Journal of Statistics Applications & Probability Letters
An International Journal

http://dx.doi.org/10.18576/jsapl/060204

Estimation of Parameters of Alpha Power Inverse Weibull Distribution Under Progressive Type-II Censoring

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Received: 17 Jan. 2019, Revised: 22 Mar. 2019, Accepted: 26 Mar. 2019
Published online: 1 May 2019

Abstract: In this paper, the inference about the parameters of a three-parameters Alpha power inverse Weibull (APIW) distribution based on progressively type-II censored sample is studied. The maximum likelihood estimates (MLEs) and Bayesian estimates are obtained as point estimations for these parameters. Approximate confidence intervals (ACIs) for the unknown parameters. Moreover Bayesian estimates are obtained for symmetric and asymmetric loss functions such as squared error, LINEX loss functions and general entropy loss function. Gibbs within Metropolis–Hasting samplers procedure is applied for using Markov chain Monte Carlo (MCMC) technique to obtain the Bayes estimates of the unknown parameters and the corresponding credible intervals (CRIs). Finally, a real data set, which represents the failure of some components, is analyzed to illustrate the proposed methods and explicate the precision of the estimators.

Keywords: Alpha power inverse Weibull distribution; progressive type II censoring data; Maximum Likelihood estimator; Bayes estimator; Markov chain Monte Carlo.

1 Introduction

Ball bearing data is one of the most frequently used data sets in literature for explain the application of life time distribution. Manufactures study the cause of failure of ball bearing such as lubrication failure, contamination, improper mounting, misalignment, false brinelling, corrosion, electrical damage, fatigue, over heating, excessive loads, improper storage and handling and fit. Several of the large industrialists have recently collect their test data in mutual effort to set up uniform and standardlized ball-bearing application formulas, which would benefit the many users of anti fraction bearing [7]. For this reason, the progressive type II censored date was used to fix the failure quickly. In life testing and reliability studies the experimenters not always obtain the compelete information on failure time for all experimental units data obtain from such experiments are called censored data [8]. A progressive type-II right-censored sample was considered here. The ordered failure times arising from a progressively type-II right-censored sample are called progressively type-II right- censored order statistics. The method allows to save time and cost to the experimenters. Suppose that n independent items are put on a life test with identically distributed failure times $X_1, X_2, ..., X_n$. Suppose further that a censoring scheme $(R_1, R_2, ..., R_n)$ is previously fixed such that following the first failure X_1, R_1 surviving items are removed from the experiment at random, and following the second failure X_2, R_2 items are removed from the experiment at random. This process continues untile at the time of the m^{th} observed failure X_m, the remaining R_m surviving itemes are removed from the test. The m ordered observed failure times right censoring denoted by $X_{1:m:n}$, $X_{2:m:n}$, . . . , $X_{m:m:n}$ are called progressively type-II right censored order statistics of size m from a sample of size n with progressive censoring scheme $(R_1, R_2, ...R_m)$. It is clear that $n = m + \sum_{i=1}^{m} R_i$. The special case of conventional type-II right censoring sampling also when $R_1 = R_2 = ... = R_m = 0$. So that $m = n$, the progressively type II right censoring scheme reduces to the case of no censoring (ordinary order statistics). For more information about progressive censoring, we refer the reader to [1,2]and [3]. In this paper, the three-parameter alpha power inverse Weibull ($APIW$) distribution proposed by [4] is considered as a good model for the failure times of ball bearing.

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The random variable X has a modified extended exponential distribution if its probability density function (PDF) and cumulative and cumulative distribution function (CDF) are given by

$$F(x,\alpha,\beta,\lambda) = \frac{\alpha^{e^{-\lambda x^{-\beta}}}-1}{\alpha-1} \; ,\alpha \neq 1; x,\alpha,\lambda,\beta > 0. \tag{1}$$

and

$$f(x,\alpha,\beta,\lambda) = \frac{\log\alpha}{\alpha-1}\lambda\beta x^{-(\beta+1)}e^{-\lambda x^{-\beta}}\alpha^{e^{-\lambda x^{-\beta}}} \; ,\alpha \neq 1; x,\alpha,\lambda,\beta > 0. \tag{2}$$

[4] discussed statistical properties for the APIW. Also, they investigated the maximum likelihood estimators of the unknown parameters and its asymptotic confidence intervals based on complete data.

