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Multi-Objective Optimization to Find The Shortest Paths Tree Problem in The Computer Networks

Mahmoud A. Mofaddel[1,*] *and Ahmed Y. Hamed*[2]

[1] Math. and Computer Science Dept., Faculty of Science, Sohag University, Sohag, Egypt
[2] College of Applied Studies and Community Services, Imam Abdul-Rahman Bin Faisal University, KSA

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Abstract: The shortest paths tree problem when considering cost and bandwidth constraints is addressed in this paper as multi-objective shortest paths tree problem. A multi-objective genetic algorithm is suitable to solve the presented problem. Therefore, this paper presents a multi-objective genetic algorithm based on Random Weighted Genetic Algorithm (RWGA) to solve the shortest paths tree problem subject to cost and bandwidth constraints. The objective of the proposed algorithm is to search the optimal set of edges connecting all nodes such that the sum of costs is minimized and the value bandwidth is maximized. The presented algorithm has been applied on two sample networks to illustrate their efficiency.

Keywords: Computer Networks, Multi-Objective Shortest Paths, Genetic Algorithms

1 Introduction

The shortest paths tree rooted at vertex s is a spanning tree T of G, such that the path distance from root v to any other vertex u in T is the shortest path distance from v to u in G [1]. In the case of single link failure, [2] proposed an algorithm to solve the optimal shortest paths tree. When considering multicast tree, [3] presented an algorithm to find the Shortest Best Path Tree (SBPT) based on labeling techniques.

Ziliaskopoulos et al. [4] proposed an algorithm to solve the shortest path trees. Also, the shortest paths tree problem has been solved by an efficient modified continued pulse coupled neural network (MCPCNN) model [5].

Ting Lu and Jie Zhu [6] presented a heuristic and approximate algorithms for multi-constrained routing (MCR) are not effective in dynamic network environment for real-time applications when the state information of the network is out of date. Also, the authors presented a genetic algorithm to solve the MCR problem subject to transmission delay and transmission success ratio.

Younes [7] proposed a genetic algorithm to determine the k shortest paths with bandwidth constraints from a single source node to multiple destinations nodes.

Liu et al. [8] presented an oriented spanning tree (OST) based genetic algorithm (GA) for solving both the multi-criteria shortest path problem (MSPP) and the multi-criteria constrained shortest path problems (MCSPP).

Younes [9] presented the genetic algorithm to find the low-cost multicasting tree with bandwidth and delay constraints.

Granat and Guerriero [10] introduced an interactive procedure for the MSPP based on a reference point labeling algorithm. The algorithm converts the multi-objective problem into a parametric single-objective problem whereby the efficient paths are found. The algorithm was tested on grid and random networks and its performance was measured based on execution time. They concluded that an interactive method, from their experimental results, is encouraging and does not require the generation of the whole Pareto-optimal set (which avoids the intractability problem).

Gen and Lin [11] used a multi-objective hybrid genetic algorithm (GA) to improve solutions of the bicriteria network design problem (finding shortest paths) with two conflicting objectives of minimizing cost and maximizing flow.

Gandibleux et al. [12] reported a concise description of the MSPP and clearly narrates the most salient issues

* Corresponding author e-mail: mmofaddel@hotmail.com

to its solution. Their study recalls Martins' labeling algorithm and attempts to improve it. Their new algorithm extends Martins' algorithm by introducing a procedure that can solve MSPP that have multiple linear functions and a max min function. Since it is an extension of Martins' algorithm, the generation of all non-dominated paths remains intractable in polynomial time. However, experimental results of their study say otherwise. Their algorithm is tested on a variety of test instances and results show that in terms of size and complexity, optimizing simultaneous linear and max min functions does not behave exponentially. They also show that their algorithm is not sensitive to different cost ranges and that density and network size increase the number of efficient solutions.

Müller-Hannemann [13] also showed that the cardinality of efficient paths in a bicriteria shortest path problem is not exponential as long as the instances are bounded by potential characteristics as defined in their experiment. They conclude with emphasis that it is still preferable to work with complete information rather than falling back on approximations.

