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The Research of based on Iterative Companding Transform Method for The Reduction of Peak-to-Average Power Ratio of OFDM Signal

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Abstract: Due to its robustness to multipath fading and high spectral utilization efficiency, Orthogonal Frequency Division Multiplexing(OFDM) is a very attractive technique for high rate wireless data communications, however, the main drawback of it is the high peak-to-average power ratio(PAPR) of the OFDM signal. In this paper, the iterative companding transform technology is presented to reduce PAPR, what kind of method is the optimal is obtained through the simulation that not only compare to the original PAPR, the PAPR of different companding transform rate, and the iterative companding transform PAPR, but also the BER performance of several methods.

Keywords: Iterative Companding-Transform, OFDM, PAPR, BER.

1 Introduction

Since the dynamic range of the signal is much larger for high PAPR [1], a high-resolution quantizer is required to reduce quantization error, which requires more bits and places a complexityand power burden on the receiver front end. Now, many solutions have been proposed to reduce PAPR such as block coding [2], selective mapping (SLM), partial transmit sequence(PTS), tone reser-vation and injection [3]. However, most of these solutions have restrictions on system parameters such as number of subcarriers, frame format, and constellation type [4]. Signal distortion solutions such as clipping [5], and companding [6,7,8,9,10] can be used without restriction on the system parameters but at the price of increased bit error rate(BER) [11] and spectral regrowth [12]. Although clipping performs very well with low modulation orders, clipping error [13] becomes very significant with higher orders and seriously degrades performance, which makes companding transform more suitable for high data rates applications.

This paper proposes the iterative companding transform technology to reduce PAPR. According to μ law compression technology [14] that imitate the voice signal to compresses the OFDM signal, the basic tenet of iterative companding transform is that compressing the high power transmission signal, and making the small power emission signal amplify, so reducing the system PAPR. This paper mainly discusses and compares the PAPR performance of iterative companding transform signal to the original companding transform signal.

The rest of this paper is outlined as follows; Section 2 analyses the PAPR of OFDM signal. Section 3 addresses the companding transform. Section 4 presents the improvement scheme. Section 5 discusses the simulation result of the companding transform and iterative companding transform. Section 6 introduces the influence of BER. The paper is concluded in Section 7.

2 PAPR ANALYSIS OF OFDM SIGNALS

In OFDM, each block of N will modulate a symbols $\{X(n), n = 0,1,2,...,N-1\}$ subcarrier of an orthogonal set whose frequency is $\{f(n), n = 0,1,2,...,N-1\}$, with $f(n) = n\Delta f$, where $\Delta f = \frac{1}{NT}$ and T is the original

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symbol period. The resulting baseband OFDM signal $x(t)$ of a block can be expressed as

$$x(t)=\sum_{n=0}^{N-1} X_n e^{j(2\pi f_n t+\beta n)}, 0\le t\le NT \qquad (1)$$

Where β_n is the initial phase of the n^{th} subcarrier, which for simplicity is assumed to be zero.

The PAPR of the transmitted signal in equation (1) is defined by,

$$PAPR=\frac{P_{PEAR}}{P_{AVG}} \qquad (2)$$

with

$$P_{PEAR}=max\left\{\,|x(t)|^2\,\right\}_{t\in[0,NT]} \qquad (3)$$

$$P_{AVG}=E\left\{|x(t)|^2\right\}=\frac{1}{NT}\int_0^{NT}|x(t)|^2\mathrm{d}t \qquad (4)$$

With N subcarriers each having normalized symbol energy, the average power $P_{AVG}=N$.

Since most systems employ discrete-time signals, instead of reducing the continuous-time peak, i.e., $max|x(t)|$, the maximum amplitude of LN samples (where L is the oversampling factor) of $x(t)$ is reduced. The case $L=1$ is known as critical sampling or Nyquist rate sampling, while $L>1$ corresponds to oversampling. It is well known that Nyquist sampling will miss some of the signal peaks and give optimistic results for PAPR. It has been pointed out[15] that oversampling factor $L=4$ is sufficient for capturing the continuous-time peaks. Oversampling can be implemented by padding $(L-1)N$ zeros on original OFDM blocks and then applying inverse Fast Fourier transform (IFFT). Therefore, the peak power can be expressed as

$$P_{PEAK}=\max_{1\le j\le(NT-1)}[|IFFT(\overline{X})|_j^2] \qquad (5)$$

with

$$\overline{X}=[X_0,X_1,...,X_{N-1},\underbrace{0+0+\cdots+0}_{(L-1)N}] \qquad (6)$$

