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Soft Separation Axioms on Nearness Approximation Spaces

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Abstract: The soft topological spaces and some their related concepts have studied by Shabir and Hussain in [1,2]. In this paper, we [3] introduce the soft sets and soft topology based on a nearness approximation space. The purpose of this paper is introduce some properties of soft topology and soft separation axioms on nearness approximation spaces and exhibit some results related to these concepts.

Keywords: Near Soft Continuous Mapping, Near Soft Separation Axioms, Near Soft Hausdorff space

1 Introduction

Near sets have been given by Peters [4] where objects, affinities are considered perceptually near to each other, i.e., objects with similar descriptions to some degrees. The near set approach leads to partitions of ensembles of sample objects with measurable information content and an approach to feature selection. The nearness of disjoint sets based on object descriptions can be seen by the discovery of near sets in approximation spaces and the introduction of a nearness approximation space.

The other notion of soft set, which is proposed by Molodtsov [5] to deal with uncertainty in a parametric manner, has been studied by many scientists[2,6,7,8,9,10]. Combine the soft sets approach with near set theory giving rise to the new concepts of soft nearness approximation space. Tasbozan [3] introduce the soft sets and soft topology based on a nearness approximation space to combine near sets approach with soft set theory. The purpose of this paper is to introduce some properties of soft topology on nearness approximation spaces. The notions of near soft point, near soft separation space are defined and their basic properties are investigated with the help of examples.

2 Preliminary

Definition 1.*Let $NAS = (\mathscr{O}, \mathscr{F}, \sim_{Br}, N_r, \upsilon_{N_r})$ be a nearness approximation space and $\sigma = (F,B)$ be a soft set over $\mathscr{O}$. The lower and upper near approximation of $\sigma = (F,B)$ with respect to NAS are denoted by $N_r{*}(\sigma) = (F_*,B)$ and $N_r^*(\sigma) = (F^*,B)$, which are soft sets over with the set-valued mappings given by $F_*(\phi) = N_r{*}(F(\phi))$=$\cup\{x \in \mathscr{O} : [x]_{Br} \subseteq F(\phi)\}$ and $F^*(\phi) = N_r^*(F(\phi))$=$\cup\{x \in \mathscr{O} : [x]_{Br} \cap F(\phi) \neq \emptyset\}$ where all $\phi \in B$. The operators $N_r{*}$ and N_r^* are called the lower and upper near approximation operators on soft sets, respectively. If $Bnd_{N_r(B)}(\sigma) \geqslant 0$, then the soft set σ is called a near soft set [3].*

Theorem 1.*A collection of partitions (families of neighborhoods) $N(\sigma)$ is a near soft set [3].*

*Proof.*Given a collection of partitions $N(\sigma)$. A partition $\xi_{\mathscr{O},B}(\sigma) \in N(\sigma)$ consists of classes $(F,B) = ([x]_B, B)$. These classes for $\phi \in B$ are near sets. Hence $\xi_{\mathscr{O},B}(\sigma)$ is a near set and $N(\sigma)$ is a near soft set.

Definition 2.*Let $\mathscr{O}$ be an initial universe set, E be the universe set of parameters and $A,B \subseteq E$ [3]*

(I) (F,A) is called a relative null near soft set (with respect to the parameters of A) if $F(\phi) = \emptyset$, for all $\phi \in A$.

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(II) (G,B) is called a relative whole near soft set (with respect to the parameters of B)if $G(\phi)=\mathscr{O}$, for all $\phi\in B$.

