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Generalized log type estimator of population mean in Adaptive cluster sampling

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Abstract: The use of functions like log and exponential in developing highly efficient estimators has been extensively studied in traditional sampling designs. Such estimators have proven to be more efficient in some studies but they cannot be used when the population under study is patchy or hidden clustered since in such situations estimators developed under the Adaptive cluster sampling design are generally used. Thus, in this paper, we have proposed a generalized log type class of estimators of population mean in the Adaptive cluster sampling design and studied various new log type estimators developed from the proposed class. The derivations of properties like bias and MSE have been presented up to the first order of approximations. To show the novelty of the new developed log type estimators over several competing estimators considered in this paper various simulation studies have been conducted. Results show that the new developed log type estimators perform better.

Keywords: Log type generalized class, Generalized class, Bias, Mean squared error, Adaptive cluster sampling, Ratio estimator, Patchy data.

1 Introduction

The debate over using a part or fraction of the population to estimate the parameters under study rather than the whole population gave birth to the field of survey sampling. As a result, researchers devised sampling methods namely Simple random sampling without replacement (SRSWOR), Simple random sampling with replacement (SRSWR), Stratified random sampling, and Cluster sampling among others to be used depending on the population under study. The guiding principle in sample surveys is to get a representative sample. This becomes difficult if we use the traditional sampling designs when the population under study is rare or clumped. For such populations, [1] proposed the Adaptive cluster sampling design where the researcher specifies a condition on the survey variable y based on which the sampling units are selected. Due to its wide applicability and flexibility, ACS has been used in a wide spectrum of studies (see [2,3,4,5,6]).

The use of auxiliary information in developing estimators caught researchers attention after Cochran's [7] proposal of ratio estimator. Since then, various ratio type estimators have been proposed (see [8,9,10,11,12]). In ACS, [13] proposed the ratio estimator and following their work various transformed ratio type estimators have been developed in the ACS design. [13] proposed their ratio estimator using one auxiliary variable in the ACS design and studied its bias and MSE. [14] proposed different ratio type estimators for the population mean using a single auxiliary variable and some known parameters of the auxiliary variable. [15] developed their improved ratio type estimators for the population mean using a single auxiliary variable with some known parameters such as coefficient of skewness and kurtosis among others of the auxiliary variable. [16] proposed their estimators by incorporating the use of robust measures namely Hodges-Lehman (HL), tri-mean (TM) and mid-range (MR) as the known parameters of the auxiliary variable to estimate the population mean.

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Studying different existing estimators for the population mean and variance in traditional sampling designs, we can see that the use of functions like log and exponential under different transformations has been studied extensively and has been shown to be more efficient (see [10,11,17,18,19]). Our aim in this paper is to study the performance and properties of log type estimators in estimating the population mean in the ACS design and for that we have proposed a generalized class of log type estimators and from this proposed generalized class we have developed various new log type estimators. For new readers, we recommend reading [1,13] for understanding the methodology of the ACS design. In Section 2 we provide a brief literature review of some related existing estimators of the population mean in ACS design. In Section 3 the proposed generalized log type class is presented along with the derivations of bias and MSE up to first order of approximations. Further, in this same section, we provide the new log type estimators developed from the proposed generalized log type class. In Section 4 we have conducted various simulation studies to highlight the precision of the new developed log type estimators over the several competing estimators presented in this paper. In Section 5 we provide the results and concluding remarks on this paper along with some future ideas for further developing the theory of ACS design.

2 Literature review

Using one auxiliary variable, [13] devised their ratio estimator as follows:

$$t_{DC} = \frac{\bar{w}_y}{\bar{w}_x}\mu_x, \tag{1}$$

The Mean Square Error of t_{DC} [13] estimator up to the first order of approximation is

$$\mathrm{MSE}(t_{DC}) = f\mu_y^2(C_{w_y}^2 + C_{w_x}^2 - 2\rho_{w_x w_y} C_{w_x} C_{w_y}), \tag{2}$$

where $f = (N-n)/Nn$.

