the model (1) has two an equilibrium points as follows

–Disease-free equilibrium point $E_1 = (U_1, V_1, W_1)$ given by the following values

$$E_1 = (\frac{\eta^\alpha - \gamma^\alpha}{\gamma^\alpha}, 0, 0).$$

–the endemic equilibrium point $E_* = (U_*, V_*, W_*)$ is

$$E_* = (\frac{(\theta^\alpha + \phi^\alpha)V_*}{\xi^\alpha V_* - \phi^\alpha}, V_*, \frac{\phi^\alpha V_*}{\theta^\alpha}),$$

where
$V_* = \frac{B}{2A}(R_0 - 1)$, $A = \xi^\alpha \theta^\alpha, B = -(\eta^\alpha N^\alpha \xi^\alpha - \gamma^\alpha N^\alpha \xi^\alpha + \phi^\alpha \theta^\alpha - \gamma^\alpha \theta^\alpha - \gamma^\alpha \phi^\alpha)$, $C = N^\alpha \phi^\alpha (\eta^\alpha - \gamma^\alpha)$ and $R_0 = \frac{\sqrt{B^2 + 4AC}}{B}$,

where R_0 is the basic reproduction number defined as the prospective measure for the disease diffusion in a target population [37]. This definition denotes to the average number of secondary cases generated by an infected individual when the susceptible population has no immunity of the disease to control the infection. Moreover, if $R_0 < 1$ the disease-free is stable and the endemic equilibrium points are unstable. When $R_0 > 1$, then the disease-free equilibrium is unstable and the endemic stable.

4.1 Stability analysis of the disease-free equilibrium point E_1

Here, we will determine the eigenvalues of model (1) at the disease-free equilibrium fixed point E_1. The Jacobian matrix of the model (1) estimated at E_1 as follows

$$J(E_1) = \begin{pmatrix} \phi^\alpha - \gamma^\alpha & \frac{-\xi^\alpha(\eta^\alpha - \gamma^\alpha)}{\gamma^\alpha} & \theta^\alpha \\ -\phi^\alpha & \frac{\xi^\alpha(\eta^\alpha - \gamma^\alpha)}{\gamma^\alpha} - (\theta^\alpha + \phi^\alpha) & 0 \\ 0 & \phi^\alpha & -\theta^\alpha \end{pmatrix},$$

and the corresponding characteristic equation $F(\lambda)$ of model 1 at the disease-free equilibrium E_1 take the form

$$(\phi^\alpha - \gamma^\alpha - \lambda_1)(\frac{\xi^\alpha(\eta^\alpha - \gamma^\alpha)}{\gamma^\alpha} - \theta^\alpha - \phi^\alpha - \lambda_2)(-\theta^\alpha - \lambda_3) = 0.$$

Then, we obtain the eigenvalues as

$$\lambda_1 = \phi^\alpha - \gamma^\alpha < 0,$$

$$\lambda_2 = \frac{\xi^\alpha(\eta^\alpha - \gamma^\alpha)}{\gamma^\alpha} - (\theta^\alpha + \phi^\alpha) < 0,$$

$$\lambda_3 = -\theta^\alpha < 0.$$

From the previous results of the eigenvalues, we deduce that the disease-free equilibrium E_1 is locally asymptotically stable.

4.2 Stability analysis of the endemic equilibrium point E_*

The Jacobian matrix at the endemic equilibrium point $E_*(U_*, V_*, W_*)$ of model (1) is

$$J(E_*) = \begin{pmatrix} -\xi^\alpha V_* + \phi^\alpha - \gamma^\alpha & -\xi^\alpha U_* & \theta^\alpha \\ \xi^\alpha V_* - \phi^\alpha & \xi^\alpha U_* - (\theta^\alpha + \phi^\alpha) & 0 \\ 0 & \phi^\alpha & -\theta^\alpha \end{pmatrix},$$

and the corresponding characteristic equation $F(\lambda)$ at an endemic equilibrium E_* is given by

$$= (-\xi^\alpha V_* + \phi^\alpha - \gamma^\alpha - \lambda)(\xi^\alpha U_* - \theta^\alpha - \phi^\alpha - \lambda)(-\theta^\alpha - \lambda) = 0.$$

Therefore, we can get the following eigenvalue as

$$\lambda_1 = -\theta^{\alpha},$$

$$\lambda_2 = -\xi^{\alpha}\frac{B(R_0-1)}{2A} + \phi^{\alpha} - \gamma^{\alpha},$$

$$\lambda_3 = \xi^{\alpha}\frac{B(\theta^{\alpha}+\phi^{\alpha})(R_0-1)}{\xi^{\alpha}B(R_0-1)-2\phi^{\alpha}A} - (\theta^{\alpha}+\phi^{\alpha}).$$

According to the previous eigenvalues, we conclude that E_* is stable when R_0 ¿ 1 and its unstable if $R_0 < 1$.

