Mathematical Sciences Letters
An International Journal

http://dx.doi.org/10.18576/msl/060213

A Note on Numerical Study of Sisko Fluid Flow Over Stretching Cylinder and Heat Transfer with Viscous Dissipation

Arif Hussain[1,*], *M.Y. Malik*[1], *S. Bilal*[1], *M. Awais*[1] *and T. Salahuddin*[2]

[1] Department of Mathematics, Quaid-i-Azam University Islamabad 44000, Pakistan.
[2] Department of Mathematics, Mirpur University of Science & Technology Mirpur, 10250, AJK, Pakistan

Received: 7 Jun. 2016, Revised: 21 Sep. 2016, Accepted: 23 Sep. 2016
Published online: 1 May 2017

Abstract: Present work describes the Sisko fluid flow characteristics over stretching cylinder and heat transfer with viscous dissipation. Governing equations are modelled and then simplified by using boundary layer approach. The scaling group of transformations is employed to transform flow govern partial differential equations into corresponding set of ordinary differential equations. Since attaining set of ordinary differential equations is nonlinear, thus numerical technique Runge–Kutta–Fehlberg method is applied to compute dimensionless velocity and temperature expressions. The influence of all parameters on momentum and heat equations is figured out with the aid of graphs. The effects of parameters on skin-friction coefficient and local Nusselt number are elaborated through graphs and tables.

Keywords: Sisko fluid model; Boundary layer flow; Stretching cylinder; Viscous dissipation; Shooting method.

1 Introduction

Viscous dissipation plays an important role in the problems involving heat transfer, because it behaves like an energy source that affected the rate of heat transfer. As yet less amount of concentration has been paid by researchers towards the viscous dissipation effect. Specifically, in laminar flow problems the effect of viscous dissipation has not yet been considered in detail. [1] is pioneer, who gave the idea of viscous dissipation. He analyzed the viscous heating effects on Newtonian fluid flows in a straight circular tube and investigated some temperature distributions at zero wall temperature. He proved that the effects are produced in the close wall region. After that many researchers extended his work and took the viscous dissipation effect in heat transfer problems. [2] studied the viscous dissipation effects in the entrance region of pipes with uniform heat flux. [3] prolonged the previous work and identified the viscous dissipation effects on the thermal entrance heat transfer in laminar pipe flows with convective boundary conditions. [4] modelled the problem on laminar forced convective flow of Newtonian fluid though a channel by taking viscous dissipation effects into account. [5] discussed the viscous dissipation effects on MHD viscous fluid flow in a porous medium. [6] extended their own work and investigated the combined effects of viscous dissipation and Joule heating on MHD viscous fluid flow past a porous stretching surface along with heat and mass transfer. [7] investigated the effects of viscous dissipation on forced convective flow of Newtonian fluid through pipes and discussed two cases i.e. hydrodynamically fully developed flow and thermally fully developed flow. [8] continued his work by discussing the forced convective thermally fully developed flow of viscous fluid through pipe along with viscous dissipation effects. [9] considered the viscous dissipation effects on non-Newtonian power law fluid flow inside the elliptical duct and obtained numerical solution of flow govern differential equations. [10] also studied the influence of viscous dissipation on boundary layer hydrodynamic flow past a porous moving vertical plate with suction and temperature dependent viscosity.

The boundary layer flow problems of Newtonian and non-Newtonian fluids have various applications in