[14] proposed their estimators incorporating some transformations using known parameters of auxiliary variables as follows:

$$t_{CH_1} = \bar{w}_y \frac{\mu_x + C_{w_x}}{\bar{w}_x + C_{w_x}}, \tag{3}$$

$$t_{CH_2} = \bar{w}_y \frac{\beta_2(w_x)\mu_x + C_{w_x}}{\beta_2(w_x)\bar{w}_x + C_{w_x}}, \tag{4}$$

$$t_{CH_3} = \bar{w}_y \frac{\mu_x + \beta_2(w_x)}{\bar{w}_x + \beta_2(w_x)}. \tag{5}$$

The expressions for MSE of $t_{CH_1} - t_{CH_3}$ [14] are as follows:

$$\mathrm{MSE}(t_{CH_1}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{\mu_x}{\mu_x + C_{w_x}}\right)^2 C_{w_x}^2 - 2\left(\frac{\mu_x}{\mu_x + C_{w_x}}\right)\rho_{w_x w_y} C_{w_y} C_{w_x}\right), \tag{6}$$

$$\mathrm{MSE}(t_{CH_2}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{\mu_x\beta_2(w_x)}{\mu_x\beta_2(w_x) + C_{w_x}}\right)^2 C_{w_x}^2 - 2\left(\frac{\mu_x\beta_2(w_x)}{\mu_x\beta_2(w_x) + C_{w_x}}\right)\rho_{w_x w_y} C_{w_y} C_{w_x}\right), \tag{7}$$

$$\mathrm{MSE}(t_{CH_3}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{\mu_x}{\mu_x + \beta_2(w_x)}\right)^2 C_{w_x}^2 - 2\left(\frac{\mu_x}{\mu_x + \beta_2(w_x)}\right)\rho_{w_x w_y} C_{w_y} C_{w_x}\right). \tag{8}$$

[15] proposed their estimator using known coefficients of auxiliary variables as follows:

$$t_{YS_1} = \bar{w}_y \frac{\beta_2(w_x)\mu_x + \beta_1(w_x)}{\beta_2(w_x)\bar{w}_x + \beta_1(w_x)}, \tag{9}$$

$$t_{YS_2} = \bar{w}_y \frac{\beta_1(w_x)\mu_x + \beta_2(w_x)}{\beta_1(w_x)\bar{w}_x + \beta_2(w_x)}. \tag{10}$$

The expressions for MSE of $t_{YS_1} - t_{YS_2}$ [15] are as follows:

$$\mathrm{MSE}(t_{YS_1}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{\beta_2(w_x)\mu_x}{\beta_2(w_x)\mu_x+\beta_1(w_x)}\right)^2 C_{w_x}^2 - 2\left(\frac{\beta_2(w_x)\mu_x}{\beta_2(w_x)\mu_x+\beta_1(w_x)}\right)\rho_{w_xw_y}C_{w_y}C_{w_x}\right), \tag{11}$$

$$\mathrm{MSE}(t_{YS_2}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{\beta_1(w_x)\mu_x}{\beta_1(w_x)\mu_x+\beta_2(w_x)}\right)^2 C_{w_x}^2 - 2\left(\frac{\beta_1(w_x)\mu_x}{\beta_1(w_x)\mu_x+\beta_2(w_x)}\right)\rho_{w_xw_y}C_{w_y}C_{w_x}\right). \tag{12}$$

[16] proposed some ratio type estimators using some robust measures and studied their properties. Their estimators are

$$t_{QK_1} = \bar{w}_y\frac{MR\mu_x+\beta_1(w_x)}{MR\bar{w}_x+\beta_1(w_x)}, \tag{13}$$

$$t_{QK_2} = \bar{w}_y\frac{MR\mu_x+TM}{MR\bar{w}_x+TM}, \tag{14}$$

$$t_{QK_3} = \bar{w}_y\frac{HL\mu_x+\beta_1(w_x)}{HL\bar{w}_x+\beta_1(w_x)}, \tag{15}$$

$$t_{QK_4} = \bar{w}_y\frac{HL\mu_x+TM}{HL\bar{w}_x+TM}. \tag{16}$$

The expressions of bias and MSE up to the first order of approximation of estimators $t_{QK_1} - t_{QK_4}$ [16] are

$$\mathrm{Bias}(t_{QK_1}) = f\mu_y\left(\left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)^2 C_{w_x}^2 - \left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{17}$$

$$\mathrm{Bias}(t_{QK_2}) = f\mu_y\left(\left(\frac{MR\mu_x}{MR\mu_x+TM}\right)^2 C_{w_x}^2 - \left(\frac{MR\mu_x}{MR\mu_x+TM}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{18}$$

$$\mathrm{Bias}(t_{QK_3}) = f\mu_y\left(\left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)^2 C_{w_x}^2 - \left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{19}$$