5 Applications and results

In this section, we exposes our used methods for solving the model (1) by the GMLFM as an analytic-approximate solution and the PCM as a numerical solution.

5.1 Generalized Mittag-Leffler function method

Now, we apply GMLFM to solve the model (1). For more details of the analysis of GMLFM (see e.g., [30,31,32,33]). Suppose that

$$\begin{aligned} U &= \sum_{n=0}^{\infty} a^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \qquad V = \sum_{n=0}^{\infty} b^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ W &= \sum_{n=0}^{\infty} c^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \qquad Z = \sum_{n=0}^{\infty} d^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \end{aligned} \tag{4}$$

according to Eq.(3) we deduces the following relation

$$\begin{aligned} {}_0^C D_t^{\alpha} U &= \sum_{n=1}^{\infty} a^n \frac{t^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)} = \sum_{n=0}^{\infty} a^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ {}_0^C D_t^{\alpha} V &= \sum_{n=1}^{\infty} b^n \frac{t^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)} = \sum_{n=0}^{\infty} b^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ {}_0^C D_t^{\alpha} W &= \sum_{n=1}^{\infty} c^n \frac{t^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)} = \sum_{n=0}^{\infty} c^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ {}_0^C D_t^{\alpha} Z &= \sum_{n=1}^{\infty} d^n \frac{t^{(n-1)\alpha}}{\Gamma((n-1)\alpha+1)} = \sum_{n=0}^{\infty} d^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \end{aligned} \tag{5}$$

by substituting from Eqs.(4) and (5) in model (1) we obtain

$$\begin{cases} \sum_{n=0}^{\infty} a^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = (\eta^{\alpha}-\gamma^{\alpha})N^{\alpha} - \xi^{\alpha}\sum_{n=0}^{\infty} l^n t^{n\alpha} + (\phi^{\alpha}-\gamma^{\alpha})\sum_{n=0}^{\infty} a^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} \\ \qquad\qquad +\theta^{\alpha}\sum_{n=0}^{\infty} c^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ \sum_{n=0}^{\infty} b^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = \xi^{\alpha}\sum_{n=0}^{\infty} l^n t^{n\alpha} - \phi^{\alpha}\sum_{n=0}^{\infty} a^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} - (\theta^{\alpha}+\phi^{\alpha})\sum_{n=0}^{\infty} b^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ \sum_{n=0}^{\infty} c^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = \phi^{\alpha}\sum_{n=0}^{\infty} b^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} - \theta^{\alpha}\sum_{n=0}^{\infty} c^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \\ \sum_{n=0}^{\infty} d^{n+1} \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} = (\gamma^{\alpha}-\eta^{\alpha})N^{\alpha} + \gamma^{\alpha}\sum_{n=0}^{\infty} a^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)} + \theta^{\alpha}\sum_{n=0}^{\infty} b^n \frac{t^{n\alpha}}{\Gamma(n\alpha+1)}, \end{cases} \tag{6}$$

where

$$l^n = \sum_{k=0}^{n} \frac{a^k b^{n-k}}{\Gamma(k\alpha+1)\Gamma((n-k)\alpha+1)}.$$