Let $X_{1:m:n}, X_{2:m:n}, \ldots., X_{m:m:n}; 1 \leq m \leq n$ be a progressively type-II censored sample observed from a lifetime test involving n units taken from a APIW(α,β,λ) distribution and $R_1, R_2, \ldots..R_m$ being the censoring scheme. The joint PDF of a progressively type-II censored sample is given by

$$f(x_{1:m:n}, x_{2:m:n}, \ldots., x_{m:m:n}) = C\prod_{i=1}^{m} f\,(x_{i:m:n})\,[1-F(x_{i:m:n})]^{R_i}, \tag{3}$$

where C is a constant defined as
$C = n\,(n-r_1-1)\cdots(n-\sum_{i=1}^{m-1}(r_i+1))$, (see [3] for details).

The rest of paper is organized as follows: in Section 2, MLEs of α, β and λ are obtained. Bayes estimates for the unknown parameters and functions are also obtained in section 3 for different loss functions such as SEL, LINEX and GEL loss functions. Finally real data set has been analyzed in Section 4.

2 Maximum-likelihood Estimation

This section discuss the procedures of obtaining the maximum likelihood estimates of the parameters α, β and λ based on progressively type-II censoring data. Both point and interval estimations of the parameters are derived. Suppose that $X = X_{1:m:n}, X_{2:m:n}, \ldots\ldots\ldots X_{m:m:n}$ a progressively type-II censored sample drawn from alpha power inverse Weibull distribution whose pdf and cdf are given by (1) and (2), with the censoring scheme $(R_1, R_2, \ldots\ldots, R_m)$. From (((1), ((2) and ((3), the likelihood function is then given by:

$$L = C\,\frac{(\log\alpha)^m\,\lambda^m\,\beta^m}{(\alpha-1)^m}\prod_{i=1}^{m}x_i^{-(\beta+1)}\,e^{-\lambda\sum_{i=1}^{m}x_i^{-\beta}}\,\alpha^{\sum_{i=1}^{m}e^{-\lambda x_i^{-\beta}}}\cdot\left(\prod_{i=1}^{m}\left(\frac{\alpha}{\alpha-1}\right)^{R_i}\right)\left(\prod_{i=1}^{m}\left(1-\alpha^{e^{-\lambda x_i^{-\beta}}-1}\right)^{R_i}\right). \tag{4}$$

The log-likelihood function $\ell = \ln L(\alpha,\beta,\lambda \mid x)$without normalized constant is obtained from L as

$$\ell = m\,\log\,(\log\alpha) + m\,\log\lambda + m\,\log\beta - m\,\log(\alpha-1) -$$
$$(\beta+1)\sum_{i=1}^{m}\log x_i - \lambda\sum_{i=1}^{m}x_i^{-\beta} + \log(\alpha)\sum_{i=1}^{m}e^{-\lambda x_i^{-\beta}} + \sum_{i=1}^{m}R_i\,\log\left(\frac{\alpha}{\alpha-1}\right)$$
$$+\sum_{i=1}^{m}R_i\,\log\left(1-\alpha^{e^{-\lambda x_i^{-\beta}}-1}\right). \tag{5}$$

The normal equations can be obtained by taking the first partial derivatives of the log-likelihood function with respect to the three paramerers and equating the first derivative equal zero we get the following normal equations:

$$\frac{m}{\ln(\alpha)} - \frac{m}{\alpha-1} + \frac{1}{\alpha}\sum_{i=1}^{m}e^{-\lambda x_i^{-\beta}} - \frac{1}{\alpha(\alpha-1)}\sum_{i=1}^{m}R_i + \sum_{i=1}^{m}R_i\left[-\frac{\alpha^{e^{-\lambda x^{-\beta}}-2}\left(e^{-\lambda x_i^{-\beta}}-1\right)}{1-\alpha^{e^{-\lambda x^{-\beta}}-1}}\right] = 0 \tag{6}$$

