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Soft semi-open sets and related properties in soft topological spaces

Bin Chen

School of Mathematical Science, University of Jinan, Jinan 250022, P.R.China.

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Abstract: D. Molodtsov (1999) [1] introduced the concept of a soft set as a new approach for modeling uncertainties. Xiao et al. [7] and Pei and Miao [8] discussed the relationship between soft sets and information systems. They showed that soft sets are a class of special information systems. So it's important to study the structures of soft sets for information systems. Shabir and Naz(2011) [20], studied the topological structures of soft sets. In this paper, we continue the study on soft topological spaces and investigate the properties of soft semi-open sets, semi-closed sets, soft semi-interior and soft semi-closure. Then discuss the relationship between soft semi-open (semi-closed) sets, soft semi-interior (semi-closure) and soft open (closed) sets, soft interior (closure) introduced in [20]. We also define and discuss the properties of soft semi-separation axioms which are important for further research on soft topology. These research not only can form the theoretical basis for further applications of topology on soft sets but also lead to the development of information systems.

Keywords: Soft sets, Soft Topology, Soft Semi-open Sets, Soft Semi-closed Sets, information systems.

1. Introduction

Soft set theory [1] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. He has shown several applications of this theory in solving many practical problems in economics, engineering, social science, medical science, etc. In recent years the development in the fields of soft set theory and its application has been taking place in a rapid pace. This is because of the general nature of parametrization expressed by a soft set.

Later Maji et al. [2] presented some new definitions on soft sets such as a subset, the complement of a soft set and discussed in detail the application of soft set theory in decision making problems [3]. D. Chen,et al. [4-5] and Kong et al. [6] introduced a new definition of soft set parameterization reduction. Xiao et al. [7] and Pei and Miao [8] discussed the relationship between soft sets and information systems. Also an attempt was made by Kostek [9] to assess sound quality based on a soft set approach. Mushrif et al. [10] presented a novel method for the classification of natural textures using the notions of soft set theory.

The algebraic nature of set theories dealing with uncertainties has been studied by some authors. Aktas and Căgman [11] compared soft sets to the related concepts of fuzzy sets and rough sets. They also defined the notion of soft groups and derived some properties. Feng et al. [12] dealt with the algebraic structure of semi-rings by applying soft set theory. The concept of a fuzzy soft group was introduced by Aygunoglu and Aygun [13]. Furthermore, İnan et al.[14] introduced and investigated the notion of fuzzy soft rings and fuzzy soft ideals. Jun et al. [15] discussed the applications of soft sets in an p-ideal theory of BCI-algebras. Also in [16,17], Jun et al. studied applications of soft sets in Hilbert algebras and ordered semigroups.

The topological structures of set theories dealing with uncertainties were first studied by Chang [18]. Chang introduced the notion of fuzzy topology and also studied some of its basic properties. Lashin et al. [19] generalized rough set theory in the framework of topological spaces. Recently, Shabir and Naz [20] introduced the notion of soft topological spaces which are defined over an initial universe with a fixed set of parameters. They also studied some of basic concepts of soft topological spaces. Later, Aygunoglu et al. [21], Zorlutuna et al.[22] and Hussain et

* Corresponding author: e-mail:jnchenbin@yahoo.com.cn

al.[23] continued to study the properties of soft topological spaces. They got many important results in soft topological spaces.

In the present study, we introduce some new concepts in soft topological spaces such as soft semi-open sets, soft semi-closed sets, soft semi-interior, soft semi-closure and soft semi-separation axioms. We also study the relationship between soft semi-interior, soft semi-closure and soft interior, soft closure. These research not only can form the theoretical basis for further applications of topology on soft sets but also lead to the development of information systems.

2. Preliminaries

Let U be an initial universe set and E_U be a collection of all possible parameters with respect to U, where parameters are the characteristics or properties of objects in U. We will call E_U the universe set of parameters with respect to U.

Definition 2.1([1]). A pair (F, A) is called a soft set over U if $A \subset E_U$ and $F : A \to P(U)$, where $P(U)$ is the set of all subsets of U.

Definition 2.2([12]). Let U be an initial universe set and E_U be an universe set of parameters. Let (F, A) and (G, B) be soft sets over a common universe set U and $A, B \subset E$. Then (F, A) is a subset of (G, B), denoted by $(F, A)\tilde{\subset}(G, B)$, if: (i) $A \subset B$; (ii)for all $e \in A$, $F(e) \subset G(e)$.

(F, A) equals (G, B), denoted by $(F, A) = (G, B)$, if $(F, A)\tilde{\subset}(G, B)$ and $(G, B)\tilde{\subset}(F, A)$.

Definition 2.3 ([2]). A soft set (F, A) over U is called a null soft set, denoted by $\emptyset$, if $e \in A$, $F(e) = \emptyset$.

Definition 2.4([2]). A soft set (F, A) over U is called an absolute soft set, denoted by $\tilde{A}$, if $e \in A$, $F(e) = U$.

Definition 2.5([2]). The union of two soft sets (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cup B$, and $\forall e \in C$,

$$H(e) = \begin{cases} F(e), & \text{if } e \in A - B; \\ G(e), & e \in B - A; \\ F(e) \cup G(e), & e \in B \cap A. \end{cases}$$

We write $(F, A) \cup (G, B) = (H, C)$.

