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Common Fixed Point Theorems in Fuzzy Metric Spaces using Concept of Conditionally Reciprocal Continuous Self Mappings

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Abstract: The purpose of this paper is to prove some new common fixed point theorems in fuzzy metric spaces. While proving our results, we utilize the idea of compatibility due to Jungck [4] together with conditionally reciprocal continuity due to R. P. Pant and R. K. Bisht [8]. Our results substantially generalize and improve a multitude of relevant common fixed point theorems of the existing literature in metric as well as fuzzy metric spaces.

Keywords: Compatible or g- compatible or f- compatible maps, conditionally reciprocal continuous maps, fuzzy metric space, common fixed point.

1 Introduction

Fixed point theory is one of the most fruitful and effective tools in mathematics which has enormous applications within as well as outside the mathematics. Despite noted improvements in computer skill and its remarkable success in facilitating many areas of research, there still stands one major short coming: computers are not designed to handle situations wherein uncertainties are involved. To deal with uncertainty, we need techniques other than classical ones wherein some specific logic is required. Fuzzy set theory is one of the uncertainty approaches wherein topological structures are basic tools to develop mathematical models compatible to concrete real life situations. To substantiate this view point, one may recall that the idea of Fuzzy set [14] and Fuzzy metric spaces [5]. As patterned in Jungck [3], a metrical common fixed point theorem generally involves conditions on commutativity, continuity, contraction and completeness (or closedeness) of the underlying space (or subspaces) along with conditions on suitable containment amongst the ranges of involved mappings. Hence, in order to prove a new metrical common fixed point theorem, one is always required to improve one or more of these conditions.

With a view to improve these conditions in common fixed point theorems, R. P. Pant and R. K. Bisht [8] introduced the notion of conditionally reciprocal continuous maps.

Fixed point theory in fuzzy metric metric spaces was initiated by Grabiec [2]. Subrahmanyam [12] gave a generalization of Jungck [3] common fixed point theorem for commuting mappings in the setting of fuzzy metric spaces, whereas Vasuki [13] gave a fuzzy version of results obtained by Pant [6]. Thereafter, many authors established fuzzy versions of a most of classical metrical common fixed point theorems (e.g.[11,13]).

In this paper, we prove some new common fixed point theorems in fuzzy metric spaces. While proving our results, we utilize the idea of compatibility due to Jungck [4] together with conditionally reciprocal continuity due to R. P. Pant and R. K. Bisht [8]. Consequently, our results improve and sharpen many known common fixed point theorems available in the existing literature of fuzzy fixed point theory.

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2 Preliminaries

Definition 1.[10] *A binary operation* $* : [0,1]\times[0,1] \to [0,1]$ *is continuous t-norm if* $*$ *satisfies the following conditions:*

(i) $*$ *is commutative and associative;*

(ii) $*$ *is continuous;*

(iii) $a*1 = a$ *for all* $a\in[0,1]$*;*

(iv) $a*b \le c*d$ *whenever* $a\le c$ *and* $b\le d$ *for all* $a,b,c,d\in[0,1]$.

Examples of t-norms are $a*b = min\{a,b\}$ and $a*b = a.b$.

Kramosil I and Michalek J. [5] introduced the concept of fuzzy metric spaces as follows:

Definition 2.[5] *The 3-tuple* $(X,M,*)$ *is called a fuzzy metric space (shortly, FM-space) if* X *is an arbitrary set,* $*$ *is a continuous t-norm and* M *is a fuzzy set in* $X^2\times[0,\infty)$ *satisfying the following conditions:*

$(FM-1)$ $M(x,y,0) = 0$,

$(FM-2)$ $M(x,y,t) = 1$, *for all* $t > 0$ *if and only if* $x = y$,

$(FM-3)$ $M(x,y,t) = M(y,x,t)$,

$(FM-4)$ $M(x,y,t) * M(y,z,s) \ge M(x,z,t+s)$ *(Triangular inequality) and*

$(FM-5)$ $M(x,y,.) : [0,1) \to [0,1]$ *is left continuous for all* $x,y,z\in X$ *and* $s,t > 0$.

