Quantum Physics Letters
An International Journal

http://dx.doi.org/10.18576/qpl/110203

An Efficient Quantum Solution for NP-Complete Problems using n Partial Measurements

Dhananjay P. Mehendale

Department of Physics, Savitribai Phule Pune University, Pune, India-411007

Received: 2 May 2022, Revised: 2 Jul. 2022, Accepted: 15 Jul. 2022
Published online: 1 Aug. 2022

Abstract: We choose the $k-$SAT problem, $k \geq 3$, involving n variables, defined by the Boolean function, Φ, having unique correct assignment, $x = t$, such that $\Phi(t) = 1$ and $\Phi(x) = 0$ for all $x \neq t$. We prepare sufficiently many copies of quantum register, $|\Psi\rangle$, containing superposition of all computational basis states representing different possible assignments. We will find the correct assignment t as a quantum state $|t\rangle = |t_1t_2t_3t_4\cdots t_{(n-1)}t_n\rangle$ through n partial measurements. We perform n partial measurements of the state, $|\Psi_1\rangle = U_\Phi|\Psi\rangle$, where U_Φ is the unitary operator corresponding to the Boolean function Φ. The action of this unitary operator, U_Φ, on $|\Psi\rangle$ inverts the phase of the solution state $|t\rangle$ and the quantum register $|\Psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle)$, where H stands for the Hadamard gate, becomes $|\Psi_1\rangle = U_\Phi|\Psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle) - \frac{2}{\sqrt{2^n}}|t_1t_2t_3t_4\cdots t_{(n-1)}t_n\rangle$. Performing the partial measurement on $|\Psi_1\rangle$ we measure its first qubit and record this result. If this result will not be equal to $|t_1\rangle$ then $|\Psi_1\rangle$ will change to state $|\Psi_2\rangle = (\prod_{i=1}^{\otimes(n-1)} H|0\rangle)$, a completely separable state with $(n-1)$ linear factors, $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ and if this result will be equal to $|t_1\rangle$ then $|\Psi_1\rangle$ will change to state $|\Psi_3\rangle = (\prod_{i=1}^{\otimes(n-1)} H|0\rangle) - \frac{2}{\sqrt{2^{n-1}}}|t_2t_3t_4\cdots t_{(n-1)}t_n\rangle$, a completely entangled state with no factors. Thus, using together (i) The knowledge about the result of the partial measurement of the first qubit, and (ii) The knowledge about which state between the experimentally distinguishable states $|\Psi_2\rangle$ and $|\Psi_3\rangle$ has arrived as a result of this partial measurement, we will know $|t_1\rangle$. By repeating the same procedure from the beginning for the remaining qubits we will know $|t_2\rangle, |t_3\rangle, \cdots, |t_n\rangle$ etc. and thus in total n such partial measurements we will know the solution state, $|t\rangle$ through performing n partial measurements.

Keywords: Inner Product, Partial Measurements, Distinguishable Quantum States

1 Introduction

In computer science, a "decision problem" is a problem with a "yes" or "no" answer. A decision problem is in NP (which stands for "Non-deterministic Polynomial" time) if a "yes" answer can be verified efficiently, i.e. in a time that grows no faster than a polynomial in the size of the problem. Hence, loosely speaking, the problems in NP are those such that if you happened to guess the solution correctly (this is the "non-deterministic" aspect) then you could verify the solution efficiently (this is the "polynomial" aspect). Hence the name "Non-deterministic Polynomial" time. A decision problem is NP-Complete if it lies in the complexity class NP and all other problems in NP can be reduced to it. Thus the NP-Complete problems are the only problems we need to study to understand the computational resources needed to solve all of the problems in NP [5,6]. Hence, the NP-Complete problems have a special place in complexity theory. In addition to their ubiquity, NP-Complete problems share a fortuitous kinship: any NP-Complete problem can be mapped into any other NP-Complete problem using only polynomial resources [3]. Thus, if a quantum algorithm were found that can solve one type of NP-Complete problem efficiently, this would immediately lead to efficient quantum algorithms for all NP-Complete problems (up to the polynomial cost of translation). So, in some sense, the three thousand or so different NP-Complete problems are really the same problem in disguise. It is therefore sufficient to focus on any one NP-Complete problem, for any progress made in

* Corresponding author e-mail: dhananjay.p.mehendale@gmail.com

© 2022 NSP
Natural Sciences Publishing Cor.

solving that problem is likely to be applicable to all the other NP-Complete problems too, so long as you do not mind paying the polynomial cost of translation.