Definition 2.6.([12]). The intersection of two soft sets of (F, A) and (G, B) over a common universe U is the soft set (H, C), where $C = A \cap B$, and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. We write $(F, A) \cap (G, B) = (H, C)$.

Now we recall some definitions and results defined and discussed in [20-23]. Henceforth, let X be an initial universe set and E be the fixed non-empty set of parameter with respect to X unless otherwise specified.

Definition 2.7. For a soft set (F, A) over U, the relative complement of (F, A) is denoted by $(F, A)'$ and is defined by $(F, A)' = (F', A)$, where $F' : A \to P(U)$ is a mapping given by $F'(e) = U - F(e)$ for all $e \in A$.

Definition 2.8. Let τ be the collection of soft sets over X, then τ is called a soft topology on X if τ satisfies the following axioms:

(1) $\emptyset, \tilde{X}$ belong to τ.

(2) The union of any number of soft sets in τ belongs to τ.

(3) The intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 2.9. Let (X, τ, E) be a soft space over X, then the members of τ are said to be soft open sets in X.

Definition 2.10. Let (X, τ, E) be a soft space over X. A soft set (F, E) over X is said to be a soft closed set in X, if its relative complement $(F, E)'$ belongs to τ.

Proposition 2.1. Let (X, τ, E) be a soft space over X. Then

(1) $\emptyset, \tilde{X}$ are soft closed sets over X.

(2) The intersection of any number of soft closed sets is a soft closed set over X.

(3) The union of any two soft closed sets is a soft closed set over X.

Definition 2.11 Let (X, τ, E) be a soft topological space and (A, E) be a soft set over X.

1) The soft interior of (A, E) is the soft set $Int(A, E) = \bigcup\{(O, E) : (O, E)$ is soft open and $(O, E)\tilde{\subset}(A, E)\}$.

2) The soft closure of (A, E) is the soft set $Cl(A, E) = \bigcap\{(F, E) : (F, E)$ is soft closed and $(A, E)\tilde{\subset}(F, E)\}$.

By property 3 for soft open sets, $Int(A, E)$ is soft open. It is the largest soft open set contained in (A, E).

By property 2 for soft closed sets, $Cl(A, E)$ is soft closed. It is the smallest soft closed set containing (A, E).

Proposition 2.2. Let (X, τ, E) be a soft topological space and let (F, E) and (G, E) be a soft set over X. Then

(1)$Int(Int(F, E)) = Int(F, E)$

(2)$(F, E)\tilde{\subset}(G, E)$ implies $Int(F, E)\tilde{\subset}Int(G, E)$

(3)$Cl(Cl(F, E)) = Cl(F, E)$

(4)$(F, E)\tilde{\subset}(G, E)$ implies $Cl(F, E)\tilde{\subset}Cl(G, E)$

Definition 2.12. Let (F, E) be a soft set over X and $x \in X$. We say that $x \in (F, E)$ read as x belongs to the soft set (F, E), whenever $x \in F(\alpha)$ for all $\alpha \in E$.

Note that for $x \in X$, $x \notin (F, E)$) if $x \notin F(\alpha)$ for some $\alpha \in E$.

Definition 2.13. Let $x \in X$, then (x, E) denotes the soft set over X for which $x(\alpha) = \{x\}$, for all $\alpha \in E$.

3. Soft semi-open sets and soft semi-closed sets

Definition 3.1. A soft set (A, E) in a soft topological space (X, τ, E) will be termed soft semi-open (writen S.S.O) if and only if there exists a soft open set (O, E) such that $(O, E)\tilde{\subset}(A, E)\tilde{\subset}Cl(O, E)$.

From the Definition of soft semi-open set, we have the following Remark.

Remark 3.1. Every soft open set in a soft topological space (X, τ, E) is S.S.O.

Now we give an example to show that the converse of above remark does not hold.

Example 3.1. Let $X = \{h_1, h_2, h_3\}$, $E = \{e_1, e_2\}$ and $\tau = \{\emptyset, \tilde{X}, (F_1, E), (F_2, E), (F_3, E), ..., (F_7, E)\}$ where $(F_1, E), (F_2, E), (F_3, E), ..., (F_7, E)$ are soft sets over X, defined as follows

$$F_1(e_1) = \{h_1, h_2\}, F_1(e_2) = \{h_1, h_2\},$$

$$F_2(e_1) = \{h_2\}, F_2(e_2) = \{h_1, h_3\},$$

$$F_3(e_1) = \{h_2, h_3\}, F_3(e_2) = \{h_1\},$$

$$F_4(e_1) = \{h_2\}, F_4(e_2) = \{h_1\},$$

$$F_5(e_1) = \{h_1, h_2\}, F_5(e_2) = X,$$

$$F_6(e_1) = X, F_6(e_2) = \{h_1, h_2\},$$

$$F_7(e_1) = \{h_2, h_3\}, F_7(e_2) = \{h_1, h_3\},$$

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. A soft set (G, E) in (X, τ, E) is defined as follows:

$$G(e_1) = \{h_2, h_3\}, G(e_2) = \{h_1, h_2\}.$$

Then, for the soft open set (F_3, E), we have $(F_3, E)\tilde{\subset}(G, E)$. Because $Cl(F_3, E) = \tilde{X}$, we have $(F_3, E)\tilde{\subset}(G, E)\tilde{\subset}Cl(F_3, E)$. Then the soft set (G, E) is S.S.O in the soft topological space (X, τ, E) but not soft open since $(G, E) \notin \tau$.

