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Heat and mass transport analysis of chemically reacting cross diffusion convection in a couple stress fluid-saturated rotating porous medium with internal heat generation

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Abstract: In the present study, non-linear stability analysis with chemical reaction is performed in pair stress fluid saturated with anisotropic porous medium with the consideration of an internal heat source. In the governing equation, the extended Darcy model has been employed in the momentum equation. The normal mode approach and truncated Fourier series methodology were adopted for linear, and non-linear stability investigations. The impact of numerous characteristics, such as the DuFour and Soret parameters, has been addressed and visually depicted.

Keywords: Rotation, Cross-diffusion, couple-stress fluid, Internal heat generation, Soret effect.

1 Introduction

The distinction between single and multicomponent systems has prompted interest in multi-component convection research. Even though density drops as height increases, i.e. when the fundamental condition is hydro-statically stable, convection occurs, in incongruity to single component systems. Applications in engineering and research encourage the theoretical and practical learning of two components convection in a rotating porous material [1,2]. Some of the key fields of application are centrifugal casting and metal solidification, chemical and food processes, bio-mechanics, petroleum industry, rotating machinery, and geophysical difficulties of this study [3,4,5]. When heat and mass transfer happen at the same time in a flowing fluid, the relationship between driving potentials and fluxes becomes much complicated.

According to research, temperature gradients, as well as composition gradients, can cause an energy flux. The Dufour or diffusion-thermo effect is the rate of energy flow caused by a concentration gradient. The thermal-diffusion or Soret effect, on the other hand, causes mass fluxes when temperature gradients exist. Every unique property gradient has a considerable impact on the flow of the other property due to cross-diffusion effects [6,7,8].

The diverse applications of double diffusive convection across porous surfaces have attracted attention in recent decades, ranging from the freezing of binary mixes to solute transport in saturated soils. The books by Vadasz [2] and Nield and Bejan [1] provide a detailed report on the convections in porous media fluids. Due to its relevance in seawater and liquefied metals, double-diffusive convection, in which concentration and temperature are two diffusive components during convection, is a very important phenomenon. Many authors, such as Poulikakos [9], and Neild et al. [13,14,15,16,17], have studied double-diffusive convection with solute concentration and temperature as the independent and dependent components. Later on Srivastava et al. [34], in the presence of a magnetic field, worked on Soret driven convection in a pair stress fluid, and Kumar et

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al.[31] researched on a thermal non-equilibrium model in a couple stress porous media fluid.

Rudraiah et al. [5], and Malashetty et al. [25] examined convection in porous media when both the Dufour and Sorer effects are present. This phenomenon has been researched by many academicians over the last several years, including Satrughan and Hutter [3], Bahloul et. al [4], Hill [7], and Bhadauria et. al [36]. Further, convection with two diffusive components in couple stress fluid was examined by Sharma [20], Malashetty et al. [18]. Gaikward,and Kamble [11] explored the cross-diffusion effect in a pair stress fluid saturated with rotating anisotropic porous medium .Recently, Shakya and Bhadauria [8] examined the impact of traverse-diffusion on thermal stability of saturated rotating anisotropic porous media with Maxwell fluid.

Physical configurations with spinning fluids produce an additional external Coriolis force, making the phenomenon of hydrodynamic stability more complicated but more interesting, and because of its importance, many researchers have investigated it. Govinder [27], and Mishra et al. [34] has published research findings on the influence of Coriolis on linear stability analysis in rotating fluid- soaked porous layer rotation.

The majority of research on thermal instabilities in porous media has focused on isotropic materials. However, the mechanical and thermal properties of porous materials are anisotropic in many real scenarios. Castinel and Carbarnous [19] were the first to explore the onset of heat convection in porous media fluid with in anisotropic property. Later on some more work has been done with anisotropic porous media, recently, Srivastva et al.[32] investigated a thermal non-equilibrium magneto-convection model in an anisotropic porous material, and Kumar [33] studied the weakly nonlinear convection in the presence of magnetic field in a visco-elastic fluid saturated porous medium.

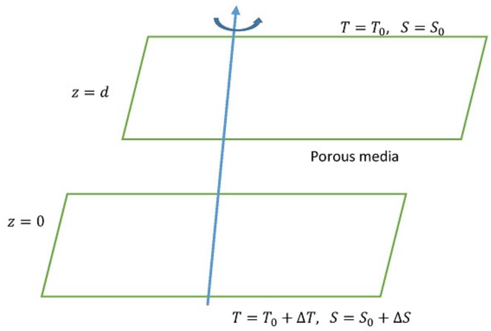
Internal heat sources have become more fascinating in recent decades as a result of their practical application in a variety of physical settings, such as nuclear heat cores and nuclear reactions. The majority of thermal stability research has been done in the absence of any internal heat source, however some study has been done using internal heat sources viz. Altawallbeh [26] investigated cross diffusion in viscoelastic fluid with saturated porous medium in presence of an internal heat source for linear, and non-linear double diffusive convection. Bhadauria [21,22,23,24] has recently investigated a number of works on internal heat sources.

