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# An Exact Scheme for the EIT for a Three-level $\Lambda$-Type Atom in a Quantum Cavity

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**Abstract:** Semi-quantum or full-quantum electromagnetically induced transparency has been fully studied for three-level atoms in a weak field approximation method, where probe field is very weaker than the coupling field. The weak field approximation is not valid in full-quantum model where the number of coupling photons is not so large. In the present article, the master equations for the interaction of two-mode photons with a three-level $\Lambda$-type atom are exactly solved and the exact dispersion and absorption spectra for the probe beam photons are also obtained analytically. The results of the exact scheme are compared with the corresponding results in the weak field approximation method in full-quantum and semi-classical models.

**Keywords:** electromagnetically induced transparency, Absorption, Dispersion, quantum electrodynamics, exact solution.

## 1 Introduction

Electromagnetically induced transparency (EIT) has been theoretically introduced by O. Kocharovskaya [1] and experimentally observed, by S. E. Harris et al. [2,3]. Recently, many authors have been interested in studying EIT and its applications [4,5,6,7,8]. EIT is widely studied for different systems, e.g. V, $\Lambda$ and cascade three-level atoms and many other atoms with more levels [9,10]. Many alkali atoms, e.g., Rydberg Rubidium atom, have been also experimentally applied for the generation of EIT [11,12]. Properties of the electromagnetic fields interacting with a three-level $\Lambda$-type atom were studied in the semi-classical [1,2,3,13,14] and full-quantum [15, 16] models by a weak field approximation (WFA) method. The EIT with the quantized fields in opto-cavity mechanics is another example for the full-quantum approach which is studied by S. Huang and G. S. Agarwal [16].

In this paper, a full-quantum model of EIT is investigated for an ensemble of $\Lambda$-type three-level atoms, in which the probe and coupling fields are quantize. Interaction of a $\Lambda$-type three-level atom with the quantized electromagnetic fields is investigated using the Jaynes-Cummings model. In this case, the exact master equations are investigated and solved in a steady-state without any WFA. An exact form of absorption and dispersion spectra are obtained for a probe fields which are not generally weaker than the coupling field. It is shown that the EIT obtained for the probe fields is either weaker or stronger than that of the coupling field. In contrast to the semi-classical model, the EIT is also obtained in the full-quantum model where the coupling field is very weak and contained a few numbers of photons. It is also shown that the EIT appeared even for a vacuum coupling field.

## 2 Proposed Setup

Suppose an ensemble of -type three-level cold circular Rydberg atoms trapped in a quantum cavity, pumped into the high quantum number excited levels (e.g., $n = 49, n = 50$ and $n = 51$), interacting (resonantly or non-resonantly) with the classical (or quantized) coupling Electromagnetic fields [17]. There are many other alkali atoms which are possible to apply in these experimental setups, e.g.: Cesium [18], Sodium [19] and so on. In this case, an ensemble of cold three-level atoms is prepared by an optical pumping initially in the state $|b\rangle$. The quantum cavity is filled with the three-level cold atoms as well as the $n_2$ number of coupling photons which are strongly coupled with the quantum cavity electrodynamics. The probe photons are individually injected into the absorptive

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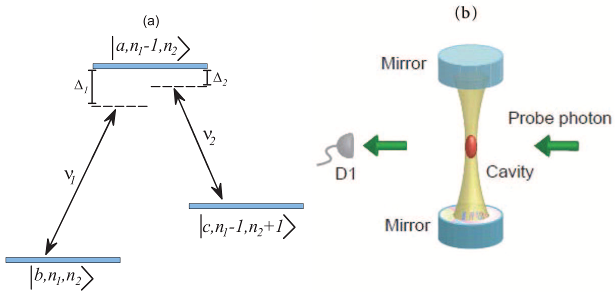


**Fig. 1:** Fig. 1. (Color online) (a) A $\Lambda$-type three-level atom interacting with two electromagnetic fields with frequencies $\nu_1$ and $\nu_2$. The red spot is an ensemble of atoms trapped and strongly coupled with the quantum cavity. The quantized probe photons are passed through the cavity and counted by D1 after interaction with the ensemble of atoms.

atoms in the cavity. Absorption of the probe photons is controlled by the number of coupling photons $n_2$. The quantum cavity illustrated in Fig. 1b, is made of superconductor mirrors to reduce the cavity loss and the absorption of the probe photons could be measured by the detector D1.

