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On Weighted Nwikpe Distribution: Properties and Applications

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Abstract: In this article, two-parameter continuous distribution has been introduced. The proposed distribution is obtained by using a weight technique and is referred to as weighted Nwikpe distribution. This distribution is a generalization of baseline distribution that is Nwikpe distribution. Some structural properties of the distribution has been derived and studied. These are density function, distribution function, and reliability function, hazard rate function, moments, moment generating function, entropies, order statistics, Bonferroni and Lorenz curves. The method of maximum likelihood technique has been established to investigate the parameters of the model. The behaviour of the parameters of the distribution has been examined by a simulation study. Real data set is used to determine whether the weighted Nwikpe distribution is better than other well-known distributions in modeling data or not.

Keywords: Weighted distribution, Nwikpe distribution, Maximum likelihood estimator, Entropies, Order statistics, Bonferroni and Lorenz curves.

1 Introduction

The theory of weighted distributions provides a collective access to the problems of model specification and data interpretation. Weighted distributions provide a technique for fitting the models to the weight functions when the samples are taken from original distribution and developed distributions. The idea of weighted distributions was first given by Fisher [1] to model the ascertainment bias. Rao [2] introduced and formulated the method of ascertainment for modeling statistical data when the standard distributions were not appropriate to record these observations with equal probabilities. Weighted distributions can be applied in various research areas related to biomedicine, reliability, and ecology and branching processes. Researchers have applied the concept of weighted distributions for different purposes. Rao [3] extended the concept of ascertainment upon the estimation of frequencies and introduced the concept of weighted distributions. Lappi and Bailey [4] used weighted distributions to analyse the HPS diameter increment data.

Castillo and Casany [5] developed weighted Poisson distribution by considering the Poisson distribution with parameter λ and weight

$$w(k) = (k+\alpha)^r .$$

The pdf of the weighted Poisson distribution is defined as

$$P_w(k,\lambda,r,\alpha) = \frac{(k+\alpha)^r \ \lambda^r \ e^{-\lambda}}{\left(E\,(x+\alpha)^r\right)k\,!},$$

$$k = 0,1,2\ldots \,, \alpha \geq 0 \,, \lambda > 0 \,, r \in \Re$$

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Gupta and Kundu [6] have introduced a new class of weighted exponential distributions by applying the method proposed by Azzalini [7] to the exponential distribution. The two-parameter weighted exponential distribution introduced by Gupta and Kundu has pdf defined as

$$f_w(x,\alpha,\beta)=\frac{\alpha+1}{\alpha!}\beta(1-\exp(-(\alpha\beta x)))\exp(-(\alpha\beta x))$$

Kersey [8] introduced the weighted inverse Weibull distribution. The proposed distribution namely weighted inverse distribution was obtained by using $w(x)=x$ as a weigh function and pdf of the inverse Weibull distribution. The pdf of the newly generated distribution is defined as

$$f_w(x,\alpha,\beta)=\frac{\beta\alpha^{1-\beta}}{\Gamma\left(1-\frac{1}{\beta}\right)}x^{-\beta}\exp(-(x\alpha))^{-\beta},x>0$$

Ye et al. [9] introduced a weighted generalized beta distribution of the second kind. The proposed distribution namely weighted generalized beta distribution was obtained by using $w(x)=x^k$ as a weigh function and pdf of the inverse Weibull distribution. The pdf of the newly generated distribution defined as

$$f_w(x,\alpha,\beta,p,q,k)=\frac{\alpha x^{(\alpha p-1)}}{\beta^{\alpha p+k}B(p+k/\alpha,q-k/\alpha)\left(1+\left(x/\beta\right)^{\alpha}\right)^{p+q}},\quad x>0$$

Aleem et al. [10] developed the class of modified Weibull distribution and its properties. The distribution was generated by considering the cumulative distribution function as weigh and pdf of the Weibull distribution in the weighted model. The pdf of the modified weighted Weibull distribution is defined as

$$f_w(x)=\beta_\gamma(c\theta^r+1)x^{\gamma-1}e^{-\beta(c\theta^\gamma+1)x^\gamma},x>0$$

Bashir and Rahul [11] introduced the weighted Lindley distribution. The pdf of the weighted Lindley distribution is given by

$$f_w(x,\theta,\Phi)=\frac{(\theta-\varphi)^2}{\theta-\varphi+1}(1+x)\exp(-(\theta-\varphi))x,x>0$$