By computing the same quantities of equation (6), we get

$$\begin{cases} \sum_{n=0}^{\infty} \left(\frac{a^{n+1}}{\Gamma(n\alpha+1)} + \xi^\alpha l^n - (\phi^\alpha - \gamma^\alpha) \frac{a^n}{\Gamma(n\alpha+1)} - \theta^\alpha \frac{c^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = (\eta^\alpha - \gamma^\alpha) N^\alpha, \\ \sum_{n=0}^{\infty} \left(\frac{b^{n+1}}{\Gamma(n\alpha+1)} - \xi^\alpha l^n + \phi^\alpha \frac{a^n}{\Gamma(n\alpha+1)} + (\theta^\alpha + \phi^\alpha) \frac{b^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0, \\ \sum_{n=0}^{\infty} \left(\frac{c^{n+1}}{\Gamma(n\alpha+1)} - \phi^\alpha \frac{b^n}{\Gamma(n\alpha+1)} + \theta^\alpha \frac{c^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0, \\ \sum_{n=0}^{\infty} \left(\frac{d^{n+1}}{\Gamma(n\alpha+1)} - \gamma^\alpha \frac{a^n}{\Gamma(n\alpha+1)} - \theta^\alpha \frac{b^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = (\gamma^\alpha - \eta^\alpha) N^\alpha. \end{cases} \tag{7}$$

By taking the first limit of equation (7) we have

$$\begin{aligned} a^1 &= (\eta^\alpha - \gamma^\alpha) N^\alpha - \xi^\alpha l^0 + (\phi^\alpha - \gamma^\alpha) a^0 + \theta^\alpha c^0, \\ b^1 &= \xi^\alpha l^0 - \phi^\alpha a^0 - (\theta^\alpha + \phi^\alpha) b^0, \\ c^1 &= \phi^\alpha b^0 - \theta^\alpha c^0, \\ d^1 &= (\gamma^\alpha - \eta^\alpha) N^\alpha + \gamma^\alpha a^0 + \theta^\alpha b^0. \end{aligned} \tag{8}$$

According to the previous Eq. (8), the equation (7) turned to

$$\begin{cases} \sum_{n=1}^{\infty} \left(\frac{a^{n+1}}{\Gamma(n\alpha+1)} + \xi^\alpha l^n - (\phi^\alpha - \gamma^\alpha) \frac{a^n}{\Gamma(n\alpha+1)} - \theta^\alpha \frac{c^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0, \\ \sum_{n=1}^{\infty} \left(\frac{b^{n+1}}{\Gamma(n\alpha+1)} - \xi^\alpha l^n + \phi^\alpha \frac{a^n}{\Gamma(n\alpha+1)} + (\theta^\alpha + \phi^\alpha) \frac{b^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0, \\ \sum_{n=1}^{\infty} \left(\frac{c^{n+1}}{\Gamma(n\alpha+1)} - \phi^\alpha \frac{b^n}{\Gamma(n\alpha+1)} + \theta^\alpha \frac{c^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0, \\ \sum_{n=1}^{\infty} \left(\frac{d^{n+1}}{\Gamma(n\alpha+1)} - \gamma^\alpha \frac{a^n}{\Gamma(n\alpha+1)} - \theta^\alpha \frac{b^n}{\Gamma(n\alpha+1)} \right) t^{n\alpha} = 0. \end{cases} \tag{9}$$

From Eq.(9), we conclude that $t^{n\alpha}$ not equal zero but the rest coefficients equal to zero and can acquire values of the constants a^n, b^n, c^n, d^n, and $n = 1, 2, 3, \cdots$.

$$\begin{cases} a^{n+1} = -\xi^\alpha l^n \Gamma(n\alpha+1) + (\phi^\alpha - \gamma^\alpha) a^n + \theta^\alpha c^n, \\ b^{n+1} = \xi^\alpha l^n \Gamma(n\alpha+1) - \phi^\alpha a^n - (\theta^\alpha + \phi^\alpha) b^n, \\ c^{n+1} = \phi^\alpha b^n - \theta^\alpha c^n, \\ d^{n+1} = \gamma^\alpha a^n + \theta^\alpha b^n. \end{cases} \tag{10}$$

At $n = 1$, Eq.(10) become

$$\begin{cases} a^2 = -\xi^\alpha l^1 \Gamma(\alpha+1) + (\phi^\alpha - \gamma^\alpha) a^1 + \theta^\alpha c^1, \\ b^2 = \xi^\alpha l^1 \Gamma(\alpha+1) - \phi^\alpha a^1 - (\theta^\alpha + \phi^\alpha) b^1, \\ c^2 = \phi^\alpha b^1 - \theta^\alpha c^1, \\ d^2 = \gamma^\alpha a^1 + \theta^\alpha b^1. \end{cases} \tag{11}$$