Theorem 3.1. A soft subset (A, E) in a soft topological space (X, τ, E) is S.S.O if and only if $(A, E)\tilde{\subset}Cl(Int(A, E))$.

Proof. Sufficiency. Let $(A, E)\tilde{\subset}Cl(Int(A, E))$. Then for $(O, E) = Int(A, E)$, we have $(O, E)\tilde{\subset}(A, E)\tilde{\subset}Cl(O, E)$.

Necessity. Let (A, E) be S.S.O. Then $(O, E)\tilde{\subset}(A, E)\tilde{\subset} Cl(O, E)$ for some soft open set (O, E). But $(O, E)\tilde{\subset} Int(A, E)$ and $Cl(O, E)\tilde{\subset}Cl(Int(A, E))$. Hence $(A, E) \tilde{\subset}Cl(O, E)\tilde{\subset}Cl(Int(A, E))$.

Theorem 3.2. Let $\{(A_\alpha, E)\}_{\alpha\in\Delta}$ be a collection of S.S.O sets in (X, τ, E). Then $\bigcup_{\alpha\in\Delta}(A_\alpha, E)$ is S.S.O.

Proof. For each $\alpha \in \Delta$, we have a soft open set (O_α, E) such that $(O_\alpha, E)\tilde{\subset}(A_\alpha, E)\tilde{\subset}Cl(O_\alpha, E)$. Then $\bigcup_{\alpha\in\Delta}$ $(O_\alpha, E) \tilde{\subset} \bigcup_{\alpha\in\Delta}(A_\alpha, E)\tilde{\subset} \bigcup_{\alpha\in\Delta} Cl(O_\alpha, E)\tilde{\subset}Cl \bigcup_{\alpha\in\Delta}$ (O_α, E).

Theorem 3.3. Let (A, E) be S.S.O in a soft topological space (X, τ, E) and suppose $(A, E)\tilde{\subset}(B, E)\tilde{\subset}Cl(A, E)$. Then (B, E) is S.S.O.

Proof. There exists a soft open set (O, E) such that $(O, E)\tilde{\subset}(A, E)\tilde{\subset}Cl(O, E)$. Then $(O, E)\tilde{\subset}(B, E)$. But $Cl(A, E)\tilde{\subset}Cl(O, E)$ and thus $(B, E)\tilde{\subset}Cl(O, E)$. Hence $(O, E)\tilde{\subset}(B, E)\tilde{\subset}Cl(O, E)$ and (B, E) is S.S.O.

Definition 3.2. A soft set (B, E) in a soft topological space (X, τ, E) will be termed soft semi-closed (writen S.S.C) if its relative complement is soft semi-open, e.g.,there exists a soft closed set (F, E) such that $Int(F, E)\tilde{\subseteq}(B, E)\tilde{\subset}$ (F, E).

Remark 3.2. Every soft closed set in a soft topological space (X, τ, E) is S.S.C.

Now we give an example to show that the converse of above remark does not hold.

Example 3.2. The soft topological space (X, τ, E) is the same as in Example 3.1. Then $(B, E) = (G, E)'$ where we have $B(e_1) = \{h_1\}, B(e_2) = \{h_3\}$ is soft semi-closed since (G, E) is S.S.O in Example 3.1. Since all the soft closed sets in (X, τ, E) are $\mathcal{F} = \{\emptyset, \tilde{X}, (H_1, E), (H_2, E), (H_3, E), ..., (H_7, E)\}$ where $(H_1, E), (H_2, E), (H_3, E), ..., (H_7, E)$ are as follows:

$$H_1(e_1) = \{h_3\}, H_1(e_2) = \{h_3\},$$

$$H_2(e_1) = \{h_1, h_3\}, H_2(e_2) = \{h_2\},$$

$$H_3(e_1) = \{h_1\}, H_3(e_2) = \{h_2, h_3\},$$

$$H_4(e_1) = \{h_1, h_3\}, H_4(e_2) = \{h_2, h_3\},$$

$$H_5(e_1) = \{h_3\}, H_5(e_2) = \emptyset,$$

$$H_6(e_1) = \emptyset, H_6(e_2) = \{h_3\},$$

$$H_7(e_1) = \{h_1\}, H_7(e_2) = \{h_2\},$$

and $(B, E) \notin \mathcal{F}$. So (B, E) is a soft semi-closed set but not a soft closed set.

Theorem 3.4. A soft subset (B, E) in a soft topological space (X, τ, E) is S.S.C if and only if $Int(Cl(B, E))$ $\tilde{\subset}(B, E)$.

Proof. Sufficiency. Let $Int(Cl(B,E))\tilde{\subset}(B,E)$. Then for $(F,E)=Cl(B,E)$, we have $Int(F,E)\tilde{\subset}(B,E)\tilde{\subset}(F,E)$.