* Corresponding author e-mail: alihassan721214@gmail.com

industry, engineering and aerodynamics. For example, glass-fiber production, polymer sheets production, paper production and plastic films etc. Thus, in last few decades the researchers took much interest in boundary layer flow problems. [11] is the first who analyzed the boundary layer flow. He discussed the boundary layer flow of two dimensional axisymmetric viscous fluid flow over a continuous solid surfaces. [12] continued the work and investigated the boundary layer two dimensional viscous fluid flow over stretching sheet. The boundary layer flow of Newtonian fluid over stretching sheet was studied by [13] by taking convective boundary condition and nanoparticles concentration effects into account. [14] studied the boundary layer flow of nanofluid over an exponentially stretching surface. [15] investigated the boundary layer flow of Casson nanofluid over a vertical exponentially stretching cylinder. [16] considered the boundary layer flow of second order fluid over a stretching sheet with uniform free stream velocity and found the non-similar solutions. [17] discussed the boundary layer flow and heat transfer of nanofluid over a nonlinearly permeable stretching/shrinking sheet. The boundary layer flow along with heat transfer was firstly inspected by [18]. They considered the heat and mass transfer on a stretching sheet with suction or blowing. [19] prolonged the previous work and investigated the heat transfer problem on a continuous stretching sheet. [20] found numerical solution of the problem addressing the viscous fluid flow over stretching cylinder along with temperature dependent thermal conductivity. [21] examined the heat transfer analysis of MHD non-Newtonian power law fluid flow passing over a stretching sheet. They found analytic solution via HAM and variations in heat transfer are deliberated against altering values of governing parameters. [22] extended their previous work and examined the heat transfer of hydrodynamic power law fluid flow over a vertical stretching surface by assuming convective boundary conditions. [23] investigated the convective heat transfer of MHD non-Newtonian Jeffrey fluid flow over a linearly stretching surface and found solution by using hypergeometric functions. The mixed convection flow of MHD non-Newtonian Eyring-Powell nanofluid was numerically deliberated by [24]. Recently, [25] discussed the heat transfer of Christove-Catteneo heat flux model for Williamson fluid flow over a stretching sheet with variable thickness. They solved modelled set of differential equations with the aid of Keller-Box method.

Many fluids which are used in industry, pharmaceutical and daily life products are non-Newtonian in nature e.g. emulsions, pints, blood flow, lubricants, starch suspensions etc. Also, as lot of varieties of non-Newtonian fluids are subsist in nature, thus different constitutive equations are suggested to examine the non-Newtonian fluid rheology. Among them, there is a Sisko fluid model which was proposed by Sisko in 1958. It is observed that the computed results of Sisko fluids corresponds to observed data of many non-Newtonian fluids such as lubricant greases, blood flow, mud and paints etc. The pioneer of this model i.e. [26] analyzed the three different lubricating greases and proved that Sisko fluid model results matched with observed data of all three greases. After that many researchers inspected the non-Newtonian flow rheology by using constitutive equations of Sisko fluid model. [27] analyzed the peristaltic flow of non-Newtonian fluid in a uniform inclined tube via Sisko fluid model. [28] extended their work and discussed the peristaltic flow of a Sisko fluid in an endoscope and obtained the both analytical and numerical solutions. [29] also investigated the problem of peristaltic Sisko nanofluid in an asymmetric channel. [30] formulated the problem of Sisko fluid in which implicit differential equations are arising. Axisymmetric flow of non-Newtonian Sisko fluid over radially stretching sheet was investigated by [31]. They found solution via analytical technique HAM and explored the effects of flow govern parameters on interesting physical quantities. [32] also discussed the analytical solution of boundary layer Sisko fluid flow over a stretching sheet. Recently, [33] calculated the numerical solution of the problem MHD Sisko fluid flow over a stretching cylinder. They recognized that fluid parameter accelerates the fluid movement. Also, [34,35] analyzed Sisko fluid model by assuming various physical assumptions.

After reviewing the aforementioned literature authors found that the effects of viscous dissipation on Sisko fluid model not discussed so far. So, present investigation focuses on the study of viscous dissipation effects on boundary layer flow of Sisko fluid over stretching cylinder. Since the governing ordinary differential equations are non-linear with complexities, thus shooting method with Runge-Kutta-Fehlberg integration scheme is used to obtain numerical solution. The impact of all physical parameters is figured out on dimensionless velocity and temperature profiles with the help of graphs. In addition, the effects of involving physical parameters on skin-friction coefficient and local Nusselt number are presented through tables and graphs.