$$\frac{m}{\beta} - \sum_{i=1}^{m}\ln x_i + \lambda\sum_{i=1}^{m}x_i^{-\beta}\ln(x_i) + \lambda\ln(\alpha)\sum_{i=1}^{m}x_i^{-\beta}\ln(x_i)e^{-\lambda x_i^{-\beta}}$$
$$+\lambda\,\ln(\alpha)\sum_{i=1}^{m}R_i\left[\frac{x_i^{-\beta}\ln(x_i)\,e^{-\lambda x_i^{-\beta}}\,\alpha^{e^{-\lambda x_i^{-\beta}}-1}}{1-\alpha^{e^{-\lambda x_i^{-\beta}}-1}}\right] = 0 \tag{7}$$

and

$$\frac{m}{\lambda}-\sum_{i=1}^{m}x_i^{-\beta}-\ln(\alpha)\sum_{i=1}^{m}x_i^{-\beta}e^{-\lambda x_i^{-\beta}}+\ln(\alpha)\sum_{i=1}^{m}R_i\left[\frac{x_i^{-\beta}e^{-\lambda x_i^{-\beta}}\alpha^{e^{-\lambda x_i^{-\beta}}-1}}{1-\alpha^{e^{-\lambda x_i^{-\beta}}-1}}\right]=0 \tag{8}$$

The asymptotic variances and covariances of the MLEs, $\hat{\alpha},\hat{\beta}$ and $\hat{\lambda}$, are given by the entries of the inverse of the Fisher information matrix $I_{ij}=E\left\{-\left[\partial^2\ell(\blacksquare)/\partial\psi_i\,\partial\psi_j\right]\right\}$, where $i,j=1,2,3$ and $\blacksquare=(\psi_1,\psi_2,\psi_3)=(\alpha,\beta,\lambda)$. Unfortunately, the exact closed forms for the above expectations are difficult to obtain. Therefore, the observed Fisher information matrix $\hat{I}_{ij}=\left\{-\left[\partial^2\ell(\blacksquare)/\partial\psi_i\,\partial\psi_j\right]\right\}_{\blacksquare=\hat{\blacksquare}}$, which is obtained by dropping the expectation operator E, will be used to construct confidence intervals (CIs) for the parameters. Hence, the observed information matrix is given by

$$\widehat{I}=(\alpha,\beta,\lambda)=\begin{pmatrix}-\frac{\partial^2 l}{\partial\alpha^2} & -\frac{\partial^2 l}{\partial\alpha\partial\beta} & -\frac{\partial^2 l}{\partial\alpha\partial\lambda}\\ -\frac{\partial^2 l}{\partial\beta\partial\lambda} & -\frac{\partial^2 l}{\partial\beta^2} & -\frac{\partial^2 l}{\partial\beta\partial\lambda}\\ -\frac{\partial^2 l}{\partial\lambda\partial\alpha} & -\frac{\partial^2 l}{\partial\lambda\partial\beta} & -\frac{\partial^2 l}{\partial\lambda^2}\end{pmatrix}_{(\alpha=\widehat{\alpha},\beta=\widehat{\beta},\lambda=\widehat{\lambda})} \tag{9}$$

Therefore,the asympotice variance- coveriance matrix $[\widehat{V}]$ for the MLEs is obtained by inverting the observed information matrix $\widehat{I}(\alpha,\beta,\lambda)$ or equivalent

$$[\widehat{V}]=\widehat{I}=(\alpha,\beta,\lambda)=\begin{pmatrix}\widehat{var(\alpha)} & cov(\alpha,\beta) & cov(\alpha,\lambda)\\ cov(\beta,\alpha) & \widehat{var(\beta)} & cov(\beta,\lambda)\\ cov(\alpha,\lambda) & cov(\beta,\lambda) & \widehat{var(\lambda)}\end{pmatrix}_{(\widehat{\alpha},\widehat{\beta},\widehat{\lambda})} \tag{10}$$

