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One-Sided Cumulative Sum Control Chart For Monitoring Shift In The Scale Parameter Lambda, (λ) Of The New Weibull-Pareto Distribution

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Abstract: In this paper, a new cumulative sum control chart has been proposed to detect shifts in one of the scale parameters of the New Weibull-Pareto Distribution using the V-Mask method of constructing cumulative sum control chart. The study showed significant variations in the parameters of the V-Mask when a small to moderate shift occurred in the scale parameter under study. The parameters observed in the V-mask included the mask angle, the lead distance and the Average Run Length.

Keywords: Cumulative sum, Control Chart, Variations, New Weibull-Pareto Distribution, lead distance, V-Mask and Average run length

1 Introduction

Quality is a perceptual, conditional and somewhat subjective attribute that people could understand it differently. Generally, it is the totality of features of a product or services, its ability to satisfy stated or implied needs. Thus, statistical quality is the use of set of statistical procedures, or techniques, intended to ensure that a manufactured product or a performed service, adheres to defined quality criteria or meets the requirements of the client or customer. Quality control is essential in building a successful business that will deliver products that consistently meet or exceed customer's expectations and forms the basis of an efficient business that minimizes waste, operates at high levels of productivity. Some of the statistical techniques used in quality control include control charts; for both variable control charts, used to monitor characteristics that can be measured and have continuum values, and control chart for attributes, used to monitor denoted characteristics that have discrete values and can be counted. These charts, developed by Dr. Walter Shewhart in the 1920's and unable to rapidly detect are referred to as Shewhart control charts, have the tendency to detect large or sustained shift in the process mean, but are unable to rapidly detect a small to moderate shift in the process mean. To mitigate this incongruity, innovative research such as the Cumulative Sum (CUSUM) control chart was introduced by Page [1] and further advanced by [2,4,3]. The main advantage of the CUSUM control charts over the Shewhart's control charts is their ability to rapidly detect, relatively small shifts in the process mean. The efficiency of CUSUM control charts largely depends on the type of statistical distribution from which it is constructed. Consequently, myriad of researchers have come out with several CUSUM control charts. In addition,[5], proposed a unified CUSUM control chart to monitor shifts in the parameters of the Erlang-Truncated exponential distribution. [6] proposed a one-sided CUSUM control chart for monitoring shifts in the shape parameter of the Pareto distribution, [8] proposed a unified CUSUM control chart for monitoring shifts in the parameters of the Pareto distribution etc. Other studies by [12] demonstrated the sensitivity of the distributions studied to the change in the process parameter. This study is built on the earlier work of [12], who proposed another one-sided CUSUM control chart using other parameters of the New Weibull-Pareto distribution.

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2 The New Weibull-Pareto Distribution

The New Weibull-Pareto Distribution (NWPD) was developed and proposed by [9] . The distribution was applied to real live data modeling and excelled both the Weibull and Pareto distributions. The NWPD was used to model the exceedances of flood peaks (in m^3/s) of the Wheaton River near carcross in Yukon Territory, Canada. This data was recently analysed by [10] and [11].
The pdf of the New Weibull Pareto distribution is given by:

$$f(x,\gamma,\delta,\lambda) = \left(\frac{\gamma\delta}{\lambda}\right)\left(\frac{x}{\lambda}\right)^{\gamma-1} e^{-\delta(x/\lambda)\gamma} \tag{1}$$

where γ is the shape parameter with δ and λ being scale parameters.
The mean and variance are given by:

$$E(X) = \lambda\delta^{\overline{\frac{-1}{\beta}}}\Gamma\left(\frac{\gamma+1}{\gamma}\right) \tag{2}$$

$$Var(x) = \lambda^2\delta^{\overline{\frac{-2}{\beta}}}\Gamma\left(\frac{\gamma+2}{\gamma}\right) - \left\{\lambda\delta^{\overline{\frac{-1}{\beta}}}\Gamma\left(\frac{\gamma+1}{\gamma}\right)\right\}^2 \tag{3}$$