$$\mathrm{Bias}(t_{QK_4}) = f\mu_y\left(\left(\frac{HL\mu_x}{HL\mu_x+TM}\right)^2 C_{w_x}^2 - \left(\frac{HL\mu_x}{HL\mu_x+TM}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{20}$$

$$\mathrm{MSE}(t_{QK_1}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)^2 C_{w_x}^2 - 2\left(\frac{MR\mu_x}{MR\mu_x+\beta_1(w_x)}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{21}$$

$$\mathrm{MSE}(t_{QK_2}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{MR\mu_x}{MR\mu_x+TM}\right)^2 C_{w_x}^2 - 2\left(\frac{MR\mu_x}{MR\mu_x+TM}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{22}$$

$$\mathrm{MSE}(t_{QK_3}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)^2 C_{w_x}^2 - 2\left(\frac{HL\mu_x}{HL\mu_x+\beta_1(w_x)}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right), \tag{23}$$

$$\mathrm{MSE}(t_{QK_4}) = f\mu_y^2\left(C_{w_y}^2 + \left(\frac{HL\mu_x}{HL\mu_x+TM}\right)^2 C_{w_x}^2 - 2\left(\frac{HL\mu_x}{HL\mu_x+TM}\right)\rho_{w_xw_y}C_{w_x}C_{w_y}\right). \tag{24}$$

3 Proposed generalized log type class

The aim of this paper is to study the performance of log type estimators in the ACS sampling design and therefore following the work of [8,17,18,19,20] we propose the following class:

$$t_G = \left(k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\alpha\mu_x+\beta}{\theta(\alpha\bar{w}_x+\beta)+(1-\theta)(\alpha\mu_x+\beta)}\right)^g\right)\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)^\eta, \tag{25}$$

where k_1 and k_2 are real constants to be optimized such that MSE of the proposed generalized log type class t_G is minimum whereas θ, g, and η are generalized scalar constants, they may assume several different combinations of real numbers. α

and β are generalized scalar constants, they may assume different combinations of parameters of auxiliary variable namely coefficient of skewness, kurtosis, variation, Mid range, population mean square among others to give special forms of the proposed generalized log type class t_G. Note that, the competing estimators considered in this paper are particular cases of the proposed generalized log type class t_G as follows:

1.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,1,0,1,0)$, t_G reduces to t_{DC} [13].
2.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,1,C_{w_x},1,0)$, t_G reduces to t_{CH_1} [14].
3.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,\beta_2(w_x),C_{w_x},1,0)$, t_G reduces to t_{CH_2} [14].
4.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,1,\beta_2(w_x),1,0)$, t_G reduces to t_{CH_3} [14].
5.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,\beta_2(w_x),\beta_1(w_x),1,0)$, t_G reduces to t_{YS_1} [15].
6.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,\beta_1(w_x),\beta_2(w_x),1,0)$, t_G reduces to t_{YS_2} [15].
7.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,MR,\beta_1(w_x),1,0)$, t_G reduces to t_{QK_1} [16].
8.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,MR,TM,1,0)$, t_G reduces to t_{QK_2} [16].
9.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,HL,\beta_1(w_x),1,0)$, t_G reduces to t_{QK_3} [16].
10.For $(k_1,k_2,\theta,\alpha,\beta,g,\eta) \rightarrow (0,1,1,HL,TM,1,0)$, t_G reduces to t_{QK_4} [16].

From this proposed generalized log type class t_G, we have developed some new estimators as follows (see Table-1):

Table 1: New developed estimators from proposed generalized log type class t_G

New developed estimator	g	θ	η
$t_{G_1} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\mu_x+\beta_2(w_x)}{\bar{w}_x+\beta_2(w_x)}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_2} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\mu_x+C_{w_x}}{\bar{w}_x+C_{w_x}}\right))(1+\log(\frac{\bar{w}_x}{\mu_x}))$	1	1	1
$t_{G_3} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\beta_1(w_x)\mu_x+\beta_2(w_x)}{\beta_1(w_x)\bar{w}_x+\beta_2(w_x)}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_4} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{MR\mu_x+\beta_2(w_x)}{MR\bar{w}_x+\beta_2(w_x)}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_5} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\beta_1(w_x)\mu_x+MR}{\beta_1(w_x)\bar{w}_x+MR}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_6} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{MR\mu_x+C^2_{w_x}}{MR\bar{w}_x+C^2_{w_x}}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_7} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{MR\mu_x+S^2_{w_x}}{MR\bar{w}_x+S^2_{w_x}}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1
$t_{G_8} = (k_1\bar{w}_y + k_2\bar{w}_y\left(\frac{\mu_x+S^2_{w_x}}{\bar{w}_x+S^2_{w_x}}\right))\left(1+\log(\frac{\bar{w}_x}{\mu_x})\right)$	1	1	1

