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Forecasting Climatic Variables Using ARIMA and VAR with Box Cox Pre- Processing

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Abstract: Climate variables like temperature, rainfall, and humidity display rich temporal behavior that demands precise handling using appropriate statistical methodology to ensure credible forecasting results. This paper undertakes a comparative study to assess univariate and multivariate time series modeling approaches to these important climate variables, employing Box-Cox transformation as a preprocessing step to enhance homogeneity. This is followed by checking the variables' stationarity and appropriateness after applying the validation test, forming ARIMA on univariate variables and a 9th-order VAR on multiple variables. The model choice is informed by information criterion functions, while forecasting comparison is assessed via error calculation and graphic representation of forecast trajectories. The empirical findings, supported by natural climate variables observed in northern Iraq, authenticate that Box-Cox pre-processing improves homogeneity to better estimate model parameters with better precision within both sets of models. The ARIMA models on each separate variable display outstanding forecasting capability, while within the multiple variables' model VAR, interactions exist but fail to outperform separate specifications uniformly. The results signify that measures of predominance within multiple variables' forecasting are crucial, being sensitive to the degree of correlations between variables. The paper demonstrates pre-processing and comparison aspects within forecasting to emphasize credible results within this work and provides a factual illustration to determine whether to prefer univariate rather than multiple variables' forecasting on natural variables.

Keywords: ARIMA, VAR, Box-Cox Transformation, Climate variables.

1. Introduction

Climatic time series data, including temperature (T), rainfall (R), and humidity (H), has complicated temporal structures relating to atmospheric circulation, hydrologic cycles, and periodic or seasonal changes. Understanding these temporal dynamics is important for climate prediction and decision-making in agriculture, water resource management, and environmental planning (Wilks, 2019). Time series have developed as the key methodology for climate sciences due to climatic processes evolving through strong autocorrelation, periodicity, and non-linear fluctuations (Shumway & Stoffer, 2017). The initial advancements in this arena mainly utilized univariate modeling, especially the ARIMA (Autoregressive Integrated Moving Average) methodology created by Box and Jenkins (1976). Univariate models generally have desirable performances, especially when most of the modeled structures of each climatic variable is self-driven (Hyndman & Athanasopoulos, 2021). Moreover, laying the groundwork for improved or better distributional properties for climatic series variables was an important guiding principle when trying to obtain stable univariate forecasts. More recently, a traditional method of applying power transformations to stabilize variance and improve the normality of a variable has been through the use of the Box–Cox transformation (BCT), proposed by Box and Cox (1964), and the Yeo–Johnson transformation (YJT), or the BCT for positive and negative values of the variable (Yeo & Johnson, 2000). Processors (Chen & Lee, 1997; Guerrero, 1993; Atkinson et al., 2021) demonstrated the efficacy of using power transformations for the stabilization of the variance and attainment of normality, and applied the BCT to time series data; they were aware of their discriminatory treatment of negative values, which is significant when dealing with climate variables. There are examples of applying the BCT or YJT and using the transformed temperature time series in both univariate settings and seasonal time series applications with improvements in ARIMA identification and accuracy of forecasting (Proietti & Lütkepohl, 2013; Othman & Mohammed Ali, 2023).

Nonetheless, the dependence of climate-related variables is inherent, T influences H, humidity affects R generation, and R impacts T via evaporative cooling. Such interdependence creates a case for multivariate time series methodologies, and particularly, VAR schemes (Lütkepohl, 2005). The empirical literature indicates that multivariate models perform better at capturing the dynamism across variables than solely univariate models. Specific benefits have been realized in the prediction of T and H (Adeyemi et al., 2017), R (Ogallo et al., 2020), and T and R models

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(Cebrián et al., 2019). More recent literature indicates that using univariate or multivariate procedures for forecasting depends on the magnitude and stability of the cross-correlations among climate variables (Živković et al., 2022). Similar findings exist in multivariate climate studies that suggest transforming the variables could enhance the signal from cross-variable synergies, reflecting that T, H and R depend on each other (Othman & Mohammed Ali, 2025).

Moreover, the use of nonparametric and functional time series methods has also increased significantly to model nonlinear climatic behavior. Kernel regression, local polynomial smoothing, and functional K-nearest neighbor forecasting models can capture flexible functional forms and can accommodate features that parametric formulations cannot (Ferraty & Vieu, 2006; Shang, 2017). The use of variance-stabilizing transformations plays an important role in smoothing and reduces bias via nonparametric estimators (Raymaekers & Rousseeuw, 2021). Many applications on functional T curves demonstrate how the adjustments to existing transformations and appropriate transformations such as BCT, YJT, or other power-based forms improve the quality of functional period estimation and short-term forecasting specifically, in the context of marked seasonality or nonlinear dynamics (He & Zheng, 2018).

In line with these conclusions, the results also indicated that custom transformation methods can decrease smoothing error in a functional climate data context (Ali & Othman, 2022), and increase the representation of seasonally varying temperature dynamics when framed in a stationary functional context (Othman & Mohammed Ali, 2021). Following up on these ideas, the recent literature has shown that when transformation methods are combined with multivariate functional time series models, they result in significant gains in forecasting accuracy in regard to climatic variables. Transformed models have been shown to outperform univariate models when modelling climatic indicators that have been jointly evolving and have nonlinear or seasonally varying dependent structures (Shang & Xu, 2021; Youngman, 2020). Likewise, applied multivariate transformation studies provided evidence of that pattern, showing preprocessing improved coherence cross series and forecasting performance in multivariate functional application (Othman & Mohammed Ali, 2025).

Despite these advances, there are relatively few studies that directly and systematically compare univariate and multivariate approaches to multiple climatic variables in the same preprocessing framework. Most works focus on individual climatic variables, model families, or transformation procedures; thus, the pragmatic issue of when climatological multivariate models actually provide predictive benefits compared to univariate counterparts remains unresolved. To support this investigation, the current study contrasts univariate ARIMA and multivariate VAR time series models on three important climatic variable time series—T, R, and H—while considering the same BCT as a common preprocessing step. The study investigates model structure, diagnostic adequacy, and forecasting accuracy under similar data-regularization conditions and ultimately seeks a somewhat transparent answer to the pressing question of when multivariate modeling is, in fact, a benefit, after all, for climatic variable forecasting when compared to univariate models.

