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On Compactness in Fuzzy Soft Topological Spaces

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Abstract: In this Paper, we introduce a new definition of the cover so-called fuzzy soft p-cover. According to this notion, we define a new type of compactness in fuzzy soft topology so- called p^*-compactness which is extension to Kandil's compactness in the fuzzy topology [7] and is avoid some Chang's deviations in the fuzzy and fuzzy soft topology [4]. Some of their basic results, properties and relations are investigated with some necessary examples.

Keywords: Fuzzy soft set, fuzzy soft point, fuzzy soft topology, fuzzy soft p-cover, fuzzy soft p-compact, fuzzy soft p^*-compact.

1 Introduction

The concept of compactness is one of the basic and the most important notions in topology**.** In soft topological space, compactness was first introduced by Zorlutuna et al. [19]. Chang first studied compactness in fuzzy topology [4], but it has many deviations. So, there are different researches for fuzzy compactness have been introduced to avoid these deviations (see [6,7,8,9,10]). Then Gain et al.[5] and Osmanoglu & Tokat [13] have introduced the notion of compactness in fuzzy soft topology as a generalization of Chang's fuzzy compactness [4]. Also, Mishra & Srivastava [12] studied fuzzy soft compactness as a generalization of Lown's fuzzy compactness [8].

Throughout this Paper, we introduce and study a new type of cover and compactness in fuzzy soft topology so-called p-cover and p^*-compactness which is extension Kandil's notion [7], development of Chang's notion [4] in fuzzy topology, and is general than that which are presented in [5,13]. Some basic results, relations and theorems related to the p^*-compactness are studied with some necessary examples.

2 Preliminaries

In this section, we recall the basic definitions which will be needed in this paper. Throughout this work, X refers to an initial universe, E be the set of all parameters for X and I^X is the set of all fuzzy sets in X, where $I = [0,1]$.

Definition 2.1. [10,18] A fuzzy set A of X is a set characterized by the membership function $A : X \longrightarrow I$ and A can be represented by ordered pairs $A = \{(x, A(x)) : x \in X, A(x) \in I\}$, $A(x)$ represents the degree of membership of x in A for $x \in X$. A fuzzy point $x_\lambda \in (0,1]$ is a fuzzy set in X given by $x_\lambda(y) = \lambda$ at $x = y$ and $x_\lambda(y) = 0$ otherwise for all $y \in X$. For $\alpha \in I$, $\underline{\alpha} \in I^X$ refers to the fuzzy constant function where, $\underline{\alpha}(x) = \alpha \ \forall x \in X$. The support of $A \in I^X$ is the crisp set $S(A) = \{x \in X : A(x) > 0\}$. For $A, B \in I^X$, the basic operations for fuzzy sets are given by Zadeh (see[18]).

Definition 2.2. [11,14] A fuzzy soft set $f_E = (f, E)$ over X with the set E of parameters is defined by the set of ordered pairs $f_E = \{(e, f(e)) : e \in E\ ,\ f(e) \in I^X\}$. Here f is a mapping given by $f : E \longrightarrow I^X$ and the value $f(e)$ is a fuzzy set called e-element of the fuzzy soft set for all $e \in E$. The family of all fuzzy soft sets over X is denoted by $FSS(X,E)$.

Definition 2.3. [1,2,11] Let f_E, g_E are two fuzzy soft sets over X. Then we have:
i) The fuzzy soft set f_E is called a null fuzzy soft set, denoted by $\tilde{0}_E$ if $f(e) = \underline{0}$ for every $e \in E$ where, $\underline{0}(x) = 0\ \forall x \in X$.
ii) If $f(e) = \underline{1}$ for every $e \in E$, then f_E is called an universal fuzzy soft set denoted by $\tilde{0}_E$ where, $\underline{1}(x) = 1\ \forall x \in X$.
iii) f_E is a fuzzy soft subset of g_E, denoted by $f_E \sqsubseteq g_E$ if $f(e) \subseteq g(e)\ \forall e \in E$.
iv) f_E and g_E are equal if $f_E \sqsubseteq g_E$ and $g_E \sqsubseteq f_E$. It is denoted by $f_E = g_E$.