We choose $k-$SAT problem, among the NP-Complete problems, involving n variables, $k \geq 3$, defined by the Boolean function, Φ, to solve with exponential speedup which was the first problem shown to be NP-Complete [7]. This $k-$SAT problem is so chosen that it has a unique correct assignment, $x = t$, that makes it true i.e. $\Phi : \{0,1\}^n \rightarrow \{0,1\}$ such that for $t = (t_1, t_2, \cdots, t_n), t_i \in \{0,1\}$, $\Phi(t) = 1$ and $\Phi(x) = 0$ for all $x \neq t$, $x = (x_1, x_2, \cdots, x_n), x_i \in \{0,1\}$. We propose a novel polynomial time quantum algorithm for finding this correct assignment $t = (t_1, t_2, \cdots, t_{(n-1)}, t_n)$ making the Boolean function true as the quantum state $|t\rangle = |t_1 t_2 t_3 t_4 \cdots t_{(n-1)} t_n\rangle$ out of all possible 2^n assignments $x = (x_1, x_2, \cdots, x_n)$ as quantum states $|x\rangle = |x_1 x_2 x_3 x_4 \cdots x_{(n-1)} x_n\rangle$. We define and operate the unitary operator, U_Φ, on the quantum register containing superposition of all possible assignments in terms of the quantum states $|x\rangle = |x_1 x_2 x_3 x_4 \cdots x_{(n-1)} x_n\rangle$ and invert the phase of the only solution present in that superposition in terms of the quantum state $|t\rangle = |t_1 t_2 t_3 t_4 \cdots t_{(n-1)} t_n\rangle$ which makes the $k-$SAT problem true. It is easy to see that the quantum register $|\Psi\rangle$ can be expressed as $|\Psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle)$, representing together all possible assignments, where H stands for Hadamard gate. After the action of the unitary operator, U_Φ, the unitary operator corresponding to the Boolean function Φ, it is easy to check that $|\Psi\rangle$ becomes $|\Psi_1\rangle = U_\Phi|\psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle) - \frac{2}{\sqrt{2^n}}|t\rangle$. We now proceed to show that we can obtain the desired solution to the $k-$SAT problem with exponentially speedup.

2 A New Efficient Quantum Algorithm for Solving $k-$SAT Problems, $k \geq 3$:

We now proceed to propose a new approach for finding the desired solution to the $k-$SAT problem which makes it true. As mentioned above, we choose the $k-$SAT problem of most difficult type, i.e. the one with only one solution. Our method for solving this $k-$SAT problem can be summarized as follows:

(1) We form the quantum register, namely the quantum state, $|\Psi\rangle$, made up of equally weighted superposition of all the computational basis states, such that each one of this computational basis state represents a possible assignment for the given $k-$SAT problem.

(2) We then compute the given Boolean function, Φ, on this quantum register, $|\Psi\rangle$, i.e. we operate the unitary operator, U_Φ, corresponding to the given Boolean function, Φ, on this register, $|\Psi\rangle$, and thus evaluate $U_\Phi|\Psi\rangle = |\Psi_1\rangle$. This action of U_Φ on $|\Psi\rangle$ causes the inversion of the phase of the only computational basis state, $|t\rangle = |t_1 t_2 \cdots t_{(n-1)} t_n\rangle$, which corresponds to the unique solution $t = (t_1, t_2, \cdots, t_n)$ to the given Boolean function corresponding to the $k-$SAT problem under consideration.