Necessity. Let (B,E) be S.S.C. Then $Int(F,E)\tilde{\subseteq}(B,E)\tilde{\subset}(F,E)$ for some soft closed set (F,E). But $Cl(B,E)\tilde{\subset}Cl(F,E)=(F,E)$ and $Int(Cl(B,E))\tilde{\subset}Int(F,E)$. Hence $Int(Cl(B,E))\tilde{\subset}Int(F,E)\tilde{\subseteq}(B,E)$.

Theorem 3.5. Let $\{(B_\alpha,E)\}_{\alpha\in\Delta}$ be a collection of S.S.C sets in (X,τ,E). Then $\bigcap_{\alpha\in\Delta}(B_\alpha,E)$ is S.S.C.

Proof. For each $\alpha\in\Delta$, we have a soft closed set (F_α,E) such that $Int(F_\alpha,E)\tilde{\subset}(B_\alpha,E)\tilde{\subset}(F_\alpha,E)$. Then $Int(\bigcap_{\alpha\in\Delta}(F_\alpha,E))\tilde{\subset}\bigcap_{\alpha\in\Delta}Int(F_\alpha,E)\tilde{\subset}\bigcap_{\alpha\in\Delta}(B_\alpha,E)\tilde{\subset}\bigcap_{\alpha\in\Delta}(F_\alpha,E)$. Because $\bigcap_{\alpha\in\Delta}(F_\alpha,E)=(F,E)$ is soft closed set by Prop 2.1(2), then $\bigcap_{\alpha\in\Delta}(B_\alpha,E)$ is S.S.C.

Theorem 3.6. Let (B,E) be S.S.C in a soft topological space (X,τ,E) and suppose $Int(B,E)\tilde{\subset}(A,E)\tilde{\subset}(B,E)$. Then (A,E) is S.S.C.

Proof. There exists a soft closed set (F,E) such that $Int(F,E)\tilde{\subseteq}(B,E)\tilde{\subset}(F,E)$. Then $(A,E)\tilde{\subset}(F,E)$. But $Int(Int(F,E))=Int(F,E)\tilde{\subset}Int(B,E)$ and thus $Int(F,E)\tilde{\subset}(A,E)$. Hence $Int(F,E)\tilde{\subseteq}(A,E)\tilde{\subset}(F,E)$ and (A,E) is S.S.C.

Now for any soft set in a soft topological space, we define the following sets.

Definition 3.3 Let (X,τ,E) be a soft topological space and (A,E) be a soft set over X.

1) The soft semi-interior of (A,E) is the soft set $Int_S(A,E)=\bigcup\{(O,E):(O,E)$ is soft semi-open and $(O,E)\tilde{\subset}(A,E)\}$.

2) The soft semi-closure of (A,E) is the soft set $Cl_S(A,E)=\bigcap\{(F,E):(F,E)$ is soft semi-closed and $(A,E)\tilde{\subset}(F,E)\}$.

By Theorem 3.2 and 3.5, we have $Int_S(A,E)$ is soft semi-open and $Cl_S(A,E)$ is soft semi-closed.

Example 3.3. Let the soft topological space (X,τ,E) and the soft set (G,E) be the same as in Example 3.1, we can get $Int_S(G,E)=(G,E)$.

Example 3.4. Let the soft topological space (X,τ,E) and the soft set (B,E) be the same as in Example 3.2, we can get $Cl_S(B,E)=(B,E)$.

From the definitions of soft semi-interior and soft semi-closure, we have the following theorem.

Theorem 3.7. Let (X,τ,E) be a soft topological space and (A,E) be a soft set over X. We have $Int(A,E)\tilde{\subset}Int_S(A,E)\tilde{\subset}(A,E)\tilde{\subset}Cl_S(A,E)\tilde{\subset}Cl(A,E)$.

Proof. By Remark 3.1, Remark 3.2 and Definition 3.3.

In the following discussion, we use $(A,E)^0$,$(A,E)^-$, $(A,E)_0$ and $(A,E)_-$ instead of $Int(A,E)$, $Cl(A,E)$, $Int_S(A,E)$ and $Cl_S(A,E)$ for short.

Theorem 3.8. Let (X,τ,E) be a soft topological space and (A,E) be a soft set over X. Then

(1)$((A,E)_-)^{'}=((A,E)^{'})_0$

(2)$((A,E)_0)^{'}=((A,E)^{'})_-$

(3)$(A,E)_0=(((A,E)^{'})_-)^{'}$

Proof.(1) $((A,E)_-)^{'}=(\bigcap\{(F,E):(F,E)$ is soft semi-closed and $(A,E)\tilde{\subset}(F,E)\})^{'}$
$=\bigcup\{(F,E)^{'}:(F,E)$ is soft semi-closed and $(A,E)\tilde{\subset}(F,E)\}=\bigcup\{(F,E)^{'}:(F,E)^{'}$ is soft semi-open and $(F,E)^{'}\tilde{\subset}(A,E)^{'}\}$
$=((A,E)^{'})_0$.

(2) can be proved similarly.

(3) By part (2).

