Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/isl/130403

Optimal control and cost-effectiveness analysis of alcohol addiction and poverty dynamics in a population

J. O. Akanni[1,5], *A. A. Ayoade*[2], *D. Enegesele*[3], *C. E. Madubueze*[4] *and F. Fatmawati*[5,*]

[1]Department of Mathematical and Computing Sciences, Koladaisi University, Ibadan, Oyo State, Nigeria
[2]Department of Mathematics, University of Lagos, Akoka, Lagos State, Nigeria
[3]Department of Stastics, Delta State University of Science and Technology, Ozoro, Delta State, Nigeria
[4]Department of Mathematics, Joseph Sarwuan Tarka University, Makurdi, Benue State, Nigeria
[5]Department of Mathematics, Universitas Airlangga, Kampus C Mulyorejo Surabaya 60115, Indonesia

Received: 7 Oct. 2023, Revised: 21 Dec. 2023, Accepted: 23 May 2024
Published online: 1 Jul. 2024

Abstract: Poverty and alcoholism are among the top challenges confronting societies worldwide and essential research subjects in the last few decades. This paper developed a deterministic model governed by a system of nonlinear differential equations to proffer a solution to the burden of poverty and alcoholism in a population. The model was studied under two different systems, namely: autonomous and non-autonomous systems. Under the autonomous system, the following analysis was carried out; Poverty and alcoholism reproduction number using the next-generation method; this was done to know all the parameters contributing to the dynamics of poverty and alcoholism in a population. Also, local and global sensitivity analysis was done using the normalized forward method and Latin hypercube sampling and partial rank correlation coefficients, respectively; this was done to identify the parameters of the model that most influence thy dynamical behaviour of the model. The non-autonomous model with three time-dependent controls was analyzed using Pontryagin's Maximum Principle to find the optimal solution to the poverty and alcoholism control problem. Cost-effectiveness analysis was conducted to further affirm the results of the optimal control problem by using the average cost-effectiveness ratio and incremental cost-effectiveness ratio methods. Numerical simulations were presented to buttress all the theoretical results. .

Keywords: Poverty and alcoholism model, Poverty and alcoholism reproduction number, Sensitivity analysis, Optimal control problem, Cost-effectiveness analysis, Numerical simulations.

1 Introduction

In this day and age, the world is confronted with a number of challenges such as pollution, climate change, inequality, corruption, illegal migration, banditry, terrorism, kidnapping, smuggling, trafficking, emergence and reemergence of deadly diseases, disasters and so on. Poverty and alcoholism are among the top challenges confronting the societies the world over and an important subject of research in the last few decades. Poverty and alcoholism remain a topical issue nowadays because they instigate other societal problems directly or indirectly especially crime and mortality.

The word poverty has a broad meaning, it refers to the state in which communities or individuals lack the resources to enjoy a minimum living standard [1,2,3,4]. It is hunger. It is lack of shelter. It is not having access to school and not being able to read. It is not having a job. It is fear for the future. It is powerlessness, lack of representation and freedom. Impoverished families and children living in the world's low-and middle-income countries are highly vulnerable, powerless and afraid [5,6,7]. The live at the very margin of human existence, their dignity is assaulted daily, and their lives are abundant in scarcity [8,9,10]. Because of lack of hope and uncertainties about the future, impoverished individuals do resort to crime and a number of unhealthy practices.

Alcoholism, on the other hand, is a social phenomenon that can have an effect on a variety of facets of life and affects people from all socioeconomic classes, educational background, and age ranges [11,12]. It is a disorder that makes an individual drink excessively; as a result, it can ruin relationships, reduce productivity, and be harmful to both physical

* Corresponding author e-mail: fatmawati@fst.unair.ac.id

and mental health [13,14,15]. Individuals who are suffering from alcohol disorder are unable to regulate their heavy or regular drinking. They know that their behavior could have detrimental or disastrous effects, but they nevertheless engage in unhealthy drinking [16,17].

Abuse of alcohol over an extended period of time can seriously harm the liver and brain. Heart diseases, wasted of money, crime, poverty, family dissolution, liver diseases, as well as ulcers, sexual issues, bone loss, suppressed immune function and an increased risk of cancer are just a few of the economic, social, and health implications of alcohol on both the individuals and society at large that require medical attention in hospitals and private or public addiction treatment facilities [13,18,19]. Alcoholism also has a significant impact on the global mortality. The World Health Organization (WHO), in its report ”World Status Report on Alcohol and Health 2018” [20], reports that alcohol kills three million people annually, or one death out of every twenty deaths. For instance, alcohol kills more people than AIDS, TB, and violence all together [13].

Poverty and alcoholism are social ills that every society strives to eradicate or at least minimize to the barest minimum to ensure peace and safety. Governments all over the world do embark on one program or the other to minimize poverty and alcoholism to have a society that is devoid of chaos and uncertainties. The major among these programs are literacy and social media advertisement.

Literacy is the capacity to read, write, talk, and listen [21]. In contrast, social media is interactive technology that makes it possible to create and share information, ideas, hobbies, and other kinds of expression through online communities and network [22,23]. Literacy allows individuals to express their thought and idea, interact with people, and learn new things. Social media, on the other hand, promote connectivity, information and updates, education, awareness, sharing of things with others, community building, mental health and noble cause [24]. Literacy and social media advertisement are no doubt agents of change and powerful tools to influence poverty and alcoholism.

Mathematical models can be employed to examine how contagious diseases propagate or how people behave in social situations [25,26,27,28,27,29,30]. As regards poverty and alcoholism, several studies have reported that alcohol abuse and dependence, as well as other risk behaviors, cluster in contexts of poverty, residential instability, and social isolation [13,31,32,33,34,35,36,37,38,39].

Chinnadurai and Athithan [39] developed a mathematical model of poverty and alcohol consumption by dividing the total population into four classes - the non-impoverished compartment N, the impoverished/poverty compartment P, the alcohol addicted class A and the recovered class R (from the two classes P and A). The stability analysis was performed via Routh-Harwitz stability criteria and the authors showed that both the poverty-alcoholism-free equilibrium and the poverty-alcoholism-endemic equilibrium existed and were locally and globally stable under some parameters conditions. The researchers discovered that the major route to alcohol addiction was poverty. They therefore urged the policy makers to address the issue of poverty holistically to forestall recruitment into alcohol addiction and to increase the recovery rate of the alcohol addicts.

A mathematical model of poverty and drug addiction proposed by Sakib and colleagues [32]. The model is made up of five compartments with the inclusion of the rehabilitation class I which made the model an extension of [39]. The model was subjected to the stability test and the researchers discovered that the poverty-addiction-free equilibrium $\mathscr{E}^{\circ}$ was locally stable when $\mathscr{R}_0 \leq 1$. However, when $\mathscr{R}_0 > 1$, the poverty-addiction endemic equilibrium $\mathscr{E}^{*}$ occured and was locally stable. Their results showed that poverty eradication had more than 50% effect on drug addiction. They therefore advocated for policies, strategies, programs and interventions which could empower the poor such as soft loan, educational opportunities, skill development, job opportunities, food for work, self employment, entrepreneurship opportunities, good governance, raising awareness, etc.

Murakami and Hashimoto [33] conducted a study on the association of income and education with problem drinking and heavy drinking among Japanese men. They discovered that lower educational attainment was significantly related with increased risk of both problem drinking and non-problematic heavy drinking. They also discovered that income was directly related to alcohol consumption with high income earners having more tendency to be frequent drinkers, probably because they have financial powers and social opportunities. A mathematical model of alcohol drinking with the effect of alcohol treatment centers was proposed in [13]. The researches derived the reproduction number for the model and performed the sensitivity analysis for the model parameters. They discovered that the most effective way of preventing alcohol addiction was to prevent exposure of underage to the moderate and heavy drinkers.

In this paper, our goal is to develop a compartmental model that considers the dynamics of poverty and alcoholism under the influence of literacy and social media advertisement. While poverty and alcoholism have been a subject of intense study with various methods (such as job creation, skill development, prohibition, stringent laws, etc.) suggested as the potential interventions [32,33,40,39], studies on the effect of literacy and social media advertisement on the dynamics poverty and alcoholism are new. To our knowledge, no study has looked at the dynamics of poverty and alcoholism under the influence of literacy and social media advertisement. The model is qualitatively analyzed using Pontryagin's maximum principle, and cost-effectiveness studies are done on the many optimal control combination schemes that have been put into practice and use at least one of the three optimal control variables. This study's remaining sections are divided as follows: A summary of the mathematical model of non-optimal control developed and studied by Chinnadurai and Athithan [39] is shown in Section 2. The non-autonomous version of the model is then developed and examined in Section 3. Additionally, the cost-effectiveness analysis and numerical simulations of the autonomous and non-autonomous models are carried out in Section 4. Finally, the study's final observation is presented in Section 5.