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v) The complement of f_E is denoted by f_E^c where, $f^c : E \longrightarrow I^X$ is a mapping defined by $f(e)^c = \underline{1} - f(e)$ for all $e \in E$. Clearly, $(f_E^c)^c = f_E$.
vi) The union of f_E and g_E is a fuzzy soft set h_E defined by, $h(e) = f(e) \cup g(e) \quad \forall e \in E$ and denoted by $f_E \sqcup g_E$.
vii) The intersection of f_E and g_E is a fuzzy soft set h_E defined by, $h(e) = f(e) \cap g(e) \quad \forall e \in E$ and denoted by $f_E \sqcap g_E$.

Definition 2.4. [2] A fuzzy soft point x_α^e over X is a fuzzy soft set over X defined as follows:

$$x_\alpha^e\left(e'\right) = \begin{cases} x_\alpha & if \quad e' = e \\ \underline{0} & if \quad e' \in E - \{e\} \end{cases} \quad \text{where,}$$

x_α is the fuzzy point in X with support x and value α, $\alpha \in (0,1]$. The set of all fuzzy soft points in X is denoted by $FSP(X,E)$. The fuzzy soft point x_α^e is called belongs to a fuzzy soft set f_E , denoted by $x_\alpha^e \tilde{\in} f_E$ iff $\alpha \leq f(e)(x)$. Every non-null fuzzy soft set f_E can be expressed as the union of all the fuzzy soft points belonging to f_E. The complement of a fuzzy soft point x_α^e is a fuzzy soft set over X.

Definition 2.5. [2,14] Let X be an universe set, E be a fixed set of parameters and δ be the collection of fuzzy soft sets over X satisfies the following conditions:
i) $\tilde{0}_E$, $\tilde{1}_E$ belong to δ,
ii) The union of any number of fuzzy soft sets in δ is in δ,
iii) The intersection of any two fuzzy soft sets in δ is in δ.

In this case (X,δ,E) is called a fuzzy soft topological space. The members of δ are called fuzzy soft open sets in X and denoted by, $FSO(X,\delta,E)$. A fuzzy soft set f_E over X is called fuzzy soft closed in X iff $f_E^c \in \delta$ and denoted by $FSC(X,\delta,E)$.

Example 2.6. 1) Let X be an universe set and E be a set of parameters for X, then the families $\delta = \{\tilde{0}_E, \tilde{1}_E\}$ and $\delta = FSS(X,E)$ are called an indiscrete fuzzy soft topology and discrete fuzzy soft topology on X, respectively.
2) Let X be an infinite set and a_t^e be any fixed fuzzy soft point on X. Then the following collections:
i) $\delta_{a_t^e} = \{\tilde{1}_E\} \cup \{f_E \in FSS(X,E) : a_t^e \tilde{q} f_E\}$,
ii) $\delta_\infty = \{f_E \in FSS(X,E) : S(f^c(e))\ is\ a\ finit\ subset\ of\ X\ and\ e \in E\} \cup \{\tilde{0}_E\}$ define the fuzzy soft topologies over X.

Notation. [15] For $x_\alpha^e \in FSP(X)$ the fuzzy soft set $O_{x_\alpha^e}$ refers to a fuzzy soft open set contains x_α^e and $O_{x_\alpha^e}$ is called a fuzzy soft neighborhood of x_α^e. The fuzzy soft neighborhood system of x_α^e denoted by, $N_E(x_\alpha^e)$ is the family of all its fuzzy soft neighborhoods. In general, for $f_E \in FSS(X,E)$ the notation O_{f_E} refers to a fuzzy soft open set contains f_E and is called a fuzzy soft neighborhood of f_E.

Definition 2.7. [2] The fuzzy soft sets f_E and g_E in $FSS(X,\ E)$ are said to be fuzzy soft quasi-coincident, denoted by $f_E q g_E$ iff there exist $e \in E$ and $x \in X$ such that $f(e)(x) + g(e)(x) > 1$. If f_E is not fuzzy soft quasi-coincident with g_E , then we write $f_E \tilde{q} g_E$, *i.e.* $f_E \tilde{q} g_E$ iff $f(e)(x) + g(e)(x) \leq 1$, that is, $f(e)(x) \leq g^c(e)(x)$ for all $x \in X$, $e \in E$.
A fuzzy soft point x_α^e is said to be soft quasi-coincident with f_E, denoted by $x_\alpha^e q f_E$ iff there exists $e \in E$, $\alpha + f(e)(x) > 1$.

