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Extended Odd Weibull Inverse Nadarajah- Haghighi Distribution with Application on COVID-19 in Saudi Arabia

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Abstract: The aim of this paper is to introduce a new suitable distribution for modeling the COVID-19 data in Saudi Arabia. A new distribution is a combination of the inverse Nadarajah-Haghighi distribution and the extended odd Weibull family to formulate the extended odd Weibull inverse Nadarajah-Haghighi (EOWINH) distribution with four parameters. A simple linear representation, hazard rate function, and moment generating function have been obtained of EOWINH distribution. To estimate the unknown parameters of the distribution of EOWINH, maximum likelihood, maximum product spacing, and Bayesian estimation methods are applied. For the Bayesian approximation, the Markov Chain Monte Carlo (MCMC) by using asymmetric loss function as a square error is obtained. To evaluate the use of estimation methods, a numerical result from the Monte Carlo simulation is obtained.

Keywords: extended odd Weibull family; Inverse Nadarajah-Haghighi distribution; Bayesian; COVID-19; maximum product spacing

1 Introduction

COVID-19, a new coronavirus disease, has expanded worldwide since December 2019, producing over 80 million illnesses and over one million deaths in over 190 countries (Johns Hopkins University). There have been about 300,000 cases of COVID-19 in Saudi Arabia (KSA) as of November 2020, with over 5000 deaths. While the exact processes of COVID-19 transmission are still being researched, it is likely to spread predominantly by airborne respiratory droplets created when an infected person coughs, sneezes, or speaks (Surveillances [1]; Hossain et al. [2]). Infection is spread by symptomatic patients, but it can also be spread by asymptomatic people before symptoms appear (Singhal [3]). COVID-19 is a propagative virus, and its trend, like that of most viruses, is influenced by the preventive measures in place.
Youssef et al. [4] were able to use the modified susceptible exposed infectious recovered (SEIR) statistical model to successfully predict that the number of COVID-19 cases would decrease to 500 per day by the beginning of October 2020 in the early stages of the epidemic in KSA (October 2020 and December 2020). Furthermore, this model could statistically establish that prevention is better than cure, and that isolating affected persons is critical for epidemic control. Therefore, we decided to find the best mathematical statistics model for modeling this data in this interval. Almetwally [5] obtained a new superior distribution for modeling of mortality rate for the COVID-19 pandemic of France which is called odd Lomax-G inverse Weibull distribution.
Modeling real-life events in the form of distributions of probability are one of the key tasks of statistics. Distributions of probability are used to model the phenomena of natural life that are characterized by uncertainty and risk. Many of the distributions of probability are derived since the phenomena of natural life are dynamic and diversified. Established distributions of probability, however, remain unable to accurately describe data for certain natural phenomena. These contribute to the extension and modification of generalized distributions of probability. With the popular existence of having added parameters, generalized probability distributions have advanced. The addition of some parameters to the known distributions of probability improved the consistency of suitability for the data of natural phenomena and improved the accuracy of the

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distribution tail shape definition.
For difficulties in reliability and survival analysis, the exponential distribution is perhaps the most extensively used statistical distribution. In the lifetime literature, this was the first model of a lifetime for which statistical methods were substantially developed. Nadarajah and Haghighi [6] presented a generalization of the exponential distribution that could be an alternative to the gamma, Weibull, and generalized-exponential distributions.
Tahir et al. [7] proposed the Inverse NH (INH) distribution, a new inverted model with decreasing and unimodal (right-skewed) density and decreasing and upside-down bathtub hazard rate shapes (UDBHR). They addressed several statistical features of the INH distribution and used several frequentist approaches to estimate model parameters. They have demonstrated the applicability of INHD in a real-life scenario using real-life data sets. They also demonstrated that the INH model fits well when compared to well-known lifetime distributions such as inverted exponential, inverted Rayleigh, inverted gamma, inverted Weibull, inverted Lindley, and inverted power Lindley. The cumulative distribution function (CDF), the PDF, the reliability function (RF), and the HRF random variable X are provided by

$$g(x;\theta,\gamma)=\theta\frac{\gamma}{x^2}\left(1+\frac{\gamma}{x}\right)^{\theta-1}e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}, \qquad (1)$$

where $x>0,\theta,\gamma>0$.

$$G(x;\theta,\gamma)=e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}, \qquad (2)$$

In different statistical writings by various authors, a generalization of a different distribution of INH distribution was discussed, mainly applied in reliability estimation. For example, a generalization of the INH distribution known as the Transmuted INH distribution with an application to lifetime data was introduced by Toumaj et al. [8]. The Marshall–Olkin INH distribution has been introduced by Raffiq et al. [9]. The parameter estimation of INH distribution based on type-II progressively censored samples has been discussed by Elshahhat and Rastogi [10]. For more example for application based on censored sample see El-Din et al. [11], Almetwaly and Almongy [12,13], and Almetwally et al. [14,15].
Alizadeh et al. [16] introduced The extended odd Weibull-G (EOW-G) family. Let $g(x;\Theta)=\frac{dG(x;\Theta)}{dx}$ denotes the survival function (S) and probability density function (PDF) of a baseline model with parameter vector $\Theta=(\theta,\gamma)$, so the CDF of the EOW-G family is given by:

$$F(x;\Omega)=1-\left\{1+\alpha\left[\frac{G(x;\Theta)}{\overline{G}(x;\Theta)}\right]^{\beta}\right\}^{\frac{-1}{\alpha}},-\infty<x<\infty. \qquad (3)$$

where $\Omega=(\alpha,\beta,\Theta)$ is a vector of parameters of EOW-G distribution. The corresponding PDF of (3) is defined by

$$f(x;\Omega)=\frac{\beta\, g(x;\Theta)\,G(x;\Theta)^{\beta-1}}{\overline{G}(x;\Theta)^{\beta+1}}\left\{1+\alpha\left[\frac{G(x;\Theta)}{\overline{G}(x;\Theta)}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \qquad (4)$$

where α,β and Θ are positive shape parameters. The random variable with PDF (4) is denoted by $X\sim$EOW-G(α,β,Θ).
In different statistical writings by various authors, a generalization of different distribution of INH distribution was discussed, mainly applied in reliability estimation as Afify and Mohamed [17] introduced the extended odd Weibull exponential distribution. Alshenawy et al. [18] discussed classical estimation methods of extended odd Weibull exponential distribution were addressed based on progressive type-II censoring schemes. Almongy [19] introduced the the extended odd Weibull Rayleigh distribution with applications of COVID-19 data. Almetwally [20] discussed extended odd Weibull inverse Rayleigh (EOWIR) distribution with application on carbon fibres.
Using three traditional estimation methods as (maximum likelihood and maximum product spacing) and Bayesian estimation method based on symmetric loss function, we will investigate the point estimate of unknown EOWINH parameters. Statistical analysis is carried out comparing these methods by the simulation to assess their efficiency and to investigate how these estimators operate for various sample sizes and parameter values. It is discussed how fit COVID-19 data can be used.
The remainder of this article is structured as follows. We define the EOWINH distribution under Section 2. In Section 3, along with some of its statistical properties for the EOWINH, is obtained. Section 4 studies three methods of point estimation. To compare the performance of these estimation methods, a simulation study is performed in Section 5. The Application of COVID-19 data is discussed in Section 6 to show the efficiency of the distribution of EOWINH with respect to other distributions. Finally, in Section 7, conclusions are provided.