The complementary cumulative distribution function (CCDF) of the PAPR is defined as follows:

$$CCDF=Pr(PAPR>PAPR_0) \qquad (7)$$

With a large number of subcarriers, the signal amplitude can be approximated as Rayleigh distributed, thus the large peaks happens only with a very small probability. Therefore, the absolute PAPR may not be meaningful for characterizing the PAPR property of OFDM signals, instead, the statistical distribution of PAPR should be taken into account [16]. However, without the use of any PAPR reduction technique, quite large PAPR (more than $7.4dB$) will happen almost with probability 1 when $N=256$.

3 Companding Transform

The technology of companding transform[16], the basic tenet of companding transform is that compressing the high power transmission signal, and making the small power emission signal amplify, so keeping the average power remainning unchanged, such not only can reduce the system PAPR, but also enhance system anti-interference capacity. Companding the signals at the Launch end, and implementing inverse operation at the receiving end. Its advantage is reducing the PAR, and enhancing the anti-interference capacity of small power signals. The basic processes of the C transform are shown in figure 1.

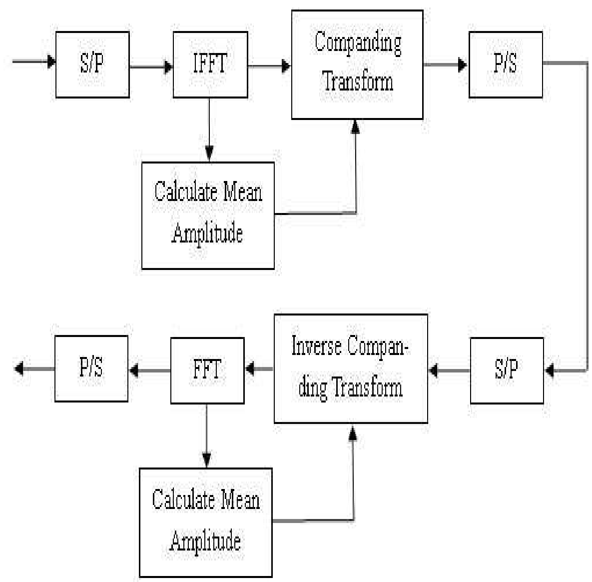

Fig. 1: The baseband block diagram of OFDM using the C transform

In OFDM system, the complex baseband signal of the output signals can be computed as follows:

$$s(t)=\sum_{-\infty}^{\infty}\sum_{k=0}^{N-1} s_{n,k}g(t-\frac{kT}{N}-nT) \qquad (8)$$

Where T is the length of OFDM signal period; k is the k^{th} samples values of a symbol cycle; n is the n^{th} OFDM signal in time domain; $g(t)$ is the impulse response that meets the Nyquist impulse filter; $s_{n,k}$ is the k^{th} samples values of the n^{th} OFDM signal i.i.e $s_{n,k}=C\left\{x_{n,k}\right\}$. Where $x_{n,k}$ is the OFDM signal of using IFFT transformation $C\{\cdot\}$, represents the companding transform, and this two kinds of transformation satisfy two conditions as follow:

1) When $|x|\le m$,$|C\{X\}|\ge|x|$; else $|C\{X\}|\le|x|$, $'m'$ is the turning point of the companding transform in the formula;

2) $E\left\{|x|^2\right\} \approx E\left\{|C\{x\}|^2\right\}$, That is the average power of companding transform roughly equals to the before. It is obvious that if the C transform form and the turning point 'm' could be choosed appropriately, the performance of PAR could be improved significantly, and the complexity of the system would not be increased too much. It's worth noting that if the average amplitude equals the turning point of C transform, and C transform can satisfy odd symmetry about turning point, which can guarantee the average power of signals through C transform basically unchanged. In order to satisfy the above-mentioned requirements, C transform can use the following formula to describe as follows:

$$s_{n,k} = C\left\{x_{n,k}\right\} = \frac{V x_{n,k}}{ln(1+\mu)|x_{n,k}|} ln(1+\frac{\mu}{V}|x_{n,k}|) \quad (9)$$

Where V is the average amplitude of the OFDM signal $x_{n,k}$The scope of μ is $\mu \leq 4$.