If $n=2$ we obtain

$$\begin{cases} a^3 = -\xi^\alpha l^2 \Gamma(2\alpha+1) + (\phi^\alpha - \gamma^\alpha)a^2 + \theta^\alpha c^2, \\ b^3 = \xi^\alpha l^2 \Gamma(2\alpha+1) - \phi^\alpha a^2 - (\theta^\alpha + \phi^\alpha)b^2, \\ c^3 = \phi^\alpha b^2 - \theta^\alpha c^2, \\ d^3 = \gamma^\alpha a^2 + \theta^\alpha b^2. \end{cases} \tag{12}$$

Also when $n=3$ we get

$$\begin{cases} a^4 = -\xi^\alpha l^3 \Gamma(3\alpha+1) + (\phi^\alpha - \gamma^\alpha)a^3 + \theta^\alpha c^3, \\ b^4 = \xi^\alpha l^3 \Gamma(\alpha+1) - \phi^\alpha a^3 - (\theta^\alpha + \phi^\alpha)b^3, \\ c^4 = \phi^\alpha b^3 - \theta^\alpha c^3, \\ d^4 = \gamma^\alpha a^3 + \theta^\alpha b^3. \end{cases} \tag{13}$$

Similarly by repeating the previous steps we obtain the rest values of constants $a^5, b^5, c^5, d^5, \cdots$.

By substituting from equations (8) and (11) into Eq.(4) we get the following expression as follows

$$\begin{aligned} U &= a^0 + a^1\frac{t^\alpha}{\Gamma(\alpha+1)} + a^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + a^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + a^4\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + a^5\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \cdots, \\ V &= b^0 + b^1\frac{t^\alpha}{\Gamma(\alpha+1)} + b^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + b^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + b^4\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + b^5\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \cdots, \\ W &= c^0 + c^1\frac{t^\alpha}{\Gamma(\alpha+1)} + c^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + c^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + c^4\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + c^5\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \cdots, \\ Z &= d^0 + d^1\frac{t^\alpha}{\Gamma(\alpha+1)} + d^2\frac{t^{2\alpha}}{\Gamma(2\alpha+1)} + d^3\frac{t^{3\alpha}}{\Gamma(3\alpha+1)} + d^4\frac{t^{4\alpha}}{\Gamma(4\alpha+1)} + d^5\frac{t^{5\alpha}}{\Gamma(5\alpha+1)} + \cdots. \end{aligned}$$

5.2 Predictor-Corrector method

Here, we analyze the PCM [38,39,40,41,42] on the fractional-order pregnant women model (1) to support the theoretical results. We investigate the solution of the fractional order differential equations (FODEs) by given the following approach.

The FODEs take the form

$$D_t^\alpha x(t) = g(t, x(t)), \qquad 0 \le t \le T, \tag{14}$$

$$x^{(m)}(0) = x_0^{(m)}, \qquad m = 0,1,2,....[\alpha]-1,$$

where

$$x(t) = \sum_{k=0}^{[\alpha]-1} y_0^{(k)}\frac{t^k}{k!} + \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(y(\tau))d\tau. \tag{15}$$

Assume that $h = \frac{T}{N}$, $tn = nh$, $n = 0,1,...,N \in \mathbb{Z}^+$. Then (15) take the form as follows:

$$y_n(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} y_0^{(k)}\frac{t_{n+1}^k}{k!} + \frac{h^\alpha}{\Gamma(\alpha+2)} f(y_n^P(t_{n+1})) + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1} f(y_n(t_j)), \tag{16}$$

where

$$a_{j,n+1} = \begin{cases} n^{\alpha+1} - (n-\alpha)(n+1)^\alpha, & if\, j = 0, \\ (n-j+2)^{\alpha+1} + (n-j)^{\alpha+1} - 2(n-j+1)^{\alpha+1}, & if 0 \le j \le n, \\ 1, & if\, j = n+1. \end{cases} \tag{17}$$