The study evaluates the practical forecasting benefits of using a multivariate analytical model over its univariate counterpart through an assessment of the selected ARIMA and VAR models to standardized T, R, and H data from northern Iraq. In addition, it will add to existing methodological knowledge to evaluate how both methodologies agree (or disagree) and the relative benefits of univariate versus multivariate modelling on forecasting climate time series. Additionally, the paper will illustrate analysis that will be of interest to the environmental, climate, and meteorological communities interested in furthering the analytical framework in the region.

The remainder of the paper is outlined, section 2 describes the theoretical components: the consideration of using variance-stabilizing transformations as well as the study formulation of univariate and multivariate ARIMA and VAR models. Section 3 describes the empirical analysis following from a description and preprocessing of the data, and estimation of the univariate and multivariate models, and forecasts for each modeling study (as well as embeddings of the entire study to a final time series of processes). Finally, section 4 has an in-depth discussion of the findings and a discussion of the metholodgocial and practical advantages of univariate and multivariate implementations for climate time series forecasting.

2. Theoretical Background and Model Specification

According to the Box–Jenkins model (Box & Jenkins, 1976), it is necessary to apply transformations at the data, so that the first steps in the modeling sequence satisfies the basic conditions of variance stationarity and suitable distributional form for appropriate model identification. Subsequently, model identification will be based on transformed or adjusted observations rather than raw observations, where the main aim is to regularize the series so ARIMA modeling can be carried out under more appropriate statistical conditions.

This makes transformation an operational step rather than a theoretical choice, applied whenever the observed series displays heteroscedasticity or non-normality. Among available transformation techniques, BCT introduced by Box and Cox (1964) remains the standard tool in practice because it provides a flexible, parametric adjustment for variance and scale. It is defined as:

$$g_{\lambda}(y) = \begin{cases} \frac{y^{\lambda}-1}{\lambda} & \lambda \neq 0 \\ ln\ y & \lambda = 0 \end{cases} \qquad (1)$$

When a series is transformed as $Z = g_{\lambda}(y)$, the likelihood of the original data can be expressed through the likelihood of the transformed values and the derivative of the transformation. Box and Cox showed that the maximum log-likelihood for a sample $y = (Y_1, Y_2, \dots, Y_n)$ takes the general form $Ln\, L_{max}(\lambda, y) = -\,(n/2)\, log\, \sigma^2(\lambda) + log\, J(\lambda, y)$**,** where $\sigma^2(\lambda)$ is the variance of the transformed series and $J(\lambda, y)$**,** is the Jacobian term. Maximizing this expression yields the optimal transformation parameter (Box & Cox, 1964).

In this study, the climatic variables T, R, and H are denoted by Y_T, Y_R and Y_H. Because climatic series often exhibit heteroscedasticity, skewness, and departures from Gaussian behavior, each series is transformed at the outset using the BCT to obtain stabilized versions $Z_T = g_{\lambda_T}(Y_T)$, $Z_R = g_{\lambda_R}(Y_R)$, and $Z_H = g_{\lambda_H}(Y_H)$. Applying a transformation before model identification follows the Box–Jenkins principle that variance stabilization and distributional regularity are prerequisites for meaningful ARIMA and VAR modeling (Box & Jenkins, 1976). By operating on the transformed scale, both univariate and multivariate structures rest on more stable stochastic foundations, improving estimation efficiency and diagnostic accuracy.

After transformation, the next step is to assess the stationarity of each series. Because a single unit-root test may be sensitive to short-run dynamics or structural variation, three complementary procedures are applied: the Augmented Dickey–Fuller test (ADFT) (Dickey & Fuller, 1979), the Phillips–Perron test (PPT) (Phillips & Perron, 1988), and the Kwiatkowski–Phillips–Schmidt–Shin test (KPSST) (Kwiatkowski et al., 1992). Where non-stationarity is detected, the transformed series is differenced until it satisfies the standard stationarity conditions. In the multivariate context, the same set of tests is applied to the vector $(Z_{T,t}, Z_{R,t}, Z_{H,t})^T$, and if the three series appear integrated of the same order, the Johansen cointegration test is used to detect long-run equilibrium relationships among them (Johansen, 1988, 1991). The combination of these tests ensures a robust characterization of the dynamic order of the climatic system.

2.1 Univariate ARIMA Modeling

Once stationarity is established, a univariate $ARIMA(p, d, q)(p, d, q)(p, d, q)$ model is constructed for each transformed series. The $ARIMA$ representation $\Phi(B)(1 - B)^d Z_t = \theta(B)\varepsilon_t$ where B is the backshift operator, $\Phi(B)$ and $\theta(B)$ are the autoregressive and moving-average polynomials of orders p and q, respectively, $(1 - B)^d$ denotes the differencing operator, and ε_t is a white-noise innovation term (Box & Jenkins, 1976). Identification of p and q is guided by the sample autocorrelation function (ACF), partial autocorrelation function (PACF), and information criteria such as the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC). Parameters are estimated by maximizing the conditional Gaussian log-likelihood of the form $\ln L = (-n/2)\ln(\sigma^2) - (1/2\sigma^2)\sum e_t^2 + C$, where e_t denotes the filtered residuals and C denotes a constant that does not depend on the parameters of the ARIMA model and therefore does not affect the maximization of the likelihood (Hamilton, 1994).

Model adequacy is assessed using the Ljung–Box Q-test for residual independence (Ljung & Box, 1978), the Jarque–Bera test for normality (Jarque & Bera, 1987), and ARCH-LM tests for heteroscedasticity (Engle, 1982). A univariate $ARIMA$ model is retained when residuals exhibit white-noise behavior, ensuring that the dynamic specification reflects genuine structure rather than noise.