2 Model without optimal control

The proble of alcoholism and poverty spread in the population was studied with the total population denoted by $N(t)$, at time t, which was sub-divided into four well-defined classes of individuals that are susceptible $S(t)$ (someone who susceptible to drinking of alcohol and likely to become poor), Alcohol addict $A(t)$ (someone who seriously indulge in drinking alcohol), poverty class $P(t)$ (someone who is financially poor), recovery class $R(t)$ (someone who stop taking alcohol and become non-impoverished). The dynamics of the model was extensively discussed in [39]. The definition of variables and parameters are presented in Table 1

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S(t) - \alpha_1 S(t)A(t) - \beta S(t)\\
\frac{dA}{dt} &= \alpha_1 S(t)A(t) - \gamma P(t) - (\mu_1+\mu)A(t) - \alpha_2 A(t)R(t)\\
\frac{dP}{dt} &= \beta S(t) + \gamma P(t) - \alpha_3 P(t)R(t) - \mu P(t)\\
\frac{dR}{dt} &= \alpha_2 A(t)R(t) + \alpha_3 P(t)R(t) - \mu R(t)
\end{aligned} \tag{1}$$

with initial conditions at $t=0$:

$$S(0) = S_0, A(0) =_0, P(0) = P_0, R(0) = R_0. \tag{2}$$

Table 1: Variables and Parameters of the model [1]

Variable	Description	Variable	Description
$S(t)$	Non-impoverished class	$A(t)$	Alcohol addict class
$P(t)$	Poverty class	$R(t)$	Recovery class
Parameter	**Description**	**Parameter**	**Description**
Λ	Recruitment rate into the susceptible	μ	Death rate
α_1	Rate at which the individual in N(t) get A(t)	β	Rate of flow from N(t) to P(t)
γ_1	Rate at which the individual in A(t) get to P	μ_1	Induced death rate
α_2	Recovery rate for A(t)	α_3	Recovery rate for P(t)

2.1 Poverty and alcoholism reproduction number

The Poverty and alcoholism reproduction number, $\mathscr{R}_0$, inferred as the anticipated number of secondary instances produced by a sole contagious person in a totally susceptible community throughout its contagious time is a boundary variable that enables us to foresee if the infection will stop or prevail [41]. Typically, $\mathscr{R}_0 < 1$ implies that the alcohol and poverty problem cannot overrun the community, $\mathscr{R}_0 > 1$ implies that each alcohol addict and poor individuals develops more than one secondary problematic person, and $\mathscr{R}_0 = 1$ respects additional scrutiny. The resolution of $\mathscr{R}_0$ is performed utilizing the next-generation matrix method [41,42,43,44]. Utilizing this procedure, we retain

$$F = \begin{pmatrix} \alpha_1 S & 0 \\ 0 & 0 \end{pmatrix} \tag{3}$$

and

$$V = \begin{pmatrix} (\mu+\mu_1) & \gamma \\ 0 & \mu-\gamma \end{pmatrix} \tag{4}$$

At the alcohol and poverty-free equilibrium, $\mathbf{D}_0$, $S_0 = \frac{\Lambda}{\mu}$. Thus, the $\mathscr{R}_0$ of model (1) is given by.

$$\mathscr{R}_0 = \frac{\alpha_1 S}{\mu + \mu_1} = \frac{\alpha_1 \Lambda}{\mu(\mu + \mu_1)}. \tag{5}$$

The alcohol and poverty problem can be eradicated from the population ($\mathscr{R}_0 < 1$) if the initial sizes of the population of the model are in the basin of attraction of the alcohol and poverty-free equilibrium.

2.2 Sensitivity analysis

In this section, sensitivity analysis of model (1) is explored. To do this, we follow the approach in previous studies [45, 46,47,48]. This helps to identify the parameters of the model that most influence the dynamical behaviour of the model. Thus, to determine the sensitivity of model (7) using the alcohol and poverty threshold quantity, the basic reproduction number $\mathscr{R}_0$ given in (5), as the response function with respect to the model parameters, the normalized forward-sensitivity index of $\mathscr{R}_0$ that depends on a parameter p is defined as

$$S_p^{\mathscr{R}_0} = \frac{\partial \mathscr{R}_0}{\partial p} \times \frac{p}{\mathscr{R}_0}. \tag{6}$$

Given the explicit formula (5) for the basic reproduction number $\mathscr{R}_0$, the analytical expressions for the sensitivity of $\mathscr{R}_0$ in respect of the parameters defining it are computed. In particular, the analytical expression for the sensitivity of $\mathscr{R}_0$ with respect to all the parameters in view of (6) is given below

$$\begin{aligned}
S_\Lambda^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \Lambda} \times \frac{\Lambda}{\mathscr{R}_0} = +1, \\
S_{\alpha_1}^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \alpha_1} \times \frac{\alpha_1}{\mathscr{R}_0} = +1, \\
S_\mu^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \mu} \times \frac{\mu}{\mathscr{R}_0} = \frac{-\mu(2\mu + \mu_1)}{\mu + \mu_1}, \\
S_{\mu_1}^{\mathscr{R}_0} &= \frac{\partial \mathscr{R}_0}{\partial \mu_1} \times \frac{\mu_1}{\mathscr{R}_0} = \frac{-\mu_1}{\mu + \mu_1}.
\end{aligned}$$

However, the sensitivity index (SI) of $\mathscr{R}_0$ for all the parameters comprising it are evaluated at the baseline parameter values given in Table 3. The signs and values of SI are presented in Table 2.

Table 2: Sensitivity index of $\mathscr{R}_0$ to each of the parameters of model (1) evaluated at the baseline parameter values given in Table 3

Parameter	Sign of SI	Numerical value of SI
α_1	$+ve$	1
Λ	$+ve$	1
μ	$-ve$	-1.4689
μ_1	$-ve$	-0.5311

From Table 2, it is observed that the sign of SI is positive for parameters (α_1, Λ), while it is negative for the others (μ, μ_1). From the set of parameters with positive SI sign, Λ is the most positive. Whereas, μ_1 is the most negative from the set of parameters with negative SI signs. The epidemiological insight from the positive sign of SI of the alcohol and poverty threshold quantity, $\mathscr{R}_0$, is that increasing or decreasing the value of any of the parameters in this category will generate an increase or decrease in the threshold $\mathscr{R}_0$ of alcohol and poverty. The negative sign of SI on the contrary suggests that increasing the value of each of the parameter set in this category will lead to a decrease in the $\mathscr{R}_0$ value, and vice-versa.

Furthermore, Latin hypercube sampling (LHS) and partial rank correlation coefficients (PRCC) are used to carry out a global sensitivity analysis to further ascertain the parameters that mostly influence the dynamics of model (7) (1) with respect to the basic reproduction number, $\mathscr{R}_0$, given in (5). LHS matrices are generated by assuming that all the model parameters obey a uniform distribution. Then, using a similar idea of the authors in [44,49], multiple runs of 1000 sample

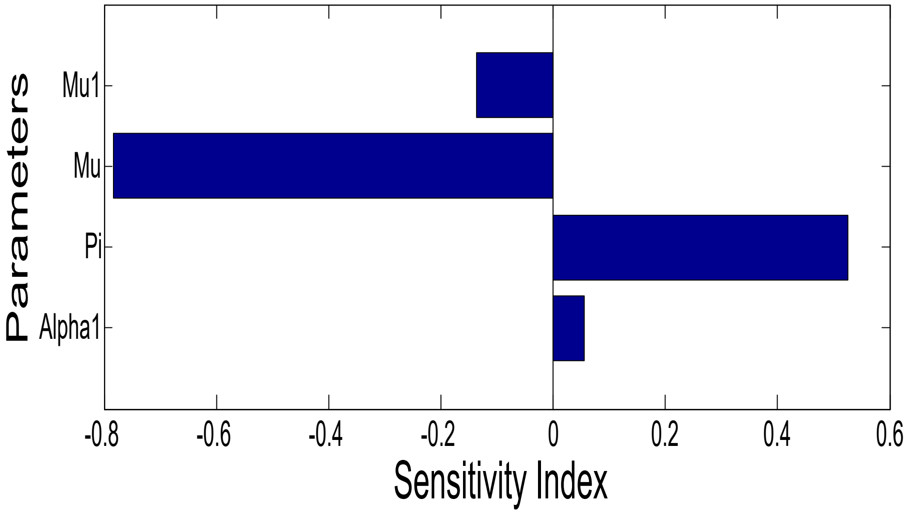

Fig. 1: PRCC values for the parameters of model (7) that make-up $\mathscr{R}_0$

size per LHS run are performed based on the model parameter baseline values provided in Table 3 with the ranges of 30% in either direction from the baseline values. Thus, the sensitivity of the parameters of model (1) is evaluated by finding the PRCC between each of the parameters and $\mathscr{R}_0$.

Figure 1 displays the PRCC values for each of the sensitive parameters of model (1). It is observed from Fig. 1 that the PRCC value is positive for some parameters, that is, those that are positively correlated (Λ, and α_1) and negative for others, that is, those that are negatively correlated (μ and μ_1). The top PRCC-ranked parameters that drive the response function, $\mathscr{R}_0$, most are the recruitment rate, Λ (see Fig. 1 and Table 2). Therefore, the result from global sensitivity analysis aligns with the result arising from the local sensitivity analysis.

3 Optimal control of alcohol and poverty model

In [39], the author work on the autonomous version of the model, but in the present work, the non-autonomous version will be exploded. Based on the analysis, the purpose of this is to recommend the best method to control the menace of illicit drug use and terrorism. The optimal control theory will be applied to the model to curtail the effect of alcohol and poverty in the community. To do this, four time-dependent control variables $u_1(t)$, $u_2(t)$, and $u_3(t)$ are introduced into model (1). The four control variables are described as follows:

(i)$u_1(t)$ represents the public awareness of the dangers, havocked and the sociality implication of involving in alcohol and poverty. Educational Campaign.
(ii)$u_2(t)$ denotes the rehabilitation control. The rate at which alcoholic addict are rehabilitate.
(iii)$u_3(t)$ is considered as the empowerment rate of poor individual, the rate at which poverty is been impoverished.

Consequently, incorporating those mentioned earlier three time-dependent control variables into the alcohol and poverty model in [39], we have model (7), which is the optimal control model given below.

$$\begin{aligned}
\frac{dS}{dt} &= \Lambda - \mu S(t) - (1-u_1(t))\alpha_1 S(t)A(t) - (1-u_1(t))\beta S(t)\\
\frac{dA}{dt} &= (1-u_1(t))\alpha_1 S(t)A(t) - \gamma P(t) - (\mu_1+\mu)A(t) - u_2 A(t)R(t)\\
\frac{dP}{dt} &= (1-u_1(t))\beta S(t) + \gamma P(t) - u_3 P(t)R(t) - \mu P(t)\\
\frac{dR}{dt} &= u_2 A(t)R(t) + u_3 P(t)R(t) - \mu R(t)
\end{aligned} \tag{7}$$

The three control variables are introduced to seek the optimal solution required to minimize the sizes of alcoholic addict, and poverty responsible for instability in the peace in the community at a minimum cost. Thus, the objective function for the optimal control problem is designed as

$$\mathfrak{J}(u_1,u_2,u_3) = \min_{0\leq u_1,u_2,u_3\leq 1} \int_0^{t_f} \left(W_1A + W_2P + \frac{1}{2}(a_1u_1^2(t) + a_2u_2^2(t) + a_3u_3^2(t) \right))dt \tag{8}$$

constrained by the system (7), where $W_1 > 0$, $W_2 > 0$, $a_1 > 0$, $a_2 > 0$, and $a_3 > 0$ are weight constants needed for balancing the associated terms in the objective functional and t_f is the final time such that $0 \leq t \leq t_f$. A quadratic cost on controls is used in accordance with the literature on optimal control problem [50,51,52,53,54,55], where $a_iu_i^2(t)$, $i = 1,2,3$, are the total costs expended on implementing the public enlightenment, empowerment rate control and rehabilitation therapy control over the interval $t \in [0,t_f]$, respectively.