2 EOWINH Distribution

A special model of the EOW-G family with INH distribution as a baseline function is the four-parameters EOWINH distribution. By substituting the INH model CDF and PDF Equations (2) and (1) of the EOW-G family (3) and (4), the EOWINH distribution CDF and PDF are obtained as;

$$F(x;\Omega)=1-\left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}}, \qquad (5)$$

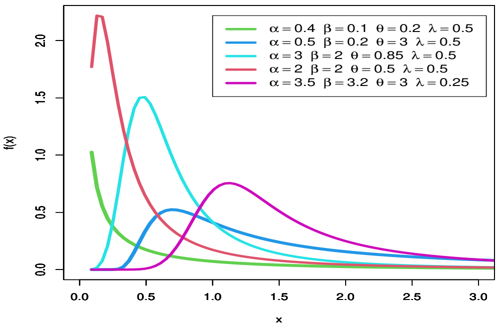

Fig. 1: pdf of EOWINH distribution.

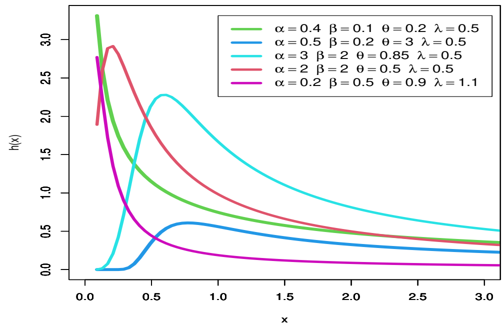

Fig. 2: HR of EOWINH distribution.

where $x > 0, \alpha, \beta, \theta, \gamma > 0$.

$$f(x;\Omega) = \theta\beta\frac{\gamma}{x^2}\left(1+\frac{\gamma}{x}\right)^{\theta-1}\frac{e^{\beta\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}}\left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \tag{6}$$

Therefore, a random variable with PDF (6) is denoted by $X \sim$EOWINH$(\alpha, \beta, \theta,$ and $\gamma)$.

The hazard rate function (HR) of the EOWINH distribution are given by

$$h(x;\Omega) = \theta\beta\frac{\gamma}{x^2}\left(1+\frac{\gamma}{x}\right)^{\theta-1}\frac{e^{\beta\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}}\left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{-1}$$

Figures 1 and 2 are different shapes of the PDF and HR of the EOWINH distribution. These figures show that the PDF of the EOWINH distribution can be right-skewed, symmetric, or decreasing curves. The HR of the EOWINH distribution has some important shapes, including constant, decreasing, and upside down curve, which are attractive characteristics for any lifetime model. It can be noticed from the application section that the EOWINH distribution possesses great flexibility and can be used to model skewed data, hence widely applied in different areas such as biomedical studies, biology, reliability, physical engineering, and survival analysis.

3 Statistical Properties of EOWINH Distribution

In this section, we observe some statistical properties of the EOWINH distribution, namely, the linear representation of PDF and CDF Also, we obtain the Quantile function to generate data.

3.1 Linear Representation

Linear representation for the EOWINH density using series techniques is useful for finding many statistical values and properties of the needed distribution. Now substituting the PDF and CDF of the INH distribution, the above equation can be written as

$$F(x) = 1 - \sum_{k,j=0}^{\infty} \Xi_{k,j} e^{(\alpha k+j)\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]}, \tag{7}$$

where $e^{(\alpha k+j)\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]}$ is a CDF of expotiatated INH distribution with parameters $(\alpha k + j, \theta, \gamma)$ and $\Xi_{k,j} = \beta^k \frac{\Gamma(\alpha k+j)\left(\frac{-1}{\beta}\right)_k}{k!j!\Gamma(\alpha k)}$ where $\left(\frac{-1}{\beta}\right)_k = \frac{-1}{\beta}\left(\frac{-1}{\beta}-1\right)\dots\left(\frac{-1}{\beta}-k+1\right)$. By differentiating (mixture), the pdf in Equation (6) can be expressed as

$$f(x) = \sum_{k,j=0}^{\infty} \xi_{k,j}\frac{(\alpha k+j)\theta\gamma}{x^2}\left(1+\frac{\gamma}{x}\right)^{\theta-1} e^{(\alpha k+j)\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]}, \tag{8}$$

where $\xi_{k,j} = -\Xi_{k,j}$.

3.2 Quantile for The EOWINH Distribution

The quantile function of the EOWINH distribution, say $x = Q(x) = F(x,\Theta)^{-1}(Q)$ is derived by inverting (5) as follows:

$$x_u=\gamma\left\{\left(1+\ln\left[1+\left(\frac{1}{\alpha}\left[(1-u)^{-\alpha}-1\right]\right)^{\frac{-1}{\beta}}\right]\right)^{\frac{1}{\theta}}-1\right\}^{-1};0<u<1 \quad (9)$$

In particular, the first quartile, say Q1, the second quartile, say Q2, and the third quartile, say Q3 are obtained by setting $Q=0.25,0.5,0.75$, respectively, in Equation (9).

4 Estimation Methods

The estimation problem of the EOWINH distribution parameters is studied in this Section using three different estimation methods called: maximum likelihood estimators (MLEs), maximum spacing product estimators (MPSEs), and Bayesian estimation based on the function of square error loss.