2.2 VAR Modeling

To capture the joint dynamics of the transformed climatic variables, the vector $\boldsymbol{Z}_t = (Z_{T,t}, Z_{R,t}, Z_{H,t})^T$ is modeled using a Vector Autoregressive process of order p. The VAR representation, $\boldsymbol{Z}_t = \sum_{i=1}^{p} A_i \boldsymbol{Z}_{t-i} + + \boldsymbol{u}_t$, where $A_1, \dots, A_p$ are coefficient matrices and $\boldsymbol{u}_t$ is a vector of innovations with covariance matrix $\sum_u$. This formulation allows each component of the climatic system to depend not only on its own past but also on the lagged values of the other transformed variables, thereby capturing cross-variable interactions between T, R, and H (Lütkepohl, 2005). Before estimating the model, the appropriate lag order p is determined by fitting preliminary VAR specifications and comparing their information criteria. The AIC, the BIC, the Hannan–Quinn Criterion (HQC), and the Final Prediction Error (FPE) are used jointly to balance statistical goodness of fit with model parsimony. These criteria penalize unnecessary increases in lag order and identify the specification that provides the optimal trade-off between fit and complexity (Wei, 2019). Once the lag order is selected, the coefficient matrices $A_1, \dots, A_p$ are estimated using ordinary least squares applied to each equation of the system. Because all equations share identical regressors, OLS estimation is asymptotically equivalent to Gaussian maximum likelihood (Hamilton, 1994). Letting the residuals be defined as $\boldsymbol{u}_t(\lambda) = \boldsymbol{Z}_t(\lambda) - A_1\boldsymbol{Z}_{t-1}(\lambda) - \cdots - A_p\boldsymbol{Z}_{t-p}(\lambda)$, the Gaussian log-likelihood of the VAR model (up to an additive constant) is expressed as

$$\ln L = \frac{-n}{2}\, ln|\Sigma_u| - \frac{1}{2}\sum_{t=1}^{n} \boldsymbol{u}_t^{'} \Sigma_u^{-1} \boldsymbol{u}_t + C \qquad (2)$$

where Σ_u denotes the covariance matrix of the innovation vector and C is a constant independent of the model parameters. Substituting the OLS estimate $\hat{\Sigma}_u$ into this expression yields the concentrated log-likelihood, $\ln L_C = (-n/2)\, ln|\hat{\Sigma}_u| + C$, which serves as a convenient criterion for comparing alternative VAR specifications. In the present study, estimation proceeds conditional on the transformed data; the Box–Cox parameters are obtained separately at the univariate level and are treated as fixed preprocessing inputs rather than being re-estimated within the multivariate system.

In the post-estimation phase, a series of diagnostic tests are implemented to assess whether the VAR model is sufficiently effective. System-wide Portmanteau tests are employed to uncover any residual vector serial correlation, while multivariate Jarque–Bera statistics assess the normality of the innovations (Doornik & Hansen, 2008). Multivariate ARCH effects are assessed for conditional heteroscedasticity. A major prerequisite for valid inference and forecasting incorporates examining dynamic stability by evaluating the unit circle eigenvalues of the VAR companion matrix. The VAR framework is appropriately termed stable if all eigenvalues are strictly in the unit circle, indicating an insufficient magnitude of the shocks to dissipate over time and well-behaved multi-step forecasts.

3. Empirical Analysis and Results: Climate Data from Northern Iraq

The theoretical approaches have been addressed and the role of variance-stabilizing transformations and the univariate ARIMA and multivariate VAR models have been given, an empirical application can occur. This section performs the empirical application of the theoretical approaches set out in previous sections by first describing the data and applying the BCT, with initial diagnostics to confirm that the transformed series is suitable for time-series modeling. This is a necessary step to provide an empirical basis for estimating and comparing the univariate and multivariate models that will be conducted in subsequent subsections.

3.1 Data Description, Transformation, and Preliminary Diagnostics

The empirical inquiry analyzes monthly climatic time series for T, R, and H utilizing data from a meteorological station in northern Iraq. The dataset ultimately comprises $n = 192$ observations per variable, which encompass a 16-year time frame. The original time-series data were acquired from the World Bank Climate Knowledge Portal (https://climateknowledgeportal.worldbank.org), and were imported into the statistical package R for preprocessing, transformation, and time series model implementation. As a first step, the raw time series for the primary climatic variables will be visually assessed by time-series plots (Figure 1).

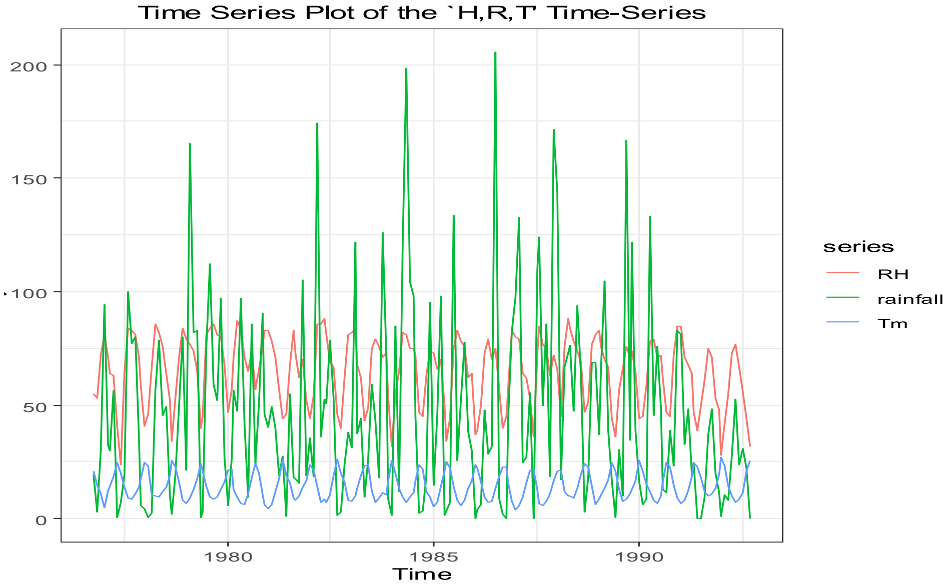


Fig. 1: Time–series graphs of the raw climatic variables: T, R, and H.