Likewise [14] suggested an interactive method that incorporates an efficient k-shortest path in identifying Pareto-optimal paths in a biobjective shortest path problem. The algorithm was tested against other MSPP algorithms on 39 network instances. They conclude that their k-shortest path algorithm performs better in terms of execution time.

Martins and Santos [15] presented a labeling algorithm for the multi-objective shortest paths problem and presents an analysis in terms of finiteness and optimality concepts and reports that any instance of the MSPP is bounded if and only if there are no absorbent cycles in the network. They showed a set of networks wherein the labeling algorithm only determines non-dominated labels.

Mooney and Winstanley [16] state that Martins' labeling algorithm works well in theory but is prohibitive in practice in terms of its implementation due to memory costs.

Guerriero and Musmanno [17] examined several label selection and node-selection methods that can find Pareto optimal solutions to the MSPP with respect to execution time. The performance of the different algorithms was measured using random and grid networks and results show that label selection methods are generally faster than node-selection methods and that parallel computing is necessary in the design of efficient methods. While some researchers focus on exhaustive solutions or on improvements thereof, other researchers are more concerned with better runtime solutions.

Tsaggouris and Zaroliagis [18] presented an improved fully polynomial time approximation scheme (FPTAS) for the multi-criteria shortest path problem and a new generic method for obtaining FPTAS to any multi-objective optimization problem with non-linear objectives. They showed how their results can be used to obtain efficient approximate solutions to the multiple constrained path problem and to the non-additive shortest path problem. Their algorithm builds upon an iterative process that extends and merges sets of node labels representing paths which departs from earlier methods using rounding and scaling techniques on the input edge costs. The algorithm resembles the Bellman-Ford method but implements the label sets as arrays of polynomial size by relaxing the requirements for strict Pareto optimality.

Fritz Bökler and Petra Mutzel, [19] presented a new pruning method which is easy to implement and performs very well on real-world road networks. In this study, we test our hypotheses on artificial MOSP instances from the literature with up to 15 objectives and real-world road networks with up to almost $160,000$ nodes.

This paper presents a multi-objective genetic algorithm based on Random Weighted Genetic Algorithm (RWGA) to solve the shortest paths tree problem subject to cost and bandwidth constraints. The objective of the proposed algorithm is to search the optimal set of edges connecting all nodes such that the sum of costs is minimized and the bandwidth value is maximized. A genetic algorithm is presented to solve the paths tree problem under cost constraint. The algorithm reads the connection matrix and the cost matrix of a given network. Also, given the source (root) node s, then the genetic operations are executed to search the minimum cost paths that construct the minimum cost paths tree rooted at the source node s.

The rest of the paper is organized as follows: Section 2 presents notations. The problem description is presented in Section 3. The proposed GA and its components are given in Section 4. Section 5 provides the pseudo code of the entire GA. Section 6 shows the illustrative examples. Finally, Section 7 presents conclusions.

2 Notations

G	A network graph
N	The number of nodes in G
E	The number of edges in G
e_{ij}	An edge between node i and node j in G
c_e	The cost of an edge e
M	The connection matrix of the given network
CM	The cost matrix of the given network
NP	The number of paths from node s to t
T_s	The shortest path rooted at node s
P_{size}	The population size
P_c	The crossover rate
P_m	The mutation rate
ng	The number of generations
RWGA	Random Weighted Genetic Algorithm
MSPP	The multi-criteria shortest path problem

3 The problem formulation and description

The optimal shortest paths tree rooted at vertex s is the collection of optimal paths from the source (root) node s to the destination nodes d_i. The path $P_{(s,d)}$ is optimal if it has minimum cost and maximum bandwidth. Let $C(P_{(s,d)})$ and $B(P_{(s,d)})$ be the cost and the bandwidth of the path $P_{(s,d)}$ respectively. The multi-objective paths tree problem is formulated as follows:

$$\begin{aligned} &Minimize \quad C(P_{(s,d_i)}) \\ &Maximize \quad B(P_{(s,d_i)}) \end{aligned}$$