4.1 Maximum Likelihood Estimators

Let $x_1,...,x_n$ be a random sample with the parameters $\alpha,\beta,\theta,\gamma$ from the EOWINH distribution. The log-likelihood feature for the distribution of EOWINH is provided by

$$\begin{aligned}l(\Omega)=&n\left[\ln(\theta)+\ln(\beta)+\ln(\gamma)\right]-2\sum_{i=1}^{n}ln(x_i)+\\&(\theta-1)\sum_{i=1}^{n}\ln\left(1+\frac{\gamma}{x_i}\right)+\beta\sum_{i=1}^{n}\left[1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}\right]-\\&(\beta+1)\sum_{i=1}^{n}\ln\left[1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}\right]-\\&\left(\frac{1}{\alpha}+1\right)\sum_{i=1}^{n}\ln\left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}\right\}\end{aligned} \quad (10)$$

The partial derivatives of $l(\Omega)$ with respect to the model parameters α,β,θ and γ are

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\alpha}=&\frac{-1}{\alpha^2}\sum_{i=1}^{n}\ln\left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}\right\}+\\&\left(\frac{1}{\alpha}+1\right)\sum_{i=1}^{n}\frac{\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}}{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}}\end{aligned} \quad (11)$$

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\beta}=&\frac{n}{\beta}+\sum_{i=1}^{n}\left\{\left[1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}\right]-\ln\left[1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}\right]\right\}-\\&(\alpha+1)\sum_{i=1}^{n}\frac{\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}ln\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]}{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}}\end{aligned} \quad (12)$$

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\theta}=&\frac{n}{\theta}+\sum_{i=1}^{n}\ln\left(1+\frac{\gamma}{x_i}\right)\left[1-\beta\left(1+\frac{\gamma}{x_i}\right)^{\theta}\right]-\\&(\beta+1)\sum_{i=1}^{n}\frac{\Delta_i(\theta,\gamma)}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}-\\&\beta(\alpha+1)\sum_{i=1}^{n}\frac{\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta-1}\Delta_i(\theta,\gamma)}{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}},\end{aligned} \quad (13)$$

and

$$\begin{aligned}\frac{\partial l(\Omega)}{\partial\gamma}=&\frac{n}{\gamma}+(\theta-1)\sum_{i=1}^{n}\frac{1}{x_i\left(1+\frac{\gamma}{x_i}\right)}+\beta\theta\sum_{i=1}^{n}\left(1+\frac{\gamma}{x_i}\right)^{\theta-1}\frac{1}{x_i}-\\&(\beta+1)\theta\sum_{i=1}^{n}\frac{\Phi_i(\theta,\gamma)}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}-\\&\beta(\alpha+1)\sum_{i=1}^{n}\frac{\frac{\Phi_i(\theta,\gamma)}{\left[1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}\right]^2}\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta-1}\frac{1}{x_i}}{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}}\right]^{\beta}},\end{aligned} \quad (14)$$

where $\Delta_i(\theta,\gamma)=\left(1+\frac{\gamma}{x_i}\right)^{\theta}ln\left(1+\frac{\gamma}{x_i}\right)e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}$, and $\Phi_i(\theta,\gamma)=\frac{1}{x_i}\left(1+\frac{\gamma}{x_i}\right)^{\theta-1}e^{1-\left(1+\frac{\gamma}{x_i}\right)^{\theta}}$.

It is possible to obtain the MLE of Ω by maximizing the last equation with respect to Ω, equal to zero. Using the Newton-Rapshon method, R packages can be used to maximize the log-likelihood function to obtain an MLE. For more information of algorithm see [?].

4.2 Maximum Product of Spacings Method

The maximum product of spacings (MPS) method is used as an alternative to the MLE method adopted by Cheng and Amin [21] to estimate the parameters of continuous univariate models. Many authors used MPS to estimate model parameters based on a complete and different censored sample by Almetwally and Almongy [22], Basu et al. [23], Almetwally et al. [22], El-Sherpieny et al. [24] and Alshenawy et al. [18]. Let $x_{(1)} < x_{(2)} < ... < x_{(n)}$ then $x_{(i)}$ is order of data.

$$D_i(\Omega) = \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_{(i-1)}}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_{(i-1)}}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}} - \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x_{(i)}}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x_{(i)}}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}}, \quad (15)$$

where $D_i(\Omega), i = 1,...,n+1$ denotes to the uniform spacings, $F(x_{(0)},\Omega) = 0$, $F(x_{(n+1)},\Omega) = 1$ and $\sum_{i=1}^{n+1} D_i(\Omega) = 1$. Further, the MPSEs of the EOWINH parameters can also be obtained by first derivatives with parameters and equal zero.

4.3 Bayesian estimation

As random and parameter uncertainties are represented by a previous joint distribution that is established prior to the data collected on the failure, the Bayesian approach deals with the parameters. The ability to incorporate prior knowledge into research makes the Bayesian method very useful in the analysis of reliability, as one of the main problems associated with reliability analysis is the limited availability of data. In the α,β,θ and γ parameters, as prior gamma distributions, we have to use the insightful before. The α,β,θ and γ independent joint prior density function can be written as follows:

$$\Pi(\Omega) \propto \alpha^{a_1-1}\beta^{a_2-1}\theta^{a_3-1}\gamma^{a_4-1}e^{-(b_1\alpha+b_2\beta+b_3\theta++b_3\gamma)} \quad (16)$$

From the likelihood function and joint prior function, the joint posterior density function of Ω is obtained. The joint posterior of the distribution of EOWINH can then be written as

$$\Pi(\Omega|x) \propto \alpha^{a_1-1}\beta^{n+a_2-1}\theta^{n+a_3-1}\gamma^{n+a_4-1}\prod_{i=1}^{n}\left(1+\frac{\gamma}{x_i}\right)^{\theta-1} \frac{e^{\beta\left(\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]-b_2\right)}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}} e^{-(b_1\alpha+b_3\theta+b_3\gamma)} \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \quad (17)$$

Using the most common loss function (symmetric), which is a squared error. Bayesian estimators of $\tilde{\Omega}$ based on the squared error loss function are defined by MCMC.

$$S(\tilde{\Omega}) = E\left(\tilde{\Omega}-\Omega\right)^2 = \int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\int_0^{\infty}\left(\tilde{\Omega}-\Omega\right)^2 \Pi(\Omega|x)d\Omega_1 d\Omega_2 d\Omega_3 d\Omega_4 \quad (18)$$

It is noted that the integrals given by (18) can not be directly obtained. As a result, we use the MCMC to find an approximate value of integrals. A significant sub-class of the MCMC techniques is the Gibbs sampling and more general Metropolis within Gibbs samplers. The Metropolis–Hastings (MH), together with the Gibbs sampling, are the two most common instances of the MCMC method. The MH algorithm, similar to acceptance-rejection sampling, believes that for each iteration of the algorithm, a candidate value from a proposal distribution can be produced. The MH algorithm, similar to acceptance-rejection sampling, believes that for each iteration of the algorithm, a candidate value from a proposal distribution can be produced. We use the MH within the Gibbs sampling steps to produce random samples of conditional posterior densities from the EOWINH distribution family:

$$\Pi(\alpha|\beta,\theta,\gamma,x) \propto \alpha^{a_1-1}e^{-b_1\alpha} \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \quad (19)$$

$$\Pi(\theta|\alpha,\beta,\gamma,x) \propto \theta^{n+a_3-1}\frac{e^{\beta\left(\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]-b_2\right)}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}} \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \quad (20)$$

$$\Pi(\beta|\alpha,\theta,\gamma,x) \propto \beta^{n+a_2-1}\frac{e^{\beta\left(\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]-b_2\right)}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}} \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}, \quad (21)$$

and

$$\Pi(\gamma|\alpha,\beta,\theta,x) \propto \gamma^{n+a_4-1}\prod_{i=1}^{n}\left(1+\frac{\gamma}{x_i}\right)^{\theta-1} \frac{e^{\beta\left(\left[1-\left(1+\frac{\gamma}{x}\right)^{\theta}\right]-b_2\right)}}{\left[1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}\right]^{\beta+1}} e^{-(b_1\alpha+b_3\theta+b_3\gamma)} \left\{1+\alpha\left[\frac{e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}{1-e^{1-\left(1+\frac{\gamma}{x}\right)^{\theta}}}\right]^{\beta}\right\}^{\frac{-1}{\alpha}-1}. \quad (22)$$