Theorem 2.8. [15] *i*) Let (X,τ) be a crisp topological space. Then the collection:
$\delta_\tau = \{\tilde{\chi}_A : A \in \tau\}$ forms a fuzzy soft topology on X induced by τ.
ii) Every fuzzy soft topological space (X,δ,E) defines crisp topology on X in the form $\tau_\delta = \{A \subseteq X : \tilde{\chi}_A \in \delta\}$ which is induced by δ.

Definition 2.9. [17] A fuzzy soft topological space (X,δ,E) is said to be:
i) Fuzzy soft R_0 (for short, FSR_0) iff for every x_α^e , $y_\beta^e \in FSP(X,E)$ with $x_\alpha^e \tilde{q} \overline{y_\beta^e}$ implies $\overline{x_\alpha^e} \tilde{q} y_\beta^e$.
ii) Fuzzy soft R_1 (for short, FSR_1) iff for every x_α^e , $y_\beta^e \in FSP(X,E)$ with $x_\alpha^e \tilde{q} \overline{y_\beta^e}$ implies there exist $O_{x_\alpha^e}$ and $O_{y_\beta^e} \in \delta$ such that $O_{x_\alpha^e} \tilde{q} O_{y_\beta^e}$.

Definition 2.10. [16] A fuzzy soft topological space (X,δ,E) is said to be:
i) Fuzzy soft T_1 (for short, FST_1) iff for every x_α^e , $y_\beta^e \in FSP(X,E)$ with $x_\alpha^e \tilde{q} y_\beta^e$ implies there exist $O_{x_\alpha^e}$ and $O_{y_\beta^e} \in \delta$ such that $O_{x_\alpha^e} \tilde{q} y_\beta^e$ and $x_\alpha^e \tilde{q} O_{y_\beta^e}$.
ii) Fuzzy soft T_2 (for short, FST_2) iff for every x_α^e , $y_\beta^e \in FSP(X,E)$ with $x_\alpha^e \tilde{q} y_\beta^e$ implies there exist $O_{x_\alpha^e}$ and $O_{y_\beta^e} \in \delta$ such that $O_{x_\alpha^e} \tilde{q} O_{y_\beta^e}$.
iii) Fuzzy soft regular (for short, FSR) iff for every $x_\alpha^e \in FSP(X,E)$ and $f_E \in \delta^c$ with $x_\alpha^e \tilde{q} f_E$ implies there exist $O_{x_\alpha^e}$ and $O_{f_E} \in \delta$ such that $O_{x_\alpha^e} \tilde{q} O_{f_E}$.
iv) Fuzzy soft normal (for short, FSN) iff for every f_E , $g_E \in \delta^c$ with $f_E \tilde{q} g_E$ implies there exist O_{f_E} , $O_{g_E} \in \delta$ such that $O_{f_E} \tilde{q} O_{g_E}$.
v) Fuzzy soft T_3 (for short, FST_3) iff it is FSR and FST_1.
vi) Fuzzy soft T_4 (for short, FST_4) iff it is FSN and FST_1.

3 Fuzzy soft p^*-compact spaces

In this section, we are going to introduce a new definition of fuzzy soft cover according to this definition we introduce a new type of compactness in fuzzy soft topological spaces so-called p^*-compactness.

Here we mention that Gain et al.[5] and Osmanoglu & Tokat [13] have introduced the definitions of cover and compactness in fuzzy soft topology as a generalization of Chang's notion [4]. Their definitions are as follows:

Definition 3.1. [5,13]
i) A family $\mathscr{A}$ of fuzzy soft sets is a cover of fuzzy soft set g_E if $g_E \sqsubseteq \sqcup\{f_{iE} : f_{iE} \in \mathscr{A}, i \in J\}$. It is open cover if each member of $\mathscr{A}$ is a fuzzy soft open set. A subcover of $\mathscr{A}$ is a subfamily of $\mathscr{A}$ which is also a cover.
ii) Let (X,δ,E) be fuzzy soft topological space and $f_E \in FSS(X,E)$. A fuzzy soft set f_E is called compact if each

fuzzy soft open cover of f_E has a finite subcover. A fuzzy soft topological space (X,δ,E) is called compact if each fuzzy soft open cover of $\tilde{1}_E$ has a finite subcover.