Note that $M(x,y,t)$ *can be thought of as the degree of nearness between x and y with respect to* t.

We can fuzzify examples of metric spaces into fuzzy metric spaces in a natural way:

Let (X,d) be a metric space. Define $a*b = a+b$ for all $a,b\in X$. Define $M(x,y,t) = \frac{t}{t+d(x,y)}$ for all $x,y\in X$ and $t>0$.Then $(X,M,*)$ is a fuzzy metric space and this fuzzy metric induced by a metric d is called the Standard fuzzy metric.

Consider M to be a fuzzy metric space with the following condition:

$(FM-6)$ $lim_{t\to\infty} M(x,y,t) = 1$ for all $x,y\in X$ and $t > 0$.

Definition 3.[5] *Let* $(X,M,*)$ *be fuzzy metric space. Then*

(a) *a sequence* $\{x_n\}$ *in* X *is said to be Cauchy sequence if, for all* $t > 0$ *and* $p > 0$, $lim_{n\to\infty} M(x_{n+p},x_n,t) = 1$ *and*

(b) *a sequence* $\{x_n\}$ *in* X *is said to be convergent to a point* $x\in X$ *if, for all* $t > 0$, $lim_{n\to\infty} M(x_n,x,t) = 1$.

Definition 4.[5] *A fuzzy metric space* $(X,M,*)$ *is said to be complete if and only if every Cauchy sequence in* X *is convergent.*

Example 1 (5)*Let* $X = \{\frac{1}{n} : n = 1,2,3,...\}\cup\{0\}$ *and let* $*$ *be the continuous t-norm and defined by* $a*b = ab$ *for all* $a,b\in[0,1]$. *For each* $t > 0$ *and* $x,y\in X$, *define* M, *by* $M(x,y,t) = \frac{t}{t+|x-y|}$ *if* $t > 0$ *and* $M(x,y,0) = 0$. *Clearly,* $(X,M,*)$ *is complete fuzzy metric space.*

Definition 5.[13] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be commuting if* $M(fgx,gfx,t) = 1$ *for all* $x\in X$ *and* $t > 0$.

Definition 6.[13] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be weakly commuting if* $M(fgx,gfx,t) \ge M(fx,gx,t)$ *for all* $x\in X$ *and* $t > 0$.

Definition 7.[7] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be compatible if* $lim_{n\to\infty} M(fgx_n,gfx_n,t) = 1$ *for all t ¿ 0, whenever* $\{x_n\}$ *is a sequence in* X *such that* $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$ *for some* $z\in X$.

Now, we can say that mappings f and g will be non-compatible if there exists at least one sequence $\{x_n\}$ such that $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$ for some $z\in X$ but either $lim_{n\to\infty} M(fgx_n,gfx_n,t) \ne 1$ or the limit does not exist.

Definition 8.[9] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be g-compatible if* $lim_{n\to\infty} M(ffx_n,gfx_n,t) = 1$, *for all* $t > 0$, *whenever* $\{x_n\}$ *is a sequence in* X *such that* $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$ *for some* $z\in X$.

Definition 9.[9] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be* f *-compatible if* $lim_{n\to\infty} M(fgx_n,ggx_n,t) = 1$, *for all* $t > 0$, *whenever* $\{x_n\}$ *is a sequence in* X *such that* $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$ *for some* $z\in X$.

Definition 10.[8] *A pair of self mappings* (f,g) *of a fuzzy metric space* $(X,M,*)$ *is said to be conditionally reciprocally continuous (CRC) if whenever set of sequences* $\{x_n\}$ *satisfying* $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n$ *is non-empty, there exist a sequence* $\{y_n\}$ *satisfying* $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = u$ *(say) such that* $lim_{n\to\infty} fgy_n = fu$ *and* $lim_{n\to\infty} gfy_n = gu$.

Before proving our main result, we state some results which are used in proving our main results:

Lemma 1.[2,5] *Let* $(X,M,*)$ *be fuzzy metric space and for all* $x,y\in X$, $t > 0$ *and if for a number* $k\in(0,1)$, $M(x,y,kt) \ge M(x,y,t)$, *then* $x = y$.