Asgharzadeh et al. [12] introduced a new weighted Lindley distribution with application to survival analysis. The pdf of a new weighted Lindley distribution is given by

$$f_w(x,\lambda,\alpha)=\frac{\lambda^2(1+\alpha)^2}{\alpha\lambda(1+\alpha)+\alpha(2+\alpha)}(1+x)$$

$$(1-\exp(-\alpha\lambda x)\exp(-\lambda x),\quad x>0$$

Para and Jan [13] introduced the Weighted Pareto type-II distribution as a new model for handling medical science data and studied its statistical properties and applications. In addition, further work on the weighted model has been done by the various researchers like Shakhatreh [14] proposed two-parameter weighted exponential distribution, Sherina et al. [15] introduced Weighted Weibull distribution, Nasiru [16] obtained weighted Weibull distribution, Domma et al. [17] introduced a new generalized weighted Weibull distribution, Ghitany [18] et al. obtained two-parameter weighted Lindley distribution, Dey et al [19] obtained weighted Weibull distribution, Badmus and Bamiduro et al. [20] proposed Lehmann Type II weighted Weibull, Alqallaf et al. [21] introduced Weighted exponential, Alsmarian [22] developed weighted Suja, Mizaal [23] discuss the mathematical study of weighted two-parameters exponential distribution. Saghir et al. [24] discussed the brief perspective and characterizations of the weighted distributions. Mahdavi [25] introduced the two weighted distributions generated by the exponential distribution. The author(s) has obtained the two distributions by incorporating the exponential distribution in Azzalini’s model to get the weighted gamma-exponential model and the weighted generalized exponential-exponential model. Shanker and Shukla [26] obtained weighted Akash distribution. Dey and Perk [27] proposed weighted exponential distribution and its properties with the application.

A class of bivariate modified weighted distributions are obtained by Hiba Zeyada [28] based on the Marshall and Olkin concept. The statistical properties of the model were obtained including joint pdf, joint cdf, joint survival function, product moments, marginal conditional density are obtained in compact forms. Shoaee [29] introduced a new class of bivariate survival distributions based on the model of dependent lives and its generalization. A new class of distributions using a functional mixture was proposed by Bouali, Chesneau et al. [30].

The weighted exponential family of distributions are introduced by Zubair et al. [31].The general expressions for the mathematical properties are studied. The length biased weighted exponentiated inverted Weibull distribution was obtained by Tazeem, Ishfaq et al. [32]. The properties of the distribution have been discussed along with parameter estimation.

Nwikpe, et al. [33] proposed a one-parameter distribution called as Nwikpe distribution. The proposed model aims to model the strength of the aircraft window data set. The distribution is a mixture model consists of the gamma distribution, and exponential distribution with θ parameter. They studied the properties of the proposed model. Parameters of the proposed distribution are presented and examined with the method of maximum likelihood estimation. The probability density function and cumulative distribution function of the Nwikpe distribution is given by

$$f(x,\theta)=\left(\frac{\theta^3}{12(\theta^2+10)}\right)\left(\theta^3 x^5+12\right)e^{-\theta x}$$

$$x>0, \theta>0 \qquad (1)$$

$$F(x,\theta)=1-\left(1+\frac{\theta^5x^5+5\theta^3x^3(4+x)+60\theta x(\theta x+2)}{12(\theta^2+10)}\right)e^{-\theta x} \qquad (2)$$

2 Weighted Nwikpe Distributions: Definition and Properties

2.1 Weighted Nwikpe Distribution

Let X be a random variable with probability density function $f(x)$. Let $w(x)$ be the non-negative weight function, then the probability density function of the weighted random variable X_w is given by

$$f_w(x)=\frac{w(x)f(x)}{E(w(x))};\ \ x>0$$

Where, $w(x)$ is a non-negative weight function and

$$E(w(x))=\int w(x)f(x)<\infty,\ 0<E\left(w(x)\right)<\infty$$

For different weighted models, we can have different choices of the weight function *w(x)*. In this paper, we study the weighted version of Nwikpe distribution, taking $w(x)=x^c$, in order to get the weighted Nwikpe distribution, the probability density function (pdf) of weighted version is given by

$$f_w(x)=\frac{x^c f(x)}{E(X^c)} \qquad (3)$$

Where,

$$E(X^c)=\int_0^\infty x^c f(x)\,dx$$

$$E(X^c)=\frac{(c+5)!+12\theta^2 c!}{(12(\theta^2+10))\theta^c} \qquad (4)$$

Substituting the values of equation (1) and (4) in equation (3), we get the probability density function (pdf) of weighted Nwikpe distribution.