Then, the predicted value $y_n^P(t_{n+1})$ is given by

$$y_n^P(t_{n+1}) = \sum_{k=0}^{[\alpha]-1} y_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{1}{\Gamma(\alpha)} \sum_{j=0}^{n} b_{j,n+1} f(y_n(t_j)), \tag{18}$$

where

$$b_{j,n+1} = \frac{h^\alpha}{\alpha}((n-j+1)^\alpha - (n-j)^\alpha). \tag{19}$$

Therefore, the PCM is an approximation for the fractional-order integration. By applying the above method, system (1) transformed to

$$\begin{cases} U_{n+1} = U_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[(\eta^\alpha - \gamma^\alpha)N^\alpha - \xi^\alpha U_{n+1}^P V_{n+1}^P + (\phi^\alpha - \gamma^\alpha)U_{n+1}^P + \theta^\alpha W_{n+1}^P\right] \\ \qquad + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1}\left[(\eta^\alpha - \gamma^\alpha)N^\alpha - \xi^\alpha U_j V_j + (\phi^\alpha - \gamma^\alpha)U_j + \theta^\alpha W_j\right], \\ V_{n+1} = V_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\xi^\alpha U_{n+1}^P V_{n+1}^P - \phi^\alpha U_{n+1}^P - (\theta^\alpha + \phi^\alpha)V_{n+1}^P\right] \\ \qquad + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1}\left[\xi^\alpha U_j V_j - \phi^\alpha U_j - (\theta^\alpha + \phi^\alpha)V_j\right], \\ W_{n+1} = W_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[\phi^\alpha V_{n+1}^P - \theta^\alpha W_{n+1}^P\right] \\ \qquad + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1}\left[\phi^\alpha V_j - \theta^\alpha W_j\right], \\ Z_{n+1} = Z_0 + \frac{h^\alpha}{\Gamma(\alpha+2)}\left[(\gamma^\alpha - \eta^\alpha)N^\alpha + \gamma^\alpha U_{n+1}^P + \theta^\alpha V_{n+1}^P\right] \\ \qquad + \frac{h^\alpha}{\Gamma(\alpha+2)}\sum_{j=0}^{n} a_{j,n+1}\left[(\gamma^\alpha - \eta^\alpha)N^\alpha + \gamma^\alpha U_j + \theta^\alpha V_j\right]. \end{cases} \tag{20}$$

6 Numerical simulations

In this section, we introduce the numerical results for the dynamical behavior of the pregnant women population model (1) to support our analytical results. Our obtained numerical simulations examined by two main methods are the GMLFM and PCM. The used parameters values to get our simulations are $\xi = 0.4, \eta = 0.3, N = 1000, \gamma = 0.2, \theta = 0.8, \phi = 0.2$, with initial conditions $U(0) = 900, V(0) = 10, W(0) = 0, Z(0) = 0$.

Fig. 1(a) and Fig. 1(b) are indicate all stages of pregnant women populations at the previous parameters such that the susceptible, infected and recovered populations of pregnant women are decreases at the given initial conditions while the population of dying is increasing until the peak. These simulations depicts for us the LHF disease has negative influence on life of the pregnant women from the others diseases by using our used methods the GMLFM and PCM.

Fig. 2 describes the pregnant women population at the different values of $\alpha = 1, 0.9, 0.8$ via GMLFM.

Fig. 3 clears that the pregnant women population at the different values of $\alpha = 1, 0.9, 0.8$ through PCM.

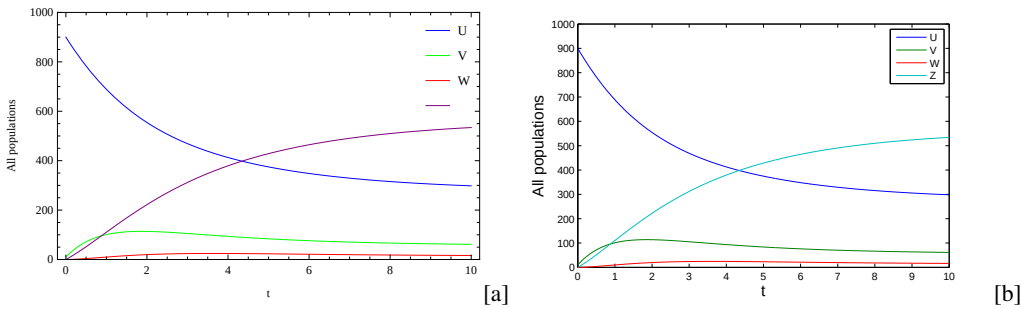

Fig. 1: The behavior of the pregnant women population of model (1) when $\alpha = 1$, (a) using GMLFM and (b) using PCM.