At receiving end, the inverse companding transform to the signals $r_{n,k}$ is implemented, that is as follows:

$$y_{n,k} = C^{-1}\left\{r_{n,k}\right\} = \frac{V' r_{n,k}}{\mu |r_{n,k}|}\left\{\mathbf{exp}[\frac{|r_{n,k}| ln(1+\mu)}{\acute{V}}] - 1\right\} \quad (10)$$

Where V' is the average amplitude of receipt signal $r_{n,k}$.

4 Improvement Method

According to the conditions of the $\mu \leq 4$, iterate the μ according to the principle of iterative, this is companding signals when $\mu = 1$, then further companding signals when $\mu = 2$, and then companding signals when $\mu = 3$, until μ meeting the maximum value of requirements. And then deal with the iterate companding transform signals.

5 Simulation of the Iterative Companding Transform

Figure 2. is simulation of companding transform and iterative companding transform of iteration times $\mu = 4$. In the figure 2, the magenta dash and dot line represents the original signal PAPR, the black line represents the PAPR of iterative companding transform of iteration times $\mu = 4$, the rest of the blue, red, black spots line and magenta line represent the PAPR of companding transform rate $\mu = 1,2,3,4$. The simulation figure is as follows:

Figure 3 is simulation of iterative companding transform. In the figure 3, the magenta line represents the original signal PAPR, the black line represents the PAPR of $\mu = 4$, the rest of the blue, red, black spots line represent the PAPR of $\mu = 1,2,3$. The simulation Figure is shown in figure 3.

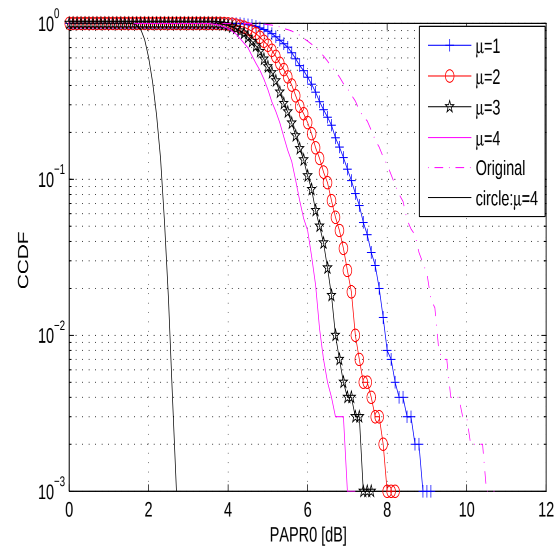

Fig. 2: Simulation of companding transform and iterative companding transform of iteration times $\mu = 4$

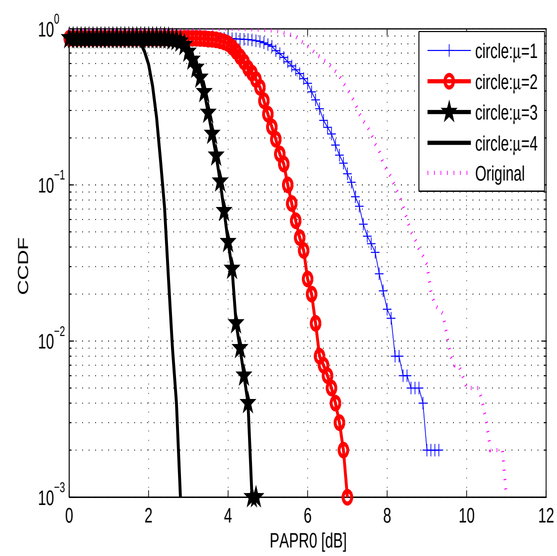

Fig. 3: Simulation of iterative companding transform

Performance analysis: When the target of CCDF is 10^{-3}, the original PAPR is about 10.7dB, the PAPR of the iterative companding transform scheme that iteration times $\mu = 4$ is about 2.7dB, the PAPRs of the companding transform scheme that companding rate $\mu = 1,2,3,4$ are separately about 8.9dB, 8.0dB, 7.6dB and 7.2dB. This is demonstrated in figure 2. When the target of CCDF is 10^{-3}, the original PAPR is about 10.7dB, the PAPRs of the iterative companding transform scheme that iteration times $\mu = 1,2,3,4$ are about 9.6dB, 6.9dB, 4.7dB and 2.7dB. This is demonstrated in figure 3. That show the PAPR is reduced effective by companding transform, in addition, with the increase of companding rate, the CCDF of OFDM signals are lower. In the condition of the same companding rate, comparing to the companding transform, the PAPR of OFDM system has been improved effectively by the iterative companding transform.