Theorem 3.9. Let (X,τ,E) be a soft topological space and let (F,E) and (G,E) be soft sets over X. Then

(1)$\emptyset_-=\emptyset$ and $\tilde{X}_-=\tilde{X}$

(2)(F,E) is S.S.C if and only if $(F,E)=(F,E)_-$

(3)$(F,E)_{--}=(F,E)_-$

(4)$(F,E)\tilde{\subset}(G,E)$ implies $(F,E)_-\tilde{\subset}(G,E)_-$

(5)$((F,E)\cap(G,E))_-\tilde{\subset}(F,E)_-\cap(G,E)_-$

Proof. (1)is obvious.

(2)If (F,E) is S.S.C, then (F,E) is itself a soft semi-closed set over X which contains (F,E). So (F,E) is the smallest soft semi-closed set containing (F,E) and $(F,E)=(F,E)_-$.

Conversely suppose that $(F,E)=(F,E)_-$. Since $(F,E)_-$ is a soft semi-closed set, so (F,E) is S.S.C.

(3)Since $(F,E)_-$ is a soft semi-closed set therefore by part (2) we have $(F,E)_{--}=(F,E)_-$.

(4)Suppose that $(F,E)\tilde{\subset}(G,E)$. Then every soft semi-closed super set of (G,E) will also contain (F,E). This means every soft semi-closed super set of (G,E) is also a soft semi-closed super set of (F,E). Hence the soft intersection of soft semi-closed super sets of (F,E) is contained in the soft intersection of soft semi-closed super sets of (G,E). Thus $(F,E)_-\tilde{\subset}(G,E)_-$.

(5)Since $((F,E)\cap(G,E))\tilde{\subset}(F,E)$ and $((F,E)\cap(G,E))\tilde{\subset}(G,E)$. So by part (4) $((F,E)\cap(G,E))_-\tilde{\subset}(F,E)_-$ and $((F,E)\cap(G,E))_-\tilde{\subset}(G,E)_-$. Thus $((F,E)\cap(G,E))_-\tilde{\subset}(F,E)_-\cap(G,E)_-$.

Theorem 3.10. Let (X,τ,E) be a soft topological space and let (F,E) and (G,E) be soft sets over X. Then

(1)$\emptyset_0=\emptyset$ and $\tilde{X}_0=\tilde{X}$

(2)(F,E) is S.S.O if and only if $(F,E)=(F,E)_0$

(3)$(F,E)_{00}=(F,E)_0$

(4)$(F,E)\tilde{\subset}(G,E)$ implies $(F,E)_0\tilde{\subset}(G,E)_0$

(5)$(F,E)_0\cup(G,E)_0\tilde{\subset}((F,E)\cup(G,E))_0$

Proof. (1)is obvious.

(2) If (F,E) is a soft semi-open set over X then (F,E) is itself a soft semi-open set over X which contains (F,E). So $(F,E)_0$ is the largest soft semi-open set contained in (F,E) and $(F,E)=(F,E)_0$.

Conversely, suppose that $(F,E)=(F,E)_0$. Since $(F,E)_0$ is a soft semi-open set, so (F,E) is a soft semi-open set over X.

(3)Since $(F,E)_0$ is a soft semi-open set therefore by part (2) we have $(F,E)_{00}=(F,E)_0$.

(4)Suppose that $(F,E)\tilde{\subset}(G,E)$. Since $(F,E)_0\tilde{\subset}(F,E)$ $\tilde{\subset}(G,E)$. $(F,E)_0$ is a soft semi-open subset of (G,E), so by definition of $(G,E)_0$, $(F,E)_0\tilde{\subset}(G,E)_0$.

(5)Since $(F,E)\tilde{\subset}((F,E)\cup(G,E))$ and $(G,E)\tilde{\subset}((F,E)\cup(G,E))$. So by (4), $(F,E)_0\tilde{\subset}((F,E)\cup(G,E))_0$ and $(G,E)_0$ $\tilde{\subset}((F,E)\cup(G,E))_0$. So that $(F,E)_0\cup(G,E)_0\tilde{\subset}((F,E)\cup(G,E))_0$, since $((F,E)\cup(G,E))_0$ is a soft semi-open set.

Theorem 3.11. Let $(X,\tau,\ E)$ be a soft topological space and let (A,E) be a soft set over X. Then
(1)$((A,E)^0)_0=((A,E)_0)^0=(A,E)^0$
(2)$((A,E)^-)_-=((A,E)_-)^-=(A,E)^-$

Proof. (1) Because $(A,E)^0$ is soft open, we have $(A,E)^0$ is soft semi-open by Remark 3.1. So we can get $((A,E)^0)_0=(A,E)^0$ by Theorem 3.10(2).

By Theorem 3.7, we have $(A,E)^0\tilde{\subset}(A,E)_0\tilde{\subset}(A,E)$, then we can get $(A,E)^0\tilde{\subset}((A,E)_0)^0$ $\tilde{\subset}(A,E)^0$ and so $((A,E)_0)^0=(A,E)^0$. This completes the proof.

(2) Because $(A,E)^-$ is soft closed, we have $(A,E)^-$ is soft semi-closed by Remark 3.2. So we can get $((A,E)^-)_-$ $=(A,E)^-$ by Theorem 3.9(2).