The graphs show evidence strong seasonal variation and considerable intra-annual variation and noticeable differences in dispersion across the observed climatic time series. Therefore, this initial visual analysis confirms that there is a multi-scale temporal structure and multi-scale variability present in these time series, which justifies the use of variance-stabilizing transformation in advance of formal time-series modeling.

Formal unit-root tests are then applied to assess stationarity. Specifically, the ADFT and PPT are used to test the null hypothesis of a unit root against a stationary alternative, while the KPSST is used to test the null of stationarity against a unit-root alternative (Dickey & Fuller, 1979; Phillips & Perron, 1988; Kwiatkowski et al., 1992). Table 1 reports the corresponding test statistics and p-values for both the original and Box–Cox transformed series.

Table 1: Unit-root and stationarity results for the original and transformed climatic series

Series	Optimal λ	ADFT		PPT		KPSST	
		Stat.	p-value	Stat.	p-value	Stat.	p-value
$Y_{T,t}$	1.00	-8.58	0.01	-67.18	0.01	0.04	0.1
$Z_{T,t}$	0.40	-6.39	0.01	-66.49	0.01	0.04	0.1
$Y_{R,t}$	1.00	-7.34	0.01	-70.58	0.01	0.26	0.1
$Z_{R,t}$	1.33	-7.23	0.01	-70.00	0.01	0.26	0.1
$Y_{H,t}$	1.00	-5.93	0.01	-154.35	0.01	0.16	0.1
$Z_{H,t}$	1.10	-5.84	0.01	-157.69	0.01	0.16	0.1

The results in Table 1 show a consistent pattern for all three climatic variables: for both the original and transformed data, the ADFT and PPGT p-values are below the 5% significance level, while the KPSS p-values exceed 0.05. This combination—rejection of the unit-root null in ADFT/PPT and non-rejection of the stationarity null in KPSST—indicates that $T, R,$ and H can be treated as stationary processes both before and after applying the BCT. In other words, the transformation is not introduced to induce stationarity, because the unit-root tests already support stationarity in the original domain; instead, the BCT is used to improve the distributional properties of the series.

To regularize the marginal distributions, each climatic variable is transformed using the BCT, with the power parameter λ estimated by maximizing the log-likelihood under approximate Gaussianity.

Because the stationarity tests are re-computed on the transformed series, the conclusions in Table 1 can be interpreted as more reliable in the transformed domain: by reducing heavy-tail behavior and variance heterogeneity, the BCT mitigates violations of test assumptions and yields test statistics that are less sensitive to outliers. Subsequent tables and figures summarize the impact of this preprocessing on model specification and diagnostics. In particular, Table 2 reports the information-criterion values (AIC, HOQ, BIC, and FPE) used to select the ARIMA and VAR orders.

Table 2: Information-Criterion Lag Selection: (a) Original Series and (b) BCT-Transformed Series

Series	AIC	HOQ	BIC	FPE
Original Series	9	8	8	9
BCT-Transformed Series	9	8	7	9

Across all four information criteria, the selected lag orders are concentrated between 7 and 9 in both the original and the Box–Cox transformed series, indicating a consistent temporal structure across the climatic variables. The observed congruence between the varying criteria indicates a relative stability of the dynamic dependence and that the chosen lag orders reflect a satisfactorily parsimonious parameterization that appropriately accounts for serial dependence.

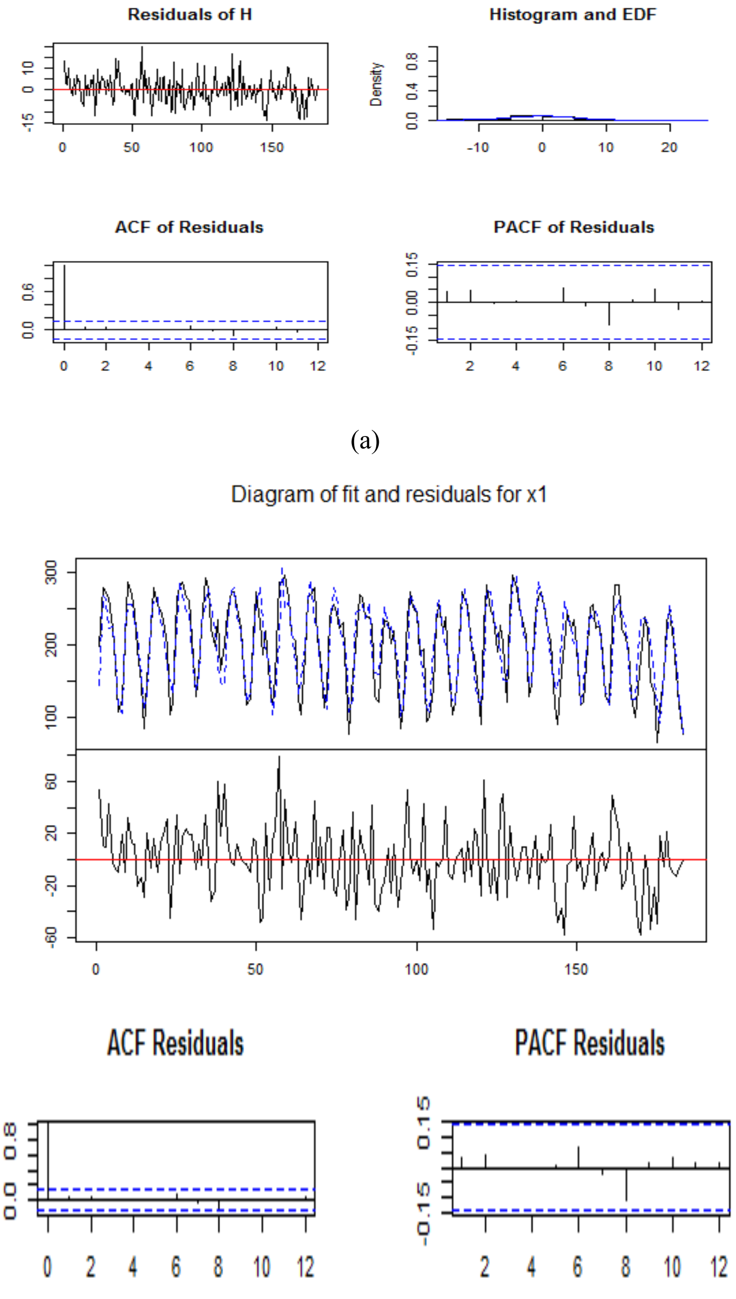
After specifying optimal lag orders, the next stage will entail estimating the univariate ARIMA and multivariate VAR models based on the transformed climatic series, followed by an assessment of model fit via standard model diagnostics, including examination of residual characteristics, autocorrelation, and so on., normality, and heteroscedasticity tests. The corresponding results are summarized in Table 3and illustrated in Figures 3, 4 and 5.