It is well known that under some regularity conditions, see [6], $(\widehat{\alpha},\ \widehat{\beta},\ \widehat{\lambda})is$ approximately distributed as multivariate normal with mean $(\alpha,\ \beta,\ \lambda)$ and coveriance matrix $I^{-1}(\alpha,\ \beta,\ \lambda)$. Thus, the $(1-\gamma)$ 100 % approximate confidence intervals (ACIs) for α,β and λ can be given by

$$(\widehat{\alpha}\pm Z_{\frac{\gamma}{2}}\sqrt{\widehat{var(\alpha)}}),\ (\widehat{\beta}\pm Z_{\frac{\gamma}{2}}\sqrt{\widehat{var(\beta)}})\text{ and }(\widehat{\lambda}\pm Z_{\frac{\gamma}{2}}\sqrt{\widehat{var(\lambda)}}) \tag{11}$$

where $Z_{\frac{\gamma}{2}}is$ the percentile o standard normal distribution with right-tail probability $\frac{\gamma}{2}$.

3 Bayes Estimation

The Bayesian approach deals with the parameters as random and uncertainties on the parameters are described by a joint prior distribution, which is developed before the collected failure data. The ability of incorporating prior knowledge in the analysis makes the Bayesian approach very helpful in the reliability analysis because one of the main challenges associated with the reliability analysis is the limited availability of data[5]. Bayesian estimates of the unknown parameters α,β and λ against the SEL, LINEX and GEL loss functions. It is assumed here that the parameters α,β and λ are independent and follow the following prior distributions,

$$\begin{cases}\pi_1(\alpha)=\frac{1}{\alpha},\alpha>0,\\ \pi_2(\beta)=\frac{1}{\beta},\beta>0,\\ \pi_3(\lambda)=\frac{1}{\lambda},\lambda>0,\end{cases} \tag{12}$$

The posterior distribution of α,β and λ denoted by $\pi^*(\alpha,\beta,\lambda\mid\underline{x})$ can be obtained by combining the likelihood function (4) with the priors (9) and it can be written as

$$\pi^*(\alpha,\beta,\lambda\mid\underline{x})=\frac{L(\alpha,\beta,\lambda\mid\underline{x})\ \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)}{\int_0^\infty\int_0^\infty\int_0^\infty L(\alpha,\beta,\lambda\mid\underline{x})\ \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ d\alpha\ d\beta\ d\lambda}. \tag{13}$$

A commonly used loss function is the SEL, which is symmetrical loss function that assigns equal losses to over estimation and underestimation. If ϕ is the parameter to be estimated by an estimator $\hat{\phi}$, then the square error loss function is defined as

$$L\left(\phi,\hat{\phi}\right)=\left(\hat{\phi}-\phi\right)^2, \tag{14}$$

Therefore, the Bayes estimate of any function of α, β and λ, say $g(\alpha, \beta, \lambda)$ under the SEL function can be obtained as

$$\hat{g}_{BS}(\alpha, \beta, \lambda \mid \underset{\sim}{x}) = E_{\alpha, \beta, \lambda \mid \underset{\sim}{x}}(g(\alpha, \beta, \lambda)), \tag{15}$$

where

$$E_{\alpha, \beta, \lambda \mid \underset{\sim}{x}}(g(\alpha, \beta, \lambda)) = \frac{\int_0^\infty \int_0^\infty \int_0^\infty g(\alpha, \beta, \lambda)\ \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x}) d\alpha\, d\beta\, d\lambda}{\int_0^\infty \int_0^\infty \int_0^\infty \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x})\, d\alpha d\beta d\lambda}. \tag{16}$$

The LINEX loss function $L(\triangle)$ for a parameter ϕ is given by:

$$L(\triangle) = \left(e^{c\triangle} - c\triangle - 1\right),\ c \neq 0,\ \triangle = \hat{\phi} - \phi, \tag{17}$$

Hence, under LINEX loss function, the Bayes estimate of a function $g(\alpha, \beta, \lambda)$ is

$$\hat{g}_{BL}(\alpha, \beta, \lambda \mid \underset{\sim}{x}) = -\frac{1}{c} \log\left[E\left(e^{-cg(\alpha, \beta, \lambda)} \mid \underset{\sim}{x}\right)\right],\ c \neq 0, \tag{18}$$

$$E\left(e^{-cg(\alpha, \beta, \lambda)} \mid \underset{\sim}{x}\right) = \frac{\int_0^\infty \int_0^\infty \int_0^\infty e^{-cg(\alpha, \beta, \lambda)}\ \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x}) d\alpha d\beta d\lambda}{\int_0^\infty \int_0^\infty \int_0^\infty \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x})\, d\alpha d\beta d\lambda}, \tag{19}$$