## 3 Master Equations

Suppose that, in cavity quantum electrodynamics, the quantized probe and coupling fields (photons) interact with a three-level $\Lambda$-type atom (see Fig. 1a). The interaction Hamiltonian of the system in the interaction picture is given by:

$$\begin{aligned}\hat{V} = & -\hbar g_1[\sigma_{ab}\hat{a}_1 e^{i\Delta_1 t} + \hat{a}_1^\dagger \sigma_{ba} e^{-i\Delta_1 t}] \\ & -\hbar g_2 \sigma_{ac}\hat{a}_2 e^{i\Delta_2 t} + \hat{a}_2^\dagger \sigma_{ca} e^{-i\Delta_2 t}],\end{aligned} \tag{1}$$

where $g_1$ and $g2$ are interaction strength of the probe and coupling fields, respectively. $\hat{a}_1(\hat{a}_1^\dagger)$ and $\hat{a}_2(\hat{a}_2^\dagger)$ are annihilation (creation) operators for the probe and coupling photons, respectively. $\sigma_{ij} = |i\rangle\langle j|$ is atomic transition operator from $|j\rangle \to |i\rangle$. In Eq. (1), $\Delta_1(\Delta_2)$ is detuning between the frequency of probe (coupling) and the corresponding atomic transition frequency.

Assume the initial state of total system is given by $|b,n_1,n_2\rangle$. By an atom-field interaction, absorption of one probe photon changes the state to $|a,n_1-1,n_2\rangle$ and a coupling photon emission, change it into $|c,n_1-1,n_2+1\rangle$. Therefore, a time evolution of initial state is given by a linear combination of these states

$$\begin{aligned}|\psi\rangle & = C_a(t)|a,n_1-1,n_2\rangle + C_b(t)|b,n_1,n_2\rangle \\ & + C_c(t)|c,n_1-1,n_2+1\rangle.\end{aligned} \tag{2}$$

Application of Eq.(2) into $\hat{\rho}_s = |\psi\rangle\langle\psi|$ gives the total density operator:

$$\begin{aligned}\hat{\rho} = & \ \rho_{aa}|a,n_1-1,n_2\rangle\langle a,n_1-1,n_2| \\ & + \rho_{bb}|b,n_1,n_2\rangle\langle b,n_1,n_2| \\ & + \rho_{cc}|c,n_1-1,n_2+1\rangle\langle c,n_1-1,n_2+1| \\ & + (\rho_{ab}|a,n_1-1,n_2\rangle\langle b,n_1,n_2| \\ & + \rho_{ac}|a,n_1-1,n_2\rangle\langle a,n_1-1,n_2+1| \\ & + \rho_{bc}|b,n_1,n_2\rangle\langle c,n_1-1,n_2+1| + C.C.),\end{aligned} \tag{3}$$

where $\rho_{ij} = C_i(t)C_j^*(t)$. The exact dynamical equations (master equations) are given by:

$$\begin{aligned}\dot{\tilde{\rho}}_{aa} = & -(\gamma_1+\gamma_2)\tilde{\rho}_{aa} + ig_1\sqrt{n_1}(\tilde{\rho}_{ab}-\tilde{\rho}_{ba}) \\ & + ig_2\sqrt{n_2+1}(\tilde{\rho}_{ca}-\tilde{\rho}_{ac}),\end{aligned} \tag{4}$$