Table 3: Comparison of Autocorrelation, Heteroscedasticity, and Normality Diagnostics for $VAR(9)$ under Original and Box–Cox Transformed Data

Indicators	Original Data		Transformed data	
	Stat.	p-value	Stat.	p-value
PT-Autocorrelation	92.06	0.01	90.95	0.01
ARCH-Heteroscedasticity	143.25	0.98	130.59	0.99
Jarque–Bera - Normality	69.98	0.00	88.37	0.00
Kurtosis	37.42	0.00	47.73	0.00
Skewness	32.56	0.00	40.65	0.00

Table 3 reports the diagnostic statistics for the $VAR(9)$ specification estimated on both the original and Box–Cox transformed series. The Portmanteau statistic yields p-values around 0.01, indicating borderline evidence of residual autocorrelation at conventional significance levels. However, the residual ACF and PACF plots in Figures 2–4 do not reveal any systematic serial structure, suggesting that the remaining dependence is of limited practical relevance for forecasting. The ARCH–LM statistics show very large p-values (≈ 0.99), providing strong support for homoscedastic residual behavior in both the original and transformed models.

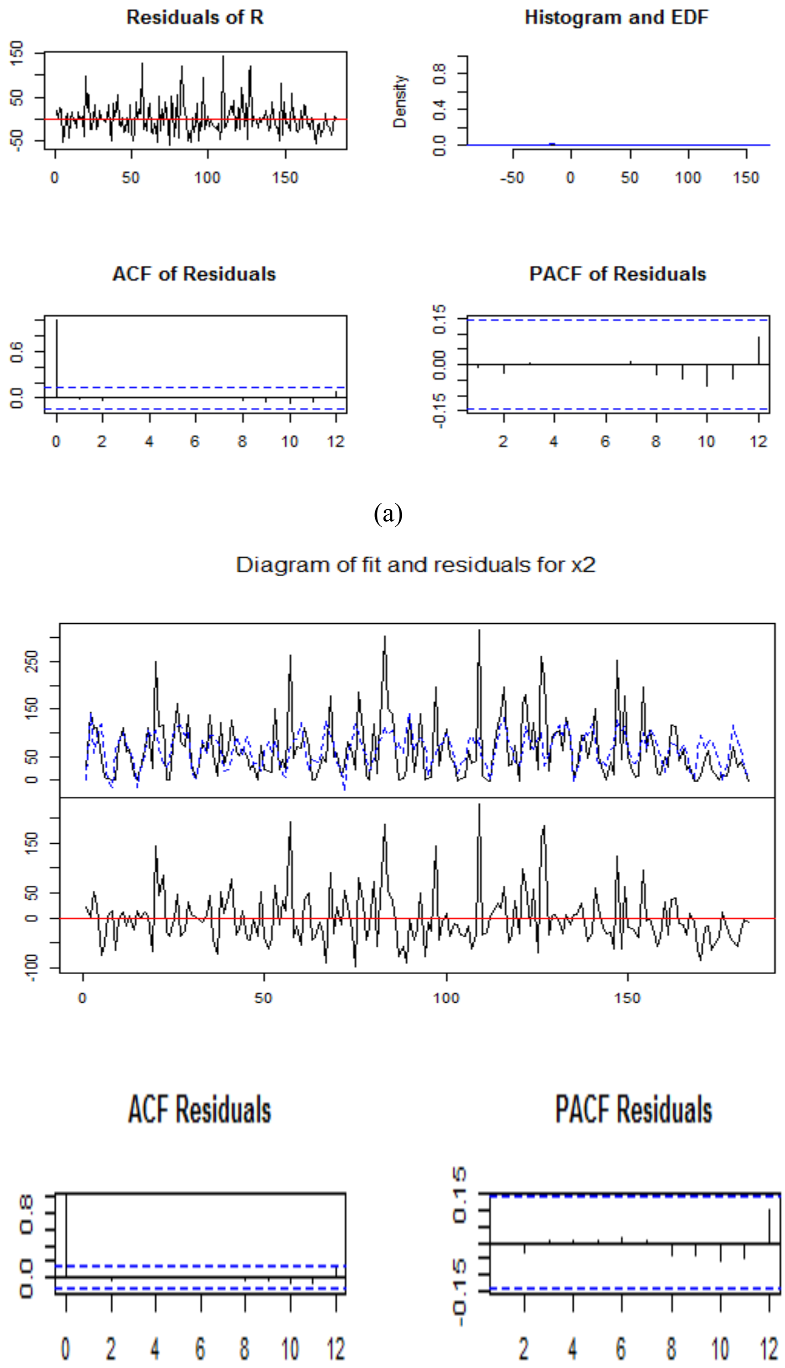
Regarding distributional properties, the Jarque–Bera test strongly rejects normality for both versions of the VAR model, and the skewness and kurtosis statistics also deviate from their Gaussian benchmarks. The conclusion is to be expected in weather applications. The typical joint modeling of variables with fundamentally different marginal distributions, T, R, and H, will produce multivariate residuals that do not normally distribute. The transformations with the BCT is successful in improving normality when testing each univariate series separately, however, once the variables are placed into a consolidated VAR system, improved univariate normality does not imply joint normality. Since the VAR residuals must be viewed as an overall error process that aggregates cross-dependent internal dynamics caused by the variables in the system, the newly embedded (cross-dependent) overall error process inherits heavy-tailed or asymmetrical components produced by the more irregular series, namely R, even when we preprocessing the univariate variables to achieve improved normality overall. Yet overall the diagnostic evidence presented indicates that $VAR(9)$ specification meets the major adequacy conditions of temporal modeling related to no substantial serial dependence and stable variance, and that evidence of departures from joint normality reflect common distributional idiosyncrasies associated with climatic variables and not mis-specification of the modeling process. Therefore, the results support the forecasting evaluation portion of the research process, which is presented in the next rotational mantle section of the dissertation.