Definition 3. *The relative complement of a near soft set (F,A) denoted by $(F,A)^c$, is defined as the near soft set (F^c,A) where $F^c(\phi)=\mathscr{O}-F(\phi)$ for all $\phi\in A$ [3].*

Definition 4. *Let $\sigma=(F,B)$ be a near soft set over $(\mathscr{O},B)$, τ be the collection of near soft subsets of σ, B is the nonempty set of parameters, and then $(\mathscr{O},B)$ is said to be a near soft topology on σ if the following conditions are met:*

i)$(\emptyset,B)$, $(\mathscr{O},B)\in\tau$ where $\emptyset(\phi)=\emptyset$ and $F(\phi)=F$, for all $\phi\in B$.
ii)The intersection of any two near soft sets in τ belongs to τ.
iii)The union of any number of near soft sets in τ belongs to τ.

The pair $(\mathscr{O},\tau)$ is called a near soft topological space [3].

Definition 5. *Let $(\mathscr{O},\tau)$ be a near soft topological space over $(\mathscr{O},B)$. A near soft subset of $(\mathscr{O},B)$ is called near soft closed if its complement is open and a member of τ [3].*

Definition 6. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$, then the members of τ are said to be near soft open sets in $\mathscr{O}$ [3].*

Definition 7. *Let (F,B) be a near soft set over $\mathscr{O}$. The near soft set (F,B) is called a near soft point, denoted by (x_e,B), if for the element $e\in B$, $F(e)=\{x\}$ and $F(e')=\emptyset$ for all $e'\in B-\{e\}$ [11].*

Proposition 1. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$, then the collection $\tau_e=\{F(e):(F,B)\in\tau\}$ for each $e\in B$, defines a topology on $\mathscr{O}$ [12].*

Definition 8. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$ and (F,B) be a near soft set over $\mathscr{O}$. Then the near soft closure of (F,B), denoted by $(F,B)^c$ is the intersection of all near soft closed super sets of (F,B). Clearly $(F,B)^c$ is the smallest near soft closed set over $\mathscr{O}$ which contains (F,B) [12].*

Definition 9. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$ and (F,B) be a near soft set over $\mathscr{O}$. Then the near soft interior of (F,B), denoted by $(F,B)^\circ$ is the collection of all near soft open super sets of (F,B). Clearly $(F,B)^\circ$ is the biggest near soft open set over $\mathscr{O}$ which remains with in (F,B) [12].*

Example 1. Let $\mathscr{O}=\{x_1,x_2,x_3,x_4,x_5\}$, $B=\{\phi_1,\phi_2\}\subseteq\mathscr{F}=\{\phi_1,\phi_2,\phi_3,\phi_4\}$ be denote a set of perceptual objects and a set of functions, respectively. Sample values of the $\phi_i, i=1,2,3,4$ functions are shown in Table 1.

	x_1	x_2	x_3	x_4	x_5
ϕ_1	0.1	0.2	0.2	0.1	0.2
ϕ_2	0.2	0.3	0.3	0.2	0.5
ϕ_3	0.3	0.4	0.3	0.3	0.4
ϕ_4	0.4	0.2	0.2	0.4	0.2

Let $\sigma=(F,B)$, $B=\{\phi_1,\phi_2\}$ be a soft set defined by $F(\phi_2)=\{x_5\}$. Then $(F,B)=(\phi_2,\{x_5\})$ is a near soft set with r=1;

$[x_1]_{\phi_1}=\{x_1,x_4\},[x_2]_{\phi_1}=\{x_2,x_3,x_5\}$,
$[x_1]_{\phi_2}=\{x_1,x_4\},[x_2]_{\phi_2}=\{x_2,x_3\},[x_5]_{\phi_2}=\{x_5\}$,

$$N_*(\sigma)=N_*(F(\phi),B)=(N_*F(\phi),B)=(F_*(\phi),B)$$

for $\phi_2\in B$, $N_*(\sigma)=(F_*(\phi_2),B)=\{(\phi_2,\{x_5\})\}$,

$$N^*(\sigma)=(F^*(\phi),B)$$

for $\phi_1,\phi_2\in B$, $N^*(\sigma)=\{(\phi_2,\{x_5\})\}$. $Bnd_N(\sigma)\geq 0$,then (F,B) is a near soft set.