2 Mathematical formulation

Consider two dimensional, steady state, boundary layer flow of axisymmetric incompressible Sisko fluid over the continuously stretching cylinder. The cylinder is stretched with velocity $U(x) = cx$ in axial direction. The heat equation is considered with viscous dissipation effects. By applying usual boundary layer approximations the governing equations reduce to

$$\frac{\partial (ru)}{\partial x} + \frac{\partial (rv)}{\partial r} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=\frac{a}{r\rho}\frac{\partial}{\partial r}(r\frac{\partial u}{\partial r})-\frac{b}{r\rho}(-\frac{\partial u}{\partial r})^{n}+\frac{nb}{\rho}(-\frac{\partial u}{\partial r})^{n-1}\frac{\partial^{2}u}{\partial r^{2}}, \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{a}{\rho C_p}\left(-\frac{\partial u}{\partial r}\right)^{2}+\frac{b}{\rho C_p}\left(-\frac{\partial u}{\partial r}\right)^{n+1}, \tag{3}$$

subject to the boundary conditions

$$u=U(x),\ v=0,\ T=T_w \text{ at } r=r_0, \tag{4}$$
$$u\to 0,\ T\to T_\infty \text{ at } r\to\infty.$$

Here x and r are axial and radial axes, u and v are velocity components of fluid along x and r directions respectively, n, a and b are the material constants, ρ is the density, T is the temperature of the fluid, T_w is the temperature of fluid at wall, T_∞ is the extreme temperature, C_p is the specific heat and α is the thermal diffusivity.

The stream function Ψ is defined such that

$$u=\frac{1}{r}\frac{\partial\Psi}{\partial r},\ v=-\frac{1}{r}\frac{\partial\Psi}{\partial x}. \tag{5}$$

To reduce the modelled partial differential equations into ordinary differential equations the following similarity transformations are employed

$$\eta=\frac{r^2-r_0^2}{2xr_0}Re_b^{\frac{1}{n+1}},\ \Psi=xr_0URe_b^{\frac{-1}{n+1}}f(\eta),\ \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{6}$$

where Re_b is defined as $Re_b=\frac{\rho x^nU^{2-n}}{b}$.

Using above defined similarity transformations in equations $(1)-(3)$, the equation (1) is identically satisfied, whereas equation (2) and equation (3) are modified to

$$A(1+2\gamma\eta)f'''+n(1+2\gamma\eta)^{\frac{n+1}{2}}(-f'')^{n-1}f'''+2A\gamma f''-$$

$$(n+1)\gamma(1+2\gamma\eta)^{\frac{n-1}{2}}(-f'')^{n}-f'^{2}+\frac{2n}{n+1}ff''=0, \tag{7}$$

$$(1+2\gamma\eta)\theta''+2\gamma\theta'+\frac{2n}{n+1}\Pr f\theta'+A(1+2\gamma\eta)Ec\Pr(-f'')^{2}+$$

$$Ec\Pr(1+2\gamma\eta)^{\frac{n+1}{2}}\left(-f''\right)^{n+1}=0, \tag{8}$$

subject to boundary conditions

$$f(0)=0,\ f'(0)=1,\ \theta(0)=1,\ f'(\infty)=0,\ \theta(\infty)=0. \tag{9}$$

The above defined dimensionless parameters i.e. curvature parameter γ, Eckert number Ec, material parameter A and Prandtl number Pr are mathematically defined as

$$\gamma=\frac{x}{r_0}Re_b^{\frac{-1}{n+1}},\ Re_a=\frac{\rho Ux}{a},\ A=\frac{Re_b^{\frac{2}{n+1}}}{Re_a},$$
$$Ec=\frac{U^2}{C_p(T_w-T_\infty)},\ \Pr=\frac{xU}{\alpha}Re_b^{\frac{-2}{1+n}}. \tag{10}$$

The interested physical quantities of problem i.e. skin friction coefficient and local Nusselt number are defined as

$$C_f=\frac{\tau_w}{\frac{1}{2}\rho U^2},\ Nu_x=\frac{xq_w}{k(T_w-T_\infty)}, \tag{11}$$

where

$$\tau_w=a(\frac{\partial u}{\partial r})_{r=r_0}-b(-\frac{\partial u}{\partial r})^{n}_{r=r_0},\ q_w=-k(\frac{\partial T}{\partial r})_{r=r_0}. \tag{12}$$

After using similarity transformations in equations $(11)-(12)$, the skin friction coefficient and local Nusselt number are converted to following form

$$\frac{1}{2}C_fRe_b^{\frac{1}{n+1}}=Af''(0)-[-f''(0)]^{n},\ Nu_xRe_b^{\frac{-1}{n+1}}=-\theta'(0). \tag{13}$$