Definition 3.2. A family $\gamma = \{f_{iE} : i \in J\}$ of a fuzzy soft sets is a p-cover of a fuzzy soft set g_E iff for all $x^e_\alpha \tilde{\in} g_E$ there exists $i_0 \in J$ such that $x^e_\alpha \tilde{\in} f_{i_0E}$. It is an open p-cover iff every member of γ is a fuzzy soft open set.
A sub p-cover of γ is a subfamily of γ which is also a fuzzy soft p-cover.

Note 1. Clearly, every fuzzy soft p-cover is a fuzzy soft cover in the sense of Definition 3.1. But the converse may not be true in general as shown by the following example.

Example 3.3 Let X be an infinite set and $\gamma = \{f_{nE} : n \in \mathbb{N}\}$ be a family of fuzzy soft sets over X defined by $f_n(e)(x) = 1 - \frac{1}{n}$ where $e \in E$, $n \in \mathbb{N}$ and $x \in X$. Then γ is a fuzzy soft cover of $\tilde{1}_E$. But γ is not a fuzzy soft p-cover of $\tilde{1}_E$. Indeed for $x^e_1 \tilde{\in} \tilde{1}_E$ there no exist any element in γ containing x^e_1.

Definition 3.4. Let (X,δ,E) be a fuzz soft topological space and $f_E \in FSS(X,E)$. Then:
i) f_E is called a fuzzy soft p-compact set iff every fuzzy soft open p-cover of f_E has a finite fuzzy soft open p-subcover. In general (X,δ,E) is a fuzzy soft p-compact space iff $\tilde{1}_E$ itself is a fuzzy soft p-compact set.
ii) (X,δ,E) is called a fuzzy soft p^*-compact space iff every fuzzy soft closed set over X is a fuzzy soft p-compact set.

Example 3.5. *i*) Every fuzzy soft set f_E with a finite support of $f(e)$, $e \in E$ is p-compact (*i.e.* every finite fuzzy soft set is a p-compact set).
ii) A cofinite fuzzy soft space (X,δ_∞,E) is a fuzzy soft p^*-compact space,where δ_∞ is defined as in the case 2) of the Example 2.6.

Proof. *i*) It is obvious.
ii) Let $f_E \in FSC(X,\delta_\infty,E)$, then f_E is finite or $f_E = \tilde{1}_E$. Now we have two cases:
If f_E is finite, then the result holds.
If $f_E = \tilde{1}_E$. Suppose γ is a fuzzy soft open p-cover of $f_E = \tilde{1}_E$. Choose $x^e_1 \tilde{\in} \tilde{1}_E$, then there is $O_{x^e_1} \in \gamma$ and so, $O^c_{x^e_1}$ is finite. Now take $g_E = \{(e, f(e)) : f(e) = y^i_1, e \in E \ and$ $y^i \in Ssup\left(O^c_{x^e_1}\right),\ i = 1,2,\dots,n\}$ which is finite, thus for all $y^{ie}_1 \tilde{\in} g_E$ there exist $O_{y^{ie}_1} \in \gamma$, $i = 1,2,\dots,n$, and so the family $\{O_{y^{ie}_1} : i = 1,2,\dots,n\} \cup \{O_{x^e_1}\}$ is a finite open p-subcover of $f_E = \tilde{1}_E$, then $\tilde{1}_E$ is p-compact. Hence (X,δ_∞,E) is fuzzy soft p^*-compact.

Note 2. Clearly, every fuzzy soft p^*-compact space is fuzzy soft compact in the sense of Definition 3.1. But the converse is not necessary true, this fact can be shown by the following example.

Example 3.6. Let X be an infinite set and $\delta = \{\tilde{1}_E\} \cup \{f_E \in FSS(X,E) : f_E \sqsubseteq \underline{0.5}_E\}$, then it is easy to check that (X,δ,E) is a fuzzy soft compact space but it is not fuzzy soft p^*-compact. Indeed the fuzzy soft set $f_E = \underline{0.5}_E \in FSC(X,\delta,E)$ and the family $\gamma = \{x^e_{0.5} : e \in E\ ,\ x \in X\}$ is a fuzzy soft open p-cover of f_E which has no a finite open p-subcover. Hence the result holds.