where

$$C(P_{(s,d_i)}) = \sum_{e \in P_{(s,d_i)}} C_e \qquad (1)$$

and

$$B(P_{(s,d_i)}) = \min_{b_e \in P_{(s,d_i)}} (b_e) \qquad (2)$$

The multi-objective problem in the case of a maximal and minimal objective is transformed into either a multi-objective minimization problem or a multi-objective maximization problem. Therefore, the original problem formulation is modified to be of the minimal type:

$$\begin{aligned} &Minimize \quad ob_1 = C(P_{(s,d_i)}) \\ &Minimize \quad ob_2 = \hat{B}(P_{(s,d_i)}) \end{aligned}$$

where $\hat{B}(P_{(s,d_i)}) = 1/B(P_{(s,d_i)})$

Therefore, the minimum-cost paths tree T_s is the collection of minimum cost paths from the source (root) node s to the destination nodes d_i:

$$C(T_s) = \sum_k C^k(P_{(s,d_i)}) \qquad (3)$$

The bandwidth of the tree T is defined as the minimum available residual bandwidth at any link along the tree:

$$B(T_s) = min(B(e), \quad e \in T_s) \qquad (4)$$

The presented method depends on reading both the connection, cost and bandwidth matrices of a given network, and then finds the shortest paths tree rooted at the source node.

4 The proposed algorithm based on RWGA

In the proposed GA, each candidate path is represented by a binary string with length N that can be used as a chromosome. Each element of the chromosome represents a node in the network topology. So, for a network of N nodes, there are N string components in each candidate solution x. Each chromosome must contain at least two none zero elements.

For example, if we consider the following network with eight nodes, shown in Figure 1, the path of Figure 2 is represented as a chromosome as shown in Figure 3.

In the following subsections, we give an explanation of different components (operations) of the presented genetic algorithm.

4.1 Initial Population

The generated chromosome in initial population must contain at least two non-zero elements to be a real candidate path. The following steps show how to generate P_{size} chromosomes of the initial population:

1. Randomly generate a chromosome x.
2. Check if x represents a real candidate path, i.e. contains at least two non-zero elements.
3. Repeat step 1 and step 2 to generate P_{size} chromosomes.

4.2 The Fitness Function

1. Step 1: Find the normalized values of ob_1 and ob_2 as follows:
   – Normalized value for ob_1:
   $Nob_1(i) = Min(ob_1(1), ob_1(2), \ldots, ob_1(P_{size})) / ob_1(i)$
   – Normalized value for ob_2:
   $Nob_2(i) = ob_2(i) / Max(ob_2(1), ob_2(2), \ldots, ob_2(P_{size}))$
2. Step 2:Calculate the Fitness value for each solution as follows:
   – Generate a random number u_k in $[0,1]$ for each objective k, $\quad k = 1,2$.
   – Calculate the random weight of each objective k as:
   $$w_k = u_k / \sum_{i=1}^{2} u_i \qquad (5)$$
   – Calculate the fitness of the solution as:
   $f(i) = w_1 * Nob_1(i) + w_2 * Nob_2(i)$
3. Step 3:Calculate the selection probability of each solution
$$Pr(i) = (f(i) - f^{min})^{-1} \sum_{j \in P_{size}} (f(j) - f^{min}), \qquad (6)$$
where $f^{min} = min\{f(i), i \in P_{size}\}$.

4.3 Genetic Crossover Operation

In the proposed GA, we use the single cut point crossover to breed a new offspring from two parents. The crossover operation will be performed if the crossover ratio $(P_c = 0.90)$ is verified. The cut point is randomly selected.