$$f_w(x,c,\theta)=\left(\frac{\theta^{c+3}\ x^c(\theta^3x^5+12)e^{-\theta x}}{12\theta^2c!+(c+5)!}\right),\ x>0,\theta>0,c>0 \qquad (5)$$

The cumulative distribution function (cdf) of the weighted Nwikpe distribution is obtained as

$$F_w(x,c,\theta)=\int_0^x f_w(x,c,\theta)\,dx$$

$$F_w(x,\theta,c)=\int_0^x\left(\frac{\theta^{c+3}\ x^c(\theta^3x^5+12)e^{-\theta x}}{12\theta^2c!+(c+5)!}\right)dx \qquad (6)$$

After simplification, using an incomplete gamma function

$$\gamma\left((c+1),\theta x\right)=\int_0^{\theta x} t^{(c+1)-1}e^{-t}\,dx$$

to equation (6). We get the cumulative distribution function of weighted Nwikpe distribution as

$$F_w(x,c,\theta)=\frac{\gamma\left(c+6;\theta x\right)+12\theta^2\gamma\left(c+1;\theta x\right)}{12\ \theta^2\ c!+(c+5)!} \qquad (7)$$

Where, $\gamma(s;x)=\int_0^x t^{s-1}e^{-t}\,dt$ is an incomplete gamma function.

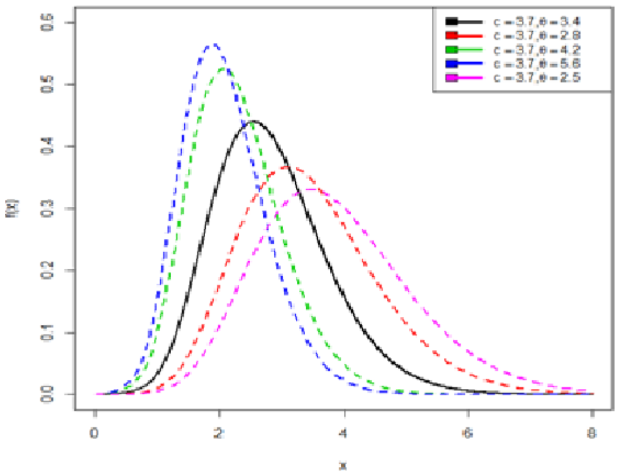

Fig.1: pdf plot of weighted Nwikpe distribution

2.2 *Reliability Function and Hazard Rate*

The reliability function of the weighted Nwikpe distribution can be obtained as

$$R_w(x,c,\theta)=1-F_w(x,c,\theta)$$

$$R_w(x,c,\theta)=1-\frac{\gamma(c+6;\theta x)+12\theta^2\gamma(c+1;\theta x)}{12\,\theta^2\,c!+(c+5)!}$$

$$;x>0,\theta>0,c>0$$

The hazard function or failure rate of weighted Nwikpe distribution is given by

$$h_w(x,c,\theta)=\frac{f_w(x,c,\theta)}{R_w(x,c,\theta)}$$

$$h_w(x,\theta,c)=\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)e^{-\theta x}}{12\theta^2c!+(c+5)!-\gamma(c+6;\theta x)+12\theta^2\gamma(c+1;\theta x)}$$