3 Numerical solution

The governing momentum and heat equations i.e. equations $(7)-(8)$ are nonlinear ordinary differential equations of order three in f and order two in θ respectively. The numerical solution of these equations is computed by using shooting method with the aid of fifth order Runge-Kutta integration scheme. In this technique, firstly given equations are reduced to a system of five first order ordinary differential equations. This system of five first order ordinary differential equations is solved by using fifth order Runge-Kutta method. To solve this system, there are five initial conditions are required but here only two initial conditions in f and one initial condition in θ at $\eta=0$ are given. The remaining two conditions in f' and θ are defined at η_∞. Now, missing two conditions at $\eta=0$ are selected. Furthermore, the appropriate finite value of η_∞ is should be chosen. To estimate the value of η_∞, some initial guess are considered and the boundary value problem of equations $(7)-(8)$ is solve to evaluate $f''(0)$ and $\theta'(0)$. The process of finding solution is repeated with another guess of η_∞ until two successive values of $f''(0)$ and $\theta'(0)$ differ only after the desired significant digits. The final value of η_∞ is taken as the finite value of the limit η_∞ for the set of physical parameters for finding velocity $f'(\eta)$ and temperature $\theta(\eta)$ in the boundary layer. After obtaining all the five initial conditions this system of simultaneous ordinary differential equations is solved by using Runge-Kutta fifth order method.

4 Graphs and Tables

Table 1: Comparison table of skin friction coefficient for different values of curvature parameter γ and $A = 0$, $n = 1$.

γ	[20]	Present Result
0	-1	-1.0002
0.25	-1.0944	-1.0956
0.50	-1.1887	-1.1889
0.75	-1.2818	-1.2845
1	-1.4593	-1.4570

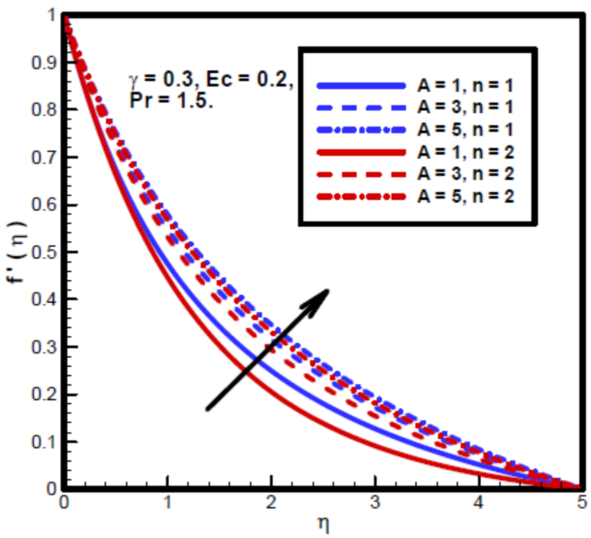

Fig. 1. Influence of Sisko parameter A on velocity profile $f'(\eta)$ for $n = 1, 2$.

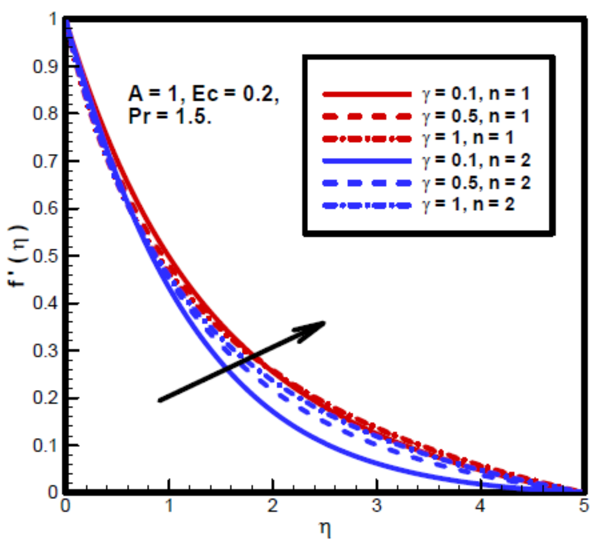

Fig. 2. Impact of curvature parameter γ on fluid velocity $f'(\eta)$ for $n = 1, 2$.