Note 3. Clearly, every fuzzy soft p^*-compact space is fuzzy soft p-compact. But the converse may not be true in general, this fact can be shown by the pervious example.

Proposition 3.7. Let (X,δ_2,E) be a fuzzy soft p-compact (p^*-compact) space and $\delta_1 \subseteq \delta_2$, then (X,δ_1,E) is fuzzy soft p-compact (p^*-compact).

Proof. For the first case, let $\{f_{iE} : i \in J\}$ be a fuzzy soft open p-cover of $\tilde{1}_E$ by fuzzy soft open sets of (X,δ_1,E). Since $\delta_1 \subseteq \delta_2$, then $\{f_{iE} : i \in J\}$ is a fuzzy soft open p-cover of $\tilde{1}_E$ by fuzzy soft open sets of (X,δ_2,E). But (X,δ_2,E) is fuzzy soft p-compact. So that, for all $x^e_\alpha \tilde{\in} \tilde{1}_E$ there exists $\{i = 1,2,3,\dots,n : i \in J\}$ such that $x^e_\alpha \tilde{\in} f_{iE}$, then $\{f_{iE} : i = 1,2,3,\dots,n\}$ is a finite fuzzy soft open p-subcover of $\tilde{1}_E$. Hence (X,δ_1,E) is a fuzzy soft p-compact space. The proof of the rest case is obvious.

Note 4. Clearly, if X be a finite set, then (X,δ,E) is fuzzy soft p^*-compact. But the convers may not be true as shown by the following example.

Example 3.8. Let X be an infinite set and $\delta_c = \{\underline{\alpha_E} : \alpha \in I\}$ then (X,δ_c,E) is a fuzzy soft p^*-compact space.

Corollary 3.9. Let (X,δ,E) be the discrete fuzzy soft topological space, then (X,δ,E) is fuzzy soft p^*-compact iff X is finite.

Proof. Let (X,δ,E) be the discrete fuzzy soft p^*-compact space. Suppose that X an infinite set. Since (X,δ,E) is fuzzy soft p^*-compact, then for every fuzzy soft closed set over X is a fuzzy soft p-compact set. Let f_E is a fuzzy soft closed set, then for all fuzzy soft open p-cover of f_E has a finite open p-subcover. Take $\gamma = \{x^e_\alpha : e \in E,\ x \in X\}$, then γ is an open p-cover of f_E which has no a finite open p-subcover. This is contradiction. Hence X is a finite set. Conversely, the proof follows direct from Note 4.

Definition 3.10. Let $\eta = \{f_{iE} : i \in J\}$ be a family of fuzzy soft sets and $g_E \in FSS(X,E)$. Then:
i) η is said to be have fuzzy soft q-intersection with respect to (*w.r.t.* , for short) g_E iff there exists $x^e_\alpha \tilde{\in} g_E$ such that $x^e_\alpha q f_{iE}$ for all $i \in J$.
ii) η is called has the fuzzy soft finite intersection property ($FSFIP$, for short) *w.r.t.* g_E iff every finite subfamily of η has fuzzy soft q-intersection *w.r.t.* g_E.

Theorem 3.11. Let (X,δ,E) be a fuzzy soft topological space. A fuzzy soft set g_E is p-compact if and only if every family of fuzzy soft closed sets over X having the $FSFIP$ *w.r.t.* g_E has fuzzy soft q-intersection *w.r.t.* g_E.

Proof. Let $g_E \in FSS(X,E)$ be fuzzy soft p-compact and let $\eta = \{f_{iE} : i \in J\}$ be the family of fuzzy soft closed sets over X which has the $FSFIP$ *w.r.t.* g_E. Now suppose η has no fuzzy soft q-intersection *w.r.t.* g_E. Then for all $x^e_\alpha \tilde{\in} g_E$ $\exists\ i \in J$ such that $x^e_\alpha \tilde{q} f_{iE}$ and so, $\eta^c = \{f^c_{iE} : i \in J\}$ is a fuzzy soft open p-cover of g_E. Since g_E is p-compact, then there is a finite open p-subcover of η^c say, $\{f^c_{sE} : s = 1,2,\dots,n \in J\}$. So, $\{f_{sE} : s = 1,2,\dots,n \in J\}$ has no fuzzy soft q-intersection *w.r.t.* g_E. Contradiction that η has the