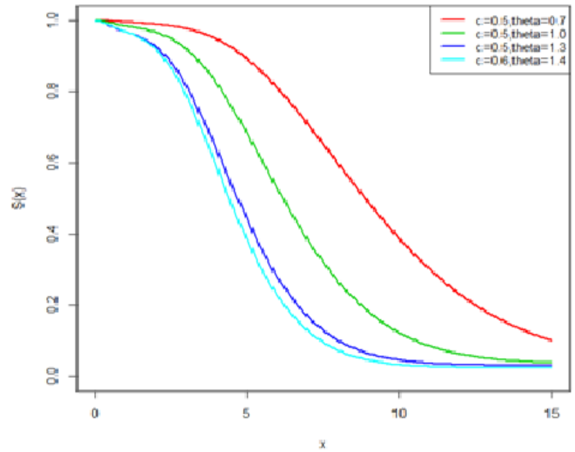

Fig.3: Reliability plot of weighted Nwikpe distribution

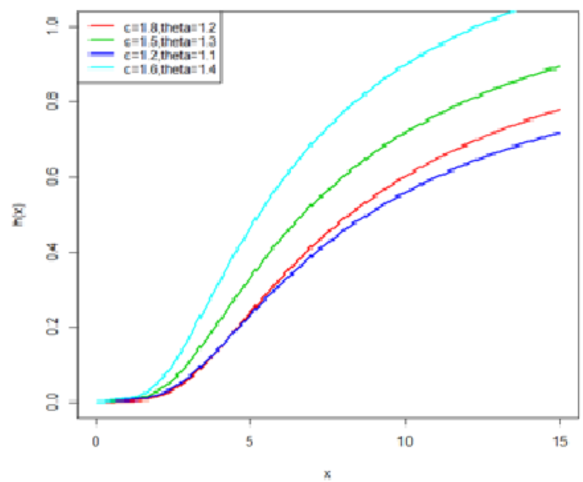

Fig.4: Hazard plot of weighted Nwikpe distribution

The figure 4 depicts that the hazard rate of the weighted distribution is increasing for the parametric value $\alpha > 1$ and $\theta > 1$.

2.3 Moments

Let X_w denote the random variable following weighted Nwikpe distribution then r^{th} order moment $E(X^r)$ is obtained a

$$E(X_w^r)=\mu_r'=\int_0^\infty x^r f_w(x,c,\theta)dx$$

$$E(X_w^r)=\left(\frac{\theta^3(c+r+5)!+12\theta^5(c+r)!}{\theta^r\left(12\theta^2c!+(c+5)!\right)}\right) \tag{8}$$

Letting $r = 1$ in equation (8), the mean of weighted Nwikpe distribution is obtained as

$$\mu_1'=\left(\frac{\theta^3(c+6)!+12\theta^5(c+1)!}{\theta\left(12\theta^2c!+(c+5)!\right)}\right)$$

$$\mu_2'=\left(\frac{\theta^3(c+7)!+12\theta^5(c+2)!}{\theta^2\left(12\theta^2c!+(c+5)!\right)}\right)$$

$$\mu_3'=\left(\frac{\theta^3(c+8)!+12\theta^5(c+3)!}{\theta^3\left(12\theta^2c!+(c+5)!\right)}\right)$$

$$\mu_4'=\left(\frac{\theta^3(c+9)!+12\theta^5(c+4)!}{\theta^4\left(12\theta^2c!+(c+5)!\right)}\right)$$

Variance = $\mu_2' - (\mu_1')^2$

$$\mu_2=\left(\frac{\theta^3(c+7)!+12\theta^5(c+2)!}{\theta^2\left(12\theta^2c!+(c+5)!\right)}\right)-\left(\frac{\theta^3(c+6)!+12\theta^5(c+1)!}{\theta\left(12\theta^2c!+(c+5)!\right)}\right)^2$$

$$S.D(\sigma)=\sqrt{\left(\frac{\left(\theta^3(c+7)!+12\theta^5(c+2)!\right)\left(12\theta^2c!+(c+5)!\right)-\left(\theta^3(c+6)!+12\theta^5(c+1)!\right)^2}{\theta^2\left(12\theta^2c!+(c+5)!\right)^2}\right)}$$

2.4 Moment Generating Function and Characteristic Function

Let X_w follows Weighted Nwikpe distribution, then the moment generating function (mgf) of X is obtained as

$$M_{X_w}(t)=E(e^{tx})=\int_0^\infty e^{tx}f_w(x,c,\theta)dx$$

$$M_{X_w}(t)=\int_0^\infty\sum_{j=0}^\infty\frac{(tx)^j}{j!}f_w(x,c,\theta)\,dx$$

$$M_{X_w}(t)=\sum_{j=0}^\infty\frac{(t)^j}{j!}\int_0^\infty x^jf_w(x,c,\theta)\,dx$$

$$M_{X_w}(t)=\sum_{j=0}^\infty\frac{t^j}{j!}\left(\frac{\theta^3(c+r+5)!+12\theta^5(c+r)!}{\theta^r\left(12\theta^2c!+(c+5)!\right)}\right)$$