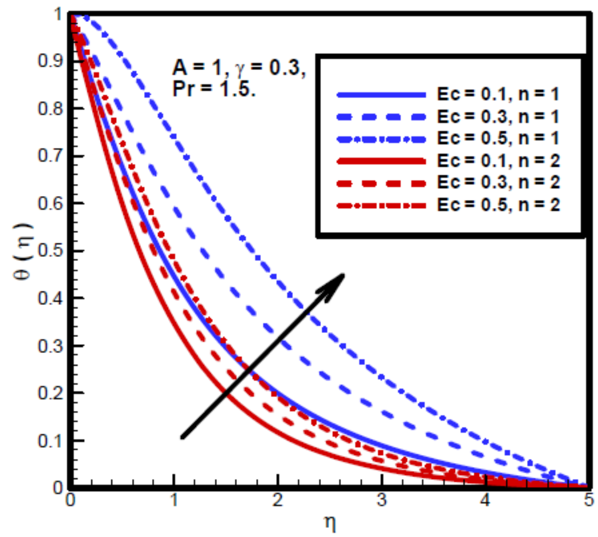

Fig. 3. Temperature profile $\theta(\eta)$ against variations in Eckert number Ec by assuming $n = 1, 2$.

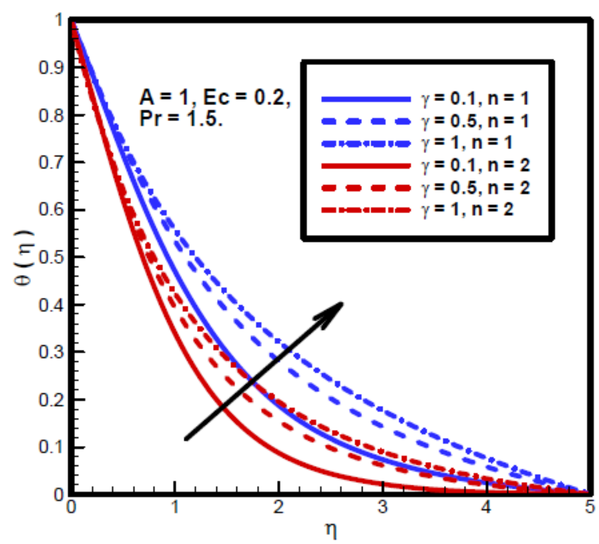

Fig. 4. Influence of curvature parameter γ on fluid temperature $\theta(\eta)$ for $n = 1, 2$.

5 Results and Discussion

In present investigation, the non-Newtonian Sisko fluid flow over stretching cylinder is analyzed under the influence of viscous dissipation. The set of differential equations is tackled with numerical technique shooting method. **Table 1** shows the comparison of skin friction coefficient for different values of curvature parameter γ with [20] results. This table presents that the calculated results have good agreement with previous literature.

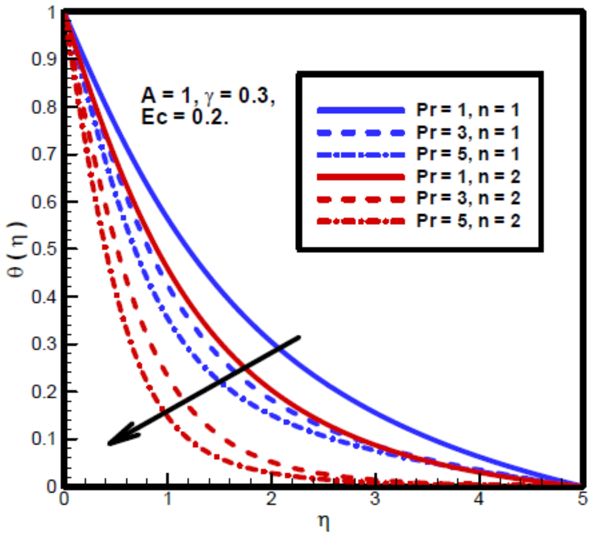

Fig. 5. Variations in fluid temperature $\theta(\eta)$ for different values of Prandtl number Pr and power law index n.