By Theorem 3.7, we have $(A,E)\tilde{\subset}(A,E)_-\tilde{\subset}(A,E)^-$, then we can get $(A,E)^-\tilde{\subset}((A,E)_-)^-$ $\tilde{\subset}(A,E)^-$ and so $((A,E)_-)^-=(A,E)^-$. This completes the proof.

4. Soft Semi-Separation Axioms

Definition 4.1. Let $(X,\tau,\ E)$ be a soft topological space over X and $x,y\in X$ such that $x\neq y$. If there exist soft semi-open sets (F,E) and (G,E) such that $x\in(F,E)$ and $y\notin(F,E)$ or $y\in(G,E)$ and $x\notin(G,E)$, then (X,τ,E) is called a soft S_0-space.

Not all soft topological spaces are soft S_0-spaces as shown by the following example:

Example 4.1. Let $X=\{h_1,h_2\}$, $E=\{e_1,e_2\}$ and $\tau=\{\emptyset,\tilde{X}\}$. Then (X,τ,E) is not a soft S_0-space because for $h_1,h_2\in X$, there only exists one soft semi-open set $\tilde{X}$ such that $h_1,h_2\in\tilde{X}$.

Definition 4.2. Let (X,τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$. If there exist soft semi-open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\notin(F,E)$ and $y\in(G,E)$, $x\notin(G,E)$, then (X,τ,E) is called a soft S_1-space.

Theorem 4.1. Let (X,τ,E) be a soft topological space over X. If (x,E) is a soft semi-closed set in (X,τ,E) for each $x\in X$ then (X,τ,E) is a soft S_1-space.

Proof. Suppose that for each $x\in X$, (x,E) is a soft semi-closed set in $(X,\tau,\ E)$, then $(x,E)^{'}$ is a soft semi-open set in $(X,\tau,\ E)$. Let $x,y\in X$ such that $x\neq y$. For $x\in X$, $(x,E)^{'}$ is a soft semi-open set such that $y\in(x,E)^{'}$ and $x\notin(x,E)^{'}$. Similarly $(y,E)^{'}$ is such that $x\in(y,E)^{'}$ and $y\notin(y,E)^{'}$. Thus $(X,\tau,\ E)$ is a soft S_1-space over X.

Lemma 4.1. Let (F,E) be a soft set over X and $x\in X$. Then:
(1)$x\in(F,E)$ if and only if $(x,E)\tilde{\subset}(F,E)$
(2)if $(x,E)\cap(F,E)=\emptyset$, then $x\notin(F,E)$.

Theorem 4.2. Let (X,τ,E) be a soft topological space over X and $x\in X$. If X is a soft S_1-space, then for each soft semi-open set (F,E) with $x\in(F,E)$:
(1) $(x,E)\tilde{\subset}\cap(F,E)$
(2) for all $z\neq x$, $z\notin\cap(F,E)$.

Proof. (1) Since $x\in\cap(F,E)$, by Lemma 4.1, it is obvious that $(x,E)\tilde{\subset}\cap(F,E)$.

(2) Let $z\neq x$ for $x,z\in X$; then there exist soft semi-open sets (H,E) such that $x\in(H,E)$ and $z\notin(H,E)$. So $z\notin H(\alpha)$ for some $\alpha\in E$, and we have $z\notin\cap_{\alpha\in E}F(\alpha)$. Consequently, $z\notin\cap(F,E)$.

Definition 4.3. Let (X,τ,E) be a soft topological space over X and $x,y\in X$ such that $x\neq y$. If there exist soft semi-open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\in(G,E)$ and $(F,E)\cap(G,E)=\emptyset$, then (X,T,E) is called a soft S_2-space.

Example 4.2. Let $X=\{h_1,h_2\}$, $E=\{e_1,e_2\}$ and τ be the soft discrete topology on X. Then $(X,\tau,\ E)$ is a soft S_2-space.

Theorem 4.3. Let (X,τ,E) be a soft topological space over X and $x\in X$. If $(X,\tau,\ E)$ is a soft S_2-space, then $(x,E)=\cap(F,E)$ for each soft semi-open set (F,E) with $x\in(F,E)$.

Proof. $\forall x\in X$ if $y\neq x$. Because $(X,\tau,\ E)$ is a soft S_2-space, there exist soft semi-open sets (H,E) and (G,E) such that $x\in(H,E)$ and $y\in(G,E)$ and $(H,E)\cap(G,E)=\emptyset$ and so $(H,E)\cap(y,E)=\emptyset$. Thus $(x,E)=\cap(F,E)$ for each soft semi-open set (F,E) with $x\in(F,E)$. This completes the proof.

Theorem 4.4. (1) Every soft S_1-space is a soft S_0-space.
(2) Every soft S_2-space is a soft S_1-space.

Proof. Let $(X,\tau,\ E)$ be a soft topological space over X and $x,y\in X$ such that $x\neq y$.

(1) If $(X,\tau,\ E)$ is a soft S_1-space then there exist soft semi-open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\notin(F,E)$ and $y\in(G,E)$, $x\notin(G,E)$. Obviously then we have $x\in(F,E)$, $y\notin(F,E)$ or $y\in(G,E)$, $x\notin(G,E)$. Thus $(X,\tau,\ E)$ is a soft S_0-space.