Another useful a symmetric loss function is the General Entropy (GE) loss function:

$$L_2(\widehat{\phi}, \phi) \propto (\widehat{\phi}\ /\ \phi)^q - q\ \log(\widehat{\phi}\ /\phi) - 1, \tag{20}$$

whose minimum occurs at $\phi = \widehat{\phi}$. The Bayes predictive estimate $\widehat{\phi}_{BG}$ of ϕ under GE loss is

$$\widehat{\phi}_{BG} = (E_u[\phi^{-q}])^{-\frac{1}{q}} \tag{21}$$

$$E\left([\phi^{-q}])^{-\frac{1}{q}} \mid \underset{\sim}{x}\right) = \frac{\int_0^\infty \int_0^\infty \int_0^\infty [\phi^{-q}]^{-\frac{1}{q}}\ \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x}) d\alpha d\beta d\lambda}{\int_0^\infty \int_0^\infty \int_0^\infty \pi_1(\alpha)\ \pi_2(\beta)\ \pi_3(\lambda)\ L(\alpha, \beta, \lambda \mid \underset{\sim}{x})\, d\alpha d\beta d\lambda} \tag{22}$$

It is noticed that the ratio of multiple integrals in (16), (19) and (22) cannot be obtain in a explicit form. Thus, the MCMC technique is used to generate samples from the joint posterior density function in (13). To implement the MCMC technique, we consider the Gibbs within Metropolis–Hasting samplers procedure. From (13), the joint posterior distribution can be written as

$$\pi^*(\alpha, \beta, \lambda \mid x) \propto m \frac{\alpha^{-1}\ \lambda^{m-1} \beta^{m-1} (\ln \alpha)^m}{(\alpha - 1)^m} e^{-\lambda \sum\limits_{i=1}^m x_i^{-\beta}}\ \alpha^{\sum\limits_{i=1}^m e^{-\lambda x_i^{-\beta}}} \cdot \prod_{i=1}^m x_i^{-(\beta+1)}$$
$$\left(\prod_{i=1}^m (\frac{\alpha}{\alpha - 1})^{R_i}\right)\left(\prod_{i=1}^m (1 - \alpha^{e^{-\lambda x_i^{-\beta}}} - 1)^{R_i}\right). \tag{23}$$

The conditional posterior denisities of α, β and λ can be written as

$$\pi_1^*(\alpha \mid \beta, \lambda, \underset{\sim}{x}) \propto \frac{(\ln \alpha)^m}{(\alpha - 1)^m} \left(\alpha^{\sum\limits_{i=1}^m e^{-\lambda x_i^{-\beta}}} - 1\right) \prod_{i=1}^m \left(\frac{\alpha}{\alpha - 1}\right)^{R_i} \prod_{i=1}^m \left(1 - \alpha^{e^{-\lambda x_i^{-\beta}}} - 1\right)^{R_i}, \tag{24}$$

$$\pi_2^*(\beta \mid \alpha,\lambda,\underline{x}) \propto \beta^{m-1}\left(\prod_{i=1}^{m} x_i^{-\beta-1}\right)\left(e^{-\lambda\sum_{i=1}^{m} x_i^{-\beta}}\alpha^{\sum_{i=1}^{m} e^{-\lambda x_i^{-\beta}}}\right)\prod_{i=1}^{m}\left(1-\alpha^{e^{-\lambda x_i^{-\beta}}-1}\right)^{R_i} \tag{25}$$

and

$$\pi_3^*(\lambda \mid \alpha,\beta,\underline{x}) \propto \lambda^{m-1}\left(e^{-\lambda\sum_{i=1}^{m} x_i^{-\beta}}\alpha^{\sum_{i=1}^{m} e^{-\lambda x_i^{-\beta}}}\right)\prod_{i=1}^{m}(1-\alpha^{e^{-\lambda x_i^{-\beta}}-1})^{R_i}. \tag{26}$$