Consider the error terms:

$$e_0 = \frac{\bar{w}_y}{\mu_y} - 1, \quad e_1 = \frac{\bar{w}_x}{\mu_x} - 1$$

such that $E(e_0) = E(e_1) = 0$,

$$E(e_0^2) = fC^2_{w_y}, \quad E(e_1^2) = fC^2_{w_x}, \quad E(e_0e_1) = f\rho_{w_yw_x}C_{w_y}C_{w_x}$$

, where $f = \frac{1}{n} - \frac{1}{N}$, $\bar{w}_y = \frac{1}{n}\sum_{i=1}^{n} w_{y_i}$, $\bar{w}_x = \frac{1}{n}\sum_{i=1}^{n} w_{x_i}$, $C^2_{w_y} = \frac{S^2_{w_y}}{\mu_y^2}$, $C^2_{w_x} = \frac{S^2_{w_x}}{\mu_x^2}$.

Using the above mentioned error terms we rewrite (25) as follows:

$$t_G = \left(k_1\mu_y(1+e_0) + k_2\mu_y(1+e_0) \times \left(\frac{\alpha\mu_x+\beta}{\theta\alpha\mu_x+\theta\alpha\mu_xe_1+\theta\beta-\alpha\mu_x+\beta-\theta\alpha\mu_x-\theta\beta}\right)^g\right) \times \left(1+\log\left(\frac{\mu_x(1+e_1)}{\mu_x}\right)\right)^\eta. \tag{26}$$

Upon simplification and subtracting μ_y from both sides we get

$$t_G - \mu_y = \mu_y\Bigg[k_1\left(1+e_0+\eta e_0e_1+\eta e_1-\eta e_1^2+\frac{\eta^2}{2}e_1^2\right) + k_2\left(1+\eta e_1-\eta e_1^2+\frac{\eta^2}{2}e_1^2+e_0+\eta e_0e_1-gv\theta e_0e_1-gv\theta e_1-gv\theta\eta e_1^2+g(g+1)/2v^2\theta^2e_1^2\right)\Bigg] - \mu_y. \tag{27}$$

Taking expectation on both sides we get,

$$Bias(t_G) = \mu_y(k_1A_4 + k_2A_5 - 1). \tag{28}$$

Squaring both sides and taking expectation of (27) we get,

$$MSE(t_G) = \mu_y^2\left(1 + k_1^2A_1 + k_2^2A_2 + 2k_1k_2A_3 - 2k_1A_4 - 2k_2A_5\right), \tag{29}$$

where

$$\begin{aligned}
A_1 &= 1 + fC_{w_y}^2 + (2\eta^2 - 2\eta)fC_{w_x}^2 + 4\eta f\rho_{w_xw_y}C_{w_x}C_{w_y},\\
A_2 &= 1 + fC_{w_y}^2 + (2\eta^2 - 2\eta + g^2\theta^2v^2 - 4\eta g\theta v + g(g+1)\theta^2v^2)fC_{w_x}^2\\
&\quad + 4(\eta - gv\theta)f\rho_{w_xw_y}C_{w_x}C_{w_y},\\
A_3 &= 1 + fC_{w_y}^2 + (2\eta^2 + g(g+1)/2\theta^2v^2 - 2\eta - 2\eta g\theta v)fC_{w_x}^2\\
&\quad + (4\eta - 2gv\theta)f\rho_{w_xw_y}C_{w_x}C_{w_y},\\
A_4 &= 1 + (\eta^2/2 - \eta)fC_{w_x}^2 + \eta f\rho_{w_xw_y}C_{w_x}C_{w_y},\\
A_5 &= 1 + (\eta^2/2 - \eta - gv\theta\eta + g(g+1)/2\theta^2v^2)fC_{w_x}^2\\
&\quad + (\eta - gv\theta)f\rho_{w_xw_y}C_{w_x}C_{w_y}.
\end{aligned}$$