Then (F,B) is a near soft set with r=2;

$[x_1]_{\phi_1,\phi_2}=\{x_1,x_4\}$,
$[x_2]_{\phi_1,\phi_2}=\{x_2,x_3\}$,
$[x_5]_{\phi_1,\phi_2}=\{x_5\}$,

$N_*(\sigma)=\{(\phi_2,\{x_5\})\}$,
$N^*(\sigma)=\{(\phi_2,\{x_5\})\}$.

$Bnd_N(\sigma)\geq 0$, then (F,B) is a near soft set. Then $\phi_2\in B$, $F(\phi_2)=\{x_5\}$ and $(\phi_2)'\in B-\{\phi_2\}$, $F((\phi_2)')=\emptyset$. (F,B) is called a near soft point and denoted by $(\phi_2,\{x_5\})$ [12].

Definition 10. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$. A near soft set (F,B) in $(\mathscr{O},\tau,B)$ is called a near soft neighborhood of the near soft point $(x_e,B)\in(F,B)$ if there exists a near soft open set (G,B) such that $(x_e,B)\in(G,B)\subset(F,B)$.*

Definition 11. *Let $(f,g):(F,B)\to(G,B')$ be a near soft mapping from (F,B) to (G,B'). A near soft mapping (f,g) said to be injective if (f,g) are both injective. A near soft mapping (f,g) said to be surjective if (f,g) are both surjective. A near soft mapping (f,g) said to be bijective if (f,g) are both bijective.*

Definition 12. *Let $(\mathscr{O}_1,\tau,B)$ and $(\mathscr{O}_2,\tau,B)$ be two near soft topological spaces. $f:(\mathscr{O}_1,\tau,B)\to(\mathscr{O}_2,\tau,B)$ be a mapping. For each near soft neighborhood (H,B) of $(f(x)_e,B)$, if there exists a near soft neighborhood $f((F,B))\subset(H,B)$ then f is said to be near soft continuous mapping (x_e,B). If f is near soft continuous mapping for all (x_e,B), then f is called near soft continuous mapping.*

Definition 13. *Let $(\mathscr{O}_1,\tau,B)$ and $(\mathscr{O}_2,\tau,B)$ be two near soft topological spaces. $f:\mathscr{O}_1\to\mathscr{O}_2$ be a mapping. $\mathscr{O}_1$ is near soft homeomorphic to $\mathscr{O}_2$ if f is a bijection, near soft continuous and f^{-1} is a near soft homeomorphism.*

Definition 14. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space and $x,y \in \mathscr{O}$ such that $x \neq y$. $(\mathscr{O},\tau,B)$ is a near soft Hausdorff space if for each near soft open sets $(F,D),(G,C) \in (\mathscr{O},B)$ such that $x \in (F,D)$, $y \in (G,C)$ and $(F,D) \cap (G,C) = \emptyset$.*

Example 2. Let $\mathscr{O} = \{x_1,x_2,x_3,x_4,x_5\}$, $B = \{\phi_1,\phi_2\} \subseteq \mathscr{F} = \{\phi_1,\phi_2,\phi_3,\phi_4\}$ be denote a set of perceptual objects and a set of functions, respectively. Sample values of the $\phi_i, i = 1,2,3,4$ functions are shown in Table 1. Let $\mathscr{O}_1 = \{x_5\}$ and $\mathscr{O}_2 = \{x_1\}$.

Let $(F,B) = (\phi_2,\{x_5\})$ is a near soft set where $F(\phi_2) = \{x_5\}$ and $(H,B) = (\phi_2,\{x_1\})$ is a near soft set where $H(\phi_2) = \{x_1\}$. Then $\tau_1 = \{(\emptyset,B),(F,B)\}$ is a near soft topology on $(\mathscr{O}_1,B)$ and $\tau_2 = \{(\emptyset,B),(H,B)\}$ is a near soft topology on $(\mathscr{O}_2,B)$.

