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Transmuted Generalized Inverse Rayleigh Distribution and Its Applications to Medical Science and Engineering

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Abstract: In this manuscript, we propose a transmuted model of the Generalized Inverse Rayleigh distribution. This article provides the detailed account of statistical properties of the new distribution. Different properties like reliability analysis, moment generating function, characteristic function, entropy and order statistics have been derived. The parameters of the newly proposed model have been estimated using the method of maximum likelihood estimation. Both the simulated as well as real life data sets are considered for making comparison between the special cases of TGIRD for the best fit.

Keywords: Transmuted Generalized Inverse Rayleigh Distribution, Reliability analysis, Entropy, Order Statistics, AIC, BIC and AICC.

1 Introduction

The Generalized Inverse Rayleigh Distribution is a very useful lifetime model which can be applied for analyzing the lifetime data. It is widely used in communication engineering, reliability analysis, applied statistics, operation research, health and biology. The Inverse Rayleigh Distribution was introduced by Trayer [1]. Gharraph, Mukarjee and Maitim [2,3] discussed the characteristic properties of the Inverse Rayleigh distribution. The modified version of Inverse Rayleigh distribution was studied in detail by Khan [4]. However, the generalized inverted scale family distributions were introduced by Potdar et. al.[5]. These newly developed models were formulated by introducing a new shape parameter to the scale family of distributions. These models provide greater flexibility in modeling complex data and the results drawn from them seems quite sound and genuine. Reshi et.al [6] estimated the scale parameter of GIRD under the different loss functions. The generalized model of Inverse Rayleigh distribution was further modified by Bakoban and Abu Baker [7]. They illustrated the different characterizing properties of the generalized Inverse Rayleigh distribution. Further, Bakoban [8] considered the estimation and the optimal design problem for GIRD under FSS- PALT using type II censored data. Also, Kawsar and Ahmad [9] compared the different informative and non informative priors for the GIRD under different Loss functions. This generalized model is appropriate for modeling the data with different hazard functions because of its flexibility. It approximates the lifetimes of several experimental units and also can be used in studying radiations, sounds and wind speed. Let the random variable X have a GIRD with parameters α and λ, where α is the shape parameter and λ is the scale parameter of the generalized model, then the probability density function (pdf) and the cumulative density function (cdf) of GIRD are respectively given as:

$$g(x)=\frac{2\alpha}{\lambda^2 x^3}e^{-(\lambda x)^{-2}}\left[1-e^{-(\lambda x)^{-2}}\right]^{\alpha-1} \quad ;x>0,\alpha,\lambda>0. \tag{1.1}$$

$$G(x)=1-\left[1-e^{-(\lambda x)^{-2}}\right]^{\alpha} \quad ;x>0,\alpha,\lambda>0. \tag{1.2}$$

This article considers an extension of generalized Inverse Rayleigh distribution using transmutation technique and checks its flexibility over its different sub models. Various statistical properties of the model have been illustrated. The paper is organized as follows: In section 2, the proposed transmuted model is introduced and obtaining its pdf and cdf. In Section 3, different special cases of the proposed model have been discussed. Section 4 and 5 are devoted to discuss the reliability analysis and various statistical properties of the proposed model. Moreover, the method of Random number generation and Renyi entropy of the new distribution are described in section 6 and 7 respectively. Further, in the sections 8 and 9, order statistics and method of maximum likelihood estimation are provided respectively. Finally, in section 10, both simulated as well as real life data sets have been considered to examine the flexibility of the newly developed model over its different

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sub models along with the concluding remarks. The paper ends with a complete bibiliography.

2 The Transmuted Generalized Inverted Rayleigh Distribution

In the past few years, many authors have generalized the known parametric models by transforming an appropriate model into a more general model by adding a shape parameter to the existing lifetime distributions. The new quadratic rank transmutation map (QRTM) technique is one of the methods that were formulated by Shaw and Buckley [10] to generalize the different theoretical models and to provide more flexible extension of these models for life testing and best fit.

Starting from an arbitrary parent cumulative density function $G(x)$, a random variable X is said to have a transmuted distribution if its cdf is given by:

$$F(x)=(1+\theta)G(x)-\theta[G(x)]^2, \quad |\theta|\le 1. \tag{2.1}$$

Where $G(x)$ is the cdf of the base distribution. It must be noted that when $\theta=0$, the proposed model reduces to base distribution.

Differentiating equation (2.1) with respect to x gives the pdf of the transmuted model as

$$f(x)=g(x)[(1+\theta)-2\theta G(x)]. \tag{2.2}$$

Here $g(x)$ is the probability density function of the base model.

Recently, Aryal and Tsokos [11,12] considered the transmuted Gumbel distribution to model climate data and the transmuted Weibull distribution and their applications to analyze real data sets. Ibrahim Elbatal [13] introduced the transmuted generalized Inverse Exponential distribution and examined its various structural properties. Faton Merovci [14] obtained the transmuted Rayleigh distribution and discussed its important properties. Further, Afaq et. al. [15] studied the transmuted inverse Rayleigh distribution and derived its different characteristic properties. More recently, Khan et. al. [16] studied the transmuted Kumaraswamy distribution and presented a comprehensive account of the mathematical properties of the new distribution.In the proposed study, we will obtain the mathematical formulation of the Transmuted Generalized Inverse Rayleigh distribution and discuss its important properties.

A random variable X is said to have a Transmuted Generalized Inverse Rayleigh distribution with parameters α,λ and θ if the cumulative density function is given by:

$$F(x)=\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\} \tag{2.3}$$

The corresponding pdf of the proposed distribution is given by:

$$f(x)=\frac{2\alpha}{\lambda^2 x^3}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha-1}\left\{1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}. \tag{2.4}$$

for $x>0,\alpha,\lambda>0$ and $|\theta|\le 1$.

Where λ and α are the scale and shape parameters of the transmuted generalized Inverse Rayleigh distribution and θ is the transmuted parameter.

By choosing various values for parameters λ, α and θ, we provide the different possible shape for the pdf of the TGIR distribution as shown in figures as below:

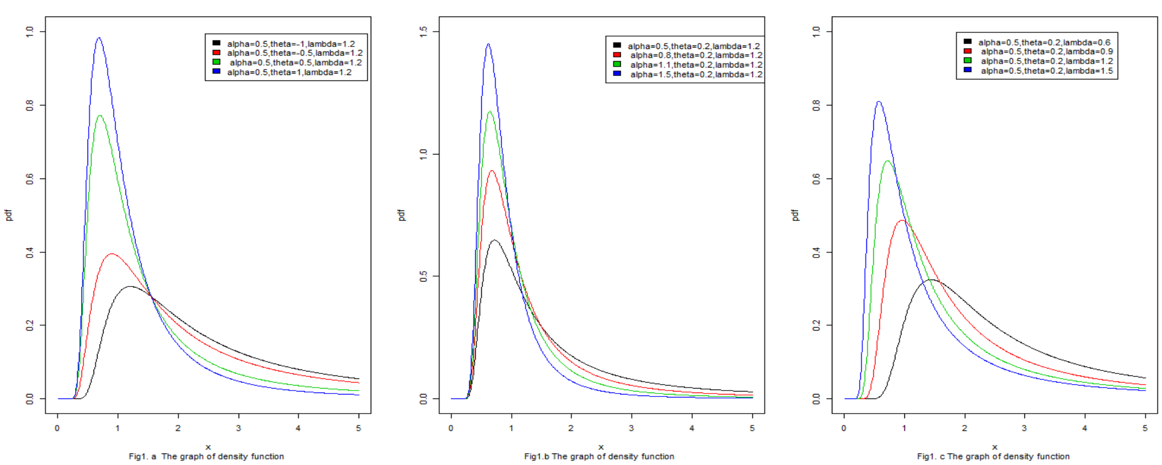

Fig. 1: The graph of density function

Figure 1 give us the detailed description of the different possible shapes of the density function for different values of the parameters. The graphs display that as the value of the three parameters is increased; the graphs of the density Functions are positively skewed and tend to be more peaked. Similarly, for the various possible values of the parameters λ, α and θ, the graphical plots for the cumulative distribution function are given as below:

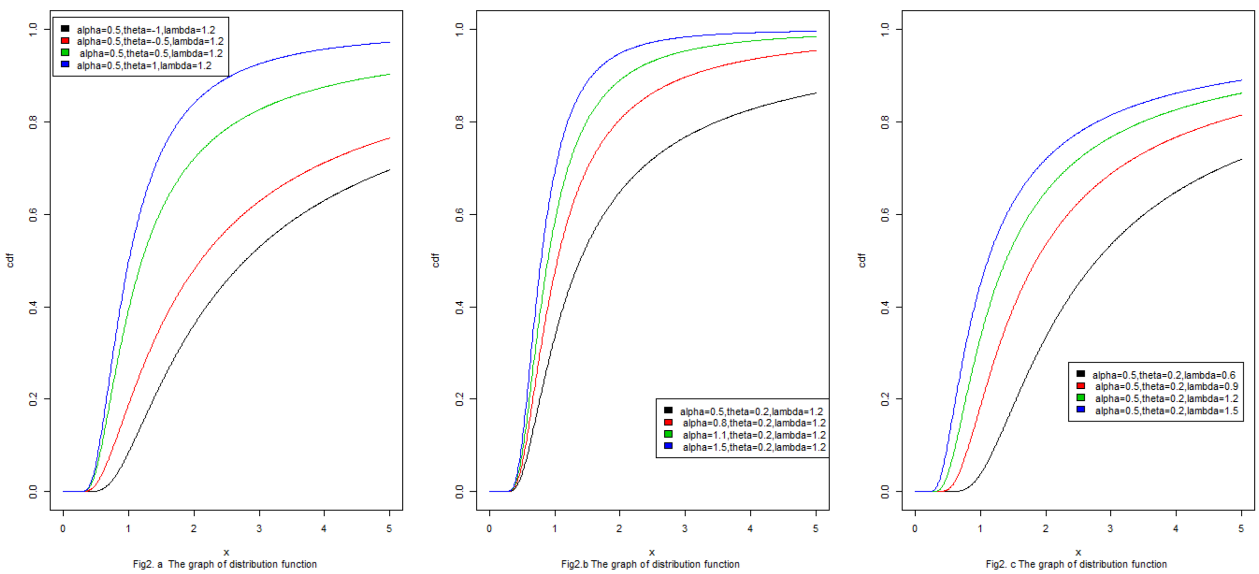

Fig. 2: The graph of distribution function

The graphical representation of the distribution function in figures 2 shows that the cumulative density functions is an increasing function with the different values of the parameters.

3Relationships with other Distributions

Some well-known theoretical distributions can be derived from the proposed TGIR distribution such as:

1) For $\theta = 0$, Equation (2.4) reduces to give the two parameter generalized Inverse Rayleigh distribution (GIRD) with probability density function as:

$$f(x) = \frac{2\alpha}{\lambda^2 x^3} e^{-(\lambda x)^{-2}} \left[1 - e^{-(\lambda x)^{-2}}\right]^{\alpha-1} \quad ; x > 0, \alpha, \lambda > 0$$

2) For $\alpha = 1$ and $\theta = 0$, Equation (2.4) reduces to give the one parameter Inverse Rayleigh distribution (IRD) with probability density function as:

$$f(x) = \frac{2}{\lambda^2 x^3} e^{-(\lambda x)^{-2}} \; ; x > 0, \lambda > 0 \, .$$

3) For $\alpha = 1$, Equation (2.4) reduces to give the two parameter Transmuted Inverse Rayleigh distribution (TIRD) with probability density function as:

$$f(x) = \frac{2}{\lambda^2 x^3} e^{-(\lambda x)^{-2}} \left[1 + \theta - 2\theta e^{-(\lambda x)^{-2}}\right] \quad ; x > 0, \theta, \lambda > 0.$$

4) For $\lambda = 1$, Equation (2.4) reduces to give the two parameter Transmuted Generalized Standard Inverse Rayleigh (TGSIR) distribution with probability density function as:

$$f(x) = \frac{2\alpha}{x^3} e^{-(x)^{-2}} \left[1 - e^{-(x)^{-2}}\right]^{\alpha-1} \left[1 + \theta - 2\theta e^{-(x)^{-2}}\right] \quad ; x > 0, \alpha, \theta > 0.$$

5) For $\lambda = 1, \theta = 0$, Equation (2.4) reduces to give the one parameter Generalized Standard Inverse Rayleigh (GSIR)distribution with probability density function as:

$$f(x) = \frac{2\alpha}{x^3} e^{-(x)^{-2}} \left[1 - e^{-(x)^{-2}}\right]^{\alpha-1} \quad ; x > 0, \alpha > 0.$$

6) For $\lambda = 1, \alpha = 1$, Equation (2.4) reduces to give the two parameter Transmuted Standard Inverse Rayleigh (TSIR) distribution with probability density function as:

$$f(x) = \frac{2}{x^3} e^{-(x)^{-2}} \left[1 + \theta - 2\theta e^{-(x)^{-2}}\right] \quad ; x > 0, \theta > 0.$$

4 Reliability Analyses

In this section, we shall discuss the reliability function, hazard rate and reverse hazard rate of the transmuted Generalized Inverse Rayleigh distribution.