Some study of convection with chemical reaction has been done by Steelberg and Brand [6], Pritchard and Richardson [29], and Srivastava [30].

In the light of above study, the present article investigate the stability of double diffusive convection under rotating anisotropic fluid flow with an internally heat source, taking into account cross diffusion where concentration and temperature interact with one another. We considered a couple stress fluid saturated with anisotropic porous media, and to our knowledge, no previous study has been done on this physical structure with chemical reaction. Finally, a non-linear stability analysis was performed to see how different flow parameters influence stability and heat mass transfer.

2 Governing Equations

Consider a rotating porous layer of infinite extent with anisotropic properties occupied by couple stress fluid of depth d. The temperature and concentration differences between the lower and upper layers have been kept constant. The presence of a heat source inside the media is taken into account. The system is modelled using extended Darcy law in the physical configurations mentioned above.



$$\nabla.\vec{q} = 0 \tag{1}$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \left(\mu - \mu_1\nabla^2 + \mu_c\nabla^4\right)K^{-1}\cdot\vec{q} + \frac{2}{\varepsilon}\Omega\times\vec{q} = -\nabla p + \rho\vec{g} \tag{2}$$

$$\varepsilon(\rho_f c_f)\frac{\partial T_f}{\partial t} + \rho_f c_f(\vec{q}.\nabla)T_f = \nabla(\varepsilon\kappa_f\nabla T_f) + D_f\nabla^2 C + Q(T_f - T_u) \tag{3}$$

$$\varepsilon\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C = \varepsilon D_m\nabla^2 C + D_T\nabla^2 T_f + k(C_{eq}(T) - C) \tag{4}$$

In this case, $C_{eq} = C_b$ is linear in T i.e. $C_{eq} = C_0 + \phi(T - T_0)$ (see Steinberg and Brand [6]), this allows for the formation of a stable basic state when the solute is in chemical balance with the solid matrix in all directions., and so the vertical flux of solute is invariant with the space.

The concentration and temperature boundary conditions in the preceding equation are as follows:

All the symbols whether constants or variables in the preceding equations have standard definitions and are mentioned in nomenclature section. To account for the influence of density fluctuations, Oberbeck-Boussinesq approximation is used. i.e.

$$\rho = \rho_0 \left[1 - \beta_T (T_f - T_u) + \beta_C (C_f - C_u)\right]. \quad (5)$$

where $\beta_C > 0$, and $\beta_T > 0$ are the coefficients of concentration, and thermal expansions respectively.

$$C = C_b(z),\ \rho = \rho_b(z),\ p = p_b(z),\ T_f = T_b(Z),\ \vec{q} = \vec{q_b} \quad (6)$$

Now we apply a minor perturbation to the fundamental state in the form

$$\vec{q} = \vec{q_b} + q',\ T_f = T_{b(z)} + T',\ C = C_b + C',\ p = p_b + p', \rho = \rho_{b(z)} + \rho' \quad (7)$$

where primes represents perturbations.
Using the equations (6) and (7) in equations (1-4) , resultant equations after excluding the primes, results in

$$\nabla.\vec{q} = 0 \quad (8)$$

$$\frac{\rho_0}{\varepsilon}\frac{\partial \vec{q}}{\partial t} + \left(\mu - \mu_1\nabla^2 + \mu_c\nabla^4\right)K^{-1}\cdot\vec{q} + \frac{2}{\varepsilon}\Omega \times \vec{q} = -\nabla p + \rho_0\left(\beta_T T - \beta_S S\right)\vec{g} \quad (9)$$

$$\gamma\frac{\partial T}{\partial t} + (\vec{q}.\nabla)T + w\frac{dT_b}{dz} = \nabla(K\nabla T) + Q(T - T_u) + \nabla^2 T_b + D_f\nabla^2 C + D\frac{d^2 C_b}{dz^2} \quad (10)$$

$$\varepsilon\frac{\partial C}{\partial t} + (\vec{q}.\nabla)C + w\frac{dC_b}{dz} = \varepsilon K_s\nabla^2 C + \varepsilon K_s\frac{d^2 C_b}{dz^2} + \varepsilon D\nabla^2 T + \varepsilon D\frac{d^2 T_b}{dz^2} \quad (11)$$