5 Simulation

For comparison between the classical estimation methods, the Monte-Carlo simulation procedure is carried out in this Section: MLE, MPS, and Bayesian estimation method under square error loss function based on MCMC, for estimation of EOWINH lifetime distribution parameters is done by R language. Monte-Carlo experiments are performed on the basis of data-generated 10000 random EOWIR distribution samples, where x has EOWINH lifetime for various parameter actual values and different sample sizes n: (50, 100, and 200). We could describe the best methods of estimators as minimizing estimators' Bias and mean squared error (MSE).

The simulation results of the methods presented in this paper for point estimation are summarized in the tables 1, 2. We consider the Bias and MSE values in order to perform the required comparison between various point estimation methods. The following remarks can be noted from these tables:

1. For fixed actual parameters of EOWINH distribution, the Bias, and MSE decrease as n increases.
2. When γ increases, the Bias and MSE for all parameters increase except for θ
3. When θ increases, the Bias and MSE for all parameters increase.
4. When β increases, the Bias and MSE for α,θ increase and the Bias and MSE for β,γ decrease.
5. When α increases, the Bias and MSE for α,θ increase and the Bias and MSE for β,γ decrease.
6. MPS estimation is the best estimation method.
7. Bayesian estimation is a better alternative method of MLE.

6 Applications of COVID-19 Data

The COVID-19 data are provided in this Section to evaluate the consistency of the EOWINH distribution. Other related models such as X-Gamma inverse Weibull (XGIW) [Ibrahim and Almetwally [25]], generalized inverse Weibull (GIW) [De Gusmao et al. [26]], Exponentiated generalized inverse Weibull (EGIW) [Elbatal and Muhammed [27]], Weibull-Lomax (WL) [Tahir et al. [28]], modified Kies inverted Topp-Leone (MKITL) [Almetwally [29]], Odd Weibull inverse Topp–Leone (OWITL) [Almetwally [30]] and Marshall–Olkin alpha power Weibull (MOAPW) [Almetwally [31]] are compared with the EOWINH model. Tables 5 and 6 provide the statistics Cramer von Mises (CVM), Anderson Darling (AD) values, and the Kolmogorov-Smirnov (KS) statistics, along with the P-value for all models fitted on the basis of real data set.

This data represents a COVID-19 data belong to Saudi Arabia of 32 days, that is recorded from 15 September 2020 to 16 October 2020. This data formed of mortality rate $\left(\frac{Dialynewdeaths}{Dialycumulativecases}\right)$. The data are as follows:0.0557 0.0559 0.0617 0.0649 0.0683 0.0709 0.0711 0.0736 0.0737 0.0739 0.0741 0.0743 0.0776 0.0782 0.0804 0.0808 0.0815 0.0818 0.0819 0.0840 0.0850 0.0864 0.0867 0.0869 0.0901 0.0904 0.0907 0.0914 0.0943 0.0946 0.1009 0.1134.

It is evident from Table 5, in comparison with other distributions as EOWIR, XGIW, GIW, EGIW, WL, MKITL, OWITL and MOAPW distributions, that EOWINH has minimum values of KS, CVM and AD statistics. This leads us to believe that EOWINH fits the real data set better. The fitted EOWINH PDF with histogram and CDF with empirical are displayed in Figure 3. The fitted EOWINH PP-plot and QQ-plot are displayed in Figure 4. By Table 6, the Bayesian estimation method of EOWINH distribution is the best estimation method. History plots, approximate marginal posterior density and MCMC convergence of α,β,θ and γ are represented in Figures 5 and 6.

Table 1: Bias and MSE of EOWINH distribution for MLE, MPS and Bayesian when $\alpha = 0.5,\ \beta = 0.5$

				MLEs		MPSEs		Bayesian	
θ	γ	n		Bias	MSE	Bias	MSE	Bias	MSE
0.5	0.5	25	α	-0.2664	0.4492	-0.1273	0.1083	-0.1302	0.3561
			β	-0.2245	0.5459	-0.0035	0.2125	0.0153	0.2980
			θ	0.1619	0.2995	0.0243	0.0398	0.0614	0.1647
			γ	-0.0147	0.2617	0.1371	0.1187	0.0791	0.2596
		70	α	-0.0777	0.2577	0.0163	0.0448	-0.0434	0.2851
			β	-0.0648	0.2702	0.0445	0.0680	-0.0071	0.2906
			θ	0.0728	0.1844	0.0043	0.0181	0.0681	0.1664
			γ	-0.0177	0.1934	0.0671	0.0503	0.0175	0.1924
		150	α	-0.0504	0.1997	0.0276	0.0237	-0.0387	0.1923
			β	-0.0362	0.1880	0.0325	0.0303	-0.0095	0.1871
			θ	0.0430	0.1242	-0.0054	0.0081	0.0316	0.1004
			γ	-0.0108	0.1557	0.0499	0.0305	0.0113	0.1389
	2	25	α	-0.1909	0.5678	-0.0878	0.1476	-0.2128	0.4018
			β	-0.1770	0.5100	0.0373	0.1569	-0.0613	0.2654
			θ	0.1232	0.2696	-0.0054	0.0021	0.0869	0.1363
			γ	-0.1308	0.5087	0.2056	0.1785	-0.1330	0.4423
		70	α	-0.0816	0.3809	0.0025	0.1221	-0.0458	0.3225
			β	-0.0943	0.3473	0.0354	0.1117	0.0045	0.2958
			θ	0.0351	0.0806	-0.0073	0.0044	0.0418	0.1023
			γ	-0.0711	0.2974	0.1202	0.1382	-0.0571	0.3955
		150	α	-0.0611	0.2630	0.0388	0.0590	-0.0194	0.1785
			β	-0.0598	0.2246	0.0373	0.0509	-0.0198	0.1843
			θ	0.0316	0.0954	-0.0108	0.0015	0.0131	0.0432
			γ	-0.0930	0.2701	0.0921	0.0813	-0.0101	0.2249
3	0.5	25	α	0.0865	0.6135	0.0845	0.3274	0.0447	0.3648
			β	-0.0901	0.5159	0.1298	0.2899	0.0684	0.3106
			θ	0.0713	0.2183	-0.0843	0.1035	-0.1019	0.3967
			γ	0.0218	0.1032	0.0366	0.0243	0.0604	0.1249
		70	α	-0.0151	0.4044	0.0027	0.1480	-0.0099	0.3158
			β	-0.0541	0.3437	0.0557	0.1131	0.0343	0.3583
			θ	0.0181	0.1586	-0.0752	0.0543	-0.0466	0.3803
			γ	0.0106	0.0604	0.0198	0.0067	0.0347	0.0984
		150	α	-0.0079	0.2920	0.0078	0.0771	-0.0111	0.1915
			β	-0.0272	0.2472	0.0369	0.0559	-0.0092	0.1675
			θ	0.0206	0.1766	-0.0388	0.0125	-0.0013	0.2308
			γ	0.0038	0.0466	0.0068	0.0015	0.0106	0.0527
	2	25	α	0.1123	0.9034	0.0982	0.4990	-0.1784	0.3653
			β	0.0690	0.9081	0.2608	0.7448	-0.0287	0.3392
			θ	0.3502	0.8558	-0.0789	0.1355	-0.0108	0.3377
			γ	0.0160	0.6918	0.0859	0.1860	0.0723	0.3076
		70	α	0.0298	0.5435	0.0572	0.2254	0.0031	0.3001
			β	-0.0245	0.4492	0.0916	0.1791	0.0282	0.2215
			θ	0.1745	0.5701	-0.1418	0.1015	0.0624	0.3427
			γ	0.0038	0.4778	0.1348	0.1202	0.0161	0.2545
		150	α	0.0319	0.3177	0.0607	0.0896	0.0099	0.1989
			β	0.0023	0.2519	0.0750	0.0643	0.0136	0.1679
			θ	0.0939	0.4967	-0.1158	0.0629	0.0035	0.2117
			γ	0.0089	0.4125	0.0850	0.0550	0.0204	0.1653