Then $f.(\mathscr{O}_1,\tau_1,B) \to (\mathscr{O}_2,\tau_2,B)$, $f(x_5) = x_1$ mapping is a near soft continuous mapping where $(x_5,\phi_2) \in (F,B)$, $(f(x_5),\phi_2) \in (H,B)$ and $f((F,B)) \subset (H,B)$.

3 Near Soft T_i $(i = 0,1,2)$ Spaces

In this section, we define near soft separation axioms using near soft points.

Definition 15. *Two near soft sets (F,B) and (G,B) in $(\mathscr{O},B)$ are said to be near soft disjoint written $(F,B) \cap (G,B) = (\emptyset,B)$, if $F(e) \cap G(e) = \emptyset$ for all $e \in B$.*

Definition 16. *Two near soft point (x_e,B) and $(y_{e'},B)$ over a common universe $\mathscr{O}$ are distinct, written $(x_e,B) \neq (y_{e'},B)$ if there corresponding near soft sets (F,B) and (G,B) are disjoint.*

Definition 17. *$(\mathscr{O},\tau,B)$ near soft topological space is a near soft T_0-space if it satisfies the T_0-axiom, i.e. for each $(x_e,B),(y_{e'},B) \in (\mathscr{O},B)$ such that $(x_e,B) \neq (y_{e'},B)$ there is a near soft open set (F_1,B) or (F_2,B) so that $(x_e,B) \in (F_1,B),(y_{e'},B) \notin (F_1,B)$ or $(y_{e'},B) \in (F_2,B),(x_e,B) \notin (F_2,B)$.*

Example 3. Let
$\mathscr{O} = \{x_1,x_2,x_3,x_4\}, B = \{\phi_1,\phi_2\}$ and
$\tau = \{(\emptyset,B),(\mathscr{O},B),(F,B),(F_1,B),(F_2,B)\}$ where
$(F,B) = \{(\phi_1,\{x_3\}),(\phi_2,\{x_2\})\}$,
$(F_1,B) = \{(\phi_1,\{x_3\})\}$, $(F_2,B) = \{(\phi_2,\{x_2\})\}$
then $(\mathscr{O},\tau,B)$ is a near soft topological space over $\mathscr{O}$. Let us take the following information table:

Table 1

	x_1	x_2	x_3	x_4
ϕ_1	1	1	2	5
ϕ_2	3	4	3	1

There are four pairs of near soft points namely $\phi_{1(x_3)}, \phi_{2(x_2)}, \phi_{1(x_4)}, \phi_{2(x_4)}$ such that
$\phi_{1(x_3)} = \{(\phi_1,\{x_3\})\}$,
$\phi_{2(x_2)} = \{(\phi_2,\{x_2\})\}$.
Since $\phi_{1(x_3)} \neq \phi_{1(x_4)}$ then there is a near soft open set (F_1,B) such that $\phi_{1(x_3)} \in (F_1,B), \phi_{1(x_4)} \notin (F_1,B)$. Similarly for the pair $\phi_{2(x_2)} \neq \phi_{2(x_4)}$ then there is a near soft open set (F_2,B) such that $\phi_{2(x_2)} \in (F_2,B), \phi_{2(x_4)} \notin (F_2,B)$. This shows that $(\mathscr{O},\tau,B)$ is a near soft T_0-space.