To discuss the non-linear theory, consider following set of equations in unknown ψ, T, C, V, which are obtained from the non-dimensionalised form of the equations (8-11) using $(x,y,z) = (x^*,y^*,z^*)d$, $(u,v,w) = (u^*,v^*,w^*)\frac{K_T}{d}$, $t = \frac{\gamma d^2}{K_{T_z}}t^*$, , $T = (\Delta T)T^*$, $p = \frac{\mu K_{T_z}}{K_T}p^*$, $C = (\Delta C)C^*$, transformations and stream function Ψ.

$$\left\{\frac{1}{P_r}\frac{\partial}{\partial t}\nabla^2 + \left(\frac{\partial^2}{\partial x^2} + \frac{1}{\xi}\frac{\partial^2}{\partial z^2}\right)\left(\frac{1}{Da} - C_1\nabla^2 + C_2\nabla^4\right)\right\}\Psi + \sqrt{T_a}\frac{\partial V}{\partial z} - \mathrm{Ra}_T\frac{\partial T}{\partial x} + \mathrm{Ra}_S\frac{\partial C}{\partial x} = 0 \quad (12)$$

$$\left\{\frac{1}{P_r}\frac{\partial}{\partial t} + \left(\frac{1}{\xi}\right)\left(\frac{1}{Da} - C_1\nabla^2 + C_2\nabla^4\right)\right\}V + \sqrt{T_a}\frac{\partial \Psi}{\partial z} = 0 \quad (13)$$

$$\left\{\frac{\partial}{\partial t} - \left(\eta\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + R_i\right)\right\}T - Du\frac{Ra_t}{Ra_s} = \frac{\partial \Psi}{\partial x}.\frac{\partial T_b}{\partial z} + \frac{\partial(\Psi,T)}{\partial(x,z)} \quad (14)$$

$$\left\{\frac{\partial}{\partial t} - \frac{1}{Le}\nabla^2\right\}C - Sr\frac{Ra_s}{Ra_t} = -\frac{\partial \Psi}{\partial x} + \frac{\partial(\Psi,C)}{\partial(x,z)} + k(T - C) \quad (15)$$

where

$$T_b = \frac{Sin\sqrt{R_i}(1-z)}{Sin\sqrt{R_i}} \quad (16)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$$

where, all the non-dimensional parameters have usual meanings, and are mentioned in nomenclature.

Now the above equations are to be solved under stress-free boundary conditions, which are

$$w = \frac{\partial^2 w}{\partial z^2} = S = T = 0 \quad \text{at} \quad z = 0, \text{ and } z = 1. \quad (17)$$

3 Weak Non-linear Stability Analysis

It is always preferable to evaluate amplitudes for controlling parameters for non-linear terms in governing equations in light of stress free boundary conditions given by (17) when analysing heat and mass transfer. For this, consider the following two-dimensional convective rolls:

$$\Psi = A_1(t)\,Sin[ax]\,Sin[\pi z] \quad (18)$$

$$T = A_2(t)\,Cos[ax]\,Sin[\pi z] + A_3(t)\,Sin[2\pi z] \quad (19)$$

$$C = A_4(t)\,Cos[ax]\,Sin[\pi z] + A_5(t)\,Sin[2\pi z] \quad (20)$$

$$V = A_6(t)\,Sin[ax]\,Cos[\pi z] + A_7(t)\,Sin[2\pi z] \quad (21)$$

where A_1, A_2, A_3, A_4, A_5, A_6, A_7 are amplitudes, which are the function of time, and are to be determined. Equating like terms after applying equations (18-21) in the set equations (12-15), we get a set of autonomous non-linear differential equations.

$$\frac{dA_1}{dt} = -\frac{Pr}{\delta^2}\left[\delta_1^2\eta_1 A_1 - \pi\sqrt{Ta}A_6 + aA_2Ra_T - aRa_SA_4\right] \quad (22)$$

$$\frac{dA_2}{dt} = \left(R_i - \delta_2{}^2\right)A_2 - Du\frac{Ra_s\delta^2 A_4}{Ra_T} - 2\pi aA_1A_3 + 2aA_1F \quad (23)$$

$$\frac{dA_3}{dt} = \left(R_i - 4\pi^2\right)A_3 + \frac{1}{2}\pi aA_1A_2 - Du\frac{4Ra_s\pi^2 A_5}{Ra_T} \quad (24)$$

$$\frac{dA_4}{dt} = -A_1a - \left(\frac{\delta^2}{Le} + k\right)A_4 - Sr\frac{Ra_T}{Ra_s}\delta^2 A_2 - 2\pi aA_1A_4 + kA_5 \quad (25)$$