$FSFIP$ $w.r.t.$ g_E. Hence we obtain the result.
Conversely, let the family $\eta = \{O^i_{x^e_\alpha} : i \in J\}$ be a fuzz soft open p-cover of g_E. Then $\eta^c = \{\left(O^i_{x^e_\alpha}\right)^c : i \in J\}$ has no fuzzy soft q-intersection $w.r.t.$ g_E. Thus η^c has no $FSFIP$ $w.r.t.$ g_E. So there are $i_1, i_2, \ldots, i_n \in J$ such that $\{\left(O^{i_s}_{x^e_\alpha}\right)^c : s = 1, 2, \ldots, n\}$ has no fuzzy soft q-intersection $w.r.t.$ g_E. Then $\{O^{i_s}_{x^e_\alpha} : s = 1, 2, \ldots, n\}$ is a finite open p-subcover of g_E. Hence g_E is p-compact.

Theorem 3.12. Every fuzzy soft closed subspace (X, δ_Y, E) of a fuzzy soft p^*-compact space (X, δ, E) is a fuzz soft p^*-compact space.

Proof. Obvious.

Theorem 3.13. Let (X, τ) be a topological space, then (X, δ_τ, E) is fuzzy soft p^*-compact if and only if (X, τ) is compact.

Proof. Let (X, δ_τ, E) be fuzzy soft p^*-compact and the family $\{A_i : i \in J\}$ is an open cover of X, then $\{\widetilde{\chi}_{A_i} : A_i \in \tau,\ i \in J\}$ is a fuzzy soft open p-cover of $\tilde{1}_E$. Since $\tilde{1}_E$ is fuzzy soft p-compact, then there is a finite open p-subcover, say $\{\widetilde{\chi}_{A_i} : i = 1, 2, \ldots, n\}$ and so, the family $\{A_i : i = 1, 2, \ldots, n\}$ is a finite fuzzy soft open subcover of X. Hence (X, τ) is compact.
Conversely, let(X, τ) is compact and $g_E \in \delta^c_\tau$, then $g_E = \widetilde{\chi}_B$, $B \in \tau^c$ implies B is compact. Since $\widetilde{\chi}_B = g_E$, then $\widetilde{\chi}_B = g_E$ is fuzzy soft p-compact. Hence (X, δ_τ, E) is p^*-compact.

Theorem 3.14. Let(X, δ, E) be a fuzz soft topological space. If (X, δ, E) is fuzzy soft p^*-compact, then (X, τ_δ) is compact.

Proof. It is similar to that of the necessity part of the above theorem.

Note 5. The converse of the above theorem may not be true in general. This is can be shown by the Example 3.6, where $\tau_\delta = \{\emptyset, X\}$ is compact.

4 Fuzzy soft separation axioms and p^*-compactness

Theorem 4.1. Let (X, δ, E) be a $FST_3 - space$ and $g_E \in FSS(X, E)$ be a p-compact set, then for all $f_E \in \delta^c$ with $f_E \tilde{q} g_E$ there are O_{f_E}, $O_{g_E} \in \delta$ such that $O_{f_E} \tilde{q} O_{g_E}$.

Proof. Let (X, δ, E) be a $FST_3 - space$, $f_E \in \delta^c$ and $g_E \in FSS(X, E)$ is fuzzy soft p-compact, then for every $y^e_\beta \widetilde{\in} g_E$ there exist $O_{y^e_\beta}$, $O_{f_E} \in \delta$ such that $O_{y^e_\beta} \tilde{q} O_{f_E}$. Clearly, $\{O_{y^e_\beta} : y^e_\beta \widetilde{\in} g_E\}$ is a fuzzy soft open p-cover of g_E. Since g_E is fuzzy soft p-compact, then there exists a finite fuzzy soft open p-subcover of g_E say, $\{O^i_{y^e_\beta} : i = 1, 2, \ldots, n\}$. One readily verifies that $O_{g_E} = \sqcup^n_{i=1} O^i_{y^e_\beta}$ and $O_{f_E} = \sqcap^n_{i=1} O^i_{f_E}$ have the required property.