(a)

(b)

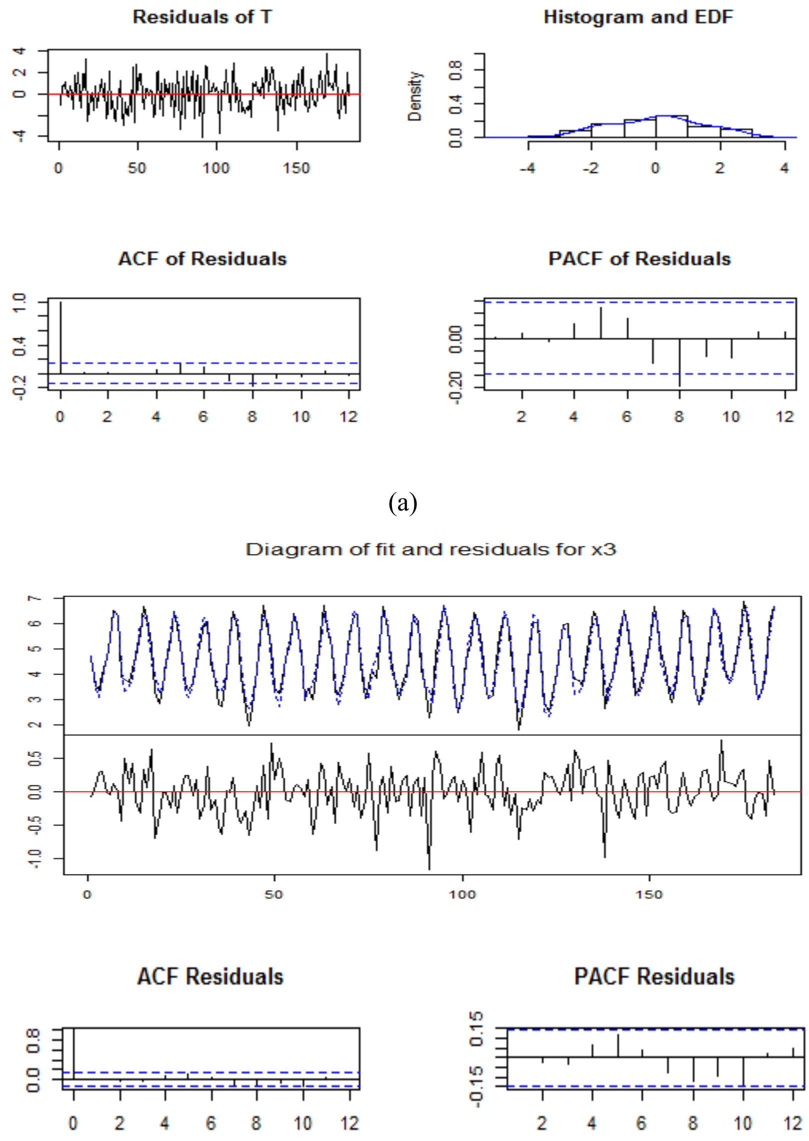
Fig. 2: Residual Plots for H: (a) Original Series and (b) BCT-Transformed Series



(a)

(b)

Fig. 3: Residual Plots for R: (a) Original Series and (b) BCT-Transformed Series



(a)

(b)

Fig. 4: Residual Plots for T: (a) Original Series and (b) BCT-Transformed Series

Figures 2-4 offer supplementary graphical diagnostics of the residuals of both the univariate models and the multivariate models. In every climatic variable, the residual series do not show observable signs of serial structure and in addition, the Box-Cox transformed models appear smoother and more homoscedastic than the untransformed models. The figures provide evidence to support the formal tests, and collectively confirm that the estimated models effectively capture the temporal dynamics of the climate variables of interest.

Now that we have concluded that the transformed series satisfy the main adequacy conditions, we will next proceed to estimate the univariate ARIMA models for each climate variable, before we model the multivariate VAR system.

3.2 Univariate and Multivariate Model Estimation

Building on the preliminary diagnostics presented in section 3.1, the transformed climatic series are now used to estimate both the univariate ARIMA specifications and the multivariate VAR model. The BCT stabilized the variance and improved the distributional properties of the T, R, and H series, providing a consistent basis for dynamic modeling. As shown earlier in Figures 2–4, the autocorrelation structure of the transformed series exhibits a rapid decay, indicating that relatively low-order AR and MA components are sufficient for capturing short-term persistence. These features support the use of parsimonious ARIMA models for the individual series and motivate the parallel estimation of a multivariate VAR structure to account for cross-variable interactions.