Table 2: Bias and MSE of EOWINH distribution for MLE, MPS and Bayesian when $\alpha = 0.5,\ \beta = 2$

				MLEs		MPSEs		Bayesian	
θ	γ	n		Bias	MSE	Bias	MSE	Bias	MSE
0.5	0.5	25	α	-0.0336	0.1579	0.0046	0.0302	0.0182	0.1400
			β	-0.2229	0.7421	0.1538	0.5308	-0.0084	0.3920
			θ	0.2116	0.4066	0.1061	0.0976	0.0346	0.1537
			γ	0.1394	0.6742	0.1849	0.4775	0.1397	0.6574
		70	α	-0.0024	0.1076	0.0299	0.0150	0.0291	0.1009
			β	-0.1079	0.4530	0.1324	0.2597	0.0742	0.4154
			θ	0.0234	0.1247	-0.0152	0.0132	0.0381	0.2305
			γ	0.1216	0.4438	0.1439	0.1583	0.0581	0.3109
		150	α	0.0038	0.0641	0.0386	0.0050	0.0286	0.0599
			β	-0.1151	0.3116	0.0951	0.0499	0.0485	0.2939
			θ	0.0347	0.1011	0.0083	0.0085	-0.0054	0.0277
			γ	0.0279	0.3249	0.0153	0.0943	0.0273	0.1328
	2	25	α	-0.0335	0.2030	0.0170	0.0387	0.0356	0.1533
			β	-0.2653	0.9298	0.2089	0.7752	-0.0003	0.4024
			θ	0.1190	0.2585	0.0369	0.0422	0.0131	0.1174
			γ	-0.0226	0.9582	0.0284	0.7704	-0.0260	0.4290
		70	α	-0.0241	0.1400	0.0263	0.0244	0.0174	0.1068
			β	-0.1889	0.6090	0.1501	0.4242	-0.0079	0.3909
			θ	0.0432	0.1111	0.0039	0.0095	0.0022	0.0815
			γ	0.0315	0.5680	0.0371	0.3826	0.0396	0.3737
		150	α	-0.0118	0.0932	0.0104	0.0074	0.0054	0.0646
			β	-0.0996	0.4456	0.0548	0.1578	-0.0209	0.3452
			θ	0.0166	0.0609	-0.0056	0.0027	0.0047	0.0328
			γ	0.0155	0.4261	0.0179	0.1068	0.0224	0.3329
3	0.5	25	α	-0.0513	0.2498	0.0013	0.0559	0.0245	0.1587
			β	-0.4064	1.1703	0.0763	1.1280	-0.0150	0.4238
			θ	0.0241	0.8328	-0.2775	0.7643	-0.0385	0.4123
			γ	0.2848	0.7217	0.2589	0.3440	0.0637	0.3488
		70	α	-0.0327	0.1753	0.0096	0.0295	0.0217	0.1369
			β	-0.2623	0.8325	0.0671	0.5846	-0.0078	0.2871
			θ	-0.0399	0.6918	-0.2356	0.5011	-0.0104	0.3046
			γ	0.1668	0.4357	0.1495	0.1488	0.0174	0.1157
		150	α	-0.0140	0.1132	0.0118	0.0126	0.0250	0.0904
			β	-0.1594	0.4751	0.0161	0.1875	0.0275	0.3446
			θ	0.0263	0.4081	-0.0813	0.1634	-0.0520	0.4764
			γ	0.0317	0.1194	0.0294	0.0147	0.0214	0.1291
	2	25	α	-0.0254	0.2814	0.0186	0.0561	0.0196	0.1215
			β	-0.2165	1.2948	0.2480	1.3070	-0.0245	0.4180
			θ	0.1662	0.8954	-0.1397	0.5800	0.0697	0.4459
			γ	0.4681	0.9976	0.2592	0.2943	-0.0404	0.3626
		70	α	0.0146	0.2225	0.0442	0.0353	0.0136	0.0929
			β	0.0308	0.9691	0.2935	0.7539	0.0129	0.3445
			θ	0.0128	0.6123	-0.2055	0.3183	0.0242	0.3729
			γ	0.2116	0.5862	0.1849	0.1787	0.0054	0.3348
		150	α	0.0006	0.1735	0.0149	0.0197	0.0187	0.0745
			β	-0.0073	0.7541	0.1249	0.4085	0.0721	0.3301
			θ	0.0142	0.5086	-0.1019	0.1899	0.0448	0.3248
			γ	0.1543	0.3961	0.1313	0.0575	0.0091	0.3295

Table 3: Bias and MSE of EOWINH distribution for MLE, MPS and Bayesian when $\alpha = 2,\ \beta = 0.5$