Similarly, we can get the characteristic function of weighted Nwikpe distribution as

$$\varphi_{X_w}(t)=M_{X_w}(it)$$

$$\varphi_{X_w}(t)=\sum_{j=0}^\infty\frac{(it)^j}{j!}\mu_j'$$

$$\varphi_{X_w}(t)=\sum_{j=0}^\infty\frac{(it)^j}{j!}\left(\frac{\theta^3(c+r+5)!+12\theta^5(c+r)!}{\theta^r\left(12\theta^2c!+(c+5)!\right)}\right)$$

3 Order Statistics

Let $x_{(1)},x_{(2)},x_{(3)}\ldots,x_{(n)}$ be the order statistics of a random sample $x_1,x_2,x_3\ldots,x_n$ drawn from the continuous population with probability density function $f_X(x)$ and cumulative distribution function with $F_X(x)$ then the pdf of r^{th} order statistics $X_{(r)}$ is given by

$$f_{Xw(r)}(x)=\frac{n!}{(r-1)!\,(n-r)!}f_X(x)\left[F_X(x)\right]^{r-1}\left[1-F_X(x)\right]^{n-r} \tag{9}$$

Inserting equations (5) and (7) in equation (9), the probability density function of r^{th} order statistic $X_{(r)}$ of Weighted Nwikpe distribution is given by

$$f_{Xw(r)}(x)=\frac{n!}{(r-1)!(n-1)!}\left(\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)\,e^{-\theta x}}{12\,\theta^2c!+(c+5)!}\right)$$

$$\left(\frac{\gamma(c+6;\theta\,x)+12\,\theta^2\gamma(c+1;\theta\,x)}{12\;\theta^2\;c!+(c+5)!}\right)$$

$$\left(1-\frac{\gamma(c+6;\theta\,x)+12\,\theta^2\gamma(c+1;\theta\,x)}{12\;\theta^2\;c!+(c+5)!}\right)^{n-r}$$

Therefore, the probability density function of highest order statistics $X_{w(n)}$ can be obtained as

$$f_{Xw(n)}(x)=n\left(\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)\,e^{-\theta x}}{12\,\theta^2c!+(c+5)!}\right)$$

$$\left(\frac{\gamma(c+6;\theta\,x)+12\,\theta^2\gamma(c+1;\theta\,x)}{12\;\theta^2\;c!+(c+5)!}\right)^{n-1}$$

and pdf of first order statistic $X_{w(1)}$ can be obtained as

$$f_{Xw(1)}(x)=\left(\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)\,e^{-\theta x}}{12\,\theta^2c!+(c+5)!}\right)$$

$$\left(1-\frac{\gamma(c+6;\theta\,x)+12\,\theta^2\gamma(c+1;\theta\,x)}{12\;\theta^2\;c!+(c+5)!}\right)^{n-1}$$

4 Entropies

4.1 Renyi Entropy

Renyi entropy [34] of order δ can be obtained as

$$R(\delta)=\frac{1}{1-\delta}\log\left(\int f_w^{\delta}(x,c,\theta)dx\right)$$

$$R(\delta)=\frac{1}{1-\delta}\log\int_0^{\infty}\left(\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)\,e^{-\theta x}}{12\,\theta^2c!+(c+5)!}\right)^{\delta}dx \qquad (10)$$

Using Binomial expansion to equation (10), we get the renyi entropy of weighted Nwikpe distribution

$$R(\delta)=\frac{1}{1-\delta}\log\left(\frac{\theta^{c+3}}{12\,\theta^2c!+(c+5)!}\right)^{\delta}\sum_{i=0}^{\infty}\binom{\delta}{i}\frac{\Gamma(\delta\,c+5i+1)}{(\theta\delta)^{(\gamma\delta\,c+5i+1)}}$$

4.2 Tsallis Entropy

Tsallis entropy [35] of order λ is given by

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\int f_w^{\lambda}(x,c,\theta)\,dx\right)$$

$$S_{\lambda}=\frac{1}{1-\lambda}\left(1-\int_0^{\infty}\left(\frac{\theta^{c+3}\,x^c(\theta^3x^5+12)\,e^{-\theta x}}{12\,\theta^2c!+(c+5)!}\right)^{\lambda}dx\right) \qquad (11)$$