4.1 Reliability Function

The reliability function is also termed as the survival or survivor function of the model. It may be defined as the probability that an item does not fail prior to sometime t. It is denoted by $R(x)$. Mathematically, the reliability function can be obtained as:

$$R(x)=1-F(x)$$

$$R(x)=1-\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}. \quad (4.1)$$

The graphical plotting of reliability of TGIR distribution for different possible values of the parameters is given as follows:

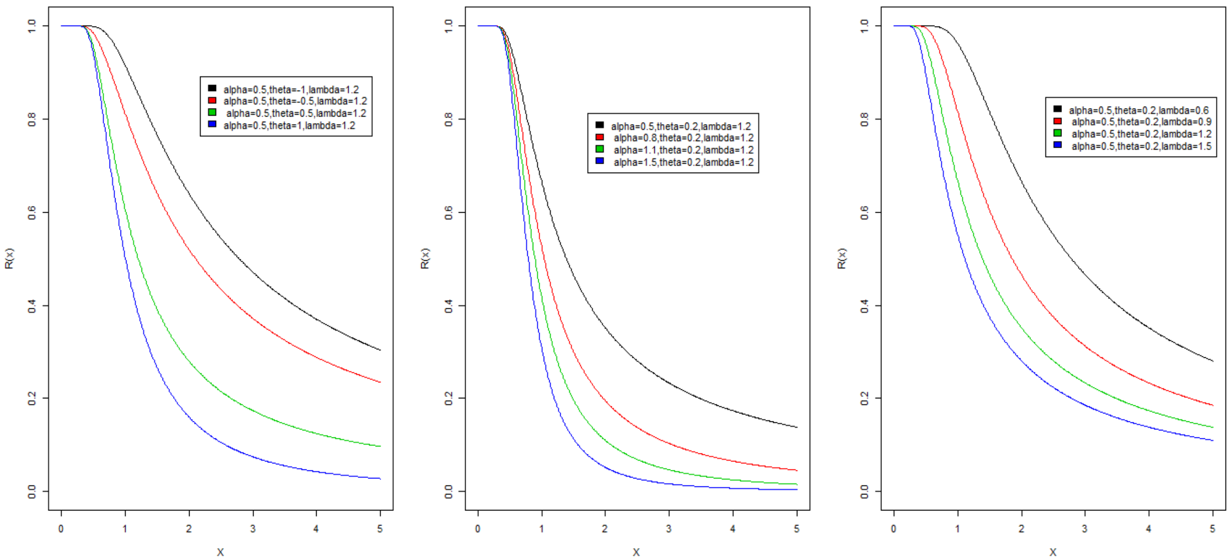

Fig3.a The graph of Reliability function Fig3.b The graph of Reliability function Fig3.c The graph of Reliability function

Fig. 3: The graph of Reliability function

The figures 3 indicate that the reliability function of a system is a decreasing function with the different possible values for parameters.

4.2 Hazard function

The hazard function of the system is also termed as the hazard rate, failure rate or force of mortality. Mathematically, it can be derived as the ratio of the probability density function and the reliability function. It is denoted by $h(x)$ and is given as:

$$h(x)=\frac{f(x)}{R(x)}=\frac{\frac{2\alpha}{\lambda^{2}x^{3}}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha-1}\left\{1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}}{1-\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}}. \quad (4.2)$$

Figures 4 illustrate that as the value of one parameter is increased keeping the other two fixed, the graphs become more peaked. Thus, the hazard rate is unimodal. It increases at initial stage and later on starts decreasing. It can also be seen that the hazard rate shows an inverted bathtub shape.

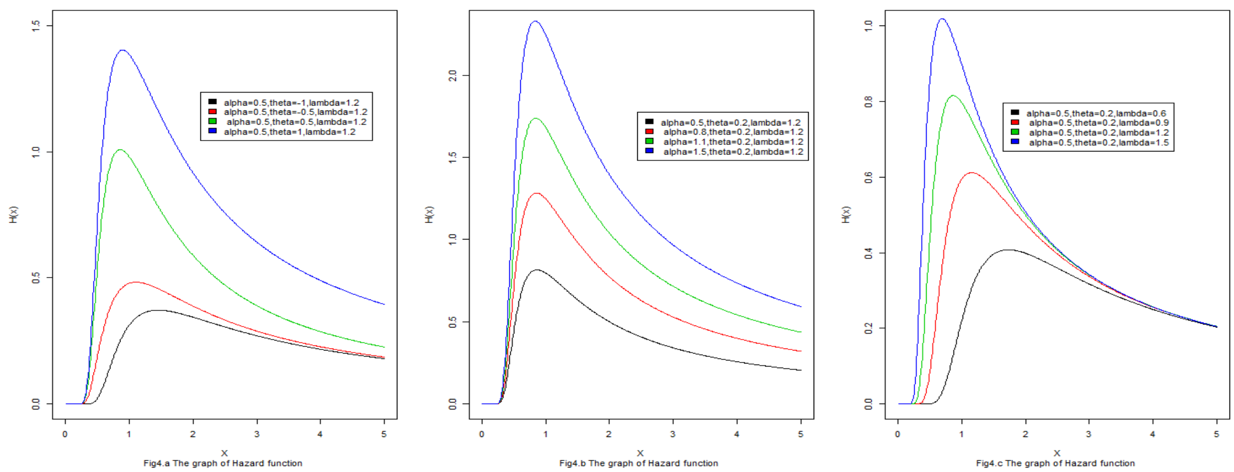

Fig. 4: The graph of Hazard function

4.3 Reverse Hazard Function

The reverse hazard rate is also an important quantity which characterizes life phenomenon. It is given as:

$$\phi(x)=\frac{f(x)}{F(x)}=\frac{\frac{2\alpha}{\lambda^2x^3}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha-1}\left\{1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}}{\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}}$$

(4.3)

5 Statistical Properties of the TGIR Distribution

This section provides some basic statistical properties of the transmuted generalized Inverse Rayleigh distribution.

5.1 The r^{th} Moment of the TGIR Distribution

Theorem 5.1: If $X \sim \mathrm{TGIR}(\alpha,\lambda,\theta)$, then the r^th^ moment of a continuous random variable X is given as follows:

$$\mu_r=E(X^r)=\frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}}+\frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}.$$

Proof: Let X is an absolutely continuous non-negative random variable with pdf $f(x)$, then r^th^ moment of X can be obtained by:

$$\mu_r=E(X^r)=\int_0^\infty x^r f(x)dx$$

From the pdf of the TGIR distribution in (2.4), then shows that $E(X^r)$ can be written as:

$$E(X^r)=\int_0^\infty x^r\frac{2\alpha}{\lambda^2x^3}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha-1}\left\{1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}dx$$

$$=(1-\theta)\int_0^\infty x^r\frac{2\alpha}{\lambda^2x^3}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha-1}dx+2\theta\int_0^\infty x^r\frac{2\alpha}{\lambda^2x^3}e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{2\alpha-1}dx$$

Making the substitution, $y=\frac{1}{\lambda^2x^2}$, $\frac{-2}{\lambda x^3}dx=dy$, so that $x=\frac{1}{\lambda y^{1/2}}$, we obtain

$$E(X^r)=\frac{(1-\theta)\alpha}{\lambda^r}\int_0^\infty y^{1-r/2-1}e^{-y}\left(1-e^{-y}\right)^{\alpha-1}dy+\frac{2\theta\alpha}{\lambda^r}\int_0^\infty y^{1-r/2-1}e^{-y}\left(1-e^{-y}\right)^{2\alpha-1}dy \qquad (5.1)$$

Using the expansion of $\left(1-e^{-y}\right)^{\alpha-1}=\sum_{j=0}^{\infty}\frac{(-1)^j\Gamma(\alpha)}{\Gamma(\alpha-j)\,j!}e^{-yj}$.