(3) We then perform partial measurement on the quantum state, $|\Psi_1\rangle$, obtained in (2) above and measure its first qubit and record its value. This measurement reduces the quantum state, $|\Psi_1\rangle$, obtained in (2) into one of the two possible **distinguishable** new quantum states, $|\Psi_2\rangle$, $|\Psi_3\rangle$. These new quantum states are having diametrically opposite properties, i.e. this partial measurement of first qubit reduces the quantum state obtained in (2) into an $(n-1)$qubit quantum state, which is either (i) a completely separable quantum state, $|\Psi_2\rangle$, with $(n-1)$ linear factors, $\frac{1}{\sqrt{(2)}}(|0\rangle + |1\rangle)$, or (ii) a completely entangled quantum state, $|\Psi_3\rangle$, with no factors of any kind, and so these states are very much distinct and distinguishable and so can be easily distinguished theoretically as well as experimentally.

(4) Using (i) the information about the value of the first qubit obtained in the partial measurement, and (ii) the information about the nature (whether a completely separable state, $|\Psi_2\rangle$, or a completely entangled state, $|\Psi_3\rangle$,) of the quantum state containing $(n-1)$ qubits that results after this partial measurement of the first qubit, we will be able to determine the first qubit $|t_1\rangle$ of the desired solution state $|t\rangle$.

(5) Going back to (1) above and repeating the same procedure from the start for the second qubit $|t_2\rangle$, the third qubit $|t_3\rangle$, $\cdots$, and the last i.e. the $n-$th qubit $|t_n\rangle$ we can determine (as we have determined the first qubit, $|t_1\rangle$) the second qubit $|t_2\rangle$, the third qubit $|t_3\rangle$, $\cdots$, the last i.e. the $n-$th qubit $|t_n\rangle$ and thus complete the job of finding the desired solution state: $|t\rangle = |t_1 t_2 \cdots t_{(n-1)} t_n\rangle$.

We now proceed to give the precise steps of the algorithm:

Algorithm:

1. We prepare the quantum register which contains a superposition of all the computational basis states, $|x\rangle$ where $x \in \{0,1,\ldots,(2^n-1)\}$, where each x is expressed in the binary notation, as follows:

$$|\Psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle) = \frac{1}{\sqrt{2^n}} \sum_{x=0}^{2^n-1} |x\rangle = \frac{1}{\sqrt{2^n}} \sum_{x_i \in \{0,1\}} |x_1 x_2 \cdots x_i \cdots x_n\rangle$$

where $i \in \{1,2,\cdots,n\}$.

2. We apply the unitary transformation, U_Φ, given below that computes Φ on the above given quantum register for which there exists a unique solution that makes Φ true which is to be found. We thus mark the only correct(true) $n-$qubit state forming the correct(true) assignment for the Boolean formula, Φ, which defines our $k-$SAT problem. Let

$$|\Omega\rangle = H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

$$U_\Phi : |x\rangle|\Omega\rangle \rightarrow |x\rangle|\Omega \oplus \Phi(x)\rangle$$

© 2022 NSP
Natural Sciences Publishing Cor.

where $\Phi(x) = 1$, if $x = t$, the only correct(true) assignment that makes the Boolean function Φ true, and $\Phi(x) = 0$, otherwise. $|\Omega\rangle$ is the ancilla qubit, and $|\Omega \oplus \Phi(x)\rangle$ is the exclusive-OR of the bit value of the ancilla and the bit value that is output from our black-box function, $\Phi(x)$.

Thus, when the unitary operator U_Φ will be operated on the entire quantum register containing superposition of all the computational basis states, representing together all possible assignments for the given $k-$SAT problem, the phase associated with the only computational basis state, $|x\rangle = |t\rangle$, (since t is the only solution for Boolean function corresponding to the $k-$SAT problem under consideration, i.e. $\Phi(t) = 1$,) will be inverted. In short, for assignment $x = t$ for which the Boolean function Φ is true, giving rise to $\Phi(t) = 1$, we will have $U_\Phi(|t\rangle) = -|t\rangle$, and for all other assignments $x(\neq t)$ $U_\Phi(|x\rangle) = |x\rangle$.