6 The Influence of Iterative Companding Transform to BER

In order to evaluate and compare the performance of the proposed transform and examine its impaction on the system, a MATLAB simulation was performed, in the simulation, the following should be used:

For convenience, system adopted BPSK modulation, subcarrier numbers for 128, OFDM signals were oversampled by a factor 4.

The channel was ideal AWGN, not considered multipath effect.

In A/D transformation, assumed the bits to represent the sample value were enough, regardless of the impaction of quantization noise, actually, the quantization noise of the system was relatively small.

In order to prevent companding signals appearring to 0 signals and could not be applied to companding transform formula, the signals which through the IFFT were added 0.0000001 respectively, the influenced to the signals amplitude could be neglected during the simulation.

The useful signal $x(n)$ whose energy was basically unchanged when through the companding transform and the inverse companding transform. The simulation of BER curves that through the companding transform, iteration times $\mu = 3$,iteration times $\mu = 2$ and the original signal BER curves were showed at figure 4 and figure 5 as follows.

Performance analysis: figure 4 and figure 5 show: the BER of the signal that goes through the iterative companding transform of iteration times $\mu = 3$ and iteration times $\mu = 4$ has little changed by compared to the BER of the original signal. In addition, the BER of the signal that goes through the iterative companding transform of iteration times $\mu = 1$ is equal to the companding transform of $\mu = 1$. Thus, the BER of the system could be neglected when the companding transform or the iterative companding transform of iteration times $\mu \leq 3$ is used.

In theory, $\mu \leq 4$, the simulation of BER curves that through the companding transform, iterative companding transform of iteration times $\mu = 4$, and the original signal BER curves were showed at figure 6 as follows.

Performance analysis: figure 6 shows: the performance of BER that through the iterative companding transform of iteration times $\mu = 4$ is significantly reduced compared to the performance of original signal, the main reason is that along with the increasing of the compamding number, the distortion of the signal is more serious, the performance will be significantly reduced when the distortion more than a certain threshold. Therefore, compare to the simulation Figures, we choose itera-tive companding transform of iteration times $\mu = 3$, it not only can make the OFDM system BER under the condition of not obviously