Also, $\left(1-e^{-y}\right)^{2\alpha-1} = \sum_{k=0}^{\infty}\frac{(-1)^k\,\Gamma(2\alpha)}{\Gamma(2\alpha-k)k!}e^{-yk}.$

Expression (5.1) takes the following form:

$$E(X^r) = \frac{(1-\theta)\alpha}{\lambda^r}w_j\int_0^{\infty}y^{1-r/2-1}e^{-y(j+1)}dy + \frac{2\theta\alpha}{\lambda^r}w_k\int_0^{\infty}y^{1-r/2-1}e^{-y(k+1)}dy.$$

After some calculations,

$$\mu_r = E(X^r) = \frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}} + \frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}. \qquad (5.2)$$

Where $w_j = \sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma(\alpha)}{\Gamma(\alpha-j)\,j!}$

And $w_k = \sum_{j=0}^{\infty}\frac{(-1)^k\,\Gamma(2\alpha)}{\Gamma(2\alpha-k)k!}.$

We observe that equation (5.2) only exists when r < 1. The implication is that the second moment and other higher order moments of the distribution do not exist.

5.2 Harmonic Mean of TGIR Distribution

This sub section deals with the derivation of harmonic mean of TGIR.

Theorem 5.2: Let X follow the TGIR distribution. Then the harmonic mean of the random variable X denoted by (H) is computed as follows:

$$\frac{1}{H} = \alpha\lambda\Gamma(3/2)\left\{\frac{(1-\theta)w_j}{(j+1)^{(3/2)}} + \frac{2\theta w_k}{(k+1)^{(3/2)}}\right\}.$$

Proof: By the definition of harmonic mean, we have:

$$\frac{1}{H} = E\left(\frac{1}{X}\right) = \int_0^{\infty}\frac{1}{x}\,f(x)dx.$$

Making the substitution as $y = \frac{1}{\lambda^2 x^2}$, $\frac{-2}{\lambda x^3}dx = dy$,

So that $x = \frac{1}{\lambda y^{1/2}}$,we obtain

$$\frac{1}{H} = (1-\theta)\alpha\lambda\int_0^{\infty}y^{(3/2)-1}e^{-y}\left(1-e^{-y}\right)^{\alpha-1}dy + 2\theta\alpha\lambda\int_0^{\infty}y^{(3/2)-1}e^{-y}\left(1-e^{-y}\right)^{2\alpha-1}dy. \qquad (5.3)$$

Expression (5.3) takes the following form:

$$\frac{1}{H} = (1-\theta)\alpha\lambda w_j\int_0^{\infty}y^{3/2-1}e^{-y(j+1)}dy + 2\theta\alpha\lambda w_k\int_0^{\infty}y^{3/2-1}e^{-y(k+1)}dy.$$

After certain calculations,

$$\frac{1}{H} = \alpha\lambda\Gamma(3/2)\left\{\frac{(1-\theta)w_j}{(j+1)^{(3/2)}} + \frac{2\theta w_k}{(k+1)^{(3/2)}}\right\}.$$

$$\Rightarrow H = \frac{1}{\alpha\lambda\Gamma(3/2)\left\{\frac{(1-\theta)w_j}{(j+1)^{(3/2)}} + \frac{2\theta w_k}{(k+1)^{(3/2)}}\right\}}.$$

5.3 Moment Generating Function

In this sub section, we will derive the

5.3 Moment Generating Function of TGIR Distribution:

Theorem 5.3: Let X have a TGIR distribution. Then moment generating function of X denoted by $M_X(t)$ is given by:

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}}+\frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}.$$

Proof: By the definition of moment generating function, we have:

$$M_X(t)=E(e^{tx})=\int_0^{\infty}e^{tx}f(x)dx.$$

Using Taylor series

$$M_X(t)=\int_0^{\infty}\left(1+tx+\frac{(tx)^2}{2!}+\Lambda\right)f(x)dx.$$

$$\Rightarrow \quad M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\int_0^{\infty}x^r f(x)dx.$$

$$\Rightarrow \quad M_X(t)=\sum_{i=0}^{\infty}\frac{t^r}{r!}E(X^r).$$

$$M_X(t)=\sum_{r=0}^{\infty}\frac{t^r}{r!}\frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}}+\frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}. \tag{5.5}$$

This completes the proof.

5.4 Characteristic Function

In this sub section, we will derive the Characteristic function of TGIR distribution.

*Theorem 5.4:*Let X have a TGIR distribution. Then characteristic function of X denoted by $\phi_X(t)$ is given by:

$$\phi_X(t)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}}+\frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}.$$

*Proof:*By definition of characteristic function we have:

$$\phi_X(t)=E(e^{itx})=\int_0^{\infty}e^{itx}f(x)dx.$$

Using Taylor series,

$$\phi_X(t)=\int_0^{\infty}\left(1+itx+\frac{(itx)^2}{2!}+\Lambda\right)f(x)dx.$$

$$\Rightarrow \quad \phi_X(t)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\int_0^{\infty}x^r f(x)dx.$$

$$\Rightarrow \phi_X(t)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}E(X^r).$$

$$\phi_X(t)=\sum_{r=0}^{\infty}\frac{(it)^r}{r!}\frac{\alpha\Gamma(1-r/2)}{\lambda^r}\left\{\frac{(1-\theta)w_j}{(j+1)^{(1-r/2)}}+\frac{2\theta w_k}{(k+1)^{(1-r/2)}}\right\}. \tag{5.6}$$

This completes the proof.

6 Quantile Function, Median and Random Number Generation

This section deals with obtaining the quantile function, median and generating random numbers of TGIR distribution.

6.1 Quantile Function and Median

*Theorem 5.5:*Let the random variable X follow TGIR distribution. Then, the q^{th} quantile $Q(u)$ of the TGIR distribution is given by:

$$Q(u)=\frac{1}{\lambda}\left[\frac{-1}{\log\left[1-\left\{\frac{(\theta-1)+\sqrt{(\theta+1)^2-4\theta u}}{2\theta}\right\}^{\frac{1}{\alpha}}\right]}\right]^{\frac{1}{2}}$$

Proof: The Quantile function is denoted by $Q(u)$ and can be mathematically calculated as follows:

$$Q(u)=F^{-1}(u), \qquad 0<u<1. \tag{6.1}$$

$\therefore$ The corresponding quantile function for the proposed model is given by:

$$Q(u)=\frac{1}{\lambda}\left[\frac{-1}{\log\left[1-\left\{\frac{(\theta-1)+\sqrt{(\theta+1)^2-4\theta u}}{2\theta}\right\}^{\frac{1}{\alpha}}\right]}\right]^{\frac{1}{2}}$$

(6.2)