The main goal is to seek an optimal control triple $u^* = (u_i^*)$, where $i = 1,2,3$, such that

$$\mathfrak{J}(u^*) = \min_{\mathfrak{U}} \{\mathfrak{J}(u_1,u_2,u_3)\}, \tag{9}$$

where $\mathfrak{U}$ is a non-empty bounded Lebesgue measurable control set given by

$$\mathfrak{U} = \left\{u_i(t), u_i(t) : [0,t_f] \to [0,1],\ i = 1,2,3\right\}. \tag{10}$$

3.1 Existence of an optimal control

Let the control set $\mathfrak{U} = [0,1]^3$, $u = (u_i) \in \mathfrak{U}$ (for $i = 1,2,3$), and $x = (S,A,P,R)$. Then, the following result in Theorem 1 gives the summary of the existence of an optimal control.

Theorem 1.*If the objective functional $\mathfrak{J}$ in (8) is as defined on the control set $\mathfrak{U}$, and constrained by the control system (7), then there exists an optimal control triple $u^* = (u_1^*,u_2^*,u_3^*)$ such that $\mathfrak{J}(u^*) = \min\{\mathfrak{J}(u_1,u_2,u_3) : u_1,u_2,u_3 \in \mathfrak{U}\}$ provided that the following properties hold [56]:*

a)The admissible control set $\mathfrak{U}$ is convex and closed.
b)The state system is bounded by a linear function in the state and control variables.
c)The integrand of the objective functional is convex in respect of the control.
d)For constants $\phi_1,\phi_2 > 0$ and $\phi_3 > 1$, the integrand is bounded below by $\phi_1\left(||u_i||^2\right)^{\frac{\phi_3}{2}} - \phi_2$.

3.2 Characterization of optimal control

Pontryagin's maximum principle [57] provides the necessary conditions that an optimal control triple of the alcoholism addict and poverty model (7) must satisfy. The principle helps in converting the alcoholism addict and poverty model (7) together with the objective functional (8) into a problem of minimizing pointwise a Hamiltonian $\mathscr{H}$, concerning the time-dependent controls $u_i(t)$, for $i = 1,\ldots,3$. The Hamiltonian for the non-autonomous system (7) is determined as

$$\mathscr{H} = \mathfrak{L}(t,x,u) + \sum_j \lambda_j f_j, \tag{11}$$

where λ_j are the respective adjoint variables for the states $x \in \{S,A,P,R\}$, and f_j is the right-hand side for jth state of the system (7). The control characterization result is claimed in Theorem 2 below.

Theorem 2. *If an optimal control triple $u^* = (u_i^*)$, $i = 1,2,3$, satisfies condition 9, then there exist adjoint (or costate) variables λ_j, $j = S,A,P,R$, satisfying the adjoint system given as*

$$\frac{d\lambda_1}{dt} = (\lambda_1-\lambda_2)(1-u_1)\alpha_1 A + (\lambda_1-\lambda_3)(1-u_1)\beta + \lambda_1\mu, \tag{12a}$$

$$\frac{d\lambda_2}{dt} = (\lambda_1-\lambda_2)(1-u_1)\alpha_1 S + (\lambda_2-\lambda_4)u_2 R + \lambda_2(\mu+\mu_1) - W_1, \tag{12b}$$

$$\frac{d\lambda_3}{dt} = (\lambda_2-\lambda_3)\gamma + (\lambda_3-\lambda_4)u_3 R + \lambda_3\mu - W_2 \tag{12c}$$

$$\frac{d\lambda_4}{dt} = (\lambda_2-\lambda_4)u_2 A + (\lambda_3-\lambda_4)u_3 P + \lambda_4\mu, \tag{12d}$$

with transversality (or terminal) conditions

$$\lambda_j(t_f) = 0 \qquad (\textit{where } j = 1,2,3,4) \tag{13}$$

and the control characterizations expressed as

$$u_1^* = \min\left\{\max\left\{0, \frac{((\lambda_2-\lambda_1)\alpha_1 A + (\lambda_3-\lambda_1)\beta)S}{a_1}\right\}, 1\right\}, \tag{14a}$$

$$u_2^* = \min\left\{\max\left\{0, \frac{(\lambda_2-\lambda_4)AR}{a_2}\right\}, 1\right\}, \tag{14b}$$

$$u_3^* = \min\left\{\max\left\{0, \frac{(\lambda_3-\lambda_4)PR}{a_3}\right\}, 1\right\}. \tag{14c}$$

Proof. First, the Hamiltonian (11) is written in an explicit form as

$$\begin{aligned}\mathscr{H} = {} & W_1 A + W_2 P + \frac{1}{2}(a_1 u_1^2(t) + a_2 u_2^2(t) + a_3 u_3^2(t)) \\ & + \lambda_1\{\Lambda - \mu S(t) - (1-u_1(t))\alpha_1 S(t)A(t) - (1-u_1(t))\beta S(t)\} \\ & + \lambda_2\{(1-u_1(t))\alpha_1 S(t)A(t) - \gamma P(t) - (\mu_1+\mu)A(t) - u_2 A(t)R(t)\} \\ & + \lambda_3\{(1-u_1(t))\beta S(t) + \gamma P(t) - u_3 P(t)R(t) - \mu P(t)\} \\ & + \lambda_4\{u_2 A(t)R(t) + u_3 P(t)R(t) - \mu R(t)\}.\end{aligned} \tag{15}$$

Then, the adjoint system (12) is derived by taking the partial derivatives of the Hamiltonian (15) with respect to the corresponding state variables, so that

$$-\frac{d\lambda_j}{dt} = \frac{\partial\mathscr{H}}{\partial j},$$

where $j = (S,A,P,R)$. Furthermore, the optimal control triple $u^* = (u_1^*, u_2^*, u_3^*)$ are derived by solving

$$\frac{\partial\mathscr{H}}{\partial u_i} = 0 \text{ (where } i = 1,2,3)$$

on the interior of the control set $\mathfrak{U}$.

By standard control arguments involving bounds on the control, we obtain

$$u_i^* = \begin{cases} 0 & \text{if } \theta_i^* \le 0 \\ \theta_i^* & \text{if } 0 \le \theta_i^* \le 1 \\ 1 & \text{if } \theta_i^* \ge 1 \end{cases}$$

for $i = 1,\ldots,3$ and where

$$\theta_1 = \frac{((\lambda_2-\lambda_1)\alpha_1 A(t) + (\lambda_3-\lambda_1)\beta)S(t)}{a_1}, \;\; \theta_2 = \frac{(\lambda_2-\lambda_4)A(t)R(t)}{a_2}, \;\; \theta_3 = \frac{(\lambda_3-\lambda_4)P(t)R(t)}{a_3}.$$

4 Numerical simulations and cost-effectiveness analysis

4.1 Numerical simulations

The present section explores the dynamical behaviours of alcoholism addict and poverty model (7) and the derived two-point boundary problem of a eight-dimensional optimality system through numerical simulations. The numerical simulations are performed in MATLAB with `ode45` routine. The model's initial conditions are taken as

$$(S(0),A(0),P(0),R(0))) = (100,10,20,10),$$

while the model parameter values and their sources are as defined in Table 3.

Table 3: Parameter values used in simulating model (7)

Parameter	Range	Baseline value	Source
Λ	5 - 20	10	[39,58]
μ	0.02 - 0.2	0.0143	[59,60]
α_1	0.0000096 - 0.00096	0.000096	[61]
β	0.00064 - 0.064	0.0064	[61]
γ	0.000008 - 0.0008	0.00008	[39,58]
μ_1	0.00162 - 0.162	0.0162	[61]

4.1.1 Non-autonomous system

An iterative forward-backwards sweep method (FBSM) based on the fourth-order Runge-Kutta method implemented in MATLAB is applied to the eight-dimensional optimality system, which is a two-point boundary problem, consisting of the state system (7) and the adjoint system (12) with the control characterisations (14) within the time interval of $[0,10]$ years to find the optimal control strategies required to minimise the menace of alcoholism addict and poverty problem in a population at minimum cost. Because the optimality system has different time orientations, the solutions of equations of the non-autonomous system (7) with initial conditions and the control initial guess are sought forward in time. Whereas the equations of the adjoint system (12) with terminal conditions (13) are solved backwards in time using the state system's current iteration solution. We refer the readers interested in the numerical procedure for the simulation of an optimality system with different time orientations using FBSM to Lenhart and Workman [62].

The weight constants values W_i and a_j (where $j = 1,2$ and $j = 1,2,3$) appearing in the objective functional (8), in addition to the parameter values given in Table 3, are taken as $W_1 = 0.0075$, $W_2 = 0.003$, $a_1 = 0.03$, $a_2 = 0.04$ and $a_3 = 0.03$. It is important to state that these weight constants values are theoretical as they are taken only to carry out the numerical illustrations of the control problem proposed and theoretically analysed in this work.