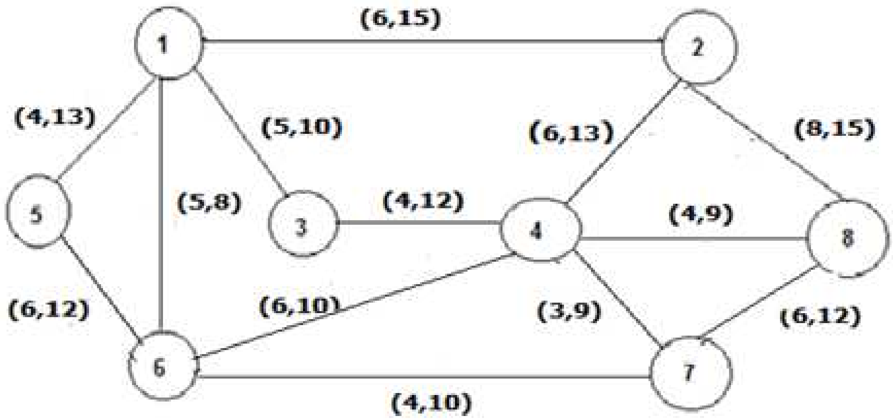

Fig. 1: The cost and bandwidth of the links (cost, bandwidth) for eight nodes network

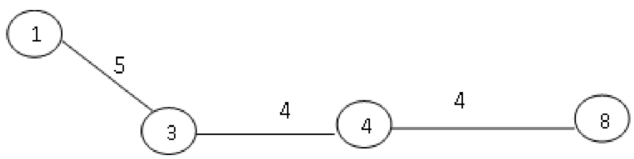

Fig. 2: A candidate Path

1	2	3	4	5	6	7	8
1	0	1	1	0	0	0	1

Fig. 3: The chromosome corresponding to the path given in Figure 2

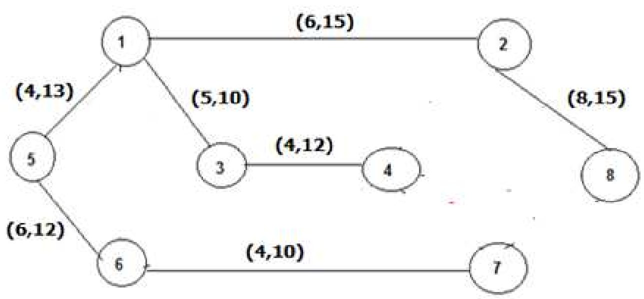

Fig. 4: The shortest paths tree

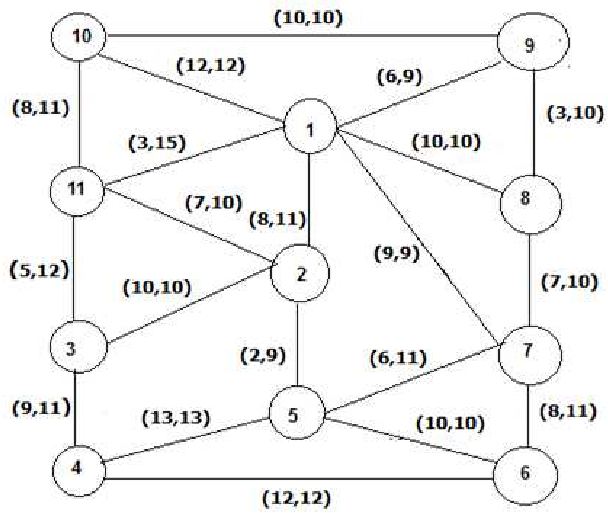

Fig. 5: The cost and bandwidth of the links (cost, bandwidth) for eleven nodes network

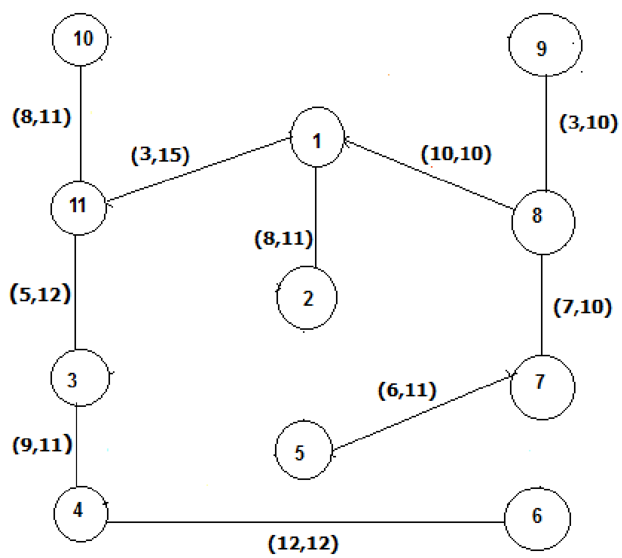

Fig. 6: The shortest paths tree rooted at node 1

4.4 Genetic Mutation Operation

The mutation operation is performed on bit-by-bit basis. In the proposed approach, the mutation operation will be performed if the mutation ratio (P_m) is verified. The P_m in this approach is chosen experimentally to be 0.02. The point to be mutated is selected randomly.