Where U has the uniform U (0,1) distribution. We obtain the median of the TGIR distribution by substituting u=0.5 in equation (6.2). Hence, the median of the proposed model is calculated as:

$$\text{Median}=F^{-1}(0.5)=\frac{1}{\lambda}\left[-\log\left[1-\left\{\frac{(\theta-1)+\sqrt{\theta^2+1}}{2\theta}\right\}^{\frac{1}{\alpha}}\right]\right]^{-\frac{1}{2}} .$$

(6.3)

6.2 Random Number Generation

In order to generate the random numbers from the transmuted generalized inverted Rayleigh distribution, the method of inversion is used as follows:

$$u=\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\},$$

Where u ~ U (0, 1). After simplification this yields

$$x=\frac{1}{\lambda}\left[\frac{-1}{\log\left[1-\left\{\frac{(\theta-1)+\sqrt{(\theta+1)^2-4\theta u}}{2\theta}\right\}^{\frac{1}{\alpha}}\right]}\right]^{\frac{1}{2}} \qquad (6.4)$$

One can use equation (6.4) to generate random numbers when the parameters are known.

7 Renyi Entropy

The entropy of a random variable X with probability density TGIR $(x;\theta,\alpha,\lambda)$ is a measure of the variation of the uncertainty. The large value of entropy is an indicator of the greater uncertainty in the data. The Renyi entropy [17], denoted by $I_R(\rho)$ for X is a measure of variation of uncertainty and is defined as:

$$I_R(\rho)=\frac{1}{1-\rho}\log\left\{\int_{-\infty}^{\infty} f(x)^{\rho}\,dx\right\}. \qquad (7.1)$$

where $\rho>0$ and $\rho\neq 1$.

Suppose X has TGIRD $(x;\theta,\alpha,\lambda)$, then by substituting equation (2.4) in (7.1) we have:

$$I_R(\rho)=\frac{1}{1-\rho}\log\left[\int_0^{\infty}\left(\frac{2\alpha}{\lambda^2}\right)^{\rho}\frac{1}{x^{3\rho}}e^{-(\lambda x)^{-2\rho}}\left[1-e^{-(\lambda x)^{-2}}\right]^{\rho(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]^{\rho}dx\right]. \qquad (7.2)$$

For the convenience, let $u(x)=\int_0^{\infty} f(x)^{\rho}\,dx$

$$\therefore u(x)=\int_0^{\infty}\left(\frac{2\alpha}{\lambda^2}\right)^{\rho}\frac{1}{x^{3\rho}}e^{-(\lambda x)^{-2\rho}}\left[1-e^{-(\lambda x)^{-2}}\right]^{\rho(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]^{\rho}dx.$$

Put $(\lambda x)^{-2}=t,\ x=\frac{1}{\lambda t^{\frac{1}{2}}},\ dx=\frac{-1}{2\lambda t^{\frac{3}{2}}}dt$.

when $x=0, t=\infty$ and $x=\infty, t=0$.

$$u(x)=\int_0^{\infty}\left(\frac{2\alpha}{\lambda^2}\right)^{\rho}\frac{1}{\left(\frac{1}{\lambda t^{\frac{1}{2}}}\right)^{-3\rho}}e^{-\rho t}\left(1-e^{-t}\right)^{\rho(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-t}\right)^{\alpha}\right]^{\rho}\frac{1}{2\lambda t^{\frac{3}{2}}}dt .$$

$$=2^{\rho-1}\alpha^{\rho}\lambda^{3(\rho-1)}\int_0^{\infty}t^{\frac{3}{2}(\rho-1)}e^{-\rho t}\left(1-e^{-t}\right)^{\rho\alpha-\rho}\left[1-\theta+2\theta\left(1-e^{-t}\right)^{\alpha}\right]^{\rho}dt .$$

$$=2^{\rho-1}\alpha^{\rho}\lambda^{3(\rho-1)}\int_0^{\infty}t^{\frac{3}{2}(\rho-1)}e^{-\rho t}\left(1-e^{-t}\right)^{(\rho\alpha-\rho)}\sum_{j=0}^{\infty}{}^{\rho}c_j\left(1-\lambda\right)^{\rho-j}(2\theta)^{j}\left(1-e^{-t}\right)^{\alpha j}dt .$$

$$=\sum_{j=0}^{\infty} {}^{\rho}c_j (1-\theta)^{\rho-j} (2\theta)^j 2^{\rho-1} \alpha^{\rho} \lambda^{3(\rho-1)} \int_0^{\infty} t^{\frac{3}{2}(\rho-1)} e^{-\rho t} \left(1-e^{-t}\right)^{(\rho\alpha-\rho)} \left(1-e^{-t}\right)^{\alpha j} dt$$

$$=\sum_{j=0}^{\infty} {}^{\rho}c_j (1-\theta)^{\rho-j} (2\theta)^j 2^{\rho-1} \alpha^{\rho} \lambda^{3(\rho-1)} \int_0^{\infty} t^{\frac{3}{2}(\rho-1)} e^{-\rho t} \left(1-e^{-t}\right)^{(\rho\alpha-\rho)+\alpha j} dt.$$

$$=\sum_{j=0}^{\infty} {}^{\rho}c_j (1-\theta)^{\rho-j} (2\theta)^j 2^{\rho-1} \alpha^{\rho} \lambda^{3(\rho-1)} \int_0^{\infty} t^{\frac{3}{2}(\rho-1)} e^{-\rho t} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma(\rho\alpha-\rho+\alpha j+1)}{(\Gamma(\rho\alpha-\rho+\alpha j+1)-k)k!} e^{-tk} dt.$$

$$=\sum_{j=0}^{\infty} {}^{\rho}c_j (1-\theta)^{\rho-j} (2\theta)^j 2^{\rho-1} \alpha^{\rho} \lambda^{3(\rho-1)} \int_0^{\infty} t^{\frac{3}{2}(\rho-1)} e^{-\rho t} \left[1+\sum_{k=1}^{\infty} a_k e^{-tk}\right] dt.$$

$$u(x)=\sum_{j=0}^{\infty} v_j 2^{\rho-1} \alpha^{\rho} \lambda^{3(\rho-1)} \left[\frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{\rho^{\frac{3\rho-1}{2}}}+\sum_{k=0}^{\infty} a_k \frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{(\rho+k)^{\frac{3\rho-1}{2}}}\right]. \qquad (7.3)$$

where $a_k=(-1)^k \dfrac{(\rho\alpha+j\alpha-\rho)(\rho\alpha+j\alpha-\rho-1)(\rho\alpha+j\alpha-\rho-2).....(\rho\alpha+j\alpha-\rho-k+1)}{k!}$.