Different combinations of optimal control strategies are considered to optimise the objective functional (8). Specifically, we focus on the combination of at least any two optimal controls. This leads us to considering four different combinations of strategies, which are described as follows: Strategy A - combination of optimal public awareness control and optimal use of rehabilitation rate control (i.e. $u_1(t)$ and $u_2(t)$ with $u_3(t) = 0$), Strategy B - combination of optimal public awareness control and optimal empowering rate control (i.e. $u_1(t)$ and $u_3(t)$ with $u_2(t) = 0$), Strategy C - combination of optimal public awareness control, optimal empowering rate control, and optimal use of rehabilitation control (i.e. $u_1(t)$, $u_2(t)$, and $u_3(t)$), Strategy D - combination of optimal empowering rate control and rehabilitation rate control (i.e. $u_2(t)$, and $u_3(t)$ with $u_1(t) = 0$),

Strategy A - combination of optimal public awareness control and optimal use of rehabilitation rate control (i.e. $u_1(t)$ and $u_2(t)$ with $u_3(t) = 0$),

In Fig. 2, the simulation of optimal control problem with the effort of public awareness measures $u_1(t)$ combined with rehabilitation rate control $u_2(t)$ (i.e., $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 = 0$) is illustrated. Fig. 2a shows the effect of Strategy A on the Alcohol addict population and it is clear that in less than first year of administering the controls we achieve an alcoholism free population, it proof that Strategy A is good in controlling Alcoholism burden in a population and in Fig 2b, Strategy

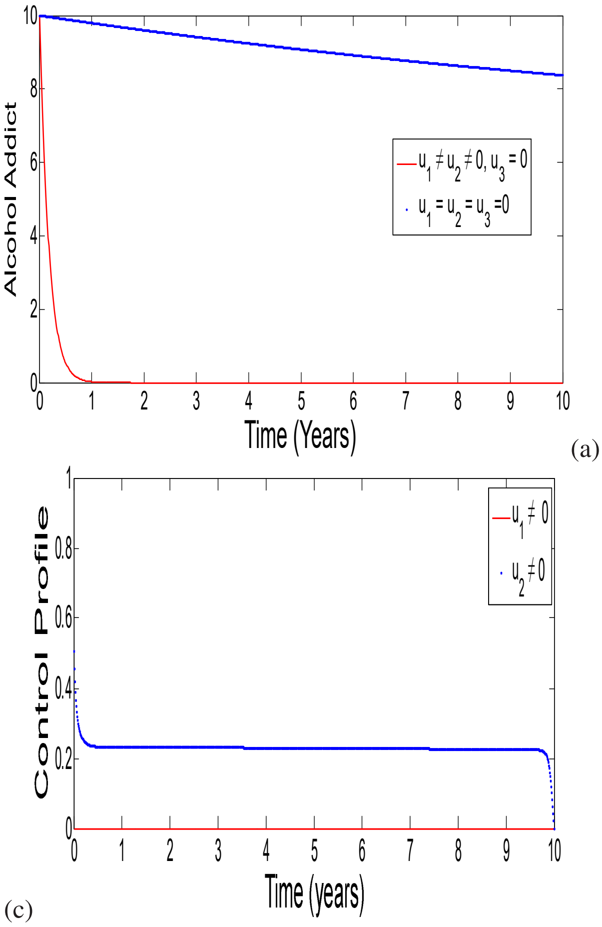

Fig. 2: Simulation showing the effect of optimal use of strategy A on the population dynamics of alcoholism addict and poverty.

A was examined on Poverty population and the impact of it was slight, meaning that it has little effect on it. In Fig 2c, the public awareness control $(u_1(t))$ is at 2% at through the period of applying the control and rehabilitation control $(u_2(t))$ is at 37% at the starting point but decreases to 24% in first 3 months of administering the control and dropped down to zero at finial time.

Strategy B - combination of optimal public awareness control and optimal empowering rate control (i.e. $u_1(t)$ and $u_3(t)$ with $u_2(t) = 0$),

The impact of the synergy of the combination of public awareness control $u_1(t)$ and empowering rate control $u_3(t)$ (i.e., $u_1 \neq 0, u_3 \neq 0$ and $u_2 = 0$) on the dynamics of poverty and alcoholism is investigated by simulating the optimal control problem. In Fig. 3, the simulation of optimal control problem with the effort of public awareness measures $u_1(t)$ with synergy of empowering rate control $u_3(t)$ (i.e., $u_1 \neq 0$, $u_3 \neq 0$ and $u_2 = 0$) is illustrated. Fig. 3a shows the effect of Strategy B on the Alcohol addict population it is obvious that administration of this controls did not have any effect on Alcoholism population and in Fig 3b, Strategy B was examined on Poverty population and it is clear that in less than first seven months of administering the controls we achieve an poverty free population, it proof that Strategy B is good in controlling poverty burden in a population. In Fig 3c, the public awareness control $(u_1(t))$ is at 9% at the beginning and reduced gradually to zero at the 8 year and remains at origin for the rest period of applying the control and empowering rate control $(u_3(t))$ is at 50% at the starting point but decreases to 30% in first 3 months of administering the control and dropped down to zero at finial time.

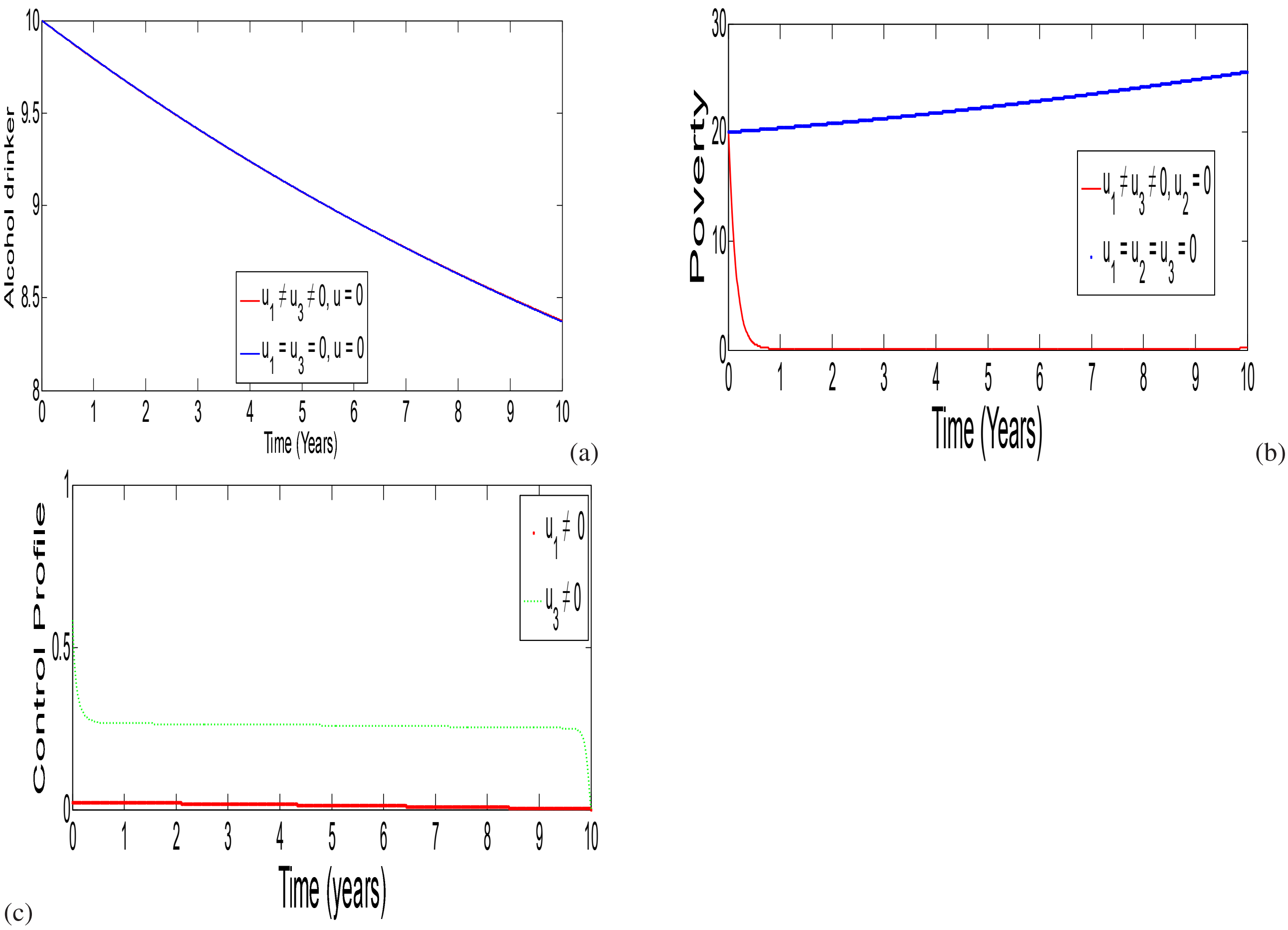

Fig. 3: Simulation showing the effect of optimal use of strategy B on the population dynamics of alcoholism addict and poverty.

Strategy C: combination of optimal awareness control, optimal empowering rate control, and optimal use of rehabilitation control (i.e. $u_1 \neq 0$, $u_2 \neq 0$, and $u_3 \neq 0$)

To demonstrate the impact of the combination of public awareness control $u_1(t)$, empowering rate control $(u_2(t))$, and rehabilitation rate control $(u_3(t))$ on the community spread of poverty and alcoholism, the optimal control problem is simulated. In Fig. 4, the simulation of optimal control problem with the effort of public awareness measures $u_1(t)$ and empowering rate control $u_2(t)$ with synergy of rehabilitation rate control $(u_3(t))$ (i.e., $u_1 \neq 0$, $u_2 \neq 0$ and $u_3 \neq 0$) is illustrated. Fig. 4a-4b shows the effect of Strategy C on the Alcohol addict and Poverty populations, it is evident that administration of this controls has a positive effect on both population. In Fig 4c, the public awareness control $(u_1(t))$ is at 2% at the beginning and reduced gradually to zero at finite time of applying the control, empowering rate control $(u_2(t))$ is at 50% at the starting point but decreases to 30% in first 3 months of administering the control and dropped down to zero at finial time, and rehabilitation rate control $(u_3(t))$ is at 80% at the starting point but decreases to 30% in first 3 months of administering the control and dropped down to zero at finial time.