5 The Entire Algorithm

1. Step 1: Set the population size (P_{size}), the crossover rate (P_c), the mutation rate (P_m) and the number of generations (ng).
2. Step 2: Generate the initial population including successful individuals $P_1, P_2, \ldots, P_{P_{size}}$.
3. Step 3: For each individual, evaluate ob_1 and ob_2 according to Eq.(1) and Eq.(2).
4. Step 4: Determine the set of non-dominated solutions E and the number of non-dominated solutions nE.

5. Step 5: Calculate the fitness value and the selection probability for each individual B in the current population as presented in Section 4.2.
6. Step 6:
   – Select parents using the selection probabilities calculated in Step 5.
   – Apply the GA operations (described in Section 4.3 to Section 4.4) to generate new populations.
   – Apply crossover on the selected parent pairs to create $P_{P_{size}}$ offspring.
   – Mutate offspring with a predefined mutation rate. Copy all offspring to P_{t+1}. Update E if necessary.
7. Step 7: Randomly remove nE solutions from the new population and add the same number of solutions from E to it.
8. Step 8: If the stopping condition is not satisfied, set $ng = ng + 1$, else, go to Step 5.

6 Experimental Results

The initial values of the parameters are: population size (P_{size}=20), maximum generation (max gen = 300), P_c = 0.90, and P_m = 0.02. The technique reads the connection, cost and bandwidth matrices of the given network. Then it generates the shortest paths tree of the network that possesses the minimum cost and maximum bandwidth. The proposed algorithm is implemented using Borland $C++$ Ver. 5.5. Three cases are used to test and validate the proposed algorithm.

6.1 Case 1:

Here, we consider the given network of eight nodes, as shown in Figure 1. The final output of the GA is shown in Table 1. Figure 4 shows the shortest paths tree rooted at node 1.

Table 1: The final output of the proposed algorithm (GA)

d_i	The shortest path	$ob_1(cost)$	$ob_2(bandwidth)$
2	{1,2}	6	15
3	{1,3}	5	10
4	{1,3,4}	9	10
5	{1,5}	4	13
6	{1,5,6}	10	12
7	{1,5,6,7}	14	10
8	{1,2,8}	14	15

The minimum-cost paths tree T_s of this network is 62 and maximum bandwidth is 10.

6.2 Case 2:

Here, the algorithm was applied on a network with 11 nodes as shown in Figure 5. The final output of the GA is shown in Table 2. Figure 6 shows the shortest paths tree rooted at node 1.

Table 2: The final output of proposed algorithm (GA)

d_i	The shortest path	$ob_1(cost)$	$ob_2(bandwidth)$
2	{1,2}	8	11
3	{1,11,3}	8	12
4	{1,11,3,4}	17	11
5	{1,8,7,5}	23	10
6	{1,11,3,4,6}	29	11
7	{1,8,7}	17	10
8	{1,8}	10	10
9	{1,8,9}	13	10
10	{1,11,10}	11	11
11	{1,11}	3	15

The minimum-cost paths tree T_s of this network is 139 and maximum bandwidth is 10.

6.3 Case 3:

Also, the GA was applied on sixteen nodes example as shown in Figure 7. The final output of the GA is shown in Table 3. Figure 8 shows the shortest paths tree rooted at node 1.