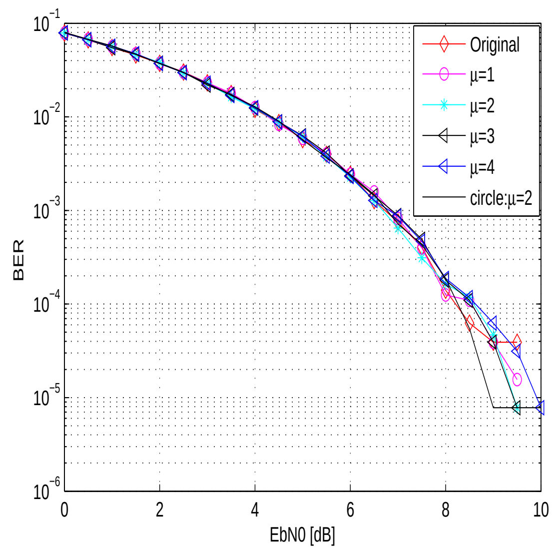

Fig. 4: BER of iteration times μ=2

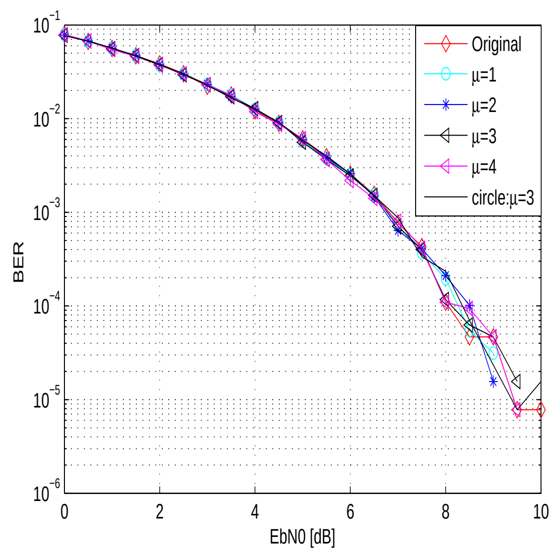

Fig. 5: BER of iteration times μ=3

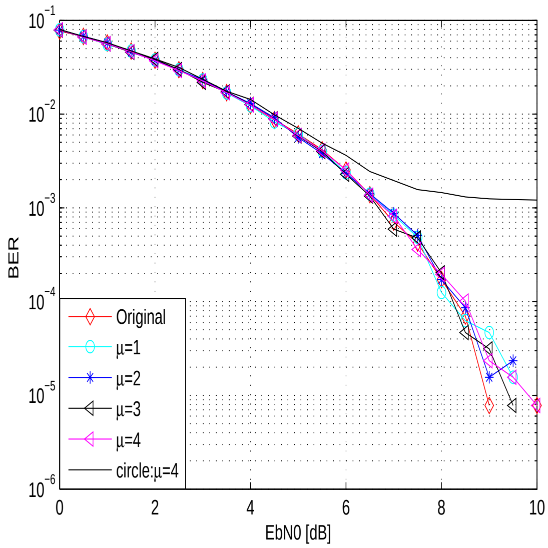

Fig. 6: BER of iteration times $\mu = 4$

increase, but also can significantly reduce OFDM signal PAPR.

7 Conclusion

OFDM technology has a lot of advantages and especially suitable for the mobile data transmission. It is a promising 4G technology, but high peak PAPR is a big obstacle to restrain it development. Through the above simulation results can be shown: the scheme of iterate companding transform of iteration times $\mu = 3$, compares to the companding transform method, it not only can make the OFDM system BER under the condition of not obviously increase, but also can significantly reduce OFDM signal PAPR, it makes the high PAPR of the OFDM system restrained to a certain extent, which has a good significance to the real communication system.

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References

[1] S. H. Han and J. H. Lee, An overview of peak-to-average power ratio reduction techniques for multicarrier transmission, IEEE Wireless Conimun. Mag., **12**, 56-65, (2005)

[2] Changchuan Yi. Broadband wireless communication technology of Multiple carriers. Beijing University of Posts and Telecommunications Press, (2004).

[3] Xiaobin WangReduction of Peak-to-Average Power Ratio of OFDM system using A Companding Technique. IEEE Transaction on Broadcasting, **45**, (1999).

[4] B. S. Krongold and D. L. Jones, PAR reduction in OFDM via active constellation extension, IEEE Trans. Broadcast., **49**,258-268, (2003) .

[5] J. Armstrong, Peak-to-average reduction for OFDM by repeated clipping and frequency domain filtering, IEE Electron. Lett., **38**, 246-247, (2002)

[6] X. Wang et al. Reduction of peak-to-average power ratio of OFDM system using a companding technique, IEEE Trans. Broadcast., **45**, 303-307, (1999).

[7] Y. Guo and J. R. Cavallaro, Reducing peak-to-average power ratio in OFDM systems by adaptive dynamic range companding, in 3G Wireless, World Wireless Congress, San Francisco, CA, 536-541, (2002).

[8] T. Jiang and G. Zhu, Nonlinear companding transform for reducing peak-to-average power ratio of OFDM signals, IEEE Trans. Broadcast., **50**,342-346, (2004).

[9] T. Jiang et al., Two novel nonlinear companding schemes with iterative receiver to reduce PAPR in multi-carrier modulation systems,IEEE Trans. Broadcast., **52**, 268-273, (2006).

[10] T. G. Pratt et al., OFDM link performance with companding for PAPR reduction in the presence of non-linear amplification, IEEE Trans.Broadcast., **52**,261-267, (2006).

[11] X. Huang et al., Companding transform for reduction in peak-to-average power ratio of OFDM signals, IEEE Trans. Wireless Commun., **3**,2030-2039, (2004).

[12] S. A. Aburakhia, E. F. Badran, and D. A. E. Mohamed, A comparison between signal clipping and -law companding schemes for the reduction of peak-to-average power ratio of OFDM signals, in Proc. 1st Int. Conf. on Digital Commun. & Computer Applications DCCA2007, Irbid, Jordan, 423-431, (2007).

[13] X. Li and L. J. Cimini, Jr., Effects of clipping and filtering on the performance of OFDM, IEEE Commun. Lett., **2**,131-133, (1998).

[14] Yumin Wang. Key technology and application of OFDM. Machinery industry press, 116-118, (2006).

[15] C. Tellambura, Computation of the continuous-time PAR of an OFDM signal with BPSK subcarriers, IEEE Commun. Lett., **5**,185-187,(2001).

[16] H. Ochiai and H. Imai, On the distribution of the peak-to-average power ratio in OFDM signals, IEEE Trans. on Commun., **49**,282-289, (2001).

[17] Changxin Fan, Lina Cao. Communication principle. Defense industry press, 268-274, (2008).

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