Lemma 2.[2,5] *Let* $(X,M,*)$ *be fuzzy metric space and* $\{y_n\}$ *be a sequence in* X. *If there exists a number* $k\in(0,1)$ *such that*

$$M(y_n,y_{n+1},kt) \ge M(y_{n-1},y_n,t)$$

for all $t > 0$ *and* $n = 1,2,3,...$ *then* $\{y_n\}$ *is a Cauchy sequence in* X.

3 Main Results

Theorem 1. *Let f and g be conditionally reciprocally continuous self-mappings of a complete fuzzy metric space $(X,M,*)$ satisfying the conditions:*

(3.1) $f(X) \subseteq g(X)$;

(3.2) *for any* $x,y \in X$, $t>0$ *and* $k \in (0,1)$ *such that* $M(fx, fy, kt) \geq M(gx, gy, t)$.

If f and g are either compatible or g- compatible or f- compatible then f and g have a unique common fixed point.

Proof. Let x_0 be any point in X. Then as $f(X) \subseteq g(X)$, there exist a sequence of points $\{x_n\}$ in X such that $fx_n = gx_{n+1}$.

Also, define a sequence y_n in X as

$y_n = fx_n = gx_{n+1}$. (3.3)

Now, we show that $\{y_n\}$ is a Cauchy sequence in X. For proving this, by (3.2), we have

$$M(y_n, y_{n+1}, kt) = M(fx_n, fx_{n+1}, kt) \geq M(gx_n, gx_{n+1}, t)$$

$$= M(y_{n-1}, y_n, t)$$

$$M(y_n, y_{n+1}, kt) \geq M(y_{n-1}, y_n, t).$$

Then, by Lemma 2.2, $\{y_n\}$ is a Cauchy sequence in X. As, X is complete, there exist a point $z \in X$ such that $lim_{n\to\infty} y_n = z$. Therefore, by (3.3), we have

$$lim_{n\to\infty} y_n = lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_{n+1} = z.$$

Since f and g be conditionally reciprocally continuous and $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$, there exist a sequence $\{s_n\}$ satisfying $lim_{n\to\infty} fs_n = lim_{n\to\infty} gs_n = u$ (say) such that $lim_{n\to\infty} fgs_n = fu$ and $lim_{n\to\infty} gfs_n = gu$. Since, $f(X) \subseteq g(X)$, for each s_n, there exist $z_n \in X$ such that $fs_n = gz_n$. Thus,

$$lim_{n\to\infty} fs_n = lim_{n\to\infty} gs_n = lim_{n\to\infty} gz_n = u.$$

By using (3.2), we get

$$M(fs_n, fz_n, kt) \geq M(gs_n, gz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(u, lim_{n\to\infty}, kt) \geq M(u, u, t) = 1.$$

This gives, $lim_{n\to\infty} fz_n = u$. Hence,

$$lim_{n\to\infty} fs_n = lim_{n\to\infty} gs_n = lim_{n\to\infty} gz_n = lim_{n\to\infty} fz_n = u.$$

Suppose that f and g are compatible mappings. Then $lim_{n\to\infty} M(fgs_n, gfs_n, t) = 1$, that is, $lim_{n\to\infty} fgs_n = lim_{n\to\infty} gfs_n$, this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. Using (3.2), we get

$$M(fu, ffu, kt) \geq M(gu, gfu, t) > M(fu, ffu, t),$$

that is, $fu = ffu$. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Now, Suppose that f and g are g-compatible mappings. Then $lim_{n\to\infty} M(ffs_n, gfs_n, t) = 1$, that is, $lim_{n\to\infty} ffs_n = lim_{n\to\infty} gfs_n = gu$. Using (3.2), we get

$$M(fu, ffs_n, kt) \geq M(gu, gfs_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, kt) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. Using (3.2), we get

$$M(fu, ffu, kt) \geq M(gu, gfu, t) > M(fu, ffu, t),$$

that is $fu = ffu$. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Finally, Suppose that f and g are f-compatible mappings. Then $lim_{n\to\infty} M(fgz_n, ggz_n, t) = 1$, that is, $lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n$. Also, $lim_{n\to\infty} gfs_n = lim_{n\to\infty} ggz_n = gu$.