(2)If $(X,\tau,\ E)$ is a soft S_2-space then there exist soft semi-open sets (F,E) and (G,E) such that $x\in(F,E)$, $y\in(G,E)$ and $(F,E)\cap(G,E)=\emptyset$. Since $(F,E)\cap(G,E)=\emptyset$, so $x\notin(G,E)$, $y\notin(F,E)$. Hence $(X,\tau,\ E)$ is a soft S_1-space.

Example 4.3. Let $X=\{h_1,h_2\}$, $E=\{e_1,e_2\}$ and $\tau=\{\emptyset,\tilde{X},(F_1,E),(F_2,E),(F_3,E)\}$ where $(F_1,E),(F_2,E)$, (F_3,E) are soft sets over X,defined as follows

$$F_1(e_1)=X,F_1(e_2)=\{h_2\},$$

$$F_2(e_1) = \{h_1\}, F_2(e_2) = X,$$

$$F_3(e_1) = \{h_1\}, F_3(e_2) = \{h_2\},$$

Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. Also (X, τ, E) is a soft S_1-space over X but not a soft S_2-space because $h_1, h_2 \in X$ and there do not exist any soft semi-open sets (F, E) and (G, E) in X such that $h_1 \in (F, E), h_2 \in (G, E)$ and $(F, E) \cap (G, E) = \emptyset$.

Example 4.4. Let $X = \{h_1, h_2\}$, $E = \{e_1, e_2\}$ and $\tau = \{\emptyset, \tilde{X}, (F_1, E)\}$ where (F_1, E) is a soft set over X, defined as follows: $F_1(e_1) = X, F_1(e_2) = \{h_2\}$. Then τ defines a soft topology on X and hence (X, τ, E) is a soft topological space over X. Also (X, τ, E) is a soft S_0-space over X but not a soft S_1-space because $h_1, h_2 \in X$ and there do not exist any soft semi-open sets (F, E) and (G, E) in X such that $h_1 \in (F, E), h_2 \notin (F, E)$ and $h_2 \in (G, E), h_1 \notin (G, E)$.

Definition 4.4. Let (X, τ, E) be a soft topological space over X, (G, E) be a soft semi-closed set in X and $x \in X$ such that $x \notin (G, E)$. If there exist soft semi-open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E), (G, E)\tilde{\subset}(F_2, E)$ and $(F_1, E) \bigcap (F_2, E) = \emptyset$, then (X, τ, E) is called a soft semi-regular space.

Lemma 4.2. Let (X, τ, E) be a soft topological space over X, (G, E) be a soft semi-closed set in X and $x \in X$ such that $x \notin (G, E)$. If (X, τ, E) is a soft semi-regular space, then there exist soft semi-open set (F, E) such that $x \in (F, E)$ and $(F, E) \bigcap (G, E) = \emptyset$.

Theorem 4.5. Let (X, τ, E) be a soft topological space over X and $x \in X$. If (X, τ, E) is a soft semi-regular space, then:

(1) For a soft semi-closed set (G, E), $x \notin (G, E)$ if and only if $(x, E) \cap (G, E) = \emptyset$.

(2) For a soft semi-open set (F, E), $x \notin (F, E)$ if and only if $(x, E) \cap (F, E) = \emptyset$.

Proof. (1) Let $x \notin (G, E)$. By Lemma 4.2, there exists a soft semi-open set (F, E) such that $x \in (F, E)$ and $(F, E) \bigcap (G, E) = \emptyset$. Since $(x, E)\tilde{\subset}(F, E)$, we have $(x, E) \cap (G, E) = \emptyset$.

The converse is obtained by Lemma 4.1(2).

(2) Let $x \notin (F, E)$. Then there are two cases: (i) $x \notin F(\alpha)$ for all $\alpha \in E$ and (ii) $x \notin F(\alpha)$ and $x \in F(\beta)$ for some $\alpha, \beta \in E$.

In case (i) it is obvious that $(x, E) \cap (F, E) = \emptyset$. In the other case, $x \in F^{'}(\alpha)$ and $x \notin F^{'}(\beta)$ for some $\alpha, \beta \in E$ and so $(F, E)^{'}$ is a soft semi-closed set such that $x \notin (F, E)^{'}$, by (1), $(x, E) \cap (F, E)^{'} = \emptyset$. So $(x, E)\tilde{\subset}(F, E)$ but this contradicts $x \notin F(\alpha)$ for some $\alpha \in E$. Consequently, we have $(x, E) \cap (F, E) = \emptyset$.

The converse is obvious.

Theorem 4.6. Let (X, τ, E) be a soft topological space over X and $x \in X$. Then the following are equivalent:

(1) (X, τ, E) is a soft semi-regular space.

(2) For each soft semi-closed set (G, E) such that $(x, E) \cap (G, E) = \emptyset$, there exist soft semi-open sets (F_1, E) and (F_2, E) such that $(x, E)\tilde{\subset}(F_1, E)$, $(G, E)\tilde{\subset}(F_2, E)$ and $(F_1, E) \bigcap (F_2, E) = \emptyset$.

Proof. The proof is obvious, obtained by Theorem 4.5(1) and Lemma 4.1(1).