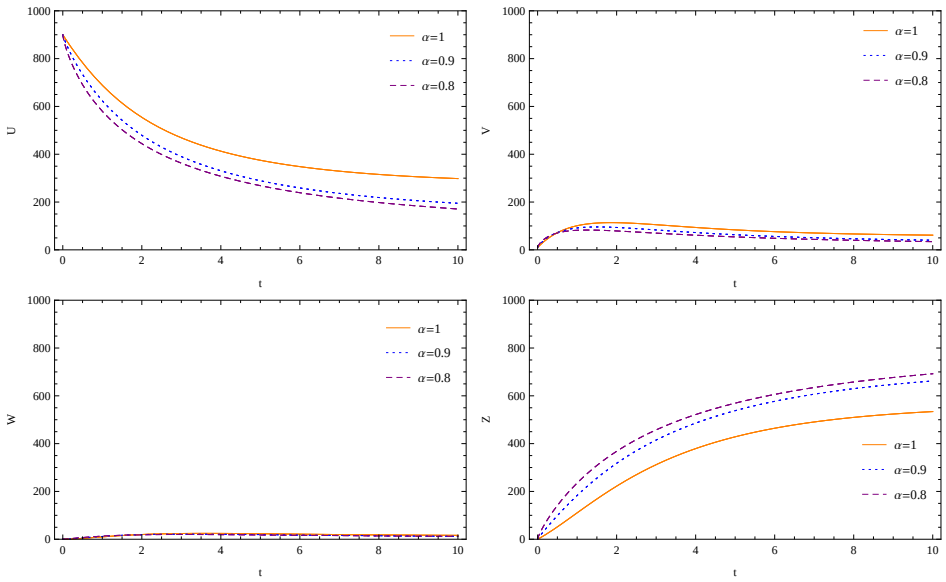

Fig. 2: The behavior of system (1) using previous parameter values at different stages U,V,W,Z for different values of $\alpha = 1, 0.9, 0.8$ by GMLFM.

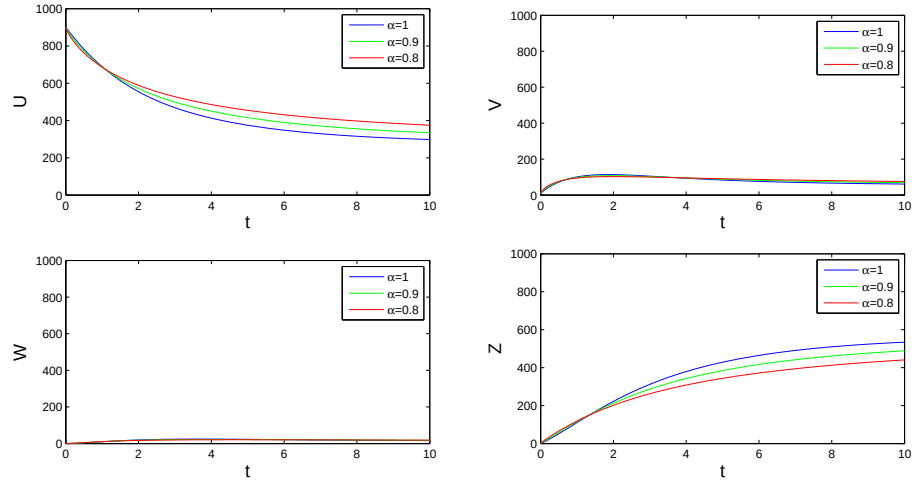

Fig. 3: The behavior of system (1) using previous parameter values at different stages U,V,W,Z for different values of $\alpha = 1, 0.9, 0.8$ by PCM.

7 Conclusion

In this manuscript, we investigated a fractional-order lassa hemorrhagic fever for pregnant women model with the high mortality rate by new parameters. We introduced some valuable definitions which help us to understand the model. Also, we elucidated the non-negative solutions which prove that the existence and uniqueness of the solution of the model (1). We presented the stability analysis of model and its impact on the diffusion of disease. Furthermore, we examined and analyzed the fractional-order lassa hemorrhagic fever for pregnant women model by using the GMLFM as an analytical method and the PCM as a numerical method. In addition to, we construed the fractional-order α play an essential role in the pregnant women population (1). Finally, the obtained numerical simulations support our theoretical analysis, these

simulations can be noticed that the used method is extremely very accurate and more suitable to analyze the various classes of nonlinear model.

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Conflict of Interests

There is no conflict of interests by authors regarding the publication of this manuscript.

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