$$\dot{\tilde{\rho}}_{bb} = \gamma_1\tilde{\rho}_{aa} + \gamma_3\tilde{\rho}_{cc} + ig_1\sqrt{n_1}(\tilde{\rho}_{ab}-\tilde{\rho}_{ba}), \tag{5}$$

$$\dot{\tilde{\rho}}_{cc} = \gamma_2\tilde{\rho}_{aa} - \gamma_3\tilde{\rho}_{cc} + ig_2\sqrt{n_2+1}(\tilde{\rho}_{ac}-\tilde{\rho}_{ca}), \tag{6}$$

$$\begin{aligned}\dot{\tilde{\rho}}_{ab} = & -\tfrac{1}{2}(\gamma_1+2i\Delta_1)\tilde{\rho}_{ab} + ig_1\sqrt{n_1}(\tilde{\rho}_{bb}-\tilde{\rho}_{aa}) \\ & + ig_2\sqrt{n_2+1}\tilde{\rho}_{cb},\end{aligned} \tag{7}$$

$$\begin{aligned}\dot{\tilde{\rho}}_{ac} = & -\tfrac{1}{2}(\gamma_1+2i\Delta_2)\tilde{\rho}_{ac} + ig_1\sqrt{n_1}\tilde{\rho}_{bc} \\ & + ig_2\sqrt{n_2+1}(\tilde{\rho}_{cc}-\tilde{\rho}_{aa}),\end{aligned} \tag{8}$$

$$\begin{aligned}\dot{\tilde{\rho}}_{bc} = & -\tfrac{1}{2}(\gamma_3-2i(\Delta_2-\Delta_1))\tilde{\rho}_{bc} + ig_1\sqrt{n_1}\tilde{\rho}_{ac} \\ & - ig_2\sqrt{n_2+1}\tilde{\rho}_{ba}.\end{aligned} \tag{9}$$

Master equations are obtained from $\dot{\hat{\rho}} = \frac{1}{i\hbar}\left[\hat{V},\hat{\rho}\right] + L(\hat{\rho}_s)$ where $L(\hat{\rho}_s) = -\frac{\Gamma_{ij}}{2}(\hat{\sigma}_{+ij}\hat{\sigma}_{-ij}\hat{\rho}_s - 2\hat{\sigma}_{-ij}\hat{\rho}_s\hat{\sigma}_{+ij} + \hat{\rho}_s\hat{\sigma}_{+ij}\hat{\sigma}_{-ij})$ is the Lindblad relaxation term and $\{ij\} \in \{\{ab\},\{ac\},\{bc\}\}$. $\hat{\sigma}_{+ij} = |i\rangle\langle j|$, $\hat{\sigma}_{-ij} = |j\rangle\langle i|$ and $\Gamma_{ij}$ are spontaneous transition rates between the states $i$ and $j$. $\gamma_1 = \Gamma_{ab}, \gamma_2 = \Gamma_{ac}$and $\gamma_3 = \Gamma_{cb}$ are also spontaneous decay rates. To obtain master equations (4)-(9), the rotating frame transformations: $\tilde{\rho}_{ab} = \rho_{ab}e^{-i\Delta_1 t}$, $\tilde{\rho}_{ac} = \rho_{ac}e^{-i\Delta_2 t}$ and $\tilde{\rho}_{cb} = \rho_{cb}e^{i(\Delta_2-\Delta_1)t}$ are applied.

In this article, a few numbers of photons are investigated for the probe and coupling fields. Hereafter, the phrase "the weak field" would mean *"the number of probe photons is very smaller than the number of coupling photons"*. Therefore, the weak field approximation (WFA) is used where the number of coupling photons are very larger than the number of probe photons; e.g. $n_2 \geq 100$. It is assumed that a probe beam is a train of individual photons so that atoms are interacted with one photon at each moment; thus, the detector would only measure absorption of one photon at each moment. Therefore the number of probe photons would be set to one for all examples. Clearly it violates the WFA.