3 Sequential Probability Ratio Test

The Sequential Probability Ratio Test (SPRT) is a procedure in which item-by-item is sampled sequentially, which was introduced by [14]. Sequential sampling is an extension of the multiple sampling phenomenon where a sequence of samples is taken and the number of samples is examined by the results of the sampling process. This procedure is based on the sequential probability ratio test. Considering a simple hypothesis $H_0 : \theta = \theta_0$ against a simple alternative $H_1 : \theta = \theta_1$

The standard Likelihood Ratio Test (LRT) has a critical region of the form:

$$\Lambda = \lambda(X_1...X_n) = log\frac{L_1(\theta_1 : X_1...X_n)}{L_0(\theta_0 : X_1...X_n)} > K \tag{4}$$

where L_1 is the likelihood function of the NWPD with a shift in the scale parameter and L_0 is the likelihood function when there is no shift in the scale parameter of the NWPD. The SPRT is used to test the null hypothesis, H_0 against the alternative hypothesis, H_1
Wald's Sequential Probability Ratio Test has the following forms:

1. If $\Lambda_n > B$, decide that H_1 is true and stop;
2. If $\Lambda_n < A$, decide that H_0 is true and stop;
3. If $A < \Lambda_n < B$, collect another observation to obtain Λ_{n+1}

It is noticeable that the SPRT is optimal in the sense that it minimizes the average sample size before a decision is taken among all sequential tests which do not have larger error probabilities than the SPRT. It can also be defined that the boundaries A and B can be calculated with the following approximation

$$A = log\frac{\beta}{1-\alpha}$$

$$A = log\frac{1-\beta}{\alpha}$$

The V-mask control scheme was proposed by [13]. The performance of the V-mask is defined by the lead distance and the angle of the mask. The mask is applied to successive values of the CUSUM statistic. The decision procedure consists of placing the V-mask on the CUSUM values with the point O of the V-mask on the last value of CUSUM values and the line OP. If all the previous CUSUM values fall within the two arms of the V-mask, the process is in control. However, if any of the CUSUM values fall outside the arms of the V-mask, then the process is considered to be out of control.

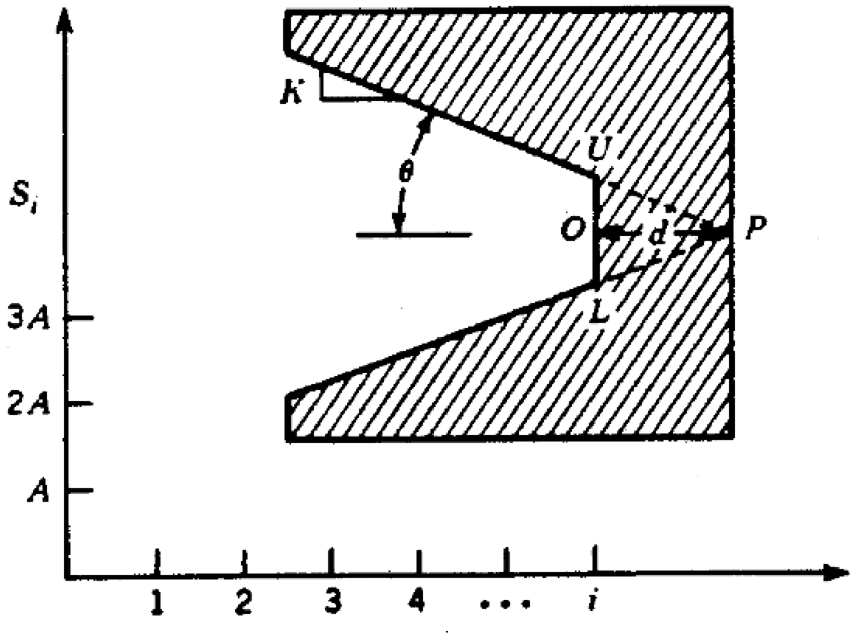

Fig. 1: V- Mask (ECE 6455: Semicondctor process control)
Where d = the lead distance, θ = the mask angle

4 The Cusum Chart Construction

Let L_1 be the likelihood function of the NWPD with a shift in the scale parameter and L_0 be the likelihood function of the NWPD with no shift in the scale parameter. Then the ratio of these likelihoods is taken as $\frac{L_1}{L_0}$. The SPRT is used for testing null hypothesis, H_0 against the alternative hypothesis, H_1.