				MLEs		MPSEs		Bayesian	
θ	γ	n		Bias	MSE	Bias	MSE	Bias	MSE
0.5	0.5	25	α	-0.0815	0.2736	0.0834	0.0910	0.0102	0.1412
			β	-0.3447	0.7432	0.2100	0.5554	0.0351	0.2968
			θ	0.1150	0.2684	0.0079	0.0510	0.0358	0.1303
			γ	0.3568	0.7913	0.2867	0.4705	0.0630	0.3474
		70	α	-0.0330	0.2035	0.0899	0.0591	0.0091	0.0927
			β	-0.1691	0.5279	0.1920	0.3304	0.0063	0.1913
			θ	0.0404	0.1329	-0.0209	0.0156	0.0143	0.0749
			γ	0.1847	0.4291	0.1463	0.1613	0.0290	0.1574
		150	α	-0.0024	0.1504	0.0791	0.0349	-0.0054	0.0576
			β	-0.0480	0.3735	0.1804	0.1962	-0.0122	0.1327
			θ	0.0149	0.0839	-0.0248	0.0075	0.0126	0.0433
			γ	0.0725	0.2521	0.0628	0.0608	-0.0001	0.1048
	2	25	α	-0.0844	0.2614	0.0825	0.0951	0.0057	0.2426
			β	-0.3128	0.6845	0.2602	0.6408	0.0142	0.2835
			θ	0.1227	0.2186	0.0090	0.0292	0.0389	0.1190
			γ	0.2331	0.9025	0.1942	0.8455	-0.0230	0.3953
		70	α	-0.0271	0.2016	0.0969	0.0625	0.0044	0.0932
			β	-0.1227	0.4919	0.2463	0.3560	-0.0010	0.1870
			θ	0.0481	0.1272	-0.0164	0.0139	0.0121	0.0542
			γ	0.1427	0.6315	0.1092	0.4237	-0.0090	0.2387
		150	α	-0.0065	0.1484	0.0752	0.0323	-0.0013	0.0562
			β	-0.0506	0.3566	0.1801	0.1775	-0.0088	0.1216
			θ	0.0188	0.0798	-0.0228	0.0062	0.0039	0.0330
			γ	0.0654	0.5041	0.0405	0.2335	0.0006	0.1584
3	0.5	25	α	-0.0566	0.3661	0.1055	0.1587	0.0163	0.1764
			β	-0.3427	0.8157	0.2054	0.7683	0.0139	0.2883
			θ	-0.1582	0.7684	-0.4618	0.9930	-0.0358	0.3880
			γ	0.3815	0.6749	0.3662	0.5541	0.0512	0.1537
		70	α	-0.0309	0.2234	0.1059	0.0756	0.0106	0.1582
			β	-0.1523	0.5378	0.2410	0.4123	-0.0012	0.1996
			θ	-0.1394	0.6117	-0.3395	0.6042	-0.0012	0.2577
			γ	0.1864	0.3972	0.1590	0.1825	0.0191	0.0971
		150	α	-0.0148	0.1637	0.0760	0.0398	-0.0020	0.0614
			β	-0.0671	0.3991	0.1884	0.2262	-0.0044	0.1318
			θ	-0.0701	0.4659	-0.1886	0.2591	-0.0108	0.1621
			γ	0.0897	0.2226	0.0521	0.0436	0.0165	0.0623
	2	25	α	-0.0099	0.4656	0.1604	0.2141	0.0371	0.1476
			β	-0.2403	0.9004	0.3548	0.9905	0.0465	0.3140
			θ	0.2166	0.8565	-0.2537	0.7211	0.0201	0.3880
			γ	0.6095	1.1555	0.3768	0.7758	-0.0057	0.3389
		70	α	-0.0038	0.3012	0.1218	0.1046	0.0123	0.0811
			β	-0.0787	0.6814	0.2956	0.5579	0.0240	0.1919
			θ	0.1286	0.6232	-0.1647	0.3587	0.0213	0.2246
			γ	0.2380	0.6723	0.0934	0.3117	-0.0194	0.2208
		150	α	0.0154	0.1929	0.1045	0.0520	0.0047	0.0546
			β	0.0097	0.4424	0.2612	0.2865	0.0043	0.1264
			θ	0.0290	0.4534	-0.1643	0.1936	0.0035	0.1503
			γ	0.0994	0.4441	0.0125	0.1442	-0.0090	0.1463

Table 4: Bias and MSE of EOWINH distribution for MLE, MPS and Bayesian when $\alpha = 2,\ \beta = 2$

				MLE		MPSEs		Bayesian	
θ	γ	n		Bias	MSE	Bias	MSE	Bias	MSE
0.5	0.5	25	α	-0.1089	0.3670	-0.0997	0.0879	-0.0263	0.3568
			β	-0.2551	0.6910	-0.0240	0.3107	-0.0289	0.4151
			θ	0.2074	0.3936	0.0986	0.1022	0.0823	0.2121
			γ	0.0369	0.4206	0.1412	0.2264	0.0810	0.4112
		70	α	-0.0730	0.2454	-0.0316	0.0425	-0.0157	0.2077
			β	-0.0863	0.3692	0.0420	0.1118	0.0004	0.2360
			θ	0.1036	0.2369	0.0372	0.0354	0.0571	0.1542
			γ	0.0002	0.2786	0.0712	0.0924	0.0095	0.2352
		150	α	-0.0058	0.1786	0.0153	0.0258	-0.0080	0.1386
			β	-0.0484	0.2670	0.0175	0.0634	-0.0027	0.1458
			θ	0.0381	0.1391	0.0058	0.0140	0.0254	0.0923
			γ	0.0182	0.2053	0.0581	0.0470	0.0047	0.1479
	2	25	α	-0.0649	0.6597	-0.0991	0.2250	-0.0537	0.3642
			β	-0.1350	1.0671	0.0954	0.7513	0.0200	0.4064
			θ	0.2177	0.4083	0.1101	0.0881	0.0473	0.1243
			γ	-0.2944	0.8297	-0.0767	0.6945	-0.0365	0.4864
		70	α	-0.0776	0.4041	-0.0554	0.0799	-0.0204	0.2990
			β	-0.0946	0.6184	0.0436	0.2391	0.0140	0.3583
			θ	0.0806	0.2104	0.0206	0.0128	0.0326	0.0854
			γ	-0.1482	0.5558	0.0165	0.2580	-0.0518	0.3709
		150	α	-0.0231	0.2709	-0.0040	0.0456	0.0014	0.1882
			β	-0.0297	0.4178	0.0434	0.1281	0.0113	0.2309
			θ	0.0341	0.1038	0.0041	0.0032	0.0103	0.0461
			γ	-0.0934	0.3872	0.0276	0.1418	-0.0092	0.2393
3	0.5	25	α	0.0599	0.9693	-0.0703	0.4033	-0.0127	0.4341
			β	-0.1907	1.3072	-0.0220	0.9034	-0.0329	0.3843
			θ	0.0905	0.4699	-0.1693	0.3473	-0.0043	0.4231
			γ	0.1199	0.3943	0.1389	0.1819	0.0421	0.1328
		70	α	-0.0507	0.5452	-0.0943	0.1603	-0.0178	0.3588
			β	-0.1661	0.8033	-0.0859	0.3605	-0.0399	0.3889
			θ	0.0136	0.4147	-0.1721	0.1921	-0.0181	0.4175
			γ	0.0630	0.2325	0.0860	0.0584	0.0408	0.1201
		150	α	-0.0094	0.3671	-0.0556	0.0919	-0.0095	0.1974
			β	-0.0373	0.5469	-0.0296	0.2004	0.0110	0.2435
			θ	0.0133	0.2140	-0.0950	0.0656	0.0094	0.2482
			γ	0.0145	0.0819	0.0343	0.0094	0.0106	0.0571
	2	25	α	0.3042	1.5283	-0.0036	0.6969	-0.0108	0.3210
			β	0.2330	2.1016	0.1653	1.7235	-0.0303	0.3939
			θ	0.4959	0.9768	-0.0262	0.1889	0.0128	0.3610
			γ	0.2081	1.0832	0.2098	0.5068	0.0278	0.3118
		70	α	0.2209	0.8648	0.0701	0.2962	0.0055	0.2057
			β	0.2591	1.2633	0.1972	0.7202	0.0225	0.2466
			θ	0.2749	0.7495	-0.0563	0.0757	0.0061	0.2221
			γ	0.0084	0.7390	0.0790	0.1800	0.0087	0.1869
		150	α	0.0305	0.4895	-0.0171	0.1341	-0.0093	0.1311
			β	0.0162	0.7115	0.0313	0.3030	-0.0079	0.1609
			θ	0.1018	0.6609	-0.0845	0.0514	0.0025	0.1416
			γ	0.1032	0.6550	0.1035	0.0903	0.0037	0.1198