In WFA, the density matrix elements in Eqs. (4)-(9) are expanded up to the first order of electric field amplitude. In this case, the population of atomic levels transfers to the lowest level $b$ which can be assumed to be as an initial state. The dispersion and absorption of probe photon are obtained from the real and imaginary parts of the probe coherence term $\tilde{\rho}_{ab}$ which could be derived

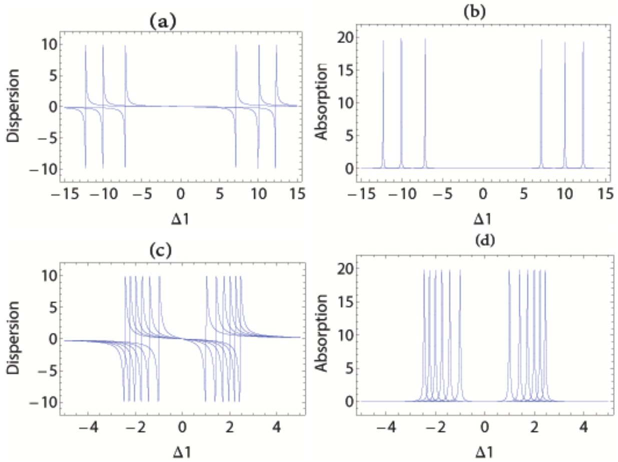


**Fig. 2:** Fig. 2. (Color online) (a) and (b) are the real and imaginary parts of $\tilde{\rho}_{ab}$ in the full-quantum (and semi-classical) model. They are, respectively, proportional to dispersion and absorption of the probe field. In this case, the WFA is used where the probe field is very weaker than the coupling field. They are plotted versus detuning of probe field $\Delta_1$ where other parameters are$\gamma_1 = \gamma_2 = 0.1$, $\gamma_3 = 0.001$, $n_1 = 1$, $\Delta_2 = 0$ and $g_1 = g_2 = 1$ and$n_2 = \{50, 100, 150\}$ for the plots (a) and (b) and $n_2 = \{0, 2, 4, 6, 8, 10\}$for (c) and (d).

from the master equations in the steady-state:

$$\begin{aligned} 0 = & -\tfrac{1}{2}(\gamma_1 + \gamma_2 + 2i\Delta_2)\tilde{\rho}_{ab} + ig_1\sqrt{n_1} \\ & + ig_2\sqrt{n_2+1}\tilde{\rho}_{cb}, \qquad (10) \\ 0 = & -\tfrac{1}{2}(\gamma_2 + \gamma_3 - 2i(\Delta_2 - \Delta_1))\tilde{\rho}_{cb} - ig_1\sqrt{n_1}\tilde{\rho}_{ac} \\ & + ig_2\sqrt{n_2+1}\tilde{\rho}_{ba}, \qquad (11) \end{aligned}$$

According to Eqs. (10) and (11), the probe coherence term is obtained as:

$$\tilde{\rho}_{ab} = \frac{2g_1\sqrt{n_1}(i\gamma_3 + 2(\Delta_2 - \Delta_1))}{(\lambda_1 + 2i\Delta_1)(\gamma_3 - 2i(\Delta_2 - \Delta_1)) + 4g_2^2(n_2+1)}. \qquad (12)$$

The real and imaginary parts of $\tilde{\rho}_{ab}$ are proportional to the dispersion and absorption of probe photons, as plotted in Figs. 2a and 2b for the full-quantum model in WFA. To obtain the coherence term in the semi-classical model in a shortcut way, insert $g_1\sqrt{n_1} \rightarrow \Omega_{pr}$and $g_2\sqrt{n_2+1} \rightarrow \Omega_{pu}$ in Eq. (12), where $\Omega_{pr}$ and $\Omega_{pu}$ are Rabi frequencies of the probe and coupling fields, respectively. The absorption and dispersion spectra of semi-classical model are exactly similar to plots in Figs. 2a and 2b, for different coupling Rabi frequencies corresponding to the large number of coupling photons. In this case, the coupling field is supposed to be stronger than the probe field and WFA is used. Both of the applied models (semi- and full-quantum models) show the effect of EIT for a $\Lambda$-type atom.