Using Binomial expansion to equation (11), we get Tsallis entropy of weighted Nwikpe distribution.

$$S(\lambda)=\frac{1}{1-\lambda}\left(1-\left(\frac{\theta^{c+3}}{12\theta^2c!+(c+5)!}\right)^{\lambda}\sum_{i=0}^{\infty}\binom{\lambda}{i}\frac{\Gamma(\lambda\,c+5i+1)}{(\theta\lambda)^{(\lambda\,c+5i+1)}}\right)$$

5 Bonferroni and Lorenz Curves

The Bonferroni [36] and Lorenz [37] curves can be obtained as

$$B(p)=\frac{1}{p\mu_1'}\int_0^q x\,f(x,c,\theta)dx$$

and

$$L(p)=\frac{1}{\mu_1'}\int_0^q x\,f(x,c,\theta)dx$$

Where,

$$\mu_1'=\left(\frac{\theta^3(c+6)!+12\,\theta^5\,(c+1)!}{\theta\left(12\,\theta^2c!+(c+5)!\right)}\right)$$

$$q=\;F^{-1}(p)$$

On simplification, we get,

$$B(p)=\left(\frac{\gamma\left((c+7),\theta q\right)+12\theta^{2}\gamma\left((c+2),\theta q\right)}{p\,(12\theta^{5}(c+1)!+\theta^{2}(c+6)!)}\right)$$

Where, $q=F^{-1}(p)$

$$L(p)=p\,B(p)$$

$$L(p)=\left(\frac{\gamma\left((c+7),\theta q\right)+12\theta^{2}\gamma\left((c+2),\theta q\right)}{(12\theta^{5}(c+1)!+\theta^{2}(c+6)!)}\right)$$

6 Estimation of Parameters

6.1 Maximum Likelihood Estimation

Consider $x_1, x_2, x_3 \ldots, x_n$ be a random sample of size n from weighted Nwikpe distribution with parameters θ, α, the Likelihood function is defined as

$$L(x;c,\theta)=\left(\left(\frac{\theta^{c+3}}{12\theta^{2}c!+(c+5)!}\right)\right)^{n}\prod_{i=1}^{n}x_i^{\,c}(\theta^{3}x_i^{5}+12)\,e^{-\theta x_i} \tag{12}$$

The log-likelihood function is

$$\log L=n\,(c+3)\log\theta-n\,\log\left(12\theta^{2}c!+(c+5)!\right) \tag{13}$$

$$-\theta\sum_{i=1}^{n}x_i+\sum_{i=1}^{n}\log x_i^{\,c}\left(\theta^{3}x_i^{\,5}+12\right)$$

The maximum likelihood estimates of θ and c can be obtained by differentiating equation (13) with respect to θ and c and must satisfy the normal equations.

$$\frac{\partial\log L}{\partial\theta}=\frac{n\,(c+3)}{\theta}-\frac{24n\,\theta c!}{12\theta^{2}c!+(c+5)!}+$$

(1

$$\sum_{i=1}^{n}\frac{3\theta^{2}x_i^{\,5}}{\left(\theta^{3}x_i^{\,5}+12\right)}-\sum_{i=1}^{n}x_i=0$$

$$\frac{\partial\log L}{\partial c}=n\log\theta-n\left(\psi(c)+\psi(c+5)\right)+\sum_{i=1}^{n}\log x_i=0 \tag{15}$$

The solution of equations (14) and (15) gives the maximum likelihood estimates of the parameters for the weighted Nwikpe distribution. However the equations cannot be solved analytically, thus we solved numerically using R programming [38] with some data set.

6.2 Simulation Study of ML Estimators

In this subsection, we study the performance of ML estimators for different sample sizes. However, we set the sample size N= 25, 50, 75,100, 250, and 400. We have employed the inverse CDF technique for data simulation for weighted Nwikpe distribution using R software. Bias, Variance and MSE for the weighted Nwikpe distribution is observed. We found that, as the sample size increase, the values of MSE are getting smaller for the parameter estimate.

7 Likelihood Ratio Test

Let $x_1, x_2, x_3 \ldots, x_n$ be a random sample from the weighted Nwikpe Distribution.