Strategy D: combination of optimal empowering rate control and rehabilitation rate control (i.e. $u_2(t) \neq 0$ with $u_3(t) \neq 0$, and $u_1 = 0$)

Strategy D - combination of optimal empowering rate control and rehabilitation rate control (i.e. $u2(t) \neq 0$, and $u3(t) \neq 0$ with $u1(t) = 0$) In Fig. 5, the simulation of optimal control problem with the effort of public awareness measures $u_1(t)$ with synergy of empowering rate control $u_3(t)$ (i.e., $u_2 \neq 0$, $u_3 \neq 0$ and $u_1 = 0$) is illustrated. Fig. 5a-5b shows the effect of Strategy C on the Alcohol addict and Poverty populations, it is evident that administration of this controls has a positive

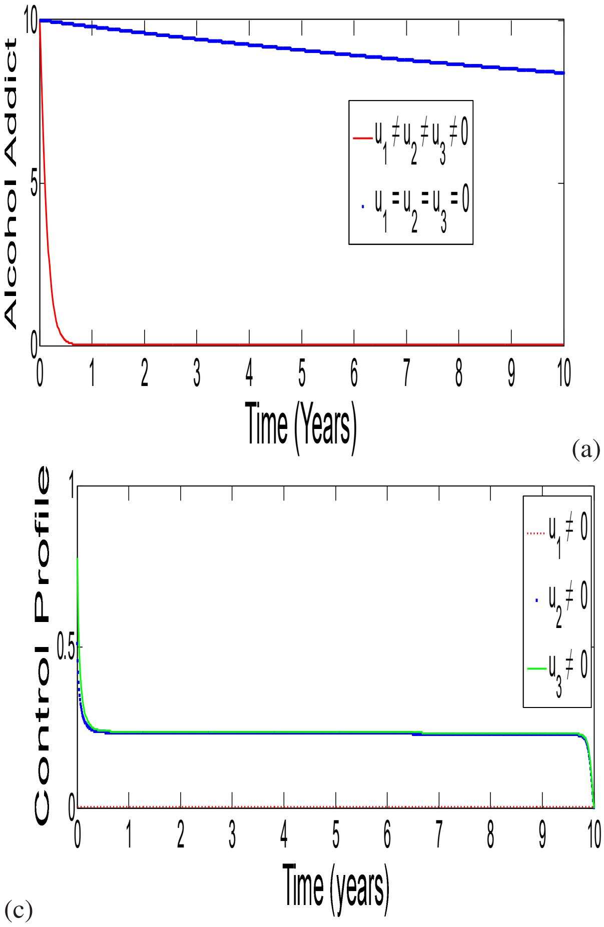

Fig. 4: Simulation showing the effect of optimal use of strategy C on the population dynamics of alcoholism addict and poverty.

effect on both population. In Fig 5c, the empowering rate control ($u_2(t)$) is at 50% at the starting point but decreases to 30% in first 3 months of administering the control and dropped down to zero at finial time, and rehabilitation rate control ($u_3(t)$) is at 80% at the starting point but decreases to 30% in first 3 months of administering the control and dropped down to zero at finial time.

4.2 Cost-effectiveness analysis

With the help of ideas from previous studies [47,50,51,53,54,63,64,65,66], we explore the cost-effectiveness analysis (economic evaluation) of the four control combination strategies under investigation using two cost analysis methods, namely, average cost-effectiveness ratio (ACER) and incremental cost-effectiveness ratio (ICER), in this section. This is necessary as there is a need to determine the most effective and least costly intervention strategy among the various combinations considered based on the results arising from the simulations of the optimal control problem.

4.2.1 Average cost-effectiveness ratio

The cost analysis method considered is the ACER. This method deals with a single intervention strategy and weighing the intervention against its baseline option. The formula to calculate ACER is given as

$$\text{ACER} = \frac{\text{Total cost produced by the intervention}}{\text{Total number of problem averted}}. \tag{16}$$

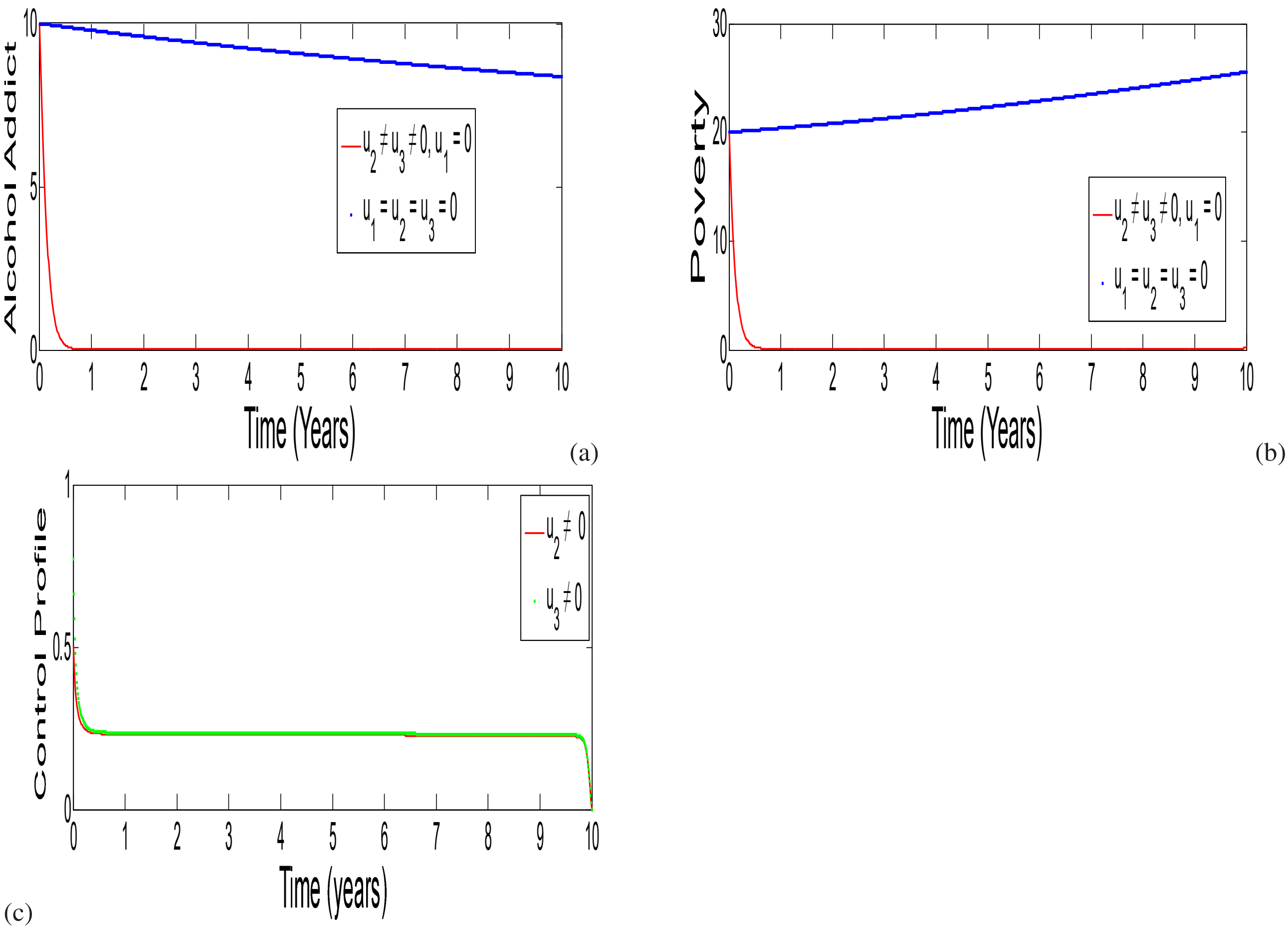

Fig. 5: Simulation showing the effect of optimal use of strategy D on the population dynamics of alcoholism addict and poverty.

In view of the objective functional (8), the total cost produced by an intervention is expressed mathematically as

$$\text{TC} = \int_0^{t_f} \sum_{i=1}^{4} a_i u_i^2 dt. \tag{17}$$

A strategy with the least ACER value is the most cost-effective according to this cost analysis approach [63,64,67]. Thus, the ACER is calculated for each of the eleven strategies using the formula (16). The numerical results obtained are presented by bar chart and pie chart in Fig. 6 and 7 respectively (see also Table 4).

Table 4: Intervention strategy, total alcoholism addict and poverty averted, total cost, and ACER

Strategy	Total alcoholism addict and poverty incidence averted	Total cost ($)	ACER
A: $u_1(t), u_2(t)$	0.0855288831	0.2033125727	2.3771218018
B: $u_1(t), u_3(t)$	0.2534248934	0.1021145709	0.402938202
C: $u_1(t), u_2(t), u_3(t)$	0.3374591813	0.1907763236	0.5653315547
D: $u_2(t), u_3(t)$	0.3374555275	0.1906495218	0.5649619172

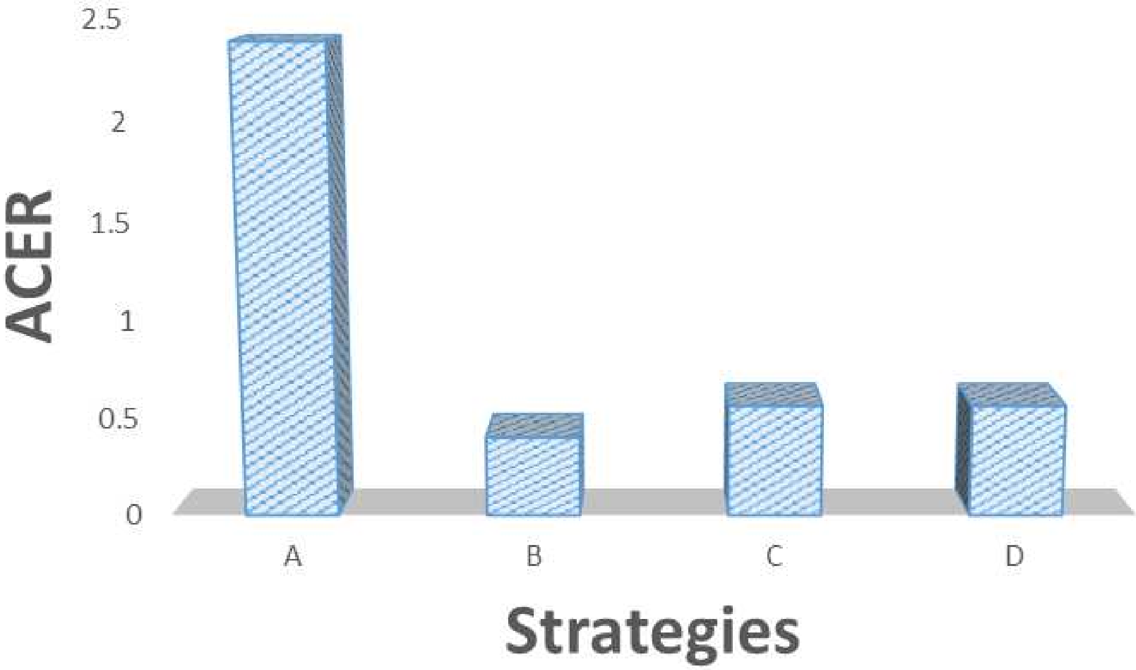

Fig. 6: ACER plots of all the implemented intervention strategies

It is shown that Strategy **B** has the least ratio, and therefore is the most cost-effective strategy using ACER cost analysis approach. This is followed by Strategy D, Strategy C, and Strategy A. The least cost-effective strategy (a strategy with the highest ratio) is Strategy A.