The pdf of the New Weibull-Pareto distribution is given by:

$$f(x,\gamma,\delta,\alpha.) = \left(\frac{\gamma\delta}{\lambda}\right)\left(\frac{x}{\lambda}\right)^{\gamma-1} e^{-\delta\left(\frac{x}{\lambda}\right)\gamma}$$

If there is a shift in λ while γ and δ are fixed, the likelihood ratio of the NWPD when a shift occurs and when there is no shift is given by:

$$\frac{L_1}{L_o} = \frac{\prod_{i=1}^{n}\left(\frac{\gamma\delta}{\lambda_1}\right)\left(\frac{\sum_{i=1}^{n}x_1}{\lambda_1}\right)^{\gamma-1} e^{-\delta\left(\frac{\sum_{i=1}^{n}x_i}{\lambda_1}\right)\gamma}}{\prod_{i=1}^{n}\left(\frac{\gamma\delta}{\lambda_0}\right)\left(\frac{\sum_{i=1}^{n}x_1}{\lambda_0}\right)^{\gamma-1} e^{-\delta\left(\frac{\sum_{i=1}^{n}x_i}{\lambda_0}\right)\gamma}}$$

This is reduced to:

$$\frac{L_1}{L_0} = \left(\frac{\lambda_0}{\lambda_1}\right)^{n}\left(\frac{\lambda_0}{\lambda_1}\right)^{\gamma-1} e^{\delta\left(\frac{\sum_{i=1}^{n}}{\lambda_0}\right)\gamma-\delta\left(\frac{\sum_{i=1}^{n}x_i}{\lambda_1}\right)\gamma} \tag{5}$$

The continuation region of the SPRT distinguishing between the two hypotheses is given by:

$$\frac{\alpha}{1-\beta} < \frac{L_1}{L_0} < \frac{1-\alpha}{\beta}$$

Where α and β are type I and II errors. Thus substituting for L_1/L_0 in 5, it gives:

$$\frac{\alpha}{1-\beta} < \left(\frac{\lambda_0}{\lambda_1}\right)^n \left(\frac{\lambda_0}{\lambda_1}\right)^{\gamma-1} e^{\delta\left(\frac{\sum_{i=1}^n}{\lambda_0}\right)\gamma-\delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_1}\right)\gamma} < \frac{1-\alpha}{\beta} \tag{6}$$

Taking logarithm of 6,it gives:

$$ln\left(\frac{\alpha}{1-\beta}\right) < ln\left[\left(\frac{\lambda_0}{\lambda_1}\right)^n \left(\frac{\lambda_0}{\lambda_1}\right)^{\gamma-1} e^{\delta\left(\frac{\sum_{i=1}^n}{\lambda_0}\right)\gamma-\delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_1}\right)\gamma}\right] < ln\left(\frac{1-\alpha}{\beta}\right)$$

this becomes:

$$ln\left(\frac{\alpha}{1-\beta}\right) < nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right) + \delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_0}\right)\gamma - \delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_1}\right)\gamma < ln\left(\frac{1-\alpha}{\beta}\right) \tag{7}$$

by setting α to zero, 7 becomes:

$$nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right) + \delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_0}\right)\gamma - \delta\left(\frac{\sum_{i=1}^n x_i}{\lambda_1}\right)\gamma < -ln\beta$$

factorizing $\sum_{i=1}^n x$

$$nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right) + \sum_{i=1}^n x_i\delta\gamma\left(\frac{\lambda_1-\lambda_0}{\lambda_1\lambda_0}\right) < -ln\beta$$

rearranging:

$$\sum_{i=1}^n x_i\delta\gamma\left(\frac{\lambda_1-\lambda_0}{\lambda_1\lambda_0}\right) < -ln\beta - nln\beta\left(\frac{\lambda_0}{\lambda_1}\right) - (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right)$$