and $v_j={}^{\rho}c_j (1-\theta)^{\rho-j}(2\theta)^j$

Substituting the value of equation (7.3) in (7.2) we get the Renyi entropy of TGIRD as follows:

$$I_R(\rho)=\frac{1}{1-\rho}\log\left(\sum_{j=0}^{\infty} v_j 2^{\rho-1}\alpha^{\rho}\lambda^{3(\rho-1)}\left[\frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{\rho^{\frac{3\rho-1}{2}}}+\sum_{k=0}^{\infty} a_k \frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{(\rho+k)^{\frac{3\rho-1}{2}}}\right]\right).$$

$$I_R(\rho)=\frac{\rho}{1-\rho}\log\alpha+3(\rho-1)\log\lambda+\frac{1}{1-\rho}\log\left(\sum_{j=0}^{\infty} v_j 2^{\rho-1}\left[\frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{\rho^{\frac{3\rho-1}{2}}}+\sum_{k=0}^{\infty} a_k \frac{\Gamma\left(\frac{3\rho-1}{2}\right)}{(\rho+k)^{\frac{3\rho-1}{2}}}\right]\right). \quad (7.4)$$

The β or q-entropy introduced by Havrda and Charvat [18] is denoted by $I_H(q)$ and can be computed as:

$$I_H(q)=\frac{1}{q-1}\left\{1-\int_{-\infty}^{\infty} f(x)^q dx\right\}, \qquad (7.5)$$

where $q>0$ *and* $q\neq 1$

Suppose X has TGIRD $(x;\theta,\alpha,\lambda)$, then by substituting (2.4) in (7.5), we get the β entropy as follows:

$$I_H(q)=\frac{1}{q-1}\left[1-\int_0^{\infty}\left(\frac{2\alpha}{\lambda^2}\right)^q \frac{1}{x^{3\rho}} e^{-(\lambda x)^{-2}q}\left[1-e^{-(\lambda x)^{-2}}\right]^{q(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]^q dx\right].$$

$$\Rightarrow I_H(q)=\frac{1}{q-1}\left(\sum_{j=0}^{\infty} v_j 2^{q-1}\alpha^q\lambda^{3(q-1)}\left[\frac{\Gamma\left(\frac{3q-1}{2}\right)}{q^{\frac{3q-1}{2}}}+\sum_{k=0}^{\infty} a_k \frac{\Gamma\left(\frac{3q-1}{2}\right)}{(q+k)^{\frac{3q-1}{2}}}\right]\right), \qquad (7.6)$$

where $a_k = (-1)^k \dfrac{(q\alpha + j\alpha - q)(q\alpha + j\alpha - q - 1)(q\alpha + j\alpha - q - 2).....(q\alpha + j\alpha - q - k + 1)}{k!}$,

and $v_j = {}^q c_j \,(1-\theta)^{q-j}(2\theta)^j$.

8 Order Statistics

Order statistics finds many applications in statistical theory and modeling. It can be applied in studying the reliability of a system and life testing. If $X_{(1)}, X_{(2)},....., X_{(n)}$ denote the order statistics obtained from the random sample $X_1, X_2,....., X_n$ drawn from TGIR distribution $(\lambda, \theta, \alpha)$ with cumulative density function and probability density function given in the equations (2.3) and (2.4) respectively, then the probability density function of the order statistics is given as below:

$$f_r(x) = \frac{n!}{(r-1)!(n-r)!}\left[F(x)\right]^{r-1}\left[1-F(x)\right]^{n-r} f(x) \qquad for\ 1 \le r \le n \qquad (8.1)$$

Using the equations (2.3) and (2.4), the pdf of the first order statistic $X_{(1)} = \min(X_1, X_2,...X_n)$ is given by:

$$f_1(x) = \frac{2\alpha n}{\lambda^2 x^3}\left[1-\left\{1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right\}\right]^{n-1} e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right] \quad (8.2)$$

Similarly, the pdf of the nth order statistic $X_{(n)} = \max(X_1, X_2,...X_n)$ is given as follows:

$$f_n(x) = \frac{2\alpha n}{\lambda^2 x^3}\left\{\left[1-\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]\left[1+\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]\right\}^{n-1} e^{-(\lambda x)^{-2}}\left(1-e^{-(\lambda x)^{-2}}\right)^{(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{a}\right] \quad (8.3)$$

8.1 Joint Distribution function of ith and jth order statistics

The joint density functions of (x_i, x_j) for $1 \le i \le j \le n$ is given by:

$$f_{i:j:n}(x_i, x_j) = C\left[F(x_i)\right]^{i-1}\left[F(x_j) - F(x_i)\right]^{j-i-1}\left[1-F(x_j)\right]^{n-j} f(x_i)\ f(x_j), \qquad (8.4)$$

where $C = \dfrac{n!}{(i-1)!(j-i-1)!(n-j)!}$.

Then the joint distribution function of the ith and jth order statistics of Transmuted Generalized Inverse Rayleigh distribution is as follows:

$$f_{i:j:n}(x) = C\left[\left(1-h_{(i)}{}^{\alpha}\right)\left(1+\theta\, h_{(i)}{}^{\alpha}\right)\right]^{i-1}\left[\left\{\left(1-h_{(j)}{}^{a}\right)\left(1+\theta\, h_{(j)}{}^{a}\right)\right\}-\left\{\left(1-h_{(i)}{}^{\alpha}\right)\left(1+\theta\, h_{(i)}{}^{\alpha}\right)\right\}\right]^{j-i-1}\left\{1-\left[\left(1-h_{(j)}{}^{a}\right)\left(1+\theta\, h_j{}^{\alpha}\right)\right]\right\}^{n-j}$$

$$\frac{2\alpha}{\lambda^2 x_{(i)}{}^3} e^{-(\lambda x_1)^{-2}} h_{(i)}{}^{(\alpha-1)}\left(1-\theta+2\theta\, h_{(i)}{}^{\alpha}\right) \frac{2\alpha}{\lambda^2 x_{(j)}{}^3} e^{-(\lambda x_j)^{-2}} h_{(j)}{}^{(\alpha-1)}\left(1-\theta+2\theta\, h_{(j)}{}^{\alpha}\right),$$

$$\text{where } h_{(k)} = \left(1-e^{-(\lambda x_k)}\right) \text{ for } k = i, j \qquad (8.5)$$

For the special case $i = 1$ and $j = n$, we get the joint distribution of minimum and maximum order statistics as follows:

$$f_{1:n}(x) = n(n-1)\left[F(x_n) - F(x_1)\right]^{n-2} f(x_1) f(x_n)$$

$$f_{1n}(x) = n(n-1)\left[\left\{1-\left(1-e^{-(\lambda x_n)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x_n)^{-2}}\right)^{\alpha}\right\}-\left\{1-\left(1-e^{-(\lambda x_1)^{-2}}\right)^{\alpha}\right\}\left\{1+\theta\left(1-e^{-(\lambda x_1)^{-2}}\right)^{\alpha}\right\}\right]^{(n-2)}$$

$$\frac{2\alpha}{\lambda^2 x_{(1)}{}^3} e^{-(\lambda x_1)^{-2}}\left(1-e^{-(\lambda x_1)^{-2}}\right)^{(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x_1)^{-2}}\right)^{\alpha}\right]$$

$$\frac{2\alpha}{\lambda^2 x_{(n)}{}^3} e^{-(\lambda x_n)^{-2}}\left(1-e^{-(\lambda x_n)^{-2}}\right)^{(\alpha-1)}\left[1-\theta+2\theta\left(1-e^{-(\lambda x_n)^{-2}}\right)^{\alpha}\right]. \qquad (8.6)$$