Theorem 4.7. Let (X, τ, E) be a soft topological space over X and $x \in X$. If (X, τ, E) is a soft semi-regular space, then:

(1) For a soft semi-open set (F, E), $x \in (F, E)$ if and only if $x \in F(\alpha)$ for some $\alpha \in E$.

(2) For a soft semi-open set (F, E), $(F, E) = \bigcup\{(x, E) : x \in F(\alpha)\}$ for some $\alpha \in E$.

Proof. (1) Assume that for some $\alpha \in E, x \in F(\alpha)$ and $x \notin (F, E)$. Then by Theorem 4.5(2), $(x, E) \cap (F, E) = \emptyset$. By the assumption, this is a contradiction and so $x \in (F, E)$.

The converse is obvious.

(2) It follows from (1) and $x \in (F, E)$ if and only if $(x, E)\tilde{\subset}(F, E)$.

Theorem 4.8. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft semi-regular space, then the following are equivalent:

(1) (X, τ, E) is a soft S_1-space.

(2) For $x, y \in X$ such that $x \neq y$, there exist soft semi-open sets (F, E) and (G, E) such that $(x, E)\tilde{\subset}(F, E)$ and $(y, E) \cap (F, E) = \emptyset$, and $(y, E)\tilde{\subset}(G, E)$ and $(x, E) \cap (G, E) = \emptyset$.

Proof. It is obvious that $x \in (F, E)$ if and only if $(x, E)\tilde{\subset}(F, E)$, and by Theorem 4.5, $x \notin (F, E)$ if and only if $(x, E) \cap (F, E) = \emptyset$. Hence we have that the above statements are equivalent.

Definition 4.5. Let (X, τ, E) be a soft topological space over X. Then (X, τ, E) is said to be a soft S_3-space if it is a soft semi-regular and soft S_1-space.

Theorem 4.9. Let (X, τ, E) be a soft topological space over X. If (X, τ, E) is a soft S_3-space, then for each $x \in X$, (x, E) is soft semi-closed.

Proof. We show that $(x, E)^{'}$ is soft semi-open. For each $y \in U - \{x\}$, since (X, τ, E) is a soft semi-regular and S_1-space, by Theorem 4.8, there exists a soft semi-open set (F_y, E) such that $(y, E)\tilde{\subset}(F_y, E)$ and $(x, E) \cap (F_y, E) = \emptyset$. So $\bigcup_{y \in U-\{x\}}(F_y, E)\tilde{\subset}(x, E)^{'}$. For the other inclusion, let $\bigcup_{y \in U-\{x\}}(F_y, E) = (H, E)$ where $H(\alpha) = \bigcup_{y \in U-\{x\}} F_y(\alpha)$ for all $\alpha \in E$. Moreover, from the definition of the relative complement and Definition 2.13, we know that $(x, E)^{'} = (x^{'}, E)$, where $x^{'}(\alpha) = U - \{x\}$ for each $\alpha \in E$. Now, for each $y \in U - \{x\}$ and for each $\alpha \in E$, $x^{'}(\alpha) = U - \{x\} = \bigcup_{y \in U-\{x\}}\{y\} =$

$\bigcup_{y \in U-\{x\}} y(\alpha) \subset \bigcup_{y \in U-\{x\}} F_y(\alpha) = H(\alpha)$. By Definition 2.2, this implies that $(x,E)^{'} \tilde{\subset} \bigcup_{y \in U-\{x\}} (F_y, E)$, and so $(x,E)^{'} = \bigcup_{y \in U-\{x\}} (F_y, E)$. Since (F_y, E) is soft semi-open for each $y \in U - \{x\}$ and by Theorem 3.2 $(x,E)^{'}$ is soft semi-open, consequently, (x,E) is soft semi-closed.

Theorem 4.10. A soft S_3-space is soft S_2-space.

Proof. Let (X, τ, E) be any soft S_3-space. For $x, y \in X$ such that $x \neq y$, by the above Theorem 4.9, (y, E) is soft semi-closed and $x \notin (y, E)$. From the soft semi-regularity, there exist soft semi-open sets (F_1, E) and (F_2, E) such that $x \in (F_1, E), y \in (y, E) \tilde{\subset} (F_2, E)$ and $(F_1, E) \bigcap (F_2, E) = \emptyset$. Hence (X, τ, E) is S_2-space.

5. Conclusion

Topology is an important and major area of mathematics and it can give many relationships between other scientific areas and mathematical models. Recently, many scientists have studied the soft set theory, which is initiated by Molodtsov and easily applied to many problems having uncertainties from social life.

In the present work, we have continued to study the properties of soft topological spaces. We introduce soft semi-interior, soft semi-closure and soft Soft semi-separation axioms and have established several interesting properties.

In our future work, we will go on studying the properties of soft semi-open sets and soft semi-closed sets such as hereditary of them. And will discuss some theorems on the equivalence of soft semi-separate spaces. Since the authors introduced topological structures on fuzzy soft sets [24-26], the semi-topological properties introduced in this paper can be generalized in the fuzzy soft sets and will be useful in the fuzzy systems. Because there exists compact connections between soft sets and information systems [7,8], we can use the results deducted from the studies on soft topological space to improve these kinds of connections.

We hope that the findings in this paper will help researcher enhance and promote the further study on soft topology to carry out a general framework for their applications in practical life.

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