making $\sum_{i=1}^n x$

$$\sum_{i=1}^n x_i < \left[-ln\beta - nln\left(\frac{\lambda_0}{\lambda_1}\right) - (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right)\right] * \frac{\lambda_1\lambda_0}{\delta\gamma(\lambda_1-\lambda_0)}$$

and finally:

$$\sum_{i=1}^n x_i < \frac{\lambda_1\lambda_0\left[ln\beta + nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right)\right]}{\delta\gamma(\lambda_1-\lambda_0)} \tag{8}$$

writing 8 in the form of a linear equation, it becomes:

$$\sum_{i=1}^n x_i < PX + Q$$

where:

$$P = \frac{\lambda_1\lambda_0\left[nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right)\right]}{\delta\gamma(\lambda_1-\lambda_0)}$$

And:

$$Q = \frac{\lambda_1\lambda_0 ln\beta}{\delta\gamma(\lambda_1-\lambda_0)}$$

On the other hand when the shift occurs such that $\lambda_0 > \lambda_1$, the 8 is written as:

$$\sum_{i=1}^n x_i \geq \frac{\lambda_1\lambda_0\left[ln\beta + nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)ln\left(\frac{\lambda_0}{\lambda_1}\right)\right]}{\delta\gamma(\lambda_1-\lambda_0)} \tag{9}$$

Also 9 can be written in a linear form as:

$$\sum_{i=1}^{n} x_i \geq P^* X + Q^*$$

$$P^* = \frac{\lambda_1 \lambda_0 \left[nln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma - 1) ln\left(\frac{\lambda_0}{\lambda_1}\right)\right]}{\delta\gamma(\lambda_1 - \lambda_0)}$$

and :

$$Q^* = \frac{\lambda_1 \lambda_0 ln\beta}{\delta\gamma(\lambda_1 - \lambda_0)}$$

Table 1 shows the simulated values of the mask angle, θ_i given the values of σ, γ, n, λ_0 and λ_1. When a positive shift exists in the parameter from λ_0 to λ_1, it can be defined from the first part of Table 1 that the value of θ increases as (λ_1 -λ_0) enlarges. On the other hand, when there is a negative shift where $\lambda_0 > \lambda_1$, the value of θ decreases marginally. Moreover, the smaller the shift in the parameter, the bigger the mask angle given that the shift in the parameter is positive. However, when the shift is negative, larger shifts in the parameter produce smaller angles of the V-Mask. The details are shown in the second part of Table 1.

Table 1: Simulated values of the mask angle

λ_0	λ_1	σ	γ	n	θ
1.5	3.0	0.1	0.2	1.0	87.2
1.5	4.5	0.1	0.2	1.0	87.7
1.5	6.0	0.1	0.2	1.0	87.9
1.5	7.5	0.1	0.2	1.0	88.1
1.5	9.0	0.1	0.2	1.0	88.2
1.5	10.5	0.1	0.2	1.0	88.3
1.5	12.5	0.1	0.2	1.0	88.4
1.5	13.5	0.1	0.2	1.0	88.5
λ_0	λ_1	σ	γ	n	θ
1.5	1.25	0.1	0.2	1.0	85.8
1.5	1.20	0.1	0.2	1.0	85.7
1.5	1.15	0.1	0.2	1.0	85.6
1.5	1.10	0.1	0.2	1.0	85.5
1.5	1.05	0.1	0.2	1.0	85.4
1.5	1.00	0.1	0.2	1.0	85.3
1.5	0.95	0.1	0.2	1.0	85.2

Table 2: Simulated values of the lead distance

λ_0	λ_1	σ	β	γ	d
0.5	0.45	0.1	0.6	0.2	76.6
0.5	0.40	0.1	0.6	0.2	57.5
0.5	0.35	0.1	0.6	0.2	51.1
0.5	0.30	0.1	0.6	0.2	47.9
0.5	0.25	0.1	0.6	0.2	46.6
0.5	0.20	0.1	0.6	0.2	44.7
0.5	0.15	0.1	0.6	0.2	43.8
0.5	0.10	0.1	0.6	0.2	43.1
λ_0	λ_1	σ	β	γ	d
1.5	1.25	0.1	0.6	0.2	-191.6
1.5	1.20	0.1	0.6	0.2	-153.2
1.5	1.15	0.1	0.6	0.2	-125.9
1.5	1.10	0.1	0.6	0.2	-105.4
1.5	1.05	0.1	0.6	0.2	-89.4
1.5	1.00	0.1	0.6	0.2	-76.6
1.5	0.95	0.1	0.6	0.2	-66.2