3. It is now clear to see that after the action of the unitary operator, U_Φ, the quantum register $|\Psi\rangle$ given above in **1** will become

$$|\Psi_1\rangle = U_\Phi|\Psi\rangle = (\prod_{i=1}^{\otimes(n)} H|0\rangle) - \frac{2}{\sqrt{2^n}}|t_1t_2\cdots t_{(n-1)}t_n\rangle.$$

4. We now proceed with the partial measurement of the above resulted state, $|\Psi_1\rangle$, and measure the "first" qubit of this state and find its bit value, which may be either 0 or 1 corresponding to 1-qubit states $|0\rangle$ or $|1\rangle$, and record the result of this partial measurement. This partial measurement of the first qubit will change the $n-$qubit state, $|\Psi_1\rangle$, to a new $(n-1)-$qubit state, where two diametrically opposite possibilities exist for this newly formed state. (i) If the first qubit is not equal to $|t_1\rangle$ then $|\Psi_1\rangle$ will change into $|\Theta\rangle = |\Psi_2\rangle = (\prod_{i=1}^{\otimes(n-1)} H|0\rangle)$, a completely separable quantum state having $(n-1)$ linear factors, $\frac{1}{\sqrt{(2)}}(|0\rangle + |1\rangle)$, (ii) If the first qubit is indeed equal to $|t_1\rangle$ then $|\Psi_1\rangle$ will change into $|\Theta\rangle = |\Psi_3\rangle = (\prod_{i=1}^{\otimes(n-1)} H|0\rangle) - \frac{2}{\sqrt{2^{(n-1)}}}|t_2t_3t_4\cdots t_{(n-1)}t_n\rangle$, a completely entangled quantum state with no factors of any kind [1]. Thus, we will arrive either at state $|\Psi_2\rangle$ or at state $|\Psi_3\rangle$. These states $|\Psi_2\rangle$, $|\Psi_3\rangle$ are quite different and so are distinguishable. For example, let us take a Hermitian operator, O, corresponding to some observable and let us denote by $\langle O\rangle_{|\chi\rangle} = \langle\chi|O|\chi\rangle$ the expectation value for the operator O in state $|\chi\rangle$, then as $|\Psi_2\rangle$, $|\Psi_3\rangle$ being distinguishable, therefore, we will observe that $\langle O\rangle_{|\Psi_2\rangle} \neq \langle O\rangle_{|\Psi_3\rangle}$.

Remark 2.1: We will discuss in brief in the next section about a theoretical [1] as well as an experimental [2] procedure to distinguish between $|\Psi_2\rangle$, $|\Psi_3\rangle$ to determine whether one has arrived at state $|\Psi_2\rangle$ or $|\Psi_3\rangle$ and for time being we will grant that we can distinguish between such states and so can determine at which of such states one has arrived at during the partial measurement of first (or any single) qubit.

5. Our aim is to determine t_1 and from above discussion it now follows that the following are the possibilities for t_1.

(i) Suppose the state $|0\rangle$ results in the partial measurement of the first qubit of $|\Psi_1\rangle$, representing as the bit value the binary number 0, and state $|\Psi_2\rangle$ has appeared as a result of this partial measurement of the first qubit then clearly $t_1 = 1$.

(ii) Suppose the state $|0\rangle$ results in the partial measurement of the first qubit of $|\Psi_1\rangle$, representing as the bit value the binary number 0, and state $|\Psi_3\rangle$ has appeared as a result of this partial measurement of the first qubit then clearly $t_1 = 0$.

(iii) Suppose the state $|1\rangle$ results in the partial measurement of the first qubit of $|\Psi_1\rangle$, representing as the bit value the binary number 1, and state $|\Psi_2\rangle$ has appeared as a result of this partial measurement of the first qubit then clearly $t_1 = 0$.

(iv) Suppose the state $|1\rangle$ results in the partial measurement of the first qubit of $|\Psi_1\rangle$, representing as the bit value the binary number 1, and state $|\Psi_3\rangle$ has appeared as a result of this partial measurement of the first qubit then clearly $t_1 = 1$.