$$\frac{dA_5}{dt} = -\left(\frac{4\pi^2}{Le} + k\right)A_5 - Sr\frac{Ra_T}{Ra_s}4\pi^2 A_3 + \frac{1}{2}\pi A_1A_5a + kA_3 \quad (26)$$

$$\frac{dA_6}{dt} = -\frac{Pr}{\xi}\eta_1 A_6 - \pi Pr\sqrt{Ta}A_1 \quad (27)$$

$$\frac{dA_7}{dt} = -\frac{Pr}{\xi}\left(\frac{1}{Da} + 4\pi^2\left(C_1 + 4\pi^2 C_2\right)\right)A_7 \quad (28)$$

Now, we analyze the above system in two cases for qualitative and quantitative analysis of heat and mass transport.

4 Steady Non Linear Analysis

In case of steady state, $\frac{dA_i}{dt}=0,\ 1\leq i\leq 7$, and so we have

$$\frac{Pr}{\delta^2}\left[\delta_1^2\eta_1A_1-\pi\sqrt{Ta}A_6+aA_2Ra_T-aRa_SA_4\right]=0 \quad (29)$$

$$\left(R_i-{\delta_2}^2\right)A_2-2\pi aA_1A_3+2aA_1F-Du\frac{Ra_s\delta^2A_4}{Ra_T}=0 \quad (30)$$

$$\left(R_i-4\pi^2\right)A_3+\frac{1}{2}\pi aA_1A_2-Du\frac{4Ra_s\pi^2A_5}{Ra_T}=0 \quad (31)$$

$$A_1a+\left(\frac{\delta^2}{Le}+k\right)A_4+Sr\frac{Ra_T}{Ra_s}\lambda^2A_2+2\pi aA_1A_4-kA_2=0 \quad (32)$$

$$\left(\frac{4\pi^2}{Le}+k\right)A_5+Sr\frac{Ra_T}{Ra_s}4\pi^2A_3-\frac{1}{2}\pi A_1A_5a-kA_3=0 \quad (33)$$

$$-\frac{\pi}{\xi}\eta_1A_6-\pi Pr\sqrt{Ta}A_1=0 \quad (34)$$

$$-\frac{Pr}{\xi}\left(\frac{1}{Da}+4\pi^2\left(C_1+4\pi^2C_2\right)\right)A_7=0 \quad (35)$$

Now, we look for an analytical solution of the above system. For this, we convert all amplitudes A_1, A_2, A_3, A_4, A_5, A_6, A_7 in terms of A_1 only for further analysis.

5 Formulation of Heat and Mass transport

Convective heat and mass transport are characterized by Nussulet number and Sherwood number, which are given by,

$$Nu=\frac{\text{Total Heat Transport}}{\text{Heat trasport by conduction only}}$$

i.e

$$Nu=1+\left[\frac{\int_0^{2\pi}\frac{\partial T}{\partial z}dx}{\int_0^{2\pi}\frac{dT_b}{dz}dx}\right]_{z=0}$$

$$Nu=1-\frac{2\pi A_3}{\sqrt{R_i}cot\left(\sqrt{(R_i)}\right)} \quad (36)$$

$$Sh=1+\left[\frac{\int_0^{2\pi}\frac{\partial C}{\partial z}dx}{\int_0^{2\pi}\frac{dC_b}{dz}dx}\right]_{z=0}$$

$$Sh=\left(1-2\pi A_5\right). \quad (37)$$

6 Result and discussion

6.1 Non-linear Stability for Steady Case

Effects of various parameters on convection over rotating anisotropic porous media with internal heat source, and cross-diffusion has been studied using several characteristic curves derived from the solution of the autonomous system of equations (29-34) with fixed values of the parameters $\xi=0.5$,$\eta=0.5$, $Le=80$, $C_1=0.005$, $C_2=0.001$, $Ta=100$, $Ras=100$, $Du=0.005$, $Sr=0.003$, $R_i=2$, and $k=1$.

Figures 1 depicts the variation of Nu with scaled Rayleigh number Ra_t for various values of Du, ξ, Sr, C_1, C_2, R_i, Le, Ra_S, and η. Further, figures 2 demonstrate the variation of Sh with scaled Rayleigh number Ra_t for different values of R_i, ξ, Ta, C_1, C_2, Sr, and Du, respectively.

It may be inferred from the figures in 1 that an increase in parameters Du, ξ, Sr, C_1, and C_2 result a decrease in Nu and hence stabilise the system. But from the figures 1(f-i), one may observe that an increase in parameters R_i, Le, Ra_S, and η results an increase in Nu and hence destabilise the system.