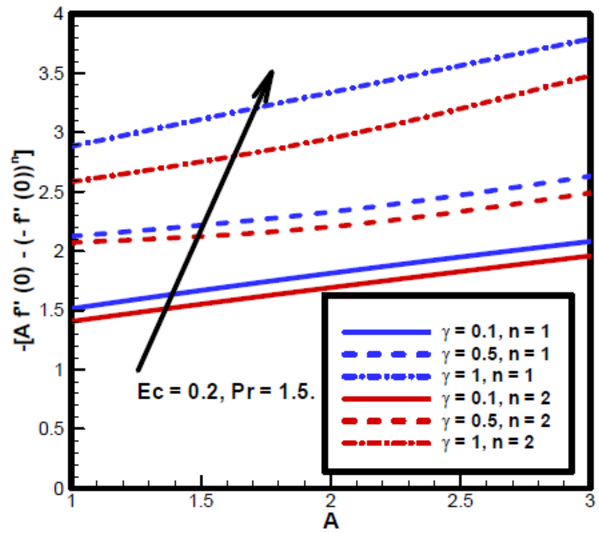

Fig. 7. Variations in local Nusselt number for different values of curvature parameter γ, Eckert number Ec and power law index n.

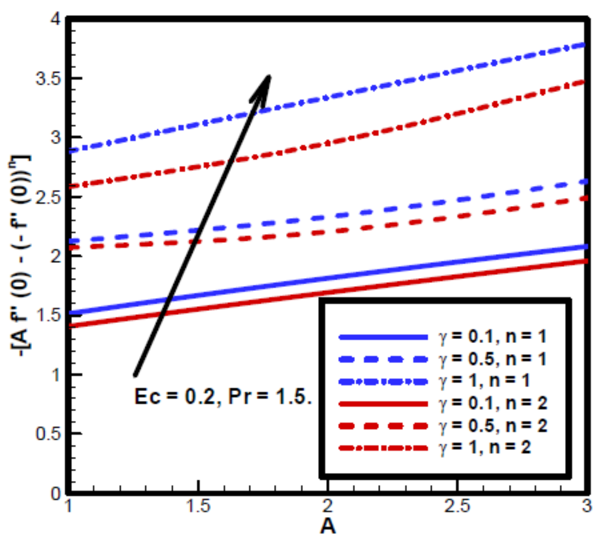

Fig. 6. Effects of curvature parameter γ, material parameter A and power law index n on wall shear stress.

Table 2: Variations in skin friction coefficient for different values of parameters γ, A and n.

		$n=1$	$n=2$
γ	A	$(A+1)f''(0)$	$Af''(0)-f''^2(0)$
0	1	-1.5984	-1.4396
0.25		-1.7734	-1.6909
0.50		-1.9630	-1.9373
0.75		-2.1612	-2.1776
0.4	1	-1.8860	-1.8395
	2	-2.4024	-2.4465
	3	-2.9461	-3.0335
	4	-3.4784	-3.6107

The effect of material parameter A on velocity profile is shown in **Fig. 1.** As, Sisko parameter A decays the fluid viscosity, so less resistance is offered to fluid motion, hence velocity increases. This fact can be validated by analyzing the current figure. In addition, it can be seen that power law index n decelerates the fluid motion.

The behavior of curvature parameter γ on velocity profile for $n = 1, 2$ is presented via **Fig. 2.** As lager values of curvature parameter reduces the radius of cylinder and hence surface area. So cylinder surface posses less resistance, the consequences of this phenomenon enhance the fluid velocity which can be analyzed from the graph.

Fig. 3 depicts fluid temperature curves against variations in Eckert number Ec for $n = 1, 2$. As Eckert number enhances the bouncy forces which accelerates the vibrations of particles. And particles collides with each other, due to collision they loses their energy i.e. some part of their mechanical energy is transformed into thermal energy which consequently enhance the temperature of fluid. Finally, one can seen from the current graph that progressing values of power law index n decline the temperature.

Fig. 4 is plotted to exhibit the effects of curvature parameter γ on temperature profile for $n = 1$ and 2. As

Table 3: Nusselt number table for different values of parameters γ, A, Ec, Pr and n.