Thus, we have managed to determine the value of t_1.

6. We now go back to step 1 and proceed again through all the steps with the following only change in step 4. In step 4, we now proceed with partial measurement of the above resulted state, $|\Psi_1\rangle$, and measure, this time, the "second" qubit of this state and proceed on exactly on similar lines and determine the value of t_2.

7. We continue on these lines and continue in succession with the partial measurement of the "third", "fourth", $\cdots$, and finally of the "$n-$th" qubit and determine the values of $t_3, t_4, \cdots, t_n$, etc. The algorithm thus determines the solution, t in n-steps.

3 A Theoretical and Corresponding Experimental Procedure to Distinguish between $|\Psi_2\rangle$, $|\Psi_3\rangle$:

We first discuss in brief the **theoretical** procedure based on the results in [1] to determine the nature of the wave-function, $|\Theta\rangle$ say, that arises in the partial measurement of the first qubit of $|\psi_1\rangle$. We want to determine whether $|\Theta\rangle = |\psi_2\rangle$ or $|\Theta\rangle = |\psi_3\rangle$. As per [1] we form the associated matrix A of size $2^1 \times 2^{(n-1)}$ associated with the state $|\Theta\rangle$ f or checking the existence of a linear factor to this state $|\Theta\rangle$.

(i) Suppose the associated matrix A of size $2^1 \times 2^{(n-1)}$ associated with the state $|\Theta\rangle$ is

$$A = \frac{1}{\sqrt{(2^n)}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & 1 & \cdots & 1 \end{bmatrix}.$$

Here, the two rows of this associated matrix A are exactly identical implying the existence of a linear factor,

© 2022 NSP
Natural Sciences Publishing Cor.

$\frac{1}{\sqrt{(2)}}(|0\rangle+|1\rangle)$, therefore we can conclude that in this case $|\psi_1\rangle$ has changed into the state $|\Theta\rangle$ where $|\Theta\rangle = |\psi_2\rangle$.

(ii) Suppose the associated matrix A of size $2^1 \times 2^{(n-1)}$ associated with the state $|\Theta\rangle$ is

$$A = \frac{1}{\sqrt{(2^n)}} \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & 1 & -1 & \cdots & 1 \end{bmatrix}.$$

Clearly, the two rows of this matrix A are not identical. The matrix element $a_{13} = 1$ while the matrix element $a_{23} = -1$, therefore, in this case there does not exist a linear factor of the type seen in (i) and here we can conclude that $|\psi_1\rangle$ has changed into $|\Theta\rangle$ where $|\Theta\rangle = |\psi_3\rangle$.

We now discuss an efficient experimental procedure (based on the efficient algorithm to find the inner product of two $N-$dimensional vectors in $Log(N)$ time [4]) to find out the nature of the wave-function, $|\Theta\rangle$, that arises in the partial measurement of the first qubit of $|\psi_1\rangle$, i.e. to determine whether the state $|\Theta\rangle = |\psi_2\rangle$ or $|\Theta\rangle = |\psi_3\rangle$.

In this experimental procedure we make use of the Cauchy-Schwartz inequality [8].

Cauchy-Schwartz Inequality: For all vectors $|P\rangle$ and $|Q\rangle$ of inner product space if $||P\rangle| = +\sqrt{\langle P|P\rangle}$, $||Q\rangle| = +\sqrt{\langle Q|Q\rangle}$ and let $\langle P|Q\rangle$ denotes the inner product of $|P\rangle$ and $|Q\rangle$ then $|\langle P|Q\rangle| \leq ||P\rangle|.||Q\rangle|$ and the equality holds if and only if vectors $|P\rangle$ and $|Q\rangle$ are proportional to each other, i.e. if and only if $|P\rangle = \alpha|Q\rangle$ for some $\alpha \in \mathbb{C}$, the underlying field of complex numbers.