To test the hypothesis we have,

$$H_o: f(x)=f(x,\theta)\ \text{ against }\ H_1: f(x)=f_w(x,c,\theta)$$

In order to test whether the random sample of size n comes from the Nwikpe distribution or weighted Nwikpe distribution, the following test statistic is used

$$\Delta=\frac{L_1}{L_0}=\prod_{i=1}^{n}\frac{f_w(x_i,c,\theta)}{f(x_i,\theta)}$$

$$\Delta = \prod_{i=1}^{n}\left(\frac{\theta^{c}\left(12\theta^{2}+10\right)}{12\theta^{2}c!+(c+5)!}\right)x_i^{\,c}$$

We reject the null hypothesis if

$$\Delta = \left(\frac{\theta^{c}\left(12\theta^{2}+10\right)}{12\theta^{2}c!+(c+5)!}\right)^{n}\prod_{i=1}^{n}x_i^{\,c} > k$$

$$\Delta = \prod_{i=1}^{n}x_i^{\,c} > k\left(\frac{12\theta^{2}c!+(c+5)!}{\theta^{c}\left(12\theta^{2}+10\right)}\right)^{n}$$

$$\Delta^{*} = \prod_{i=1}^{n}x_i^{\,c} > k^{*},$$

Where, $k^{*} = k\left(\frac{12\theta^{2}c!+(c+5)!}{\theta^{c}\left(12\theta^{2}+10\right)}\right)^{n}$

For large sample size n, $2\log\Delta$ is distributed as chi-square variate with one degree freedom. Thus we reject the null hypothesis, when the probability value is given by

$p\left(\Delta^{*} > \alpha^{*}\right)$, where $\alpha^{*} = \prod_{i=1}^{n}x_i^{c}$ is less than level of significance and $\prod_{i=1}^{n}x_i^{c}$ is the observed value of the statistic Δ^{*}.

8 llustration

In this section, we determine the flexibility of the generalization of the Nwikpe distribution by fitting the model to two data sets over the Nwikpe, Exponential, Shanker, Akash, Amarendra, and Lindley distribution. The results and the performance of the weighted Nwikpe distribution are determined by using Akaike Information Criterion, Bayesian Information Criterion, Akaike Information Criterion Corrected and -2ln L. The distribution with the lowest AIC, BIC, AICC and -2ln L is considered as the most flexible distribution for a given data set.

8.1 Data Set 1

The following data represent the tensile strength, measured in GPA, of 69 carbon fibers tested under tension at gauge lengths of 20mm, Bader and Priest [39].

1.312, 1.314, 1.479, 1.552, 1.700, 1.803,1.861, 1.865, 1.944, 1.958, 1.966, 1.997, 2.006, 2.021, 2.027, 2.055, 2.063, 2.098, 2.14, 2.179, 2.224, 2.240, 2.253,2.270, 2.272, 2.274, 2.301, 2.301, 2.359, 2.382, 2.382, 2.426, 2.434, 2.435, 2.478, 2.490, 2.511, 2.514, 2.535, 2.554, 2.566, 2.57, 2.586, 2.629, 2.633, 2.642, 2.648, 2.684, 2.697, 2.726,2.770,2.773, 2.800, 2.809, 2.818, 2.821, 2.848, 2.88, 2.954, 3.012, 3.067, 3.084, 3.090, 3.096, 3.128, 3.233, 3.433, 3.585, 3.585.

8.2 Data Set 2

The data set represents the strength of glass of the aircraft windows given by Fuller et al. [40].

18.83, 20.80, 21.657, 23.03, 23.23, 24.05, 24.321, 25.50, 25.52, 25.80, 26.69, 26.77, 26.78, 27.05, 27.67, 29.90, 31.11, 33.20, 33.73, 33.76, 33.89, 34.76, 35.75, 35.91, 36.98, 37.08, 37.09, 39.58, 44.045, 45.29, 45.381.

It can be easily seen from table 2 and 3 that the weighted Nwikpe distribution gives better fit than all the considered distributions and hence it can be considered as an important two-parameter lifetime distribution for modeling lifetime data.

Table 1: Bias, Variance and ML Estimators for Different Sample Sizes.