Therefore, $lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n = gu$.

Using (3.2), we get

$$M(fu, fgz_n, kt) \geq M(gu, ggz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, kt) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. Using (3.2), we get

$$M(fu, ffu, kt) \geq dM(gu, gfu, t) > M(fu, ffu, t),$$

that is, $fu = ffu$. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Uniqueness of the common fixed point theorem follows easily in each of the three cases.

We now give an example to illustrate Theorem 1.

Example 1. Let $(X,M,*)$ be a fuzzy metric space, where $X = [2,20]$, with continuous t-norm $a * b = min\{a,b\}$ for all $a,b \in [0,1]$ and $M(x,y,t) = \frac{t}{t+|x-y|}$ if $t > 0$ and $M(x,y,0) = 0$ for all $x,y \in X$.

Define $f,g : X \to X$ by $fx = 2$ if $x = 2$ or $x > 5$, $fx = 6$ if $2 < x \leq 5$, $g2 = 2$, $gx = 11$ if $2 < x \leq 5$, $gx = \frac{(x+1)}{3}$ if $x > 5$. Let $\{x_n\}$ be a sequence in X such that $x_n = 2$ and $\{y_n\}$ be a sequence in X such that $y_n = 5 + \frac{1}{n}$ for each n. Then clearly, f and g satisfy all the conditions of Theorem 1 and have a unique common fixed point at $x = 2$.

Theorem 2. *Let f and g be non-compatible self-mappings of a fuzzy metric space $(X,M,*)$ satisfying the conditions:*

(3.4) $f(X) \subseteq g(X)$;

(3.5) $M(fx, fy, t) \geq M(gx, gy, t)$;

(3.6) $M(fx, ffx, t) \geq M(gx, ggx, t)$ *whenever* $gx \neq ggx$ *for all* $x,y \in X$ *and* $t > 0$.

Suppose f and g be conditionally reciprocally continuous If f and g are either g- compatible or f-compatible then f and g have common fixed point.

*Proof.*Since f and g are non-compatible maps, there exists a sequence $\{x_n\}$ in X such that $fx_n \to z$ and $gx_n \to z$ for some $z \in X$ as $n \to \infty$ but either $lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ or the limit does not exist. Also, since f and g be conditionally reciprocally continuous and $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$, there exist a sequence $\{y_n\}$ satisfying $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = u$ (say) such that $lim_{n\to\infty} fgy_n = fu$ and $lim_{n\to\infty} gfy_n = gu$. Since, $f(X) \subseteq g(X)$, for each y_n, there exist z_n in X such that $fy_n = gz_n$. Thus, $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = lim_{n\to\infty} gz_n = u$. By using (3.5), we get

$$M(fy_n, fz_n, t) \geq M(gy_n, gz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(u, lim_{n\to\infty} fz_n, t) \geq M(u, u, t) = 1.$$

This gives, $lim_{n\to\infty} fz_n = u$. Therefore, we have

$$lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = lim_{n\to\infty} gz_n = lim_{n\to\infty} fz_n = u.$$

Now, Suppose that f and g are g-compatible mappings. Then $lim_{n\to\infty} M(ffy_n, gfy_n, t) = 1$, that is, $lim_{n\to\infty} ffy_n = lim_{n\to\infty} gfy_n = gu$. Using (3.5), we get

$$M(fu, ffy_n, t) \geq M(gu, gfy_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, t) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. If $fu \neq ffu$, using (3.6), we get

$$M(fu, ffu, t) \geq M(gu, ggu, t) \geq M(fu, ffu, t),$$

a contradiction. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Finally, Suppose that f and g are f-compatible mappings. Then $lim_{n\to\infty} M(fgz_n, ggz_n, t) = 1$, that is, $lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n$. Also, $lim_{n\to\infty} gfy_n = lim_{n\to\infty} ggz_n = gu$.

Therefore,$lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n = gu$.