We now discuss in brief the corresponding **experimental** procedure based on the results in [2] to determine the nature of the wave-function, $|\Theta\rangle$ say, that arises in the partial measurement of the first qubit of $|\psi_1\rangle$. We create sufficiently many copies of the state $|\Theta\rangle$. We then determine whether $|\Theta\rangle = |\psi_2\rangle$ or $|\Theta\rangle = |\psi_3\rangle$ as follows:

As per [2] we partially measure the first qubit of the state $|\Theta\rangle$ on $|0\rangle$ and arrive at the quantum state $|\Theta_0\rangle$ say. It is easy to check that the coefficients of $|\Theta_0\rangle$ are identical to the first row of the associated matrix, A, given above in the discussion of the theoretical procedure. Also, we then partially measure the first qubit (in the other copy) of $|\Theta\rangle$ (this time) on $|1\rangle$ and arrive at the quantum state $|\Theta_1\rangle$ say. It is easy to check that the coefficients of $|\Theta_1\rangle$ are identical to the second row of the associated matrix, A, given above in the discussion of the theoretical procedure.

We then evaluate $|\langle\Theta_0|\Theta_1\rangle|$, as well as $||\Theta_0\rangle|$, and $||\Theta_1\rangle|$. Now, using the results in [2] it is easy to see that

(i) If $|\langle\Theta_0|\Theta_1\rangle| = ||\Theta_0\rangle| \cdot ||\Theta_1\rangle|$ then $|\Theta\rangle = |\psi_2\rangle$, and

(ii) If $|\langle\Theta_0|\Theta_1\rangle| < ||\Theta_0\rangle| \cdot ||\Theta_1\rangle|$ then $|\Theta\rangle = |\psi_3\rangle$.

Remark 3.1: The above experimental procedure is efficient because it makes use of the "quantum" speedup available for finding the "inner product" of two $N-$dimensional state vectors in $Log(N)$ time [4].

4 An example

To illustrate our approach we discuss steps to find the solution to a $k-$SAT problem, $k \geq 3$, containing four variables and defined by the Boolean function: $\Phi(x_1, x_2, x_3, x_4)$.

For the sake of simplicity we have chosen a $k-$SAT problem, $k \geq 3$, to solve containing only four variables. We have

$$|\Psi\rangle = H^{\otimes 4}|0\rangle^{\otimes 4} = \frac{1}{\sqrt{2^4}} \sum_{x=0}^{2^4-1} |x\rangle,$$

Let

$$|\Omega\rangle = H|1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).$$

By applying the unitary operator

$$U_\Phi : |x\rangle|\Omega\rangle \rightarrow |x\rangle|\Omega \oplus \Phi(x)\rangle$$

on the entire quantum register $|\Psi\rangle$ where $\Phi(x) = 1$, if $x = t$, the only correct(true) assignment that makes the Boolean function $\Phi(x)$ true, and $\Phi(x) = 0$, otherwise.

Now, suppose the unique solution, $t = (t_1, t_2, t_3, t_4)$ to the above problem be $t = (t_1, t_2, t_3, t_4) = (1, 1, 0, 0)$ then by operating the unitary operator, U_Φ, on the above wave-function $|\Psi\rangle = H^{\otimes 4}|0\rangle^{\otimes 4}$ it will change to $|\Psi_1\rangle$, where $|\Psi_1\rangle = U_\Phi \Psi\rangle$, is as follows:

$$\begin{aligned} |\Psi_1\rangle = \frac{1}{\sqrt{2^4}}(&|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle \\ &+|0100\rangle + |0101\rangle + |0110\rangle + |0111\rangle \\ &+|1000\rangle + |1001\rangle + |1010\rangle + |1011\rangle \\ &-|1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle). \end{aligned}$$

Clearly, the phase of the only quantum state which represents the solution state, $|t\rangle = |1100\rangle$, has got inverted.

We now make the partial measurement of the first qubit of $|\Psi_1\rangle$.