Now we partially differentiate (29) with respect to k_1 and k_2 and equate the resultant equations to zero we get,

$$k_{1_{\text{opt}}} = \frac{A_2A_4 - A_3A_5}{A_1A_2 - A_3^2}, \tag{30}$$

$$k_{2_{\text{opt}}} = \frac{A_1A_5 - A_4A_3}{A_1A_2 - A_3^2}. \tag{31}$$

Using the optimum values of k_1 and k_2 in (29) we get the minimum MSE as

$$MSE_{\min}(t_G) = \mu_y^2\left(1 + k_{1_{\text{opt}}}^2A_1 + k_{2_{\text{opt}}}^2A_2 + 2k_{1_{\text{opt}}}k_{2_{\text{opt}}}A_3 - 2k_{1_{\text{opt}}}A_4 - 2k_{2_{\text{opt}}}A_5\right). \tag{32}$$

4 Simulation study

To show that the new log type estimators developed from the proposed generalized log type class t_G are better than the related competing estimators presented in this article, we have conducted four simulation studies. The comparison among the estimators is made using the MSE (mean square error).

The following algorithm is used to conduct the simulation studies in R software:

1. Using the models:

$$y = \frac{1}{2}x + e, \quad y = \frac{1}{3}x + e, \quad y = \frac{1}{4}x + e, \quad \text{and} \quad y = \frac{1}{5}x + e$$

where auxiliary variable x has been taken from [21] and $e \sim N(0,x)$, the populations of survey variable y for population-1 to population-4 are generated respectively.
2. Using sample sizes 145, 150, 155, and 160, the sampling procedure of ACS is repeated 20000 times to calculate several values of all estimators for population-1.
3. For each sample size, values of MSEs are obtained for each estimator for population-1 using the formula:

$$\text{MSE}(t_*) = \frac{1}{20000}\sum_{i=1}^{20000}(t_* - \mu_y)^2,$$

where i is the iteration and t_* is an appropriate estimator.
4. Steps 2 and 3 are repeated to obtain MSEs for each estimator for the rest of the populations under study.

The results of the simulation studies are tabulated in Tables- [2, 5].

Table 2: MSEs of all estimators for population-1.

Estimators	n=145	n=150	n=155	n=160
t_{DC}	0.00113	0.00109	0.00103	0.00097
t_{CH_1}	0.00124	0.00119	0.00114	0.00108
t_{CH_2}	0.00112	0.00108	0.00103	0.00097
t_{CH_3}	0.00129	0.00124	0.00119	0.00111
t_{YS_1}	0.00112	0.00108	0.00103	0.00097
t_{YS_2}	0.00122	0.00118	0.00113	0.00106
t_{QK_1}	0.00113	0.00109	0.00104	0.00098
t_{QK_2}	0.00113	0.00109	0.00103	0.00097
t_{QK_3}	0.00112	0.00108	0.00103	0.00097
t_{QK_4}	0.00113	0.00109	0.00103	0.00097
t_{G_1}	0.00128	0.00123	0.00115	0.00110
t_{G_2}	0.00121	0.00117	0.00110	0.00105
t_{G_3}	0.00118	0.00114	0.00108	0.00103
t_{G_4}	0.00113	0.00109	0.00104	0.00099
t_{G_5}	0.00116	0.00112	0.00106	0.00101
t_{G_6}	0.00116	0.00112	0.00106	0.00101
t_{G_7}	0.00152	0.00147	0.00139	0.00133
t_{G_8}	0.00102	0.00100	0.00096	0.00091

Table 3: MSEs of all estimators for population-2.

Estimators	n=145	n=150	n=155	n=160
t_{DC}	0.00687	0.00653	0.00610	0.00588
t_{CH_1}	0.00612	0.00575	0.00550	0.00528
t_{CH_2}	0.00620	0.00587	0.00556	0.00535
t_{CH_3}	0.00619	0.00581	0.00556	0.00534
t_{YS_1}	0.00621	0.00589	0.00557	0.00536
t_{YS_2}	0.00610	0.00574	0.00548	0.00526
t_{QK_1}	0.00615	0.00582	0.00552	0.00531
t_{QK_2}	0.00687	0.00653	0.00610	0.00588
t_{QK_3}	0.00625	0.00592	0.00560	0.00539
t_{QK_4}	0.00687	0.00653	0.00610	0.00588
t_{G_1}	0.00470	0.00449	0.00436	0.00424
t_{G_2}	0.00470	0.00449	0.00435	0.00423
t_{G_3}	0.00470	0.00449	0.00435	0.00423
t_{G_4}	0.00469	0.00448	0.00435	0.00423
t_{G_5}	0.00469	0.00448	0.00435	0.00423
t_{G_6}	0.00469	0.00448	0.00435	0.00423
t_{G_7}	0.00493	0.00474	0.00460	0.00449
t_{G_8}	0.00472	0.00446	0.00433	0.00420

Table 4: MSEs of all estimators for population-3.