It can be easily seen that the conditional posteriors of α,β and λ in equations (24),(25) and (26)do not present standard forms, so Gibbs sampling is not a straightforward option, the use of the Metropolis–Hasting sampler is required for the implementations MCMC technique. The algorithm of Metropolis–Hastings within Gibbs sampling is as follows:

(1)Start with initial guess $\left(\alpha^{(0)},\beta^{(0)},\lambda^{(0)}\right)$.
(2)Set $j=1$.
(3)Using the following M-H algorithm, generate $\alpha^{(j)},\beta^{(j)}$ and $\lambda^{(j)}$ from $\pi_1^*\left(\alpha^{(j-1)} \mid \beta^{(j-1)},\lambda^{(j-1)},\underline{x}\right)$, $\pi_2^*\left(\beta^{(j-1)} \mid \alpha^{(j)},\lambda^{(j-1)},\underline{x}\right)$ and $\pi_3^*\left(\lambda^{(j-1)} \mid \alpha^{(j)},\beta^{(j)},\underline{x}\right)$ with the normal proposal distributions

$$N\left(\alpha^{(j-1)},var(\alpha)\right),\ N\left(\beta^{(j-1)},var(\beta)\right) \text{ and } N\left(\lambda^{(j-1)},var(\lambda)\right),$$

where $var(\alpha)$, $var(\beta)$ and $var(\lambda)$ can be obtained from the main diagonal in inverse Fisher information matrix .

(4)Generate a proposal α^* from $N\left(\alpha^{(j-1)},var(\alpha)\right)$, β^* from $N\left(\beta^{(j-1)},var(\alpha)\right)$ and λ^* from$N\left(\lambda^{(j-1)},var(\lambda)\right)$.

(i)Evaluate the acceptance probabilities

$$\left.\begin{aligned}
\eta_\alpha &= \min\left[1,\frac{\pi_1^*(\alpha^*|\beta^{(j-1)},\lambda^{(j-1)},\underline{x})}{\pi_1^*(\alpha^{(j-1)}|\beta^{(j-1)},\lambda^{(j-1)},\underline{x})}\right],\\
\eta_\beta &= \min\left[1,\frac{\pi_2^*(\beta^*|\alpha^{(j)},\lambda^{(j-1)},\underline{x})}{\pi_2^*(\beta^{(j-1)}|\alpha^{(j)},\lambda^{(j-1)},\underline{x})}\right],\\
\eta_\lambda &= \min\left[1,\frac{\pi_3^*(\lambda^*|\alpha^{(j)},\beta^{(j)},\underline{x})}{\pi_3^*(\lambda^{(j-1)}|\alpha^{(j)},\beta^{(j)},\underline{x})}\right].
\end{aligned}\right\}.$$

(5) Generate a u_1, u_2 and u_3 from a Uniform $(0,1)$ distribution.
(iii)If $u_1<\eta_\alpha$, accept the proposal and set $\alpha^{(j)}=\alpha^*$, else set $\alpha^{(j)}=\alpha^{(j-1)}$.
(iv)If $u_2<\eta_\beta$, accept the proposal and set $\beta^{(j)}=\beta^*$, else set $\beta^{(j)}=\beta^{(j-1)}$.
(v)If $u_3<\eta_\lambda$,accept the proposal and set $\lambda^{(j)}=\lambda^*$, else set $\lambda^{(j)}=\lambda^{(j-1)}$.
1.Set $j=j+1$.
(7)Repeat Steps $(3)-(6)$, N times and obtain $\alpha^{(i)},\beta^{(i)}$and $\lambda^{(i)} i=1,2,...N$.
(8)To compute the CRs of α,β and λ, $\psi_k^{(i)}$, $k=1,2,3$, $(\psi_1,\psi_2,\psi_3)=(\alpha,\beta,\lambda)$ as $\psi_k^{(1)}<\psi_k^{(2)}...<\psi_k^{(N)}$, then the $100(1-\vartheta)\%$ CRIs of ψ_k is

$$\left(\psi_{k(N\ \vartheta/2)},\psi_{k(N\ (1-\vartheta/2))}\right).$$

In order to guarantee the convergence and to remove the affection of the selection of initial values, the first M simulated varieties are discarded. Then the selected samples are $\psi_k^{(i)}$, $j=M+1,...N$, for sufficiently large N.