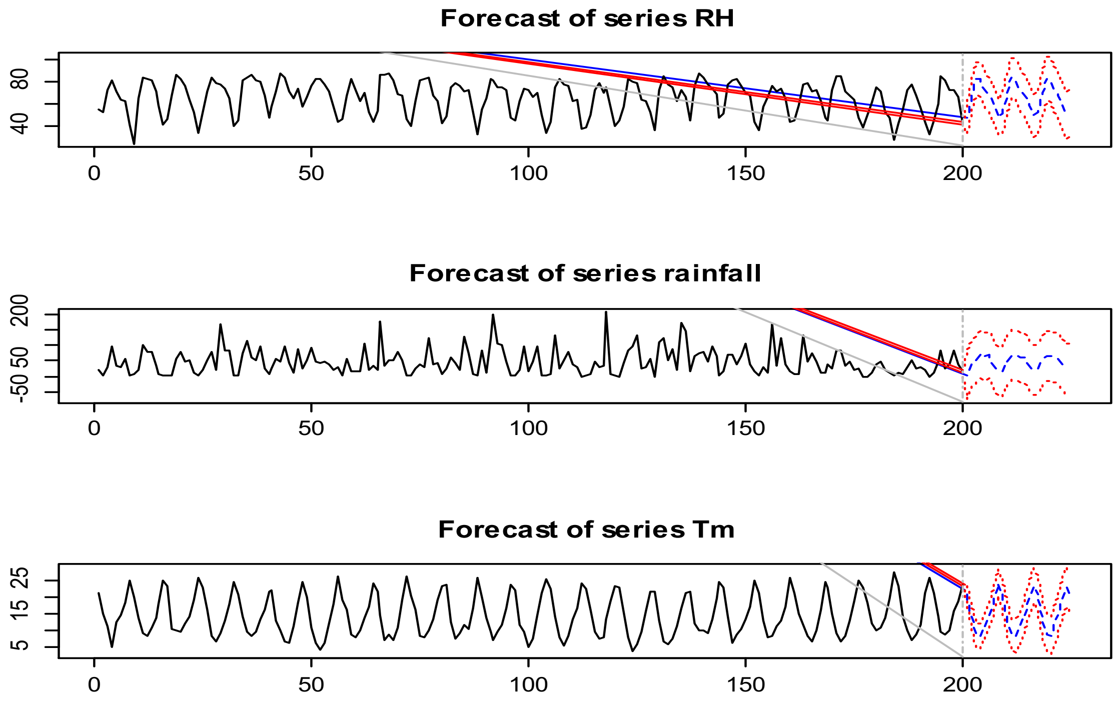
The model orders were selected using standard information criteria, balancing goodness of fit against model parsimony. Once the ARIMA orders and the VAR lag length (nine lags) were determined, the parameters of both modeling frameworks were estimated by maximum likelihood. The results, including the estimated coefficients and the mean squared errors (MSE) before and after the BCT, are summarized below.

Table 4: Estimated Parameters and MSE of the Univariate ARIMA and Multivariate VAR Models for the Box–Cox Transformed Climatic Series (H, R, T).

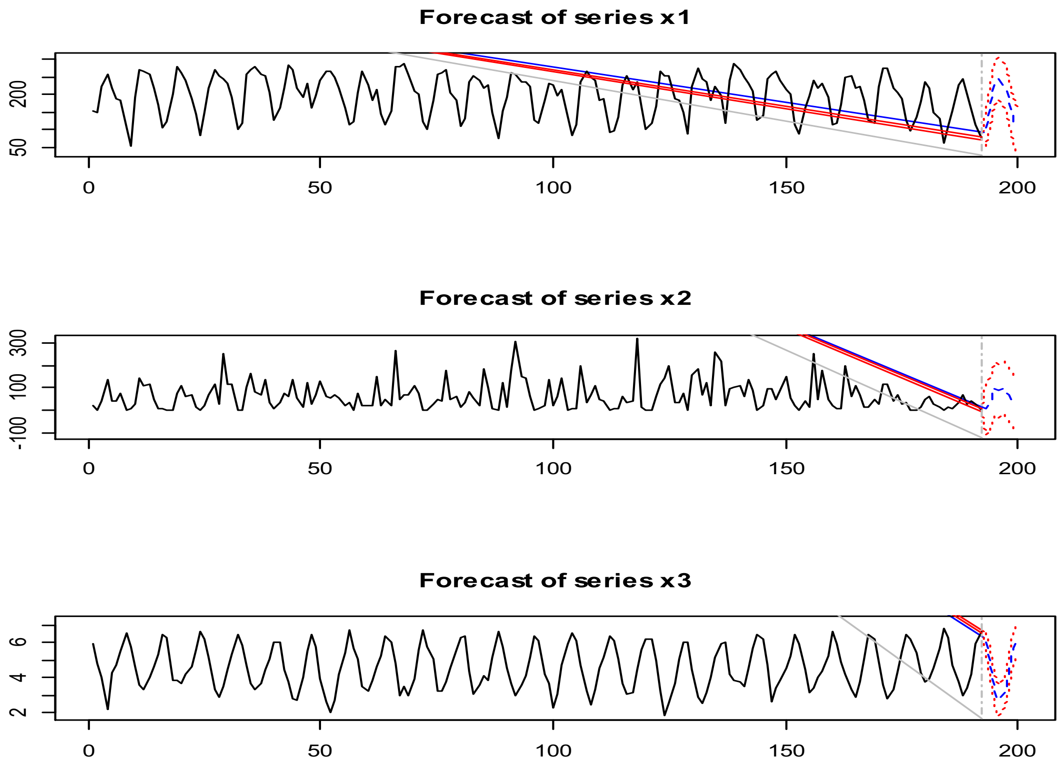
Series	Original Data $\lambda_H = \lambda_R = \lambda_T = 1$		Transformed Data $\lambda_H = 1.3,\ \lambda_R = 1.1,\ \ \lambda_T = 0.4$	
	$ARIMA$ (1,0,0)(1,1,1)8	$VAR(9)$	$ARIMA$ (1,0,0)(1,1,1)8	$VAR(9)$
T	2.2405	3.2624	2.1668	3.033
R	366.388	497.190	379.5328	536.5995
H	15.6396	49.2073	15.1095	44.6282

Table 4 provides direct numerical evidence for the comparative performance of the univariate and multivariate models. The univariate ARIMA specifications exhibit MSE that are consistently lower than the $VAR(9)$ model, both prior to and after the BCT was applied. Although the multivariate structure is supposed to take advantage of the cross-variable dependencies in the forecasts, the fairly mild correlations among T, R and H limit most of the forecasting capabilities that VAR could provide. Conversely, the transformation achieved even more significant improvements in the univariate framework, where each of series is benefitting individually from variance stabilization and more favorable marginals. This reinforces the conclusion that univariate modeling is the better forecasting option for this dataset, compared to multivariate modeling.