Sample Size	θ =1.2			c = 0.3		
N	Bias	Variance	MSE	Bias	Variance	MSE
25	20.43670	257.7957	675.4567	18.79052	364.8161	717.8996
50	17.16548	94.49843	389.152	14.8353	129.2258	349.3110
75	10.85597	31.54542	149.3978	7.58014	43.3548	100.8133
100	16.14025	58.48277	318.9883	13.6211	79.0009	264.5340
250	11.62651	5.990382	141.1662	8.30251	7.27722	76.20384
400	10.13205	2.411737	105.0701	6.61653	3.384882	47.16331
	θ = 0.3			c = 0.5		
25	78.13097	11.13845	8.184896	33.49206	260.9222	1382.640
50	8.17184	7.249752	74.02872	33.02830	178.8993	1269.768
75	8.019381	2.337856	66.64833	32.42125	56.44626	1107.584
100	7.305664	5.308559	58.68128	28.84660	60.7805	954.9066
250	6.978366	2.820293	51.51788	27.53613	65.42063	823.6592
400	6.843113	1.098669	47.92686	26.56396	25.49759	731.1413
	θ = 2			c = 1		
25	18.71878	157.2793	507.6720	36.78858	329.0645	1682.464
50	54.52185	599.4488	3572.081	31.68343	306.4980	1310.338
75	37.15250	256.7097	1637.018	19.57602	123.1012	506.3216
100	38.81826	87.44561	1594.303	20.53530	41.38000	463.0787
250	40.65167	84.95280	1737.511	21.95189	38.94486	520.8301
400	37.46984	32.12828	1436.117	19.74412	16.22667	406.0568

Table 2: MLEs AIC, BIC AICc and -2 log L of the fitted distribution for the given data set 1

Distribution	ML Estimates	-2 log L	AIC	BIC	AICc
Weighted Nwikpe	$\hat{\theta} = 9.5402$ $\hat{c} = 17.3879$	100.0697	104.0697	108.5379	104.4389
Nwikpe	$\hat{\theta} = 2.1555$	198.8053	200.8053	200.6442	200.8621
Exponential	$\hat{\theta} = 0.4079$	261.7432	263.7411	265.9655	263.8011
Shanker	$\hat{\theta} = 0.6582$	233.0054	235.0054	237.2376	235.0135
Amarendra	$\hat{\theta} = 1.2445$	207.947	209.947	209.7750	210.0121
Sujatha	$\hat{\theta} = 0.9363$	221.6088	223.6088	225.8355	223.6688
Akash	$\hat{\theta} = 0.9647$	224.2798	226.2797	228.5132	226.3424
Lindley	$\hat{\theta} = 0.6590$	238.3667	240.3659	242.6134	240.4400

Table 3: MLEs AIC, BIC AICc and -2 log L of the fitted distribution for the given data set 2

Distribution	ML Estimates	-2 log L	AIC	BIC	AICc
Weighted Nwikpe	$\hat{\theta} = 0.6145$ $\hat{c} = 12.9342$	208.2313	212.2313	215.0992	213.1203
Nwikpe	$\hat{\theta} = 0.1945$	223.3051	225.3051	224.7964	225.4402
Exponential	$\hat{\theta} = 0.1640$	454.9130	456.9100	459.3044	456.959
Shanker	$\hat{\theta} = 0.3084$	408.9220	410.9216	410.8300	410.9722
Amarendra	$\hat{\theta} = 0.6015$	373.9708	375.9707	375.8792	376.0212
Sujatha	$\hat{\theta} = 0.4403$	392.3864	394.3863	394.2949	394.4369
Akash	$\hat{\theta} = 0.4605$	388.6078	390.6073	390.5162	390.5668
Lindley	$\hat{\theta} = 0.2910$	418.5780	420.5781	420.4782	420.6286

9 Conclusion

In this article, we have introduced a new generalization of the two- parameter continuous distribution termed as Weighted Nwikpe distribution. The new distribution is generated by using the weighting technique. The method of maximum likelihood estimation is used for estimating the parameters of the model. Some statistical properties along with reliability measures has been discussed.The simulation study is carried out to know the performance of the parameters of the model. The result observed from table 1 depicts that the bias, variance and MSE decrease when the sample size tends to increase. The applicability of proposed model is demonstrated by using a goodness of fit criterion such as AIC, BIC, AICC and -2ln L. It has been noted from table 2 and 3 that the weighted Nwikpe distribution provides a better fit than competing models Nwikpe distribution, Exponential, Shanker, Amarendra, Sujatha, Akash, Lindley distributions

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Competing interests
The authors declare that they have no competing interests.

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