Table 5: MLE estimates, SE, KS with P-Value, CVM, and AD for COVID-19 data of Saudi Arabia

		α	β	θ	γ	KS	P-Value	CVM	AD
EOWINH	estimate	8.4380	0.4905	0.5296	0.1397	0.0766	0.9845	0.0231	0.1717
	se	4.6558	0.4273	0.5702	0.2369				
EOWIR	estimate	0.1190	29.6826	0.2010		0.0832	0.9665	0.0381	0.2771
	se	0.3498	13.1644	0.5910					
XGIW	estimate	35.6375	1.9116	0.0312		0.0820	0.9703	0.0362	0.2657
	se	23.6521	0.2936	0.0275					
GIW	estimate	0.1481	0.0141	6.0669		0.1572	0.3693	0.1717	1.1273
	se	0.0458	0.0248	0.7592					
EGIW	estimate	0.3176	133.8462	1.1396	1.6659	0.0870	0.9513	0.0333	0.2475
	se	0.1583	198.9590	0.2794	1.3955				
WL	estimate	9.2046	6.9983	0.9057	0.1034	0.0874	0.9498	0.0439	0.3498
	se	66.5627	1.9754	0.9009	0.0961				
MKITL	estimate	2.6210	109.3216			0.1042	0.8425	0.0630	0.4871
	se	0.3422	5.5756						
OWITL	estimate	3.1836	36.0380	44.7220		0.0932	0.9197	0.0500	0.3943
	se	0.4773	103.3311	30.9857					
MOAPW	estimate	0.0046	10.9296	0.0574	0.1218	0.0768	0.9839	0.0297	0.2149
	se	0.0218	2.1310	0.1519	0.0259				

Table 6: MPS, and Bayesian estimates, SE for COVID-19 data of Saudi Arabia

	MPSEs		Bayesian	
	estimate	se	estimate	se
α	6.5772	12.6377	8.6243	0.3306
β	0.5598	2.5751	0.4782	0.2904
θ	0.8539	3.6818	0.4396	0.1337
γ	0.0696	0.4022	0.2407	0.1281

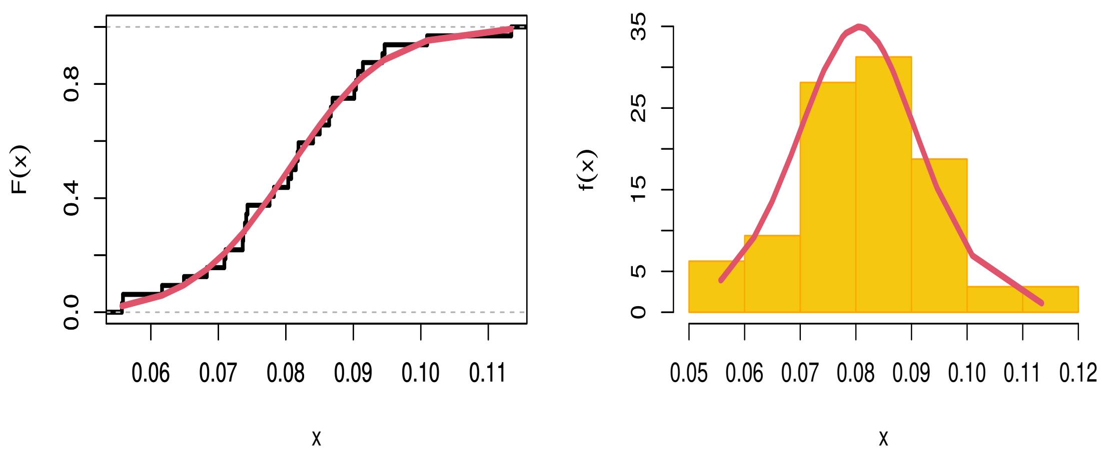

Fig. 3: Estimated PDF, and histogram plot of EOWINH PP-plot and QQ-plot of EOWIR for COVID-19 data of Saudi Arabia