To further affirm this result, we compute the ICER values for the various control combination strategies.

4.2.2 Incremental cost-effectiveness ratio

On the other hand, the **ICER** has to do with the comparison of the differences between the costs and health outcomes of the alternative intervention strategies (usually two or more) competing for the same resources. To compare competing intervention strategies (usually two or more) incrementally, one intervention strategy is compared with the next less-effective alternative strategy. Simply put, the **ICER** is stated as

$$\text{ICER} = \frac{\triangle \text{ in costs expended on control strategies implementation}}{\triangle \text{ in total number of infections averted by the competing strategies}}. \tag{18}$$

In Table 5, the total numbers of alcoholism addict and poverty averted by strategies A, B, C, and D are arranged from the least to the highest alongside the respective associated costs expended on the implementation of the four strategies. Thus, using (18), we obtain the numerical results in Table 5 following the computation of ICER as follows:

$$\begin{aligned}
\text{ICER(A)} &= \frac{0.2033125727}{0.0855288831} = 2.3771218018, \\
\text{ICER(B)} &= \frac{0.1021145709 - 0.2033125727}{0.2534248934 - 0.0855288831} = -0.6027421475, \\
\text{ICER(C)} &= \frac{0.1907763236 - 0.1021145709}{0.3374591813 - 0.2534248934} = 34.7040888937, \\
\text{ICER(D)} &= \frac{0.1906495218 - 0.1907763236}{0.3374555275 - 0.3374591813} = 1.0536032704.
\end{aligned}$$

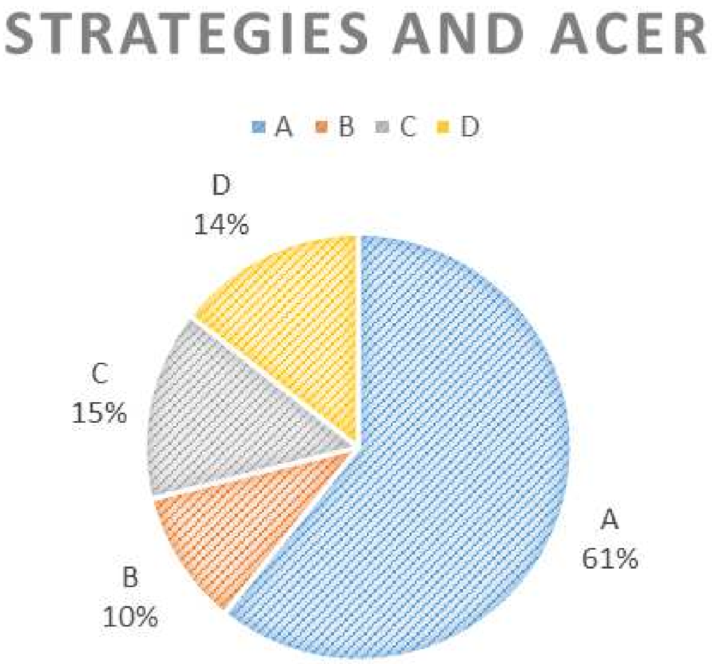

Fig. 7: ACER pie of all the implemented intervention strategies

Table 5: Intervention strategy, total alcoholism addict and poverty averted, total cost, and ICER

Strategy	Total alcoholism addict and poverty incidence averted	Total cost ($)	ICER
A: $u_1(t), u_2(t)$	0.0855288831	0.2033125727	2.3771218018
B: $u_1(t), u_3(t)$	0.2534248934	0.1021145709	-0.6027421475
C: $u_2(t), u_3(t)$	0.3374555275	0.1906495218	1.0536032704
D: $u_1(t), u_2(t), u_3(t)$	0.3374591813	0.1907763236	34.7040888937

Comparing Strategy B and Strategy A in Table 5, it is seen that ICER(B) is less than ICER(A). This indicates that Strategy B strongly dominates Strategy A. Thus, Strategy B has greater effectiveness at cheaper cost compared to Strategy A. Therefore, Strategy A is left out from further consideration. Next, Strategy B is further compared with Strategy C. Hence, using (18), the summary of ICER is given in Table 6. A look at Table 6 reveals that Strategy B strongly dominates

Table 6: Comparison of strategies B and C

Strategy	Total alcoholism addict and poverty incidence averted	Total cost ($)	ICER
B: $u_1(t), u_3(t)$	0.2534248934	0.1021145709	0.4029382020
C: $u_2(t), u_3(t)$	0.3374555275	0.1906495218	1.0536032704
D: $u_1(t), u_2(t), u_3(t)$	0.3374591813	0.1907763236	34.7040888937

Strategy C as ICER(C) is greater than ICER(B). This means that strategy C is less effective and more expensive to implement than Strategy B. Therefore, it is best to remove Strategy C from the set of intervention strategies to implement

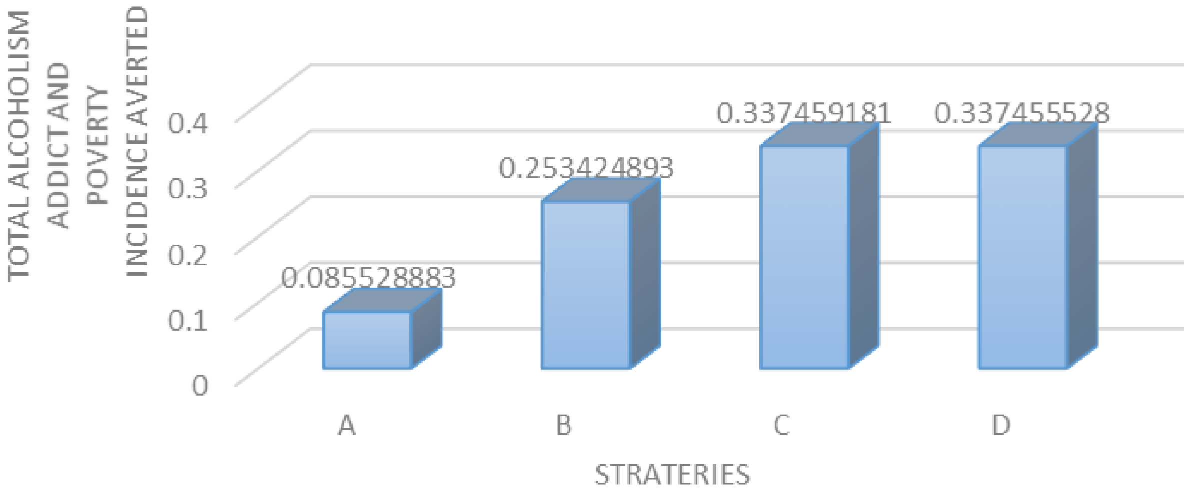

Fig. 8: Plots of the total poverty and alcoholism incidence averted by all the implemented intervention strategies

in order to preserve the limited resources. Consequently, Strategy C is excluded and Strategy B is further compared with Strategy D. Thus, using (18), the summary of ICER is given in Table 7.

Table 7: Comparison of strategies B and D

Strategy	Total alcoholism addict and poverty incidence averted	Total cost ($)	ICER
B: $u_1(t), u_3(t)$	0.2534248934	0.1021145709	0.4029382020
D: $u_1(t), u_2(t), u_3(t)$	0.3374591813	0.1907763236	1.0550663892

Upon the comparison of Strategy B and Strategy D, it is shown that ICER(B) is lower than ICER(D). This means that strategy B strongly dominates Strategy D. Thus, Strategy B is more effective and less costly than Strategy D. A look at Table 7 suggests that Strategy B is strongly dominated by Strategy D, which comes from the comparison of ICER(B) and ICER(D), where ICER(D) is greater than ICER(B). Thus, Strategy B is more effective and less costly than Strategy D. Consequently, the most cost-effective strategy according to ICER cost analysis approach is Strategy B, confirming the results of the ACER methods obtained earlier. It therefore follows from the cost-effectiveness analysis that combination of the effort of public awareness measures with empowerment rate control is the most effective and least costly control intervention strategy capable of diminishing the burden of alcoholism addict and poverty optimally in the population.

It is worthy of note that Strategy D produces the least total cost required to achieve the optimal control of illicit drug use and terrorism in the population as given in Table 5. This is clearly presented in Fig. 8, 9, 10 and 11, which also reaffirms the results obtained from the ACER and ICER methods that Strategy D is the most cost-effective strategy.