The dispersion and absorption of the probe photons are also plotted in Figs. 2c and 2d for small number of coupling photons, it is known that Eqs. (12) and (13) are not valid for the coupling field, which are not very stronger than probe field, because the WFA is used in their derivations.

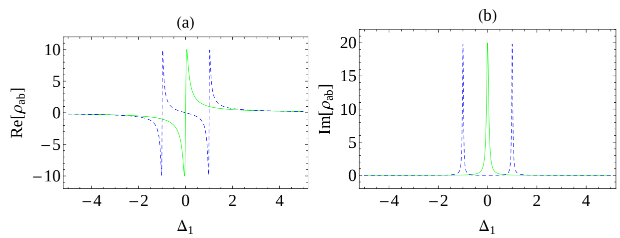


**Fig. 3:** Fig. 3. (Color online) A comparison between dispersions (a) and absorptions (b) of the probe field for vacuum coupling fields is shown. The solid (green) line and dashed (blue) line are plotted for the semi-classical and full-quantum models of atom-field interaction, respectively. The EIT appears even where the coupling field is in the vacuum state (where$n_1 = 1$ and $n_2 = 0$); but, in the semi-classical model (where$\Omega_{pu} = 0$ and $\Omega_{pr} = 1$), it is disappeared. In this case other conventional parameters are the same as what is selected in Fig. 2.

There is a main difference between the absorption spectra of probe field for the coupling field in the vacuum state and zero coupling field strength in semi-classical model. In the vacuum coupling field the EIT is appeared in full-quantum model but it is disappeared for the zero coupling field strength. It is due to the interaction of atom with the vacuum electromagnetic field. This difference is shown in Figs. 3a and 3b.

To obtain the more correct dispersion and absorption spectra, where the coupling field is not stronger than the probe field, the exact form of the coherence term for the probe photons must be obtained without the WFA.

## 4 Exact Solution

The exact solution of master equations is found for the coherence term in the steady-state without weak field approximation. The master equations (4)-(9) are exactly solved in the steady-state to obtain the exact coherence term $\tilde{\rho}_{ab}$. Using MATHEMATICA software, the coherence term was obtained analytically. It was more complicated than the corresponding one in WFA. The numerator and denominator of coherence term are individually expanded in terms of the probe detuning while the coupling detuning is set to be zero. A compact form of the exact coherence term is obtained as:

$$\tilde{\rho}_{ab} = \frac{2g_1\sqrt{n_1}(iZ_0 + Z_1\Delta_1 + iZ_2\Delta_1^2 + Z_3\Delta_1^3)}{K_0 + K_2\Delta_1^2 + K_4\Delta_1^4}, \qquad (13)$$

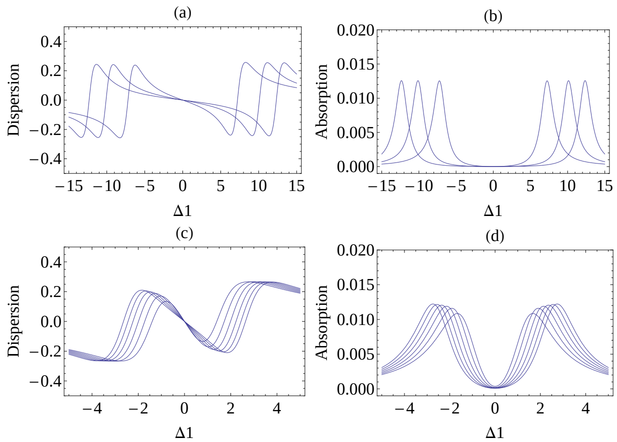


**Fig. 4:** Fig. 4. (Color online) (a) and (b) are the exact dispersion and absorption of the probe field in terms of its detuning for large number of coupling photons ($n_2 = 50, 100, 150$); (c) and (d) for small number of coupling photons ($n_2 = 0, 1, 2, 3, 4, 5$). Other parameters are chosen similar to the corresponding one in WFA.