Using (3.5), we get

$$M(fu, fgz_n, t) \geq M(gu, ggz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, t) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. If $fu \neq ffu$, using (3.6), we get

$$M(fu, ffu, t) \geq M(gu, ggu, t) \geq M(fu, ffu, t),$$

a contradiction. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

We now give an example to illustrate Theorem 2.

*Example 2.*Let $(X, M, *)$ be a fuzzy metric space, where $X = [2, 20]$, with continuous t-norm $a * b = min\{a, b\}$ for all $a, b \in [0, 1]$ and $M(x, y, t) = \frac{t}{t+|x-y|}$ if $t > 0$ and $M(x, y, 0) = 0$ for all $x, y \in X$.

Define $f, g : X \to X$ by $fx = 2$ if $x = 2$ or $x > 5$, $fx = 6$ if $2 < x \leq 5$, $g2 = 2, gx = 11$ if $2 < x \leq 5$, $gx = \frac{(x+1)}{3}$ if $x > 5$. Let $\{x_n\}$ be a sequence in X such that $x_n = 2$ and $\{y_n\}$ be a sequence in X such that $y_n = 5 + \frac{1}{n}$ for each n. Then clearly, f and g satisfy all the conditions of Theorem 2 and have a common fixed point at $x = 2$. Also, f and g are non compatible for $x_n = 5 + \frac{1}{n}$.

Theorem 3.*Let f and g be non-compatible self-mappings of a fuzzy metric space $(X, M, *)$ satisfying the conditions:*

(3.7) $f(X) \subseteq g(X)$;

(3.8) $M(fx, fy, kt) \geq M(gx, gy, t)$ for all $k \geq 0$;

(3.9) $M(fx, ffx, t) > M(gx, ggx, t)$ whenever $gx \neq ggx$ for all $x, y \in X$ and $t > 0$.

Suppose f and g be conditionally reciprocally continuous If f and g are either g- compatible or f-compatible then f and g have common fixed point.

*Proof.*Since f and g are non-compatible maps, there exists a sequence $\{x_n\}$ in X such that $fx_n \to z$ and $gx_n \to z$ for some $z \in X$ as $n \to \infty$ but either $lim_{n\to\infty} M(fgx_n, gfx_n, t) \neq 1$ or the limit does not exist. Also, since f and g be conditionally reciprocally continuous and $lim_{n\to\infty} fx_n = lim_{n\to\infty} gx_n = z$, there exist a sequence $\{y_n\}$ satisfying $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = u$ (say) such that $lim_{n\to\infty} fgy_n = fu$ and $lim_{n\to\infty} gfy_n = gu$. Since, $f(X) \subseteq g(X)$, for each y_n, there exist z_n in X such that $fy_n = gz_n$. Thus, $lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = lim_{n\to\infty} gz_n = u$. By using (3.8), we get

$$M(fy_n, fz_n, kt) \geq M(gy_n, gz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(u, lim_{n\to\infty} fz_n, kt) \geq M(u, u, t) = 1.$$

This gives, $lim_{n\to\infty} fz_n = u$. Therefore, we have

$$lim_{n\to\infty} fy_n = lim_{n\to\infty} gy_n = lim_{n\to\infty} gz_n = lim_{n\to\infty} fz_n = u.$$

Now, Suppose that f and g are g-compatible mappings. Then $lim_{n\to\infty} M(ffy_n, gfy_n, t) = 1$, that is, $lim_{n\to\infty} ffy_n = lim_{n\to\infty} gfy_n = gu$. Using (3.8), we get

$$M(fu, ffy_n, kt) \geq M(gu, gfy_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, kt) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. If $fu \neq ffu$, using (3.9), we get

$$M(fu, ffu, t) \geq M(gu, ggu, t) \geq M(fu, ffu, t),$$

a contradiction. Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Finally, Suppose that f and g are f-compatible mappings. Then $lim_{n\to\infty} M(fgz_n, ggz_n, t) = 1$, that is, $lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n$. Also, $lim_{n\to\infty} gfy_n = lim_{n\to\infty} ggz_n = gu$.