Given values of $\lambda_0, \lambda_1, \sigma, \beta$ and γ with a positive shift in the parameter λ, where $\lambda_1 > \lambda_0$; the first part of the Table 2 shows that the lead distance decreases as the size of the shift increases. Also, the smaller the shift in the parameter, the larger the value of the lead distance and vice-versa. On the other hand, if the shift is negative, $\lambda_1 < \lambda_0$, the value of d decreases. The negative shift indicates a shift in the left-hand side of the process parameter. The details are shown in the second part of Table 2.

5 The Average Run Length

The Average Run Length (ARL) is the number of points that on average will be plotted on a control chart before an out of control condition is indicated (a point plotted outside the control limits). If the system is in control (all the plotted points are within the arms of the control limits), the ARL $=\frac{1}{\alpha}$. On the other hand, if the process parameter is out of control, ARL $= \frac{1}{(1-\beta)}$., where α and β are probabilities of Type I and Type II errors. The ARL is given by:

$$ARL = \frac{-ln\alpha}{E[lnZ]} \tag{10}$$

where

$$Z = \frac{f(x,\gamma,\lambda,\delta_1)}{f(x,\gamma,\lambda,\delta_0)}$$

then:

$$Z = \frac{\lambda_0}{\lambda_1}\left(\frac{\lambda_0}{\lambda_1}\right)^{\gamma-1} e^{\delta\left(\frac{x}{\lambda_0}\right)^{\gamma}-\delta\left(\frac{x}{\lambda_0}\right)^{\gamma}} \tag{11}$$

taking natural logarithm of 11

$$lnZ = ln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma-1)\left(\frac{\lambda_0}{\lambda_1}\right) + \delta\left(\frac{x}{\lambda_0}\right)\gamma - \delta\left(\frac{x}{\lambda_1}\right)\gamma \tag{12}$$

Then expectation of 12 becomes:

$$E[lnZ] = ln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma - 1)\left(\frac{\lambda_0}{\lambda_1}\right) + E[x]\left(\frac{\gamma\delta}{\lambda_0} - \frac{\gamma\delta}{\lambda_1}\right) \tag{13}$$

$$E[x] = \int_{\lambda}^{\infty} x f(x,\gamma,\delta,\lambda)dx$$

this becomes:

$$E[x] = \lambda\delta^{\frac{-1}{\gamma}}\Gamma\left(\frac{1+\gamma}{\gamma}, u\right)$$

As obtained in 8 and substituting into 13, it yields:

$$E[lnZ] = ln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma - 1)\left(\frac{\lambda_0}{\lambda_1}\right) + \lambda\delta^{\frac{-1}{\gamma}}\Gamma\left(\frac{1+\gamma}{\gamma}, u\right)\left(\frac{\gamma\delta}{\lambda_0} - \frac{\gamma\delta}{\lambda_1}\right) \tag{14}$$

Hence substituting 14 into:

$$ARL = \frac{-ln\alpha}{ln\left(\frac{\lambda_0}{\lambda_1}\right) + (\gamma - 1)\left(\frac{\lambda_0}{\lambda_1}\right) + \lambda\delta^{\frac{-1}{\gamma}}\Gamma\left(\frac{1+\gamma}{\gamma}, u\right)\left(\frac{\gamma\delta}{\lambda_0} - \frac{\gamma\delta}{\lambda_1}\right)} \tag{15}$$

where $\Gamma(a,b)$ is an upper incomplete gamma function.
Given the values of $\lambda_0, \lambda_1, \sigma, \gamma, \mu$ *and* α with a shift in λ_0 such that $\lambda_0 < \lambda_1$, with the other parameters fixed, the ARL increases as the value of $(\lambda_1 - \lambda_0)$ enlarges. On the other hand, if the shift in λ is negative, the ARL decreases. Moreover, the larger the shift in the process parameter, the bigger the value of the ARL when the shift in the parameter is positive. However, when the shift in the parameter is negative, larger shifts produce smaller values and vice-versa. These are shown in Table 3. The first part of the table indicates a positive shift with α having a value of $\alpha = 0.05$. The second part of the table shows a negative shift.