9 Maximum Likelihood Estimation

In this section maximum likelihood estimators and inference for TGIRD $(x;\lambda,\theta,\alpha)$ are discussed.
In order to estimate the unknown parameters of the TGIR distribution we use the technique of maximum likelihood Estimation. The maximum likelihood estimates (MLE's) of the parameters that are inherent within the Transmuted Generalized Inverse Rayleigh distribution function are obtained as follows:

Let $x_1, x_2, \ldots . x_n$ be the random sample drawn from the Transmuted Generalized Inverse Rayleigh distribution. Then the likelihood function is given by:

$$L(\lambda,\theta,\alpha \mid x)=\left(\frac{2\alpha}{\lambda^2}\right)^n \prod_{i=1}^{n}\frac{1}{x_i^{3}}\exp\left(-\sum_{i=1}^{n}(\lambda x_i)^{-2}\right)\prod_{i=1}^{n}\left(1-\exp(\lambda x_i)^{-2}\right)^{(\alpha-1)}\prod_{i=1}^{n}\left[1-\theta+2\theta\left(1-\exp(-\lambda x_i)^{-2}\right)^{\alpha}\right]$$

(9.1)

By taking the logarithm of equation (9.1), the Log Likelihood function $L(\lambda,\theta,\alpha \mid x)$ can be obtained as:

$$Log\, L(\lambda,\theta,\alpha \mid x)=n\log 2+n\log\alpha-2n\log\lambda+\sum_{i=1}^{n}\log\frac{1}{x_i^{3}}-\sum_{i=1}^{n}(\lambda x_i)^{-2}+(\alpha-1)\sum_{i=1}^{n}\log\left(1-e^{-(\lambda x_i)^{-2}}\right)$$
$$+\sum_{i=1}^{n}\log\left(1-\theta+2\theta\left(1-e^{(\lambda x_i)^{-2}}\right)^{\alpha}\right).$$

(9.2)

Differentiating the Log Likelihood function in equation (9.2) with respect to the unknown parameters λ,θ and α and equating them to zero, we get the respective normal equations as follows:

$$\frac{d\log L}{d\lambda}=\frac{-2n}{\lambda}+\frac{2}{\lambda^3}\sum_{i=1}^{n}x_i^{-2}-(\alpha-1)\sum_{i=1}^{n}\frac{2}{\lambda^3x^2}\frac{e^{-(\lambda x)^{-2}}}{(1-e^{-(\lambda x)^{-2}})}-\sum_{i=1}^{n}\frac{4\theta\alpha\left(1-e^{-(\lambda x)^{-2}}\right)^{(\alpha-1)}}{\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]}\frac{e^{-(\lambda x)^{-2}}}{\lambda^3x^2} \quad (9.3)$$

$$\frac{d\log L}{d\theta}=\sum_{i=1}^{n}\frac{\left(2\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}-1\right)}{\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]}. \quad (9.4)$$

$$\frac{d\log L}{d\alpha}=\frac{n}{\alpha}+\sum_{i=1}^{n}\log\left[1-e^{-(\lambda x)^{-2}}\right]+\sum_{i=1}^{n}\frac{2\theta\left[1-e^{-(\lambda x)^{-2}}\right]^{\alpha}\log\left[1-e^{-(\lambda x)^{-2}}\right]}{\left[1-\theta+2\theta\left(1-e^{-(\lambda x)^{-2}}\right)^{\alpha}\right]}$$

(9.5)

The solutions obtained by solving the three normal equations simultaneously represent the maximum likelihood estimates of unknown parameters (λ,θ,α) of the proposed distribution.

Fisher Information Matrix

For the three parameters of TGIR $(x;\lambda,\theta,\alpha)$ all the second order derivatives of the log- likelihood function exist. Thus, the inverse dispersion matrix is given by:

$$\begin{pmatrix}\hat{\alpha}\\ \hat{\lambda}\\ \hat{\theta}\end{pmatrix} \sim N\left(\begin{pmatrix}\alpha\\ \lambda\\ \theta\end{pmatrix},\begin{pmatrix}\hat{V}_{\alpha\alpha} & \hat{V}_{\alpha\lambda} & \hat{V}_{\alpha\theta}\\ \hat{V}_{\lambda\alpha} & \hat{V}_{\lambda\lambda} & \hat{V}_{\lambda\theta}\\ \hat{V}_{\theta\alpha} & \hat{V}_{\theta\lambda} & \hat{V}_{\theta\theta}\end{pmatrix}\right) \quad (9.6)$$

$$V^{-1}=-E\begin{pmatrix}V_{\alpha\alpha} & V_{\alpha\lambda} & V_{\alpha\theta}\\ V_{\lambda\alpha} & V_{\lambda\lambda} & V_{\lambda\theta}\\ V_{\theta\alpha} & V_{\theta\lambda} & V_{\theta\theta}\end{pmatrix} \quad (9.7)$$

where $V_{\alpha\alpha}=\frac{\partial^2 L}{\partial\alpha\partial\alpha}, V_{\alpha\lambda}=\frac{\partial^2 L}{\partial\alpha\partial\lambda}, V_{\alpha\theta}=\frac{\partial^2 L}{\partial\alpha\partial\theta}$ and so on.
By deriving the inverse dispersion matrix, the asymptotic variances and covariances of the ML estimators for α, λ and θ are obtained.

10 Applications

In this section, we consider both the simulated as well as real life data sets to compare the flexibility of the proposed transmuted model of Generalized Inverse Rayleigh Distribution over its different sub models. For comparing

The different models we have used the criteria like AIC (Akaike Information Criterion) and BIC (Bayesian Information Criterion). The distribution which provides us lesser values of AIC and BIC is considered as best. The values of AIC and BIC can be computed as follows:
AIC=2k-2logL and BIC=klogn-2logL,
Where k is the number of parameters in the statistical model, n is the sample size and -2logL is the maximized value of the log-likelihood function under the considered model. The analysis of both the data sets is performed through R software. The MLEs of the parameters are obtained with standard errors shown in parentheses.
Further, the corresponding log-likelihood values, AIC and BIC are displayed in Table 1and 2.