				$n=1$	$n=2$
γ	A	Pr	Ec	$-\theta'(0)$	$-\theta'(0)$
0	1	1	0.2	0.6221	0.5312
0.25				0.7179	0.6125
0.5				0.8139	0.7072
0.75				0.9017	0.8033
0.4	1			0.7761	0.6689
	2			0.7388	0.6237
	3			0.6937	0.5745
	4			0.5237	0.5237
	1	1		0.7761	0.6689
		2		0.9906	0.7863
		3		1.1743	0.8918
		4		1.3284	0.9813
		1	0	0.9102	0.8308
			0.2	0.7761	0.6689
			0.4	0.6420	0.5070
			0.6	0.5080	0.3451

curvature parameter enhances velocity as well as kinetic energy and so fluid temperature. Because, it is well-known fact that temperature of any fluid is defined as average kinetic energy of the fluid.

Fig. 5 displays the impact of Prandtl number Pr on temperature profile for $n=1$ and 2. For larger values of Prandtl number Pr diminishes the thermal diffusivity i.e. the capability of the fluid to possesses heat, the sequel of this phenomenon declines temperature.

The variations in wall shear stress against pertinent parameters A, γ and n are deliberated in **Fig. 6.** It is found that the both Sisko parameter and curvature parameter enlarges numerical values of skin friction coefficient while effects of power law index are reverse on it.

The local Nusselt number for different values of curvature parameter γ, Eckert number Ec and power law index n is depicted in **Fig. 7.** This graph explains that the Eckert number reduces numerical values of $-\theta'(0)$ while it enlarges against higher values of both curvature parameter γ and power law index n.

Table 2 presents the effects of physical parameters A, γ and n on skin-friction coefficient. It can be seen that both parameters A and γ the escalates the values of skin-friction coefficient increase while it decreases for larger values of power law index n in absolute sense.

Table 3 depicts the numerical values of Nusselt number against variations in physical parameters γ, Ec, n and Pr. Current table shows that the effects of Pr, n and γ are same i.e. they all increase the local Nusselt number while Ec decreases the numerical values of local Nusselt number absolutely.

6 Concluding Remarks

The present investigation deals with the numerical treatment of Sisko fluid flow over stretching cylinder and heat transfer with viscous dissipation. The influence of pertinent parameters on fluid velocity and temperature is shown via graphs. Additionally, skin friction coefficient and local Nusselt number are discussed by varying fluid parameters. The major outcomes of present study are listed below:

- Material parameter A substantially grows the both fluid velocity and coefficient of skin friction.
- The curvature parameter γ has same effects on fluid velocity, temperature, wall shear stress and local Nusselt number i.e. all the interested quantities are enhance against larger values of curvature parameter.
- The impact of Eckert number Ec on fluid temperature and local Nusselt number is opposite.
- Prandtl number Pr falls down the fluid temperature, on the other hand it accelerates the rate of heat transfer from the surface.

Acknowledgements

Authors are thankful to Higher Education Commission (HEC) of Pakistan.

References

[1] H.C. Brinkman, Appl. Sci. Res. **2,** 120-124 (1951).
[2] W.J. Ou and K.C. Cheng, Appl. Sci. Res. **28**, 289-301 (1973).
[3] T.F. Lin, K.H. Hawks and W. Leidenfrost, Warm. Stoffubertrag, **17**, 97-105 (1983).
[4] V.P. Tyagi, J. Heat Transfer, **88,** 161-167 (1996).
[5] S.P. Anjali Devi and B. Ganga, Int. J. Math. Mech. **5**, 45-59 (2009).
[6] S.P. Anjali Devi and B. Ganga, Nonlinear Anal. Mod. Control, **14,** 303-314 (2009).
[7] O. Aydin, Energy Convr. Manag. **46,** 757-769 (2005).
[8] O. Aydin, Energy Convr. Manag. **46,** 3091-3102 (2005).
[9] H. Ragueb and K. Mansouri, Energy Convr. Manag. **68,** 124-132 (2013).
[10] P.K. Singh, Int. J. Eng. Sci. Tech. **4**, 2647-2656 (2012).
[11] B.C. Sakiadis, AIChE J. **7,** 26-28 (1961).
[12] L.J. Crane, J. Appl. Math. Phy. (ZAMP) **21,** 645-647 (1970).
[13] O.D. Makinde and A. Aziz, Int. J. Therm. Sci. **50,** 1326-1332 (2011).