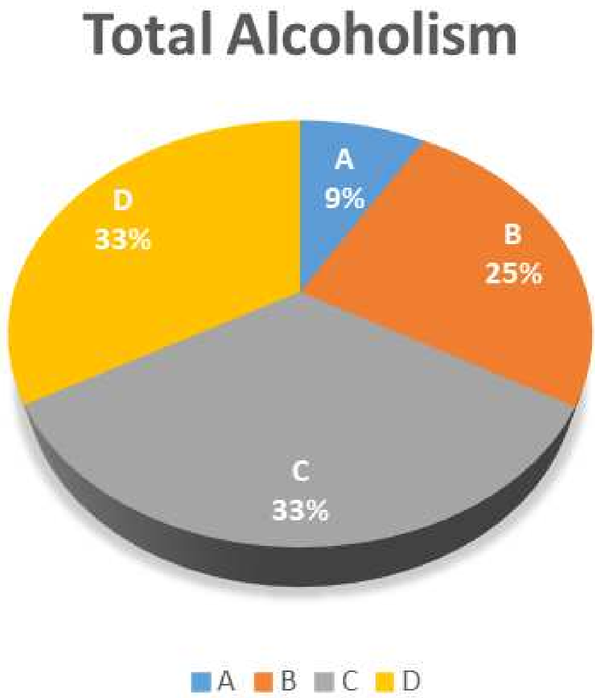

Fig. 9: Pie chart of the total poverty and alcoholism incidence averted by all the implemented intervention strategies

5 Conclusion

In this research, we extend the work of [39] by considering some analysis under the autonomous system, which they did not consider and primary extents it to the non-autonomous system. It is because poverty and alcoholism are among the top challenges confronting societies worldwide and an essential subject of research in the last few decades. A deterministic model governed by a system of nonlinear differential equations was developed in this work. The primary purpose of this work is to proffer a solution to the burden of poverty and alcoholism in a population. The model was studied under two different systems, namely: autonomous and non-autonomous systems. Under the autonomous system, the following analysis was carried out; Poverty and alcoholism reproduction number $\mathscr{R}_0$ using the next-generation method, which was done to know all the parameters contributing to the dynamics of poverty and alcoholism in a population.

The local and global sensitivity analysis was done using the normalized forward method and Latin hypercube sampling and partial rank correlation coefficients, respectively; this was done to identify the parameters of the model that most influence thy dynamical behaviour of the model. The following parameters has a positive index (i.e σ_1 and Λ) while μ and μ_1 has a negative index. The two sensitivity analysis used established worked in agreement. The non-autonomous model with three time-dependent controls; the controls are public awareness control, rehabilitation control and empowerment rate control were used. Pontryagin's Maximum Principle is to find the optimal solution to the poverty and alcoholism control problem.

Cost-effectiveness analysis was conducted to further affirm the results of the optimal control problem by using the average cost-effectiveness ratio and incremental cost-effectiveness ratio methods. The cost-effectiveness analysis shows that Strategy B, which combines optimal public awareness control and empowering rate control, is the most cost-effective. It further affirms the saying that precaution is better than cure. Numerical simulations were presented to buttress all the qualitative results.

Conflict of interest

None.

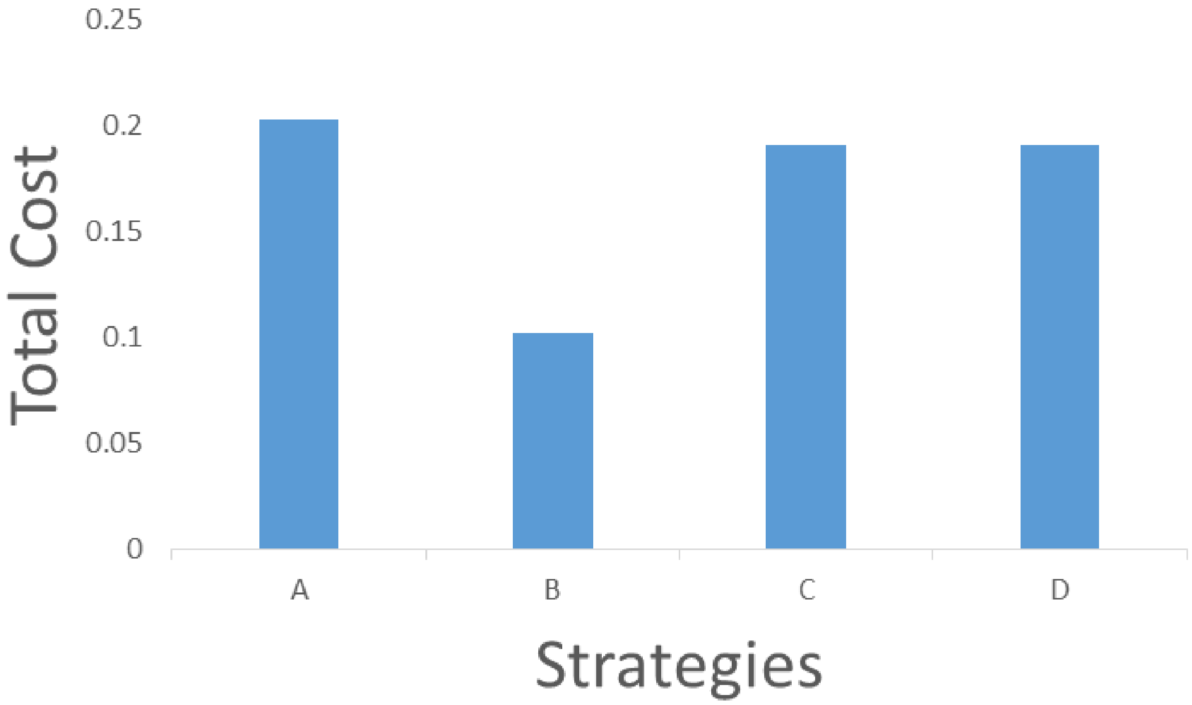

Fig. 10: Plots of the total cost expended on all the implemented intervention strategies

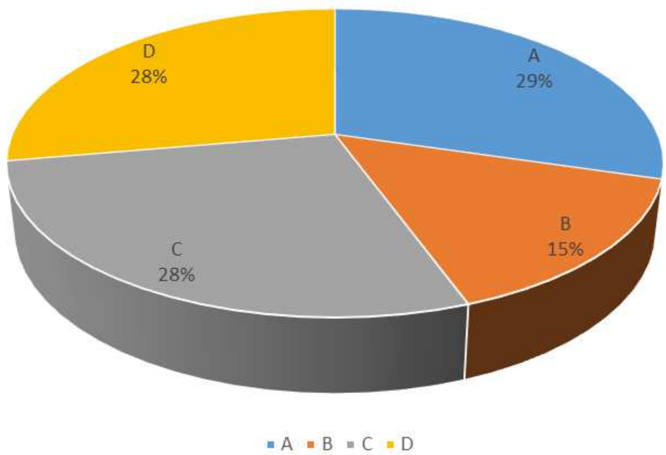

Fig. 11: Pie chart of the total cost expended on all the implemented intervention strategies

Data availability statement

All data generated or analysed during this study are included in this article.

Acknowledgement

The authors are grateful to the anonymous referee for a careful checking of the details and for helpful comments that improved this paper.

References

[1] M.S. Awan, N. Malik, H. Sarwar and M. Waqas, Impact of education on poverty reduction, Int J Acad Res 3(1) (2011) 659–664.
[2] M.I. Jabir, L. Mensah, A. Gyeke-Dako, Financial inclusion and poverty reduction in sub-saharan africa, Afr Fin J 19 (2017) 1–22.
[3] World Bank Group, Washington DC, Poverty and shared prosperity 2016: taking on inequality, https :// openk nowle dge.world bank.org/bitst ream/handl e/10986 /25078 /97814 64809 583.pdf.
[4] C. Neilson, D. Conteras, R. Cooper and J. Hermann, The dynamics of poverty in chile, Journal of Latin American Studies 40(2) (2008) 251–273.
[5] M. Chibba, Financial inclusion, poverty reduction and the millennium development goals, Eur J Dev Res 21 (2009) 213–230.
[6] C. Park, R.V. Mercado, Financial inclusion, poverty, and income inequality in developing asia. adb economics working paper series 426/2015, Manila, Philippines. https ://www.adb.org/sites /defau lt/files /publi catio n/153143/ewp-426.pdf (2015) 13 pages.
[7] N. Apergis, O. Dincer and J. E. Payne, On the dynamics of poverty and income inequality in us states, Journal of Economic Studies 38(2) (2011) 132–143.
[8] M. Desai, Human development: concepts and measurement, Eur Econ Rev 35 (1991) 350–357.
[9] J. Schmied, A. Marr, Financial inclusion and poverty: the case of peru., Region Sect Econ Stud 16(2) (2016) 29–43.
[10] Z. A. Hatta and I. Ali, Poverty reduction policies in malaysia: trends, strategies and challenges, Asian Culture and History 5(2) (2013) 48–56.
[11] K. B. Carey, Heavy drinking contexts and indices of problem drinking among college students, Journal of Studies on Alcohol 56(3) (1995) 287–292.
[12] V. M. Gonzalez, R. L. Collins, and C. M. Bradizza, Solitary and social heavy drinking, suicidal ideation, and drinking motives in underage college drinkers, Addictive Behaviors 34(12) (2009) 993–999.
[13] B. Khajji, A.Labzai, O. Balatif, M. Rachik, Mathematical modeling and analysis of an alcohol drinking model with the influence of alcohol treatment centers,, International Journal of Mathematics and Mathematical Sciences Article ID 4903168 (2020) 13 pages.
[14] B. A. Reboussin, E.-Y. Song, and M. Wolfson, Social influences on the clustering of underage risky drinking and its consequences in communities,, Journal of Studies on Alcohol and Drugs 73(6) (2012) 890–898.
[15] L. A. Crawford, K. B. Novak, and R. R. Jayasekare, Volunteerism, alcohol beliefs, and first-year college students' drinking behaviors: implications for prevention,, The Journal of Primary Prevention 40(4) (2019) 429–448.
[16] National cancer institute, https://www.cancer.gov/publications/dictionaries/cancer-terms/def/alcoholism,.
[17] E. A. P. Poelen, R. H. J. Scholte, G. Willemsen, D. I. Boomsma, and R. C. M. E. Engels, Drinking by parents, siblings, and friends as predictors of regular alcohol use in adolescents and young adults: a longitudinal twin-family study,, Alcohol and Alcoholism 42(4).
[18] R. H. J. Scholte, E. A. P. Poelen, G. Willemsen, D. I. Boomsma, and R. C. M. E. Engels, Relative risks of adolescent and young adult alcohol use: the role of drinking fathers, mothers, siblings, and friends,, Addictive Behaviors 33(1) (2008) 1–14.
[19] H. Xiang, C.-C. Zhu, and H.-F. Huo, Modelling the effect of immigration on drinking behaviour,, Journal of Biological Dynamics 11(1) (2017) 275–298.
[20] World health organization,
https://apps.who.int/iris/bitstream/handle/10665/274603/9789241565639-eng.pdf,.
[21] K.E. Stanovich, Studying the consequences of literacy within a literate society: The cognitive correlates of print exposure,, Memory & Cognition 20(1) (1992) 51–68.
[22] J. H. Kietzmann, K. Hermkens, I.P. McCarthy, B.S. Silvestre, Social medial get serious! understanding the functional building blocks of social media,, Business Horizons 54(3) (2011) 241–251.
[23] J.A. Obar and S. Wildman, Social media definition and the governance challenge: an introduction to the special issue,, Telecommunications Policy 39(3) (2015) 745–750.
[24] D. R. Olson, Mindand media: The epistemic functions of literacy,, Journal of Communication 38 (1988) 27–36.
[25] J.O. Akanni, Mathematical analysis of the role of detection rate on dynamical spread of ebola virus disease,, Journal of Modeling and Simulation Materials 3(1) (2020) 37–52.
[26] J.O. Akanni, Asymptotic stability of illicit drug dynamics with banditry compartment,, Appl. Math. Inf. Sci. 14(5) (2020) 791–800.
[27] A. Kouidere, A. Labzai, H. Ferjouchia, O. Balatif and M. Rachik, A new mathematical modeling with optimal control strategy for the dynamics of population of diabetics and its complications with effect of behavioral factors,, J. of Applied Mathematics https://doi.org/10.1155/2020/1943410.
[28] A.A Ayoade, O. Odetunde and B. Falodun, Modeling and analysis of the impact of vocational educationon the unemployment rate in nigeria,, Application and Applied Mathematics: An International Journal (AAM) 15(1) (2020) 550–564.
[29] A. Gambo and M.O. Ibrahim, Mathematical modeling of dynamics behavior of terrorism and control,, Capsian Journal Mathematical Sciences 9(1) (2020) 68–89.