Therefore,$lim_{n\to\infty} fgz_n = lim_{n\to\infty} ggz_n = gu$.

Using (3.8), we get

$$M(fu, fgz_n, kt) \geq M(gu, ggz_n, t).$$

Taking limit as $n \to \infty$, we get

$$M(fu, gu, kt) \geq M(gu, gu, t) = 1,$$

this gives, $fu = gu$. Also, $fgu = ffu = fgu = gfu$. If $fu \neq ffu$, using (3.9), we get

$$M(fu, ffu, t) > M(gu, ggu, t) = M(fu, ffu, t),$$

a contradiction.Hence, $fu = ffu = gfu$ and fu is a common fixed point of f and g.

Now, We give an example to illustrate Theorem 3.

*Example 3.*Let $(X, M, *)$ be a fuzzy metric space, where $X = [2, 20]$, with continuous t-norm $a * b = min\{a, b\}$ for all $a, b \in [0, 1]$ and $M(x, y, t) = \frac{t}{t+|x-y|}$ if $t > 0$ and $M(x, y, 0) = 0$ for all $x, y \in X$.

Define $f, g : X \to X$ by $fx = 2$ if $x = 2$ or $x > 5$, $fx = 4$ if $2 < x \leq 5$, $g2 = 2$, $gx = 4$ if $2 < x \leq 5$, $gx = \frac{(x+1)}{3}$ if $x > 5$. Let $\{x_n\}$ be a sequence in X such that $x_n = 2$ and $\{y_n\}$ be a sequence in X such that $y_n = 5 + \frac{1}{n}$ for each n and $k = \frac{3}{2}$. Then clearly, f and g satisfy all the conditions of Theorem 3.3 and have two common fixed point at $x = 2$ and $x = 4$. Also, f and g are non compatible for $x_n = 5 + \frac{1}{n}$.

4 Conclusion

As an application of conditional reciprocal continuity, we proved common fixed point Theorems 1, 2 and 3 that extend the scope of the study of common fixed point theorems from the class of compatible continuous mappings to a wider class of mappings which also includes non-compatible and discontinuous mappings.

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References

[1] A. George and P. Veeramani, On some results in fuzzy metric spaces. Fuzzy Sets and Systems, **64**, 395399 (1994).
[2] M. Grabiec, Fixed point on fuzzy metric spaces. Fuzzy Sets and Systems, **27**, 385389 (1998).
[3] G. Jungck, Commuting mappings and fixed points. American Mathematical Monthly,**83**, 261263 (1976).
[4] G. Jungck, Compatible mappings and common fixed points. Int. J. Math.& Math. Sci.,**9**, 771779 (1986).
[5] I. Kramosil and J. Michalek, Fuzzy metric and statistical metric spaces, Kybernetica,**11**, 326-334 (1975).
[6] R. P. Pant, Common fixed point for non commuting mappings. J. Math. Anal. Appl., **188**, 436440 (1994).
[7] R. P. Pant, Common fixed points of four mappings. Bull. Calcutta Math. Soc., **90**, 281286 (1998).
[8] R. P. Pant and R.K. Bisht, Common fixed point theorems under a new continuity condition, Ann Univ Ferrara, **58**, 127141 (2012).
[9] H.K. Pathak and M.S. Khan, A comparison of various types of compatible maps and common fixed points, Indian J. Pure Appl. Math., **28**(4), 477485 (1997).
[10] B. Schweizer and A. Sklar, Probabilistic metric spaces. North Holland, New York (1983).
[11] B. Singh and M.S. Chauhan, Common fixed point of compatible mappings in fuzzy metric spaces, Fuzzy Sets and Systems, **115**, 471475 (2000).
[12] P.V. Subrahmanyam, Common fixed point theorems in fuzzy metric spaces. Inform. Sci., **83**(4), 109112 (1995).
[13] R. Vasuki, Common fixed points for R-weakly commuting maps in fuzzy metric spaces, Indian J. Pure Appl. Math., **30**, 419423 (1999).
[14] A. L. Zadeh, Fuzzy sets, Information and control, **89**,338-353 (1965).

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