Estimators	n=145	n=150	n=155	n=160
t_{DC}	0.00687	0.00653	0.00610	0.00588
t_{CH_1}	0.00599	0.00564	0.00538	0.00517
t_{CH_2}	0.00625	0.00592	0.00560	0.00539
t_{CH_3}	0.00599	0.00564	0.00537	0.00517
t_{YS_1}	0.00626	0.00594	0.00561	0.00541
t_{YS_2}	0.00599	0.00565	0.00538	0.00518
t_{QK_1}	0.00619	0.00587	0.00555	0.00535
t_{QK_2}	0.00687	0.00653	0.00610	0.00588
t_{QK_3}	0.00630	0.00597	0.00564	0.00544
t_{QK_4}	0.00687	0.00653	0.00610	0.00588
t_{G_1}	0.00259	0.00248	0.00244	0.00238
t_{G_2}	0.00256	0.00246	0.00242	0.00237
t_{G_3}	0.00255	0.00245	0.00241	0.00237
t_{G_4}	0.00253	0.00244	0.00240	0.00235
t_{G_5}	0.00254	0.00245	0.00241	0.00236
t_{G_6}	0.00254	0.00245	0.00241	0.00236
t_{G_7}	0.00255	0.00245	0.00241	0.00236
t_{G_8}	0.00253	0.00244	0.00241	0.00236

Table 5: MSEs of all estimators for population-4.

Estimators	n=145	n=150	n=155	n=160
t_{DC}	0.00549	0.00522	0.00492	0.00460
t_{CH_1}	0.00551	0.00523	0.00493	0.00465
t_{CH_2}	0.00530	0.00504	0.00475	0.00448
t_{CH_3}	0.00563	0.00534	0.00502	0.00474
t_{YS_1}	0.00531	0.00505	0.00476	0.00448
t_{YS_2}	0.00547	0.00519	0.00489	0.00462
t_{QK_1}	0.00530	0.00504	0.00475	0.00448
t_{QK_2}	0.00549	0.00522	0.00492	0.00460
t_{QK_3}	0.00531	0.00505	0.00476	0.00448
t_{QK_4}	0.00549	0.00522	0.00492	0.00460
t_{G_1}	0.00335	0.00334	0.00319	0.00311
t_{G_2}	0.00335	0.00334	0.00320	0.00311
t_{G_3}	0.00335	0.00335	0.00320	0.00311
t_{G_4}	0.00336	0.00336	0.00321	0.00312
t_{G_5}	0.00335	0.00335	0.00321	0.00312
t_{G_6}	0.00335	0.00335	0.00321	0.00312
t_{G_7}	0.00344	0.00347	0.00335	0.00328
t_{G_8}	0.00340	0.00337	0.00321	0.00311

5 Results and Discussion

The aim of this article was to study the performance of log type estimators in ACS design. Thus, motivated from the work of [8,17,18,19,20], we proposed a generalized log type class t_G and developed some new log type estimators from this proposed class. The new developed estimators used some known parameters of auxiliary variable namely mid-range, coefficient of skewness, coefficient of kurtosis, coefficient of variation among others. The derivations of properties like bias and MSE of the proposed log type class t_G have been presented up to first order of approximations. To show that the new developed log type estimators are better than the competing estimators presented in this paper, four simulation studies have been conducted.

From the results obtained in the simulation study (see Tables- [2, 5]), we can clearly see that some of the new developed estimators resulted in lower MSE as compared to all the existing related estimators presented in this article in different populations considered. Among the new developed estimators t_{G_1} to t_{G_8}, the estimator t_{G_8} resulted in the lowest MSE and hence it is recommended that when the population under study is rare or hidden clustered, the ACS design should be used and the developed log type estimators are recommended to be used to estimate the population mean. For future areas of studies, we recommend using different transformations and functions to study the performance of resultant estimators.

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