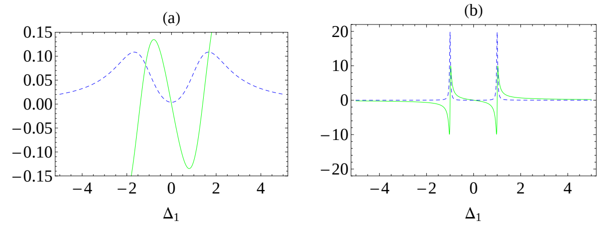


**Fig. 5:** Fig.5. (Color online) Comparison between exact dispersions and absorptions of the probe field for the vacuum coupling fields in a) exact methods and b) WFA, showing their differences. The solid (green) line and dashed (blue) line show the dispersion and absorption, respectively.

where

$$
\begin{aligned}
Z_0 &= \gamma_3(4g_1^2 n_1\gamma_1 + 4g_2^2\gamma_2(n_2+1) + \gamma_1\gamma_2\gamma_3)\\
&\quad \times(4g_2^2(\gamma_1+\gamma_3)(n_2+1) + (\gamma_1+\gamma_2)(4g_1^2 n_1 + \gamma_2\gamma_3), \quad (14)\\
Z_1 &= (-32g_2^4(n_2+1)^2\gamma_2(\gamma_1+\gamma_3) + 2\gamma_3(\gamma_1+\gamma_2)\\
&\quad \times(4g_1^2 n_1 + \gamma_2\gamma_3)^2 8g_2^2(n_2+1)(\gamma_2(\gamma_2+\gamma_3)\gamma_3(\gamma_1+\gamma_2+\gamma_3)\\
&\quad \times 4g_1^2 n_1(-\gamma_2^2 + \gamma_3(\gamma_1+\gamma_2) + \gamma_3^2))), \quad (15)\\
Z_2 &= 4\gamma_1\gamma_2(\gamma_2\gamma_3(\gamma_1+\gamma_2) + 4g_2^2(n_2+1)(\gamma_1+\gamma_3)), \quad (16)\\
Z_3 &= 8\gamma_2(-\gamma_2\gamma_3(\gamma_1+\gamma_2) - 4g_2^2(n_2+1)(\gamma_1+\gamma_3)), \quad (17)
\end{aligned}
$$

$$
\begin{aligned}
K_0 &= (4g_1^2 n_1\gamma_1 + 4g_2^2\gamma_2(n_2+1) + \gamma_1\gamma_2\gamma_3)\\
&\quad \times(16g_2^4(n_2+1)^2(\gamma_1+\gamma_3) + (4g_1^2 n_1 + \gamma_2\gamma_3)(\gamma_1\gamma_3(\gamma_1+\gamma_2)\\
&\quad + 4g_1^2 n_1(\gamma_2+2\gamma_3)) + 4g_2^2(n_2+1)(4g_1^2 n_1(\gamma_1+\gamma_2)\\
&\quad + \gamma_3(\gamma_1^2+\gamma_2^2+\gamma_1(\gamma_2+\gamma_3))))), \quad (18)\\
K_2 &= 4(16g_1^4 n_1^2\gamma_3(\gamma_1+\gamma_2) - 32g_2^4(n_2+1)^2\gamma_2(\gamma_1+\gamma_3)\\
&\quad + \gamma_2^2(\gamma_1+\gamma_2)\gamma_3(\gamma_1^2+\gamma_3^2) + 4g_1^2 n_1\gamma_2(\gamma_2^2\gamma_1 + 2\gamma_1\gamma_2\gamma_3\\
&\quad + 2(\gamma_1+\gamma_2\gamma_3^2) + 4g_2^2(n_2+1)(\gamma_2(\gamma_1^3+\gamma_1^2\gamma_3 - 2\gamma_2^2\gamma_3 + \gamma_3^3)\\
&\quad + \gamma_1\gamma_3(-2\gamma_2+\gamma_3)) + 4g_1^2 n_1(-\gamma_2^2 + 3\gamma_2\gamma_3 + \gamma_3^2\\
&\quad + \gamma_1(2\gamma_2+\gamma_3)))), \quad (19)\\
K_4 &= 16\gamma_2(\gamma_2\gamma_3(\gamma_1+\gamma_2) + 4g_2^2(n_2+1)(\gamma_1+\gamma_3)), \quad (20)
\end{aligned}
$$