Since it is given to us that the unique solution for the $k-$SAT problem under consideration is, $t = (t_1, t_2, t_3, t_4) = (1, 1, 0, 0)$, the following will be the possibilities:

(1) If the result of the partial measurement of the "first" qubit of $|\Psi_1\rangle$ is equal to $|0\rangle$ corresponding to binary number 0 then the resulting state after this partial measurement will change to $|\Psi_2\rangle = (\prod_{i=1}^{\otimes(3)} H|0\rangle)$, a completely separable state having three linear factors, $\frac{1}{\sqrt{(2)}}(|0\rangle+|1\rangle)$, therefore, as per the possibility (i) in the step 5 of the algorithm, given the section II above, we have determined that $t_1 = 1$.

We now go back and again prepare $|\Psi_1\rangle = U_\Phi|\Psi\rangle$ and proceed this time with the measurement of the "second" qubit:

(2) If the result of the partial measurement of the "second" qubit is equal to $|1\rangle$ corresponding to binary

© 2022 NSP
Natural Sciences Publishing Cor.

number 1 then it is easy to see in the present case (by looking at the given solution, $t = (1,1,0,0)$), that the resulting state after this partial measurement will be $|\Psi_3\rangle$, i.e. $(\prod_{i=1}^{\otimes(3)} H|0\rangle) - \frac{2}{\sqrt{2^3}}|100\rangle$ a completely entangled state having no factors of any kind, therefore, as per the possibility (iv) given in the step 5 of the algorithm given the second section above, $t_2 = 1$.

By proceeding on similar lines as above:

(3) If we will now make the partial measurement of the "third" qubit of the state $|\Psi_1\rangle$ and will determine the value of the measured qubit and the nature of the resulted quantum state of this partial measurement, (whether $|\Psi_2\rangle$, or,$|\Psi_3\rangle$) we will verify that the value of the "third" bit, t_3, as $t_3 = 0$.

(4) Subsequently, if we will now make the partial measurement that of the "fourth" qubit of the state $|\Psi_1\rangle$ and will determine the value of the measured qubit and the nature of the resulted quantum state of this partial measurement, (whether $|\Psi_2\rangle$, or,$|\Psi_3\rangle$) we will verify the value of the "fourth" bit, t_4, as $t_4 = 0$.

Thus will fully determine the desired quantum state $|t\rangle$ as $|1100\rangle$ which corresponds to the desired unique solution, $t = (t_1,t_2,t_3,t_4) = (1,1,0,0)$ which was to be found for the given $k-$SAT problem. Note that this solution was found in steps of the order, $O(n)$.

Acknowledgement

I thank Dr. M. R. Modak, S. P. College, Pune-411030, India, for encouragement.

References

[1] D. P. Mehendale, P. S. Joag, A Simple Algorithm for Complete Factorization of an N-Partite Pure Quantum State, *Quant. Phys. Lett.* 6, No. 1, pp 73-77,(2017).
[2] D. P. Mehendale, Two Simple Algorithms for Testing the Entanglement Status of an N-qubit Pure Quantum State, *Quant.Phys. Lett.*, No. 1, 1-4 (2022).
[3] L. Berman and J. Hartmanis, On Isomorphisms and Density of NP and Other Complete Sets, *SIAM Journal on Computing*, Volume 6, Issue 2, pp. 305-322, (1977).
[4] S. Lloyd, Masoud Mohseni, Patrick Rebentrost, Quantum algorithms for supervised and unsupervised machine learning, *arXiv quant-ph*, 1307.0411v2, (2013).
[5] C. P. Williams, *Explorations in Quantum Computing*, second edition, Springer-Verlag London Limited, 293-296, (2011).
[6] M. Nielsen and I. Chuang, *Quantum Computation and Quantum Information*, Cambridge university Press, 40-42, (2000).
[7] S. Cook, *The Complexity of Theorem Proving Procedures*, Theory of Computing, Third ACM Symposium, (1971).
[8] Cauchy-Schwartz Inequality, Wikipedia Article, *Wikipedia*, https://en.m.wikipedia.org/Cauchy-Schwartz-inequality-Wikipedia.html, (2019).

© 2022 NSP
Natural Sciences Publishing Cor.