Table 3: Simulated values of the ARL

λ_0	λ_1	σ	γ	μ	α	ARL
0.5	0.55	1.1	0.2	2.0	0.05	3.6
0.5	0.75	1.1	0.2	2.0	0.05	4.4
0.5	0.80	1.1	0.2	2.0	0.05	5.5
0.5	0.85	1.1	0.2	2.0	0.05	7.1
0.5	0.90	1.1	0.2	2.0	0.05	9.6
0.5	0.95	1.1	0.2	2.0	0.05	14.1
0.5	1.00	1.1	0.2	2.0	0.05	24.8
0.5	0.05	1.1	0.2	2.0	0.05	82.9
λ_0	λ_1	σ	γ	μ	α	ARL
0.5	0.45	1.1	0.2	2.0	0.05	2.5
0.5	0.40	1.1	0.2	2.0	0.05	2.1
0.5	0.35	1.1	0.2	2.0	0.05	1.7
0.5	0.30	1.1	0.2	2.0	0.05	1.4
0.5	0.25	1.1	0.2	2.0	0.05	1.2
0.5	0.20	1.1	0.2	2.0	0.05	0.9
0.5	0.15	1.1	0.2	2.0	0.05	0.7
0.5	0.10	1.1	0.2	2.0	0.05	0.5

6 Practical demonstration with some random values of the NWPD

Table 4 shows some random numbers generated with the NWPD. The first ten values are random numbers generated from the distribution with the scale parameter fixed at 7, while the last five numbers represent some random numbers of the NWPD when the scale parameter shifts from 7 to 12. The V-Mask with a mask angle of 68^0 and a lead distance of 1.3 cm was placed on the last plotted point. The process graph was then plotted with the CUSUM value to the nearest whole number against the sample number. The graph showed that sample 1 to 10 fell outside the arms of the V-Mask, indicating a shift in the process parameter. The details are shown in Table 4 and Figure 2

Table 4: Random numbers of the NWPD

$sample$	$Data(x)$	lnx	$CUSUM$
1	17.78	2.88	3.00
2	6705.99	8.81	12.00
3	63.04	4.14	16.00
4	16.07	2.78	19.00
5	476.83	6.17	25.00
6	1478.83	7.30	32.00
7	3871.31	8.26	40.00
8	42.70	3.75	44.00
9	2970.00	8.00	52.00
10	90.77	4.51	57.00
11	3.52	1.26	58.00
12	364.63	5.90	64.00
13	989.97	6.90	71.00
14	1905.45	7.55	79.00
15	944.50	6.85	86.00

Fig. 2: Graphical illustration of the behaviour of the random numbers from the NWPD

7 Conclusion

In this study, we proposed a new one-sided CUSUM control chart for the NWPD. The results showed that when a positive shift exists in the parameter delta λ, the value of the mask angle θ increases as $(\lambda_1 - \lambda_0)$ enlarges. On the other hand, when there is a negative shift (i.e. to the left) of the process parameter, the value of the mask angle θ decreases marginally. Generally, the smaller the shift in the parameter, the bigger the mask angle given that the shift in the parameter is positive. However, when the shift is negative, larger shifts in the parameter produce smaller angles of the V-Mask.The study also showed that the lead distance decreased as the size of the shift increased. In addition, the smaller the shift in the parameter, the larger the value of the lead distance and vice-versa. On the other hand, if the shift is negative, the value of the lead distance, d decreases. Generally, the ARL increases as the value of $(\lambda_1 - \lambda_0)$ increases. However, if the shift in λ is negative, the ARL decreases. Moreover, the larger the shift in the process parameter, the bigger the value of the ARL. In other words when the shift in the parameter is positive the larger ARL but when the shift in the parameter is negative, the larger shift produces smaller values.

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