Definition 18. *$(\mathscr{O},\tau,B)$ near soft topological space is a near soft T_1-space if it satisfies the T_1-axiom, i.e. for each $(x_e,B),(y_{e'},B) \in (\mathscr{O},B)$ such that $(x_e,B) \neq (y_{e'},B)$ there is a near soft open set (F_1,B) or (F_2,B) so that $(x_e,B) \in (F_1,B),(y_{e'},B) \notin (F_1,B)$ and $(y_{e'},B) \in (F_2,B),(x_e,B) \notin (F_2,B)$.*

Definition 19. *A near soft topological space $(\mathscr{O},\tau,B)$ is a near soft T_2-space if it satisfies the T_2-axiom, i.e. for each $(x_e,B),(y_{e'},B) \in (\mathscr{O},B)$ such that $(x_e,B) \neq (y_{e'},B)$ there are near open sets $(F_1,B),(F_2,B) \subset (\mathscr{O},B)$ so that $(x_e,B) \in (F_1,B),(y_{e'},B) \in (F_2,B)$ and $(F_1,B) \cap (F_2,B) = (\emptyset,B)$.*

Proposition 2. *Every near soft T_1-space is a near soft T_0-space. Every near soft T_2-space is a near soft T_1-space.*

Definition 20. *Let $(\mathscr{O},\tau,B)$ be a near soft topological space over $\mathscr{O}$, (G,B) be a near soft closed set in $(\mathscr{O},\tau,B)$ and $(x_e,B) \in (F,B)$ such that $(x_e,B) \notin (G,B)$. If there exist near soft open sets (F_1,B) and (F_2,B) such that $(x_e,B) \in (F_1,B),(G,B) \subseteq (F_2,B)$ and $(F_1,B) \cap (F_2,B) = (\emptyset,B)$ then $(\mathscr{O},\tau,B)$ is called a near soft regular space. A regular near soft T_1-space is called a T_3-space.*

Definition 21. *A near soft topological space $(\mathscr{O},\tau,B)$ is normal if for each pair $(G,B),(H,B)$ of disjoint closed near soft sets of $(\mathscr{O},B)$, there is a pair (F_1,B) and (F_2,B) of disjoint open subsets of $(\mathscr{O},B)$ so that $(G,B) \subseteq (F_1,B),(H,B) \subseteq (F_2,B)$ and $(F_1,B) \cap (F_2,B) = (\emptyset,B)$. A normal near soft T_1-space is called a T_4-space.*

Example 4. Let $\mathscr{O} = \{x_1,x_2,x_3,x_4\}, B = \{\phi_1,\phi_2\}$ and
$\tau = \{(\emptyset,B),(\mathscr{O},B),(F,B),(F_1,B),(F_2,B)\}$
where
$(F,B) = \{(\phi_1,\{x_3\}),(\phi_2,\{x_2\})\}$,
$(F_1,B) = \{(\phi_1,\{x_3\})\}$,
$(F_2,B) = \{(\phi_2,\{x_2\})\}$
then $(\mathscr{O},\tau,B)$ is a near soft topological space over $\mathscr{O}$ and its complement
$K = \{(\emptyset,B),(\mathscr{O},B),(F_K,B),(F_{1_K},B),(F_{2_K},B)\}$ where
$(F_K,B) = \{(\phi_1,\{x_1,x_2,x_4\}),(\phi_2,\{x_1,x_3,x_4\})\}$,
$(F_{1_K},B) = \{(\phi_1,\{x_1,x_2,x_4\})\}$,
$(F_{2_K},B) = \{(\phi_2,\{x_1,x_3,x_4\})\}$
are near soft closed sets. $(F_{1_K},B) = \{(\phi_1,\{x_1,x_2,x_4\})\} \in (\mathscr{O},B)$ near soft closed set and $(\phi_1,\{x_3\}) \notin (F_{1_K},B)$. There exist near soft open sets $(\mathscr{O},B)$ and (F_{1_K},B) such that $(F_{1_K},B) \in (\mathscr{O},B)$, $(\phi_1,\{x_3\}) \in (F_1,B)$ and $(\mathscr{O},B) \cap (F_1,B) \neq (\emptyset,B)$ then $(\mathscr{O},\tau,B)$ is not a near soft regular space.

4 Conclusions

In this study, we introduce near soft point, near soft continuous mapping, near soft separation axioms and near soft Hausdorff space.

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