[30] A.A. Ayoade, P.A. Ikpechukwu, S. Thota, O.J. Peter, Modeling the effect of quarantine and hospitalization on the spread of covid-19 during the toughest period of the pandemic,, J. Mahani Math. Res. 12(1) (2023) 339–361.
[31] S.C. Duncan, T.E. Duncan, L.A. Strycker, A multilevel analysis of neighborhood context and youth alcohol and drug problems,, Prev Sci. 3 (2002) 125–133.
[32] M.A. Sakib, M.A. Islam, P. Shahrear and U. Habiba, Dynamics of poverty and drug addiction in sylhet, bangladesh,, Journal of Multidisciplinary Engineering Science and Technology 4(2) (2017) 6562–6569.
[33] G. Murakami and K. Hashimoto, Associations of education and income with heavy drinking and problem drinking among men: evidence from a populationbased study in japan,, BMC Public Health 19 (2019) 420,https://doi.org/10.1186/s12889–019–6790–5.
[34] H. F. Huo and N. N. Song, Global stability for a binge drinking model with two stages,, Discrete Dynamics in Nature and Society Article ID 829386.
[35] H.-F. Huo, Y.-L. Chen, and H. Xiang, Stability of a binge drinking model with delay,, Journal of Biological Dynamics 11(1) (2017) 210–225.
[36] S. H. Ma, H. F. Huo, and X. Y. Meng, Modelling alcoholism as a contagious disease: a mathematical model with awareness programs and time delay,, Discrete Dynamics in Nature and Society Article ID 260195.
[37] S. Sharma and G. P. Samanta, Drinking as an epidemic: a mathematical model with dynamics behavior,, Journal of Applied Mathematics Informatics 31(1-2) (2013) 1–25.
[38] S. Sharma and G. P. Samanta, Analysis of a drinking epidemic model,, International Journal of Dynamics and Control 3(3) (2015) 288–305.
[39] K. Chinnadurai and S. Athithan, Mathematical modelling of poverty dynamics with the effect of alcohol consumption,, AIP Conference Proceedings 2277, 130001 (2020) 12 pages, https://doi.org/10.1063/5.0025263.
[40] U. A. Mohd Roslan, S. Z., A. A. and S. M. Abd. Malik, A mathematical model on the dynamics of poverty, poor and crime in west malaysia,, Far East Journal of Mathematical Sciences (FJMS) 107(2) (2018) 309–319.
[41] P. van den Driessche, J. Watmough, Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission, Mathematical biosciences 180 (2002) 29–48.
[42] J.K. Kanyaa, S. Osman and M. Wainaina, Mathematical modelling of substance abuse by commercial drivers, Global Journal of Pure and Applied Mathematics, ISSN 0973-1768 Volume 14, Number 9, pp. 1149-1165 Research India Publications, 14:9.
[43] J. O. Akanni, Mathematical assessment of the role of illicit drug use on terrorism spread dynamics, Journal of Applied Mathematics and Computing DOI: 10.1007/s12190-021-01674-y (2022) 28.
[44] O. D. Makinde, J. O. Akanni, A. Abidemi, Modelling the impact of drug abuse on a nation's education sector, Journal of Applied Nonlinear Dynamics.
[45] A. Abidemi, M. I. Abd Aziz, R. Ahmad, Vaccination and vector control effect on dengue virus transmission dynamics: Modelling and simulation, Chaos, Solitons & Fractals 133 (2020) 109648.
[46] J. Akanni, F. Akinpelu, S. Olaniyi, A. Oladipo, A. Ogunsola, Modelling financial crime population dynamics: optimal control and cost-effectiveness analysis, International Journal of Dynamics and Control 8 (2) (2020) 531–544.
[47] J. P. Romero-Leiton, J. E. Castellanos, E. Ibargüen-Mondragón, An optimal control problem and cost-effectiveness analysis of malaria disease with vertical transmission applied to San Andrés de Tumaco (Colombia), Computational and Applied Mathematics 38 (3) (2019) 1–24.
[48] H. W. Berhe, O. D. Makinde, Computational modelling and optimal control of measles epidemic in human population, Biosystems 190 (2020) 104102.
[49] D. Okuonghae, A. B. Gumel, B. O. Ikhimwin, E. Iboi, Mathematical assessment of the role of early latent infections and targeted control strategies on syphilis transmission dynamics, Acta Biotheoreticadoi:https://doi.org/10.1007/s10441-018-9336-9.
[50] M. Ghosh, S. Olaniyi, O. S. Obabiyi, Mathematical analysis of reinfection and relapse in malaria dynamics, Applied Mathematics and Computation 373 (2020) 125044.
[51] H. W. Berhe, O. D. Makinde, D. M. Theuri, Optimal control and cost-effectiveness analysis for dysentery epidemic model, Applied Mathematics and Information Sciences 12 (6) (2018) 1183–1195.
[52] A. Abidemi, R. Ahmad, N. A. B. Aziz, Global stability and optimal control of dengue with two coexisting virus serotypes, MATEMATIKA: Malaysian Journal of Industrial and Applied Mathematics 35 (4) (2019) 149–170.
[53] A. Abidemi, N. A. B. Aziz, Optimal control strategies for dengue fever spread in Johor, Malaysia, Computer Methods and Programs in Biomedicine 196 (2020) 105585.
[54] A. Abidemi, J. Akanni, Dynamics of illicit drug use and banditry population with optimal control strategies and cost-effectiveness analysis, Computational and Applied Mathematics 41 (1) (2022) 1–37.
[55] S. F. Abimbade, S. Olaniyi, O. A. Ajala, M. O. Ibrahim, Optimal control analysis of a tuberculosis model with exogenous re-infection and incomplete treatment, Optimal Control Applications and Methods 41 (6) (2020) 2349–2368.
[56] W. H. Fleming, R. W. Rishel, Deterministic and stochastic optimal control, Springer, New York, 1975.
[57] L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze, E. F. Mishchenko, The Mathematical Theory of Optimal Processes, Interscience, New York, 1962.
[58] F. Nyabadza and L. Coetzee, A systems dynamic model for drug abuse and drug-related crime in western cape province of south africa,, Hindawi Computational and Mathematical Methods in Medicine https://doi.org/10.1155/2017/4074197 (2017) 13 pages.
[59] A. S. Kalula and F. Nyabadza, A theoretical model for substance abuse in the presence of treatment,, South Africa Journal of Science htp://doi.org/10.4102/sajs.v108i3/4.654 (2012) 12 pages.

[60] A. Pluddemann,S. Dada and C. Parry, Monitoring alcohol and substance abuse trends in south africa, SACENDU Research brief 13(2) (2010) 1–16.

[61] K. Peltzer, S. Ramlagan, G. Mohlala and G. Matseke, Illicit drug use trends in south africa. human sciences research council, Available at: http://www.ncbi.nlm.nih-.gov/pmc/articles/PMC3010753/. [Online; accessed on August 15, 2020].

[62] S. Lenhart, J. T. Workman, Optimal control applied to biological models, CRC Press, London, 2007.

[63] F. B. Agusto, I. M. ELmojtaba, Optimal control and cost-effective analysis of malaria/visceral leishmaniasis co-infection, PLoS One 12 (2) (2017) e0171102.

[64] J. K. K. Asamoah, E. Yankson, E. Okyere, G.-Q. Sun, Z. Jin, R. Jan, Optimal control and cost-effectiveness analysis for dengue fever model with asymptomatic and partial immune individuals, Results in Physics 31 (2021) 104919.

[65] J. K. K. Asamoah, Z. Jin, G.-Q. Sun, B. Seidu, E. Yankson, A. Abidemi, F. Oduro, S. E. Moore, E. Okyere, Sensitivity assessment and optimal economic evaluation of a new COVID-19 compartmental epidemic model with control interventions, Chaos, Solitons & Fractals 146 (2021) 110885.

[66] J. O. Akanni, S. Ajao, S. F. Abimbade, F. Fatmawati, Dynamical analysis of covid-19 and tuberculosis co-infection using mathematical modelling approach, Mathematical Modelling and Control (2024), 4(2) 208–229.

[67] S. Olaniyi, O. S. Obabiyi, K. O. Okosun, A. T. Oladipo, S. O. Adewale, Mathematical modelling and optimal cost-effective control of COVID-19 transmission dynamics, The European Physical Journal Plus 135 (11) (2021) 1–20.