[14] S. Nadeem and C. Lee, Nanoscl. Res. Lett. **7,** (2012), DOI:10.1186/1556-276X-7-94.
[15] M.Y. Malik, M. Naseer, S. Nadeem and A. Rehman, Appl. Nanosci. **4,** 869-873 (2014).
[16] K.R. Rajagopal, T.Y. Na and A.S. Gupta, J. Math. Phy. Sci. **21,** 189-200 (1987).
[17] K. Zaimi, A. Ishak and I. Pop, Sci. Reports, **4,** (2014), DOI: 10.1038/srep04404.
[18] P.S. Gupta, and A.S. Gupta, Can. J. Chem. Eng. **55,** 744-746 (1977).
[19] P. Carragher and L.J. Crane, J. Appl. Math. Mech. (ZAMM) **62,** 564-565 (1982).
[20] R.R. Rangi and N. Ahmad, Appl. Math. **3,** 205-209 (2012).
[21] A. Shehzad and R. Ali, Can. J. Appl. Sci. **2,** 202-215 (2012).
[22] A. Shehzad and R. Ali, Walailak J. Sci. Tech. **10,** 43-56 (2013).
[23] J. Ahmed, A. Shahzad, M. Khan and R. Ali, AIP Adv. **5,** (2015), http://dx.doi.org/10.1063/1.4935571.
[24] M.Y. Malik, Imad Khan, Arif Hussain and T. Salahuddin, AIP Adv. **5,** (2015), http://dx.doi.org/10.1063/1.4935639.
[25] T. Salahuddin, M.Y. Malik, Arif Hussain, S. Bilal and M. Awais, J. Magn. Magn. Mater. **401,** 991-997 (2016).
[26] A.W. Sisko, Ind. Eng. Chem. Res. **50,** 1789–1792 (1958).
[27] S. Nadeem and N.S. Akbar, Acta. Mech. Sin. **26,** 675-683 (2010).
[28] S. Nadeem, N.S. Akber and K. Vajravelu, Int. J. Comp. Math. **88,** 1013-1023 (2011).
[29] N.S. Akber, Appl. Nanosci. **4,** 663-673 (2014).
[30] F.T. Akyildiz, K. Vajravelu, R.N. Mohapatra, E. Sweet and R.A. Van Gorder, Appl. Math. Comp. **210,** 189-196 (2009).
[31] M. Khan and A. Shahzad, Int. J. Non-Linear Mech. **47,** 999-1007 (2012).
[32] M. Khan and A. Shahzad, Qustns. Mathcae. **36,** 137-151 (2013).
[33] M.Y. Malik, Arif Hussain, T. Salahuddin and M. Awais, Int. J. Numer. Methods Heat Fluid Flow, **26,** 1787-1801 (2016).
[34] M.Y. Malik, Arif Hussain, T. Salahuddin and M. Awais, **32,** 593-601 (2016).
[35] M.Y. Malik, Arif Hussain, T. Salahuddin, M. Awais and S. Bilal, AIP Adv. **6**, (2016), http://dx.doi.org/10.1063/1.4942476.

Arif Hussain is working as a PhD Scholar in the Department of Mathematics at Quaid-i-Azam University, Islamabad under the supervision of Prof. Dr. M.Y. Malik. He is working in computational fluid dynamics and published a lot of research papers in many international journals.

M. Y. Malik is Professor and Chairman at Department of Mathematics, Quaid-i-Azam University Islamabad. He has working in computational fluid dynamics since last few decades and published many papers in various well reputed journals.

S. Bilal is working as a PhD Scholar in the Department of Mathematics at Quaid-i-Azam University, Islamabad under the supervision of Prof. Dr. M.Y. Malik. He is working in computational fluid dynamics and published a lot of research papers in many international journals.

M. Awais is working as a PhD Scholar in the Department of Mathematics at Quaid-i-Azam University, Islamabad under the supervision of Prof. Dr. M.Y. Malik. He is working in computational fluid dynamics and published a lot of research papers in many international journals.

T. Salahuddin is working as Assistant Professor in the Department of Mathematics, Mirpur University of Science and Technology, Mirpur AJK. He has completed his Ph. D under the supervision of Prof. Dr. M.Y. Malik. He has published a lot of research article.