are real parameters. Dispersion and absorption of the coherence term are proportional to

$$Re[\tilde{\rho}_{ab}] = \frac{2g_1\sqrt{n_1}(Z_1\Delta_1 + Z_3\Delta_1^3)}{K_0 + K_2\Delta_1^2 + K_4\Delta_1^4}, \quad (21)$$

$$Im[\tilde{\rho}_{ab}] = \frac{2g_1\sqrt{n_1}(Z_0\Delta_1 + Z_2\Delta_1^2)}{K_0 + K_2\Delta_1^2 + K_4\Delta_1^4}, \quad (22)$$

respectively.

The exact real and imaginary parts of $\tilde{\rho}_{ab}$ are plotted in Figs. 4a-4d for large and small numbers of coupling photons. Similar to the full-quantum model with WFA, for the large and small numbers of coupling photons, the EIT effect is also appeared at zero detuning of the probe field. The comparison between the exact dispersion and absorption spectra in Fig. 4 and the approximate dispersion and absorption spectra in Fig. 2 demonstrate that the exact dispersion and absorption peaks are smaller and their breadth of peaks are wider that the corresponding one in WFA. Furthermore, the EIT effect not only appears for the larger number of coupling photons but also it appears for the small (or even zero) number of coupling photons. Detuning of the absorption peaks$\Delta_1$, increases with increasing the number of coupling photons. Although the EIT is appeared either in the semi-classical and full-quantum models in WFA; or the exact methods in full-quantum model, they have a serious difference where coupling photons are in the vacuum state. A comparison between the exact and WFA plots for the absorption and dispersion spectra, is presented in Fig. 5. Furthermore, for the exact method, the absolute value of the dispersion slope at zero detuning is more than the WFA method.

## 5 Conclousions

In this paper, the master equations for $\Lambda$-type three-level atom interacting with two-mode quantized electromagnetic field was investigated. The coherence terms for the probe photons were analytically and exactly obtained. The EIT effect was obtained for the strong, weak and even vacuum coupling photons. The absorption and dispersion spectra of the system were compared in the exact and WFA methods. The following results were obtained: 1- The weak field approximation is not suitable for full-quantum model where the number of coupling photons is small or where it is in the vacuum state. Therefore an exact scheme is needed. The master equations for three-level -type system is solved exactly in the steady-state where the detuning of coupling photons are vanishes. 2- The profile of the absorption and dispersion spectra for the exact and WFA methods are compared. Their magnitudes in the exact method were weaker and their peaks were wider than the corresponding one in the WFA. 3- As well as the WFA method, in the exact scheme the EIT is also appeared in the absorption and dispersion spectra. 4- In spite of the semi-classical model, the EIT appeared even for the vacuum coupling photons in full-quantum model either in exact or WFA methods. 5- Furthermore, for the exact method, the absolute value of the dispersion slope at zero detuning is very larger than the WFA method.

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