10.1 Simulated Data

In the simulation study, three data sets of size 50, 60 and 70 have been generated from R software to examine the performance of the new model over its sub models. The values of the parameters are chosen $\alpha = 2.0, \lambda = 1.5$, and transmutation parameter $\theta = 0.3$. The data sets are obtained by using the inverse cdf method as discussed in section 6 and the summary of results is presented in the table 1 below:

Table 1: MLEs of the model parameters using generated data sets, the resulting SEs in a side and Criteria for Comparison

n	Distribution	α	θ	λ	Log-likelihood	AIC	BIC
50	TGIRD	6.41425 (2.11894)	0.76958 (0.21623)	0.42464 (0.02401)	-8.290532	22.58106	28.31713
	GIRD	7.97079 (2.07549)	–	0.44757 (0.02599)	-10.43216	24.86432	28.68837
	IRD	–	–	0.72809 (0.05148)	-36.83595	75.6719	77.58392
	TIRD	–	1.00000 (0.41498)	0.60304 (0.03841)	-22.79343	49.58686	53.41091
	TGSIRD	1.51747 (0.23394)	-1.00000 (0.64012)	–	-29.82228	63.64455	67.4686
	GSIRD	1.03968 (0.14703)	–	–	-45.03757	92.07513	93.98716
	TSIRD	–	-1.00000 (0.71012)	–	-36.92314	75.84627	77.75829
60	TGIRD	7.07457 (2.18266)	0.77689 (0.19778)	0.42338 (0.02156)	-6.670248	19.3405	25.62353
	GIRD	8.78948 (2.12186)	–	0.44596 (0.02326)	-9.311974	22.62395	26.81264
	IRD	–	–	0.73729 (0.04759)	-42.88901	87.77801	89.87236
	TIRD	–	1.00000 (0.41498)	0.61087 (0.03841)	-25.70763	55.41526	59.60395
	TGSIRD	1.55752 (0.21692)	-1.00000 (0.57689)	–	-33.55117	71.10233	75.29102
	GSIRD	1.067599 (0.13782)	–	–	-51.95221	105.9044	107.9988
	TSIRD	–	--1.00000 (0.67927)	–	-43.10461	88.20922	90.30356
70	TGIRD	7.28036 (2.19588)	0.76587 (0.20522)	0.42177 (0.02001)	-7.367017	20.73403	27.47952
	GIRD	9.10859 (2.06116)	–	0.44308 (0.02142)	-10.12916	24.25833	28.75532
	IRD	–	–	0.73731 (0.04406)	-49.92224	101.8445	104.093
	TIRD	–	1.00000 (0.39859)	0.61088 (0.03599)	-29.80085	63.60171	68.0987
	TGSIRD	1.55898 (0.20058)	-1.00000 (0.53249)	–	-38.98326	81.96652	86.46351
	GSIRD	1.06887 (0.12775)	–	–	-60.48912	122.9782	125.2267
	TSIRD	–	-1.00000 (0.62881)	–	-50.17406	102.3481	104.5966

10.2 Real Life Data

Here we consider the two real data sets pertaining to medical science and engineering field respectively as given under and the results are presented in the table 2.

Data set I: The first real data set is a subset of the data reported by Bekker et al. [19], which corresponds to the survival times (in years) of a group of patients given chemotherapy treatment alone. The data consisting of survival times (in years) for 46 patients are: 0.047,0.115, 0.121, 0.132, 0.164, 0.197,0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641,0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178,2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.978, 4.003, 4.033.

Data set II: The second data set is the failure times of 84 Aircraft Windshield. The windshield on a large aircraft is a complex piece of equipment, comprised basically of several layers of material, including a very strong outer skin with a heated layer just beneath it, all laminated under high temperature and pressure. Failures of these items are not structural failures. Instead, they typically involve damage or delamination of the nonstructural outer ply or failure of the heating system. These failures do not result in damage to the aircraft but do result in replacement of the windshield. These data on failure times are reported in the book “Weibull Models” by Murthy et al. [20]. The failure times of 84 Aircraft Windshield is

0.040, 1.866, 2.385, 3.443, 0.301, 1.876, 2.481, 3.467, 0.309, 1.899, 2.610, 3.478, 0.557, 1.911, 2.625, 3.578, 0.943, 1.912, 2.632, 3.595, 1.070, 1.914, 2.646, 3.699, 1.124, 1.981, 2.661, 3.779,1.248, 2.010, 2.688, 3.924, 1.281, 2.038, 2.82,3, 4.035, 1.281, 2.085, 2.890, 4.121, 1.303, 2.089, 2.902, 4.167, 1.432, 2.097, 2.934, 4.240, 1.480, 2.135, 2.962, 4.255, 1.505, 2.154, 2.964, 4.278, 1.506, 2.190, 3.000, 4.305, 1.568, 2.194, 3.103, 4.376, 1.615, 2.223, 3.114, 4.449, 1.619, 2.224, 3.117, 4.485, 1.652, 2.229, 3.166, 4.570, 1.652, 2.300, 3.344, 4.602, 1.757, 2.324, 3.376, 4.663.

Table 2: MLEs of the model parameters using real data sets, the resulting SEs in a side and Criteria for Comparison

Data	Distribution	α	θ	λ	Log-likelihood	AIC	BIC
Data Set I	TGIRD	0.346832 (0.04858)	-0.87077 (0.13338)	9.14404 (1.71163)	-67.22199	140.444	145.864
	GIRD	0.27152 (0.04546)	–	7.44040 (1.14802)	-72.14705	148.2941	151.9074
	IRD	–	–	4.31796 (0.32184)	-115.0868	232.1737	233.9803
	TIRD	–	-0.83225 (0.12189)	4.85539 (0.42737)	-103.9305	211.861	215.4743
	TGSIRD	1.05375 (0.21481)	0.37705 (0.23203)	–	-775.1399	1554.28	1557.893
	GSIRD	1.21617 (0.18129)	–	–	-776.6437	1555.287	1557.094
	TSIRD	–	0.41394 (0.18805)	–	-775.1712	1552.342	1554.149
Data set II	TGIRD	0.26838 (0.02557)	-0.96344 (0.03638)	6.41096 (0.87017)	-225.6284	457.2569	464.5848
	GIRD	0.18960 (0.02263)	–	6.20165 (0.82896)	-248.8307	501.6614	506.5467
	IRD	–	–	2.80331 (0.15203)	-403.4537	808.9073	811.35
	TIRD	–	-0.97405 (0.02573)	2.88943 (0.15686)	-355.1746	714.3492	719.2345
	TGSIRD	0.75629 (0.06506)	-0.84393 (0.08236)	–	-779.0335	1562.067	1566.952
	GSIRD	0.54498 (0.05911)	–	–	-791.8212	1585.642	1588.085
	TSIRD	–	-0.88319 (0.06227)	–	-784.8918	1571.784	1574.226

11Conclusions

This manuscript deals with the introduction of transmuted generalized inverse Rayleigh distribution which is the generalization of many distributions viz GIRD, IRD, TIRD, TGSIRD, GSIRD and TSIRD. The main aim of the paper is to study its different statistical properties like moments, harmonic mean, survival function, hazard rate, Renyi entropy and maximum likelihood estimation. Further, the postulated distribution is compared with its different sub models in terms of fitting. This newly proposed model has been applied to both the generated as well as the real life data sets. The results obtained are displayed in table 1 and 2 respectively which show that the proposed distribution has lesser values of AIC and BIC than its different special cases. This proves that the newly developed model provides better fit than its sub models.

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