Table 3: The final output of the proposed algorithm (GA)

d_i	The shortest path	$ob_1(cost)$	$ob_2(bandwidth)$
2	{1,3,2}	11	12
3	{1,3}	3	12
4	{1,7,8,4}	28	10
5	{1,3,7,10,9,5}	34	10
6	{1,3,2,5,9,10,7,6}	43	10
7	{1,3,2,5,9,10,7}	35	10
8	{1,3,7,11,12,8}	30	10
9	{1,2,5,9}	21	10
10	{1,3,2,5,9,10}	26	12
11	{1,3,2,7,8,12,11}	30	10
12	{1,3,7,11,12}	18	10
13	{1,3,2,5,9,13}	28	10
14	{1,3,7,11,14}	21	12
15	{1,3,7,11,14,15}	24	12
16	{1,3,7,11,14,16}	28	12

The minimum-cost paths tree T_s of this network is 380 and maximum bandwidth is 10.

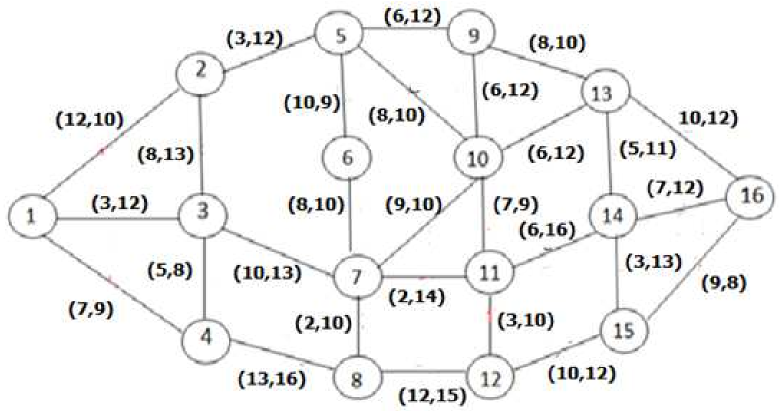

Fig. 7: The cost and bandwidth of the links (cost, bandwidth) of sixteen nodes network

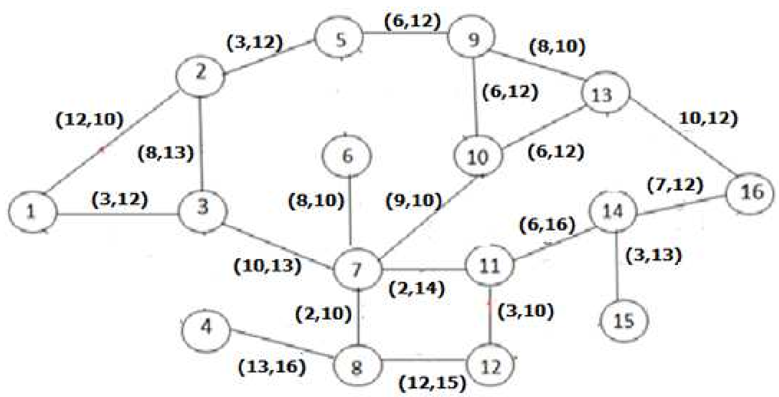

Fig. 8: The minimum-cost paths tree rooted at node 1

7 Conclusions

The paper presented a multi-objective genetic algorithm based on Random Weighted Genetic Algorithm (RWGA) to solve the shortest paths tree problem subject to cost and bandwidth constraints. The objective of the proposed algorithm is to search the optimal set of edges connecting all nodes such that the sum of costs is minimized and the value bandwidth is maximized. The GA has been applied on three cases, the results proved efficiency of the proposed GA.

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Mahmoud A. Mofaddel received his B. Sc. and M. Sc. Degrees from ASSIUT University in 1985 and 1991 respectively, and his Ph. D. degree from Rostock University, GERMANY in 1999. He authored and co-authored more than 23 scientific papers. His research interests include high performance computing, and image processing.

Ahmed Younes Hamed received his PhD degree in Sept. 1996 from South Valley University, Egypt. His research interests include Artificial Intelligence and genetic algorithms; specifically in the area of computer networks. Recently, he has started conducting a research in the area of Image Processing. Currently, he works as an Associate Professor in University of Dammam, KSA. Younes always publishes the outcome of his research in international journals and conferences.