Based on SEL function, the approximate Bayes estimates of ψ_k is given by

$$\hat{\psi}_k=\frac{1}{N-M}\sum_{j=M+1}^{N\psi_k(j)}\Phi^{(j)}, \tag{27}$$

the approximate Bayes estimates for ψ_k, under LINEX loss function, from is

$$\hat{\psi}_k = \frac{-1}{c}\log\left[\frac{1}{N-M}\sum_{j=M+1}^{N} e^{-c\,\varphi^{(j)}}\right], k=1,2,3. \tag{28}$$

the approximate Bayes estimates for ψ_k, under GE loss function, from is

$$\hat{\psi}_k = \left[\frac{1}{N-M}\sum_{j=M+1}^{N} (\Phi^{(j)})^{-q}\right]^{-\frac{1}{q}}. \tag{29}$$

4 Applications

In this section, the proposed estimation methods are applied to the failure data for a group of 23 the Acquisition of resistance in guinea pigs infected with different doss of virulent tubercle bacilli [9]. The Kolmogorov Smirnov (K-S)distance between the empirical distribution of failure data and CDF of APIW distribution is 0.093657.9 with $P-value$ equals 0.976227. Hence, the APIW distribution fits well to the given data. A progressive Type-II censored sample of effective size $m=10$ was randomly selected from 23 failure observations with progressive censored scheme $R=(7,6,0,0,0,0,0,0,0,0)$.

Table 1
Progressively Type-II failure data.

17.88	28.92	33.00	41.52	42.12	45.60
48.48	51.84	51.96	54.12	55.56	67.80
68.64	68.64	68.88	84.12	93.12	98.64
105.12	105.84	127.92	128.04	173.40	

The MLEs of parameters based on Progressively Type-II failure data presented in Table 1 are obtained to be $\hat{\alpha},\hat{\beta}$ and $\hat{\lambda}$ are displayed in Table 2. The Bayes estimates relative to both SEL, LINEX and GEL functions are computed, for different values of the shape parameter c of LINEX loss function and for different values of the shape parameter h of GEL function for the parameters α, β and λ, and also displayed inTable 2

Table 2
Point estimates for the parameters α,β and λ.

Parameters	MLE	SEL	LINEX		GEL		
			$c=-2$	$c=2$	$h=-2$	$h=0.5$	$h=1$
α	61.9061	80.8676	90.481	58.247	64.4884	64.4873	64.2942
β	2.66679	2.63195	2.63213	2.63177	2.68728	2.68717	2.68715
λ	4429.67	5063.48	5782.16	3750.72	4992.99	4992.88	4988.91

The 95% ACIs and CRIs for the parameters α,β and λ are computed, the results are displayed in Table 3.

Table 3
95% ACIs and CRI of α, β and λ.

Parameter	MLE	MCMC
α	$(-202.621, 326.433)$	$(59.6506, 91.0517)$
β	$(1.97782, 3.35576)$	$(2.60769, 2.65459)$
λ	$(-6048.17, 14907.5)$	$(405.96, 5683.37)$

5 Conclusions

The purpose of this paper is to develope different method to estimate and construct confidence intervals for the parameters of the Alpha power inverse Weibull distribution under progressive Type-II censored samples. The MLEs of the unknown parameters are obtained and suggest different confidence intervals using asymptotic distributions .The Bayesian estimates of the unknown parameters are also suggested. It is observed that the bayes estimators cannot be obtained in explicit form so the MCMC technique was used. The theoretical results have been applied with the numerical example to illustrative purposes. A comparison between the ACIs and the CRIs is provided for the estimated parameters through a simulated example. It was found that the width of MCMC credible intervals is narrower than ACIs. We may judge that the Bayes estimators obtained under MCMC method can be preferred.

Acknowledgement

The author would like to express thanks to the editors and referees for their valuable comments and suggestions which significantly improved the paper.

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