To explore the forecasting implications of the multivariate structure, one-step-ahead predictions from the $VAR(9)$ model are compared for original series and transformed series. The figure below illustrates the forecast paths for T, R, and H with both series sets.



(a)



(b)

Fig. 5: Comparative Forecasts from the $VAR(9)$ Framework: Original Data (a) versus Box–Cox Transformed Data (b).

The forecasts created from the adjusted series are more aligned with the observed series, especially during the periods of large variability, thus demonstrating the usefulness of variance stabilization and enhancing the predictive performance of the multivariate series. The aforementioned estimation results led to the comparative analysis discussed in the subsequent section, where the follow-up time, critical for preventing a loss of power, was unit level.

4. Discussion and Implications

The aim of this research was to assess forecasting performance associated with univariate and multivariate models for the three major climate variables, T, R, and H, after doing a BCT as a preprocessing step. The BCT was not intended as a theoretical contribution but rather a procedural device to stabilize variance and to enhance the distributional properties of the series in accordance Box-Jenkins encouragements using transformation is often necessary for reliable identification and estimation of time-series models (Box & Jenkins, 1976).

A first finding is, while the raw climate series were viewed as stationary based on ADF, PP, and KPSS statistical tests, the BCT improved their marginal distributional properties while reducing heavy tails. This improvement ultimately is not just superficial. In this case, the reduced skewness and kurtosis produced cleaner residual behavior and stable parameter estimates. These improvement trends reflect the findings of Guerrero (1993) and Atkinson et al. (2021), which found that power transformations significantly enable the reliability of inferences concerning climatic and environmental series. Hence, we provide evidence of these preceding claims.

The singular univariate ARIMA models provided an outstanding performance after we had applied some form of transformation, and in all cases, the forecasts errors were lower than those for the pre-transformation time series. This directly supports the findings of Othman & Mohammed Ali (2023), who showed that BCT and related transformations lead to gains for ARIMA based forecasting of T curves. Furthermore, the persistent autoregressive nature of T and H series, aligns with the idea advanced by Hyndman & Athanasopoulos (2021): When a variable is primarily determined by itself, a properly specified univariate model often outperforms a more complex one. The evidence in this study supports this claim by being apparent decrease in MSE and favorable diagnostics.

The multivariate $VAR(9)$ model inherently sought to take advantage of the inter-variable relationships. This approach worked well in determining the common patterns of the climatic variables over the seasons. However, the approach showed only moderate levels of inter-variable correlations. Though the transformation of the VAR residuals worked well, it had forecasting errors that were not better than the ARIMA models. This result corroborates the fact that the CARMA results of Adeyemi et al. (2017) and Cebrián et al. (2019) stated that multivariate models will prove superior only if the correlations between the variables are sufficiently strong and affect the dynamics of the series. This study's observations agree with their result because the levels of the inter-correlation of T, R, and H were not strong enough.

One of the observations that can be made regarding the results is the normality of the residuals of the VAR. Despite the variable transformation, the residuals of the VAR series still remained un-normal due to the nature of the series being modeled with different marginal distributions. This result matches the observations made by Živković et al. (2022), who claimed that the multivariate models tend to retain the irregularities of the series that are not normally distributed; thus, obtaining normal distribution might remain challenging. Our result serves as an empirical replication of their observations. Overall, these findings support an important methodological point: there is no presumption, in multivariate analysis, that multivariate models are preferable. Rather, as Ogallo et al. (2020) and Youngman (2020) have each suggested, such choices between univariate and multivariate analysis must be grounded in empirical, not purely theoretical, considerations. On these grounds, in applying these methods in this particular context, univariate models showed improved forecast accuracy, while the VAR model facilitated interesting structural analysis, albeit with no improved accuracy.

The final implication is procedural in nature. By applying an identical preprocessing to all variables, we could ensure an equitable comparison for model types. It identifies with what Guerrero (1993) had to say regarding model comparison with different structural hypotheses, stating transformation is necessary for testing those differences. Our findings support its application: By means of BCT, we could guarantee differences in scales will not interfere with model comparison.

In light of these results, there is no doubt that there is concrete empirical support for claiming that, with regard to these particular types of climate data, univariate ARIMA forecasts outperform those from multivariate models like those examined in a variance-stabilized context, simply because while there was certainly value in understanding relationships between different data categories, these relationships were not pronounced enough to provide improved forecasts. These conclusions not only support other research, but they provide value in their own regard regarding considerations for climate forecasts.

5. Conclusion

In this section, it is shown that maximizing the LLF results in a power parameter estimate that does not always move the data enough to make it normal. When the data is normalized, it is obvious that the quality of other modeling efficiency criteria will be compromised, and vice versa. As a result, it is possible to conclude that no feasible solution region exists for all quality standards and transformation goals. The researcher believe that these difficulties stem from the use of power transformation to transform the response variable before the model fitting. While the use of classical efficiency criteria after modeling the transformed datasets contributed to the success in choosing a simple model. The analysis proved that the boxcox model was more flexible in smoothing the data and contributes to accessing to a simple model with good forecast ability. In the multivariate method we found that the transformed results had a slight improvement in the error value. Also, in the parameter, it showed that the ARIMA method's univariate prediction was better than the multivariate.

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