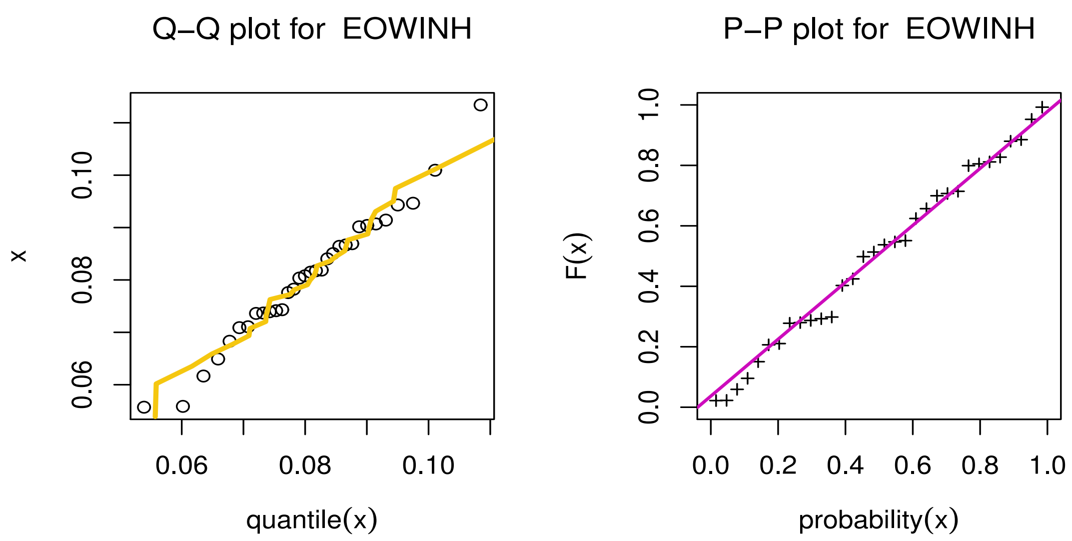

Fig. 4: PP-plot and QQ-plot of EOWINH for COVID-19 data of Saudi Arabia

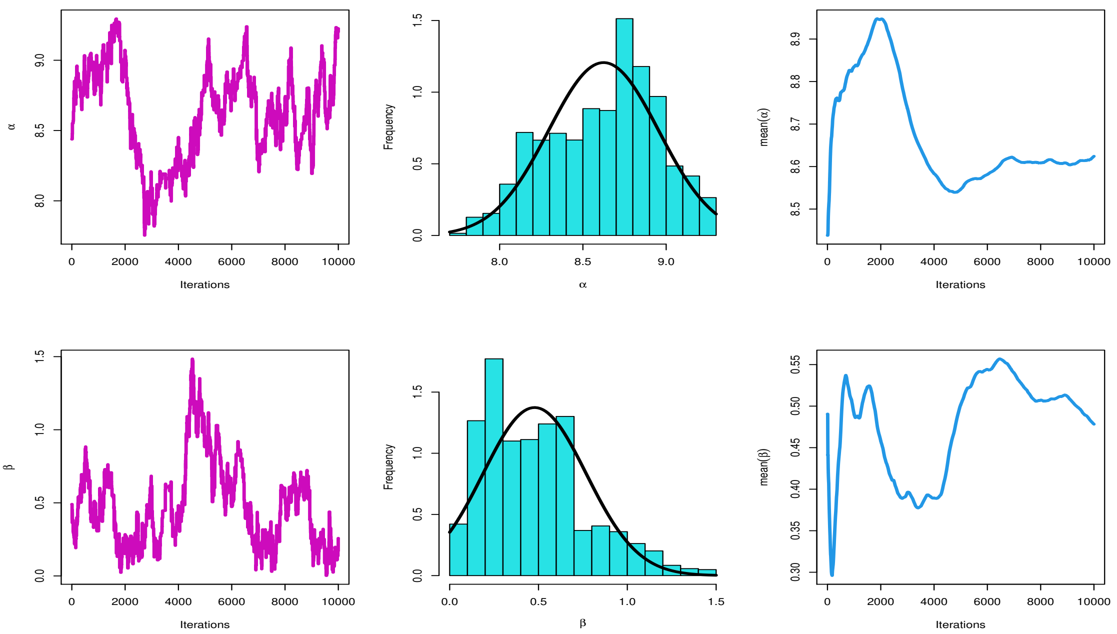

Fig. 5: Convergence of MCMC estimation of EOWINH (α, β) for COVID-19 data of Saudi Arabia

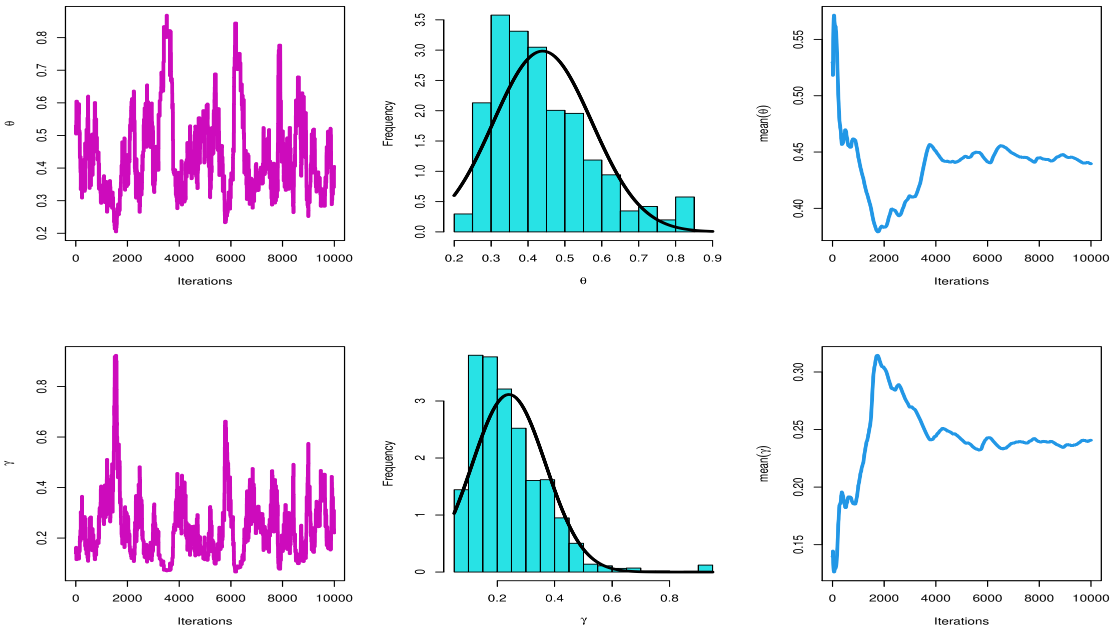

Fig. 6: Convergence of MCMC estimation of EOWINH (θ, γ) for COVID-19 data of Saudi Arabia

7 Conclusion

A new generalization of INH and Weibull distributions called EOWINH distribution is formulated in this paper. We studied its statistical properties and obtained a linear representation for its pdf that was successful in finding the function and quantile function of moments and moment generation. Point estimation of the EOWINH unknown parameters α, β, θ, and γ was considered by MLE, MPS, and Bayesian estimation methods. To distinguish the performance of different estimation methods, a comparison was carried out through simulation analysis using the R package. For that reason, the MCMC approach was used, real data set were also considered, and EOWINH was shown to match these data of COVID-19 better compared to other competitive distributions. Bayesian estimation is the best estimation method for estimate parameters of EOWINH distribution.

Conflicts of Interest:

The author declares that there is no conflict of interest regarding the publication of this paper.

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Author contributions:

1. New suitable distribution for modeling the COVID-19 data in Saudi Arabia.
2. Extended odd Weibull inverse Nadarajah-Haghighi (EOWINH) distribution.
3. Maximum likelihood, maximum product spacing, and Bayesian estimation methods are applied for parameters of EOWINH distribution.

Data Availability:

All data are available in the paper.

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