Theorem 4.2. Let (X, δ, E) be a FST_2-space, $x^e_\alpha \in FSP(X, E)$ and g_E be fuzzy soft p-compact with $x^e_\alpha \tilde{q} g_E$, then there are $O_{x^e_\alpha}$ and $O_{g_E} \in \delta$ such that $O_{x^e_\alpha} \tilde{q} O_{g_E}$. Moreover, if f_E and g_E are fuzzy soft p-compact with $f_E \tilde{q} g_E$, then there are O_{f_E} and $O_{g_E} \in \delta$ such that $O_{f_E} \tilde{q} O_{g_E}$.

Proof. It is similar to that of the above theorem.

Theorem 4.3. Every fuzzy soft p-compact set in FST_2-space is a fuzzy soft closed set.

Proof. Let g_E be a fuzzy soft p-compact set in FST_2-space (X, δ, E), then from the above theorem we have for every $x^e_\alpha \tilde{q} g_E$ there exists $O_{x^e_\alpha} \in \delta$ such that $O_{x^e_\alpha} \tilde{q} g_E$ that is, for all $x^e_\alpha \widetilde{\in} g^c_E$ there exists $O_{x^e_\alpha} \in \delta$ such that $O_{x^e_\alpha} \widetilde{\in} g^c_E$. Therefore, g^c_E is fuzzy soft open. Hence the result holds.

Theorem 4.4. Every fuzzy soft p^*-compact T_2-space (X, δ, E) is a fuzzy soft T_4-space.

Proof. Let (X, δ, E) be a fuzzy soft p^*-compact T_2-space and f_E, $g_E \in \delta^c$ with $f_E \tilde{q} g_E$. Since (X, δ, E) is fuzzy soft p^*-compact, then f_E, g_E are fuzzy soft p-compact and so, from Theorem 4.2, there exist O_{f_E}, $O_{g_E} \in \delta$ such that $O_{f_E} \tilde{q} O_{g_E}$. Hence (X, δ, E) is a fuzzy soft T_4-space.

Theorem 4.5. Let (X, δ, E) be a FSR_1-space. Then (X, δ, E) is a FST_2-space if and only if every fuzzy soft p-compact set is closed.

Proof. The necessity follows from Theorem 4.3 . Conversely, let every fuzzy soft p-compact set is closed, then (X, δ, E) is a FST_1-space (see [16], Theorem 3.4). Since (X, δ, E) are FSR_1 and FST_1, then (X, δ, E) is a FST_2-space (see [16], Theorem 3.9).

Theorem 4.6. Let (X, δ, E) be a fuzzy soft topological space, then every p^*-compact FSR_1-space is a fuzzy soft regular(normal) space.

Proof. Let (X, δ, E) be a p^*-compact FSR_1-space and $f_E \in \delta^c$ with $x^e_\alpha \tilde{q} f_E$. Then for all fuzzy soft point $y^e_\beta \widetilde{\in} f_E$ we have, $x^e_\alpha \tilde{q} \overline{y^e_\beta}$. Since (X, δ, E) is FSR_1, then there exist $O_{x^e_\alpha}$ and $O_{y^e_\beta} \in \delta$ such that $O_{x^e_\alpha} \tilde{q} O_{y^e_\beta}$. Then the family $\{O_{y^e_\beta} : y^e_\beta \widetilde{\in} f_E\}$ is a fuzzy soft open p-cover of f_E. Since (X, δ, E) is fuzzy soft p^*-compact, then f_E is fuzzy soft p-compact, and so there exists $\{O^i_{y^e_\beta} : y^e_\beta \widetilde{\in} f_E, i = 1, 2, \ldots, n\}$ is a finite fuzzy soft p-subcover of f_E. Now take $O^*_{x^e_\alpha} = \sqcap^n_{i=1} O^i_{x^e_\alpha}$ and $O_{f_E} = \sqcup^n_{i=1} O^i_{y^e_\beta}$, then $O^*_{x^e_\alpha}$, $O_{f_E} \in \delta$ and $O^*_{x^e_\alpha} \tilde{q} O_{f_E}$. Hence the result holds. The proof of the rest case is similar.

Corollary 4.7. Let (X, δ, E) be fuzzy soft p^*-compact, then the following statements are equivalent:
i) (X, δ, E) is fuzzy soft R_1 .
ii) (X, δ, E) is fuzzy soft regular,
iii) (X, δ, E) is FSR_0 and fuzzy soft normal.

Proof. Obvious.

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