Further, from the figures 2, it is found that an increase in the parameters R_i, and ξ, results an increase in Sh and thus destabilise the system. Similarly, we can observe from the figures in 2 that C_1, C_2, Sr, and Du stabilise the system.

6.2 Non Linear Stability of Unsteady case

The truncated representation of the Fourier series approach is applied to detect heat, and mass transfer events in non-linear stability analysis. Nu and Sh oscillate with time at first, eventually reaching a stable state. For unsteady case , autonomous system of differential equation (22-28) solved numerically for fixed values of $\xi=0.5$,$\eta=0.5$, $Le=80$, $C_1=0.005$, $C_2=0.001$, $Ta=100$, $Ras=100$, $Du=0.005$, $Sr=0.003$, $R_i=2$, and $k=1$.

Figures 3 and 4 demonstrate the influence of various parameters on the Nusselt number Nu and the Sherwood number Sh with time t, respectively.

From the figures 4, one may observe that initially Sherwood number takes value 1, which means mass transport happens only due to conduction. Furthermore, we can see that when the values of Ta, Le, C_1, C_2 grow, the curves tend to move down, implying that there is less heat and mass exchange, and therefore these parameters have a stabilizing influence. However, as Ras and ξ grow, curves tend to shift higher, indicating that there is more heat and mass movement and hence these parameters have a destabilising influence.

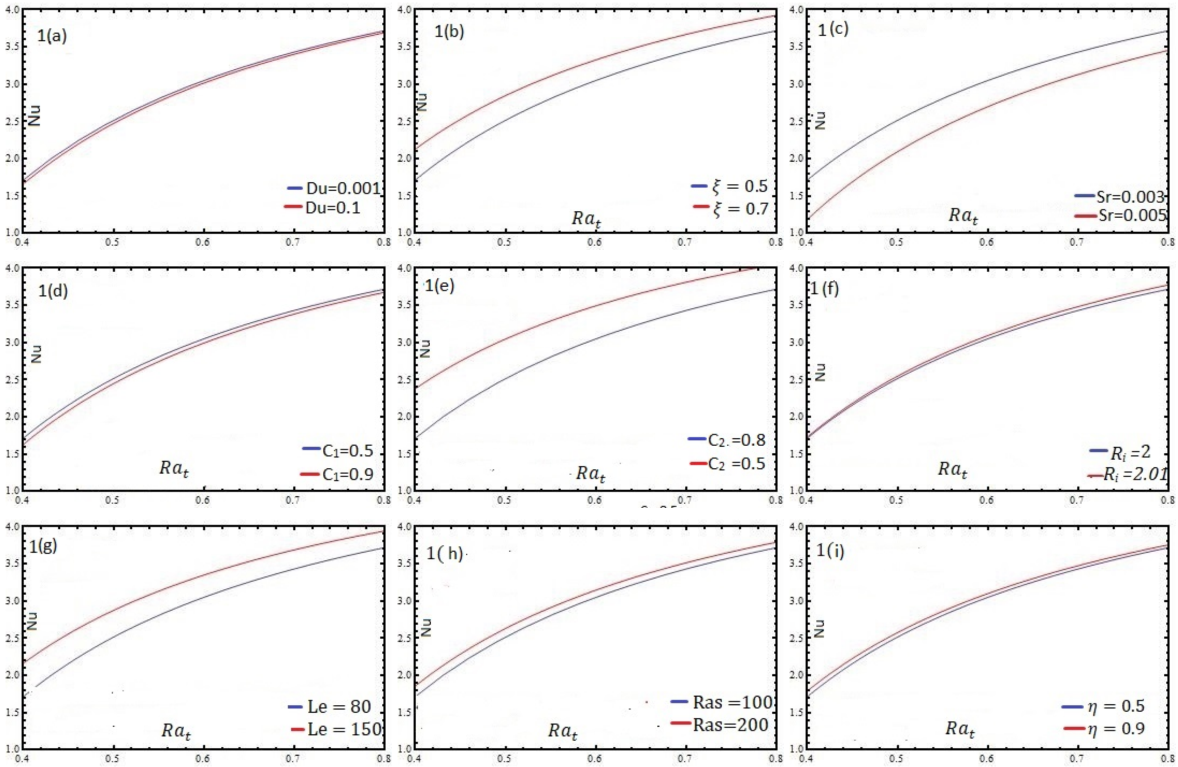


Fig. 1: Variability of Nu with Ra_t for different values of Du, ξ, Sr, C_1, C_2, R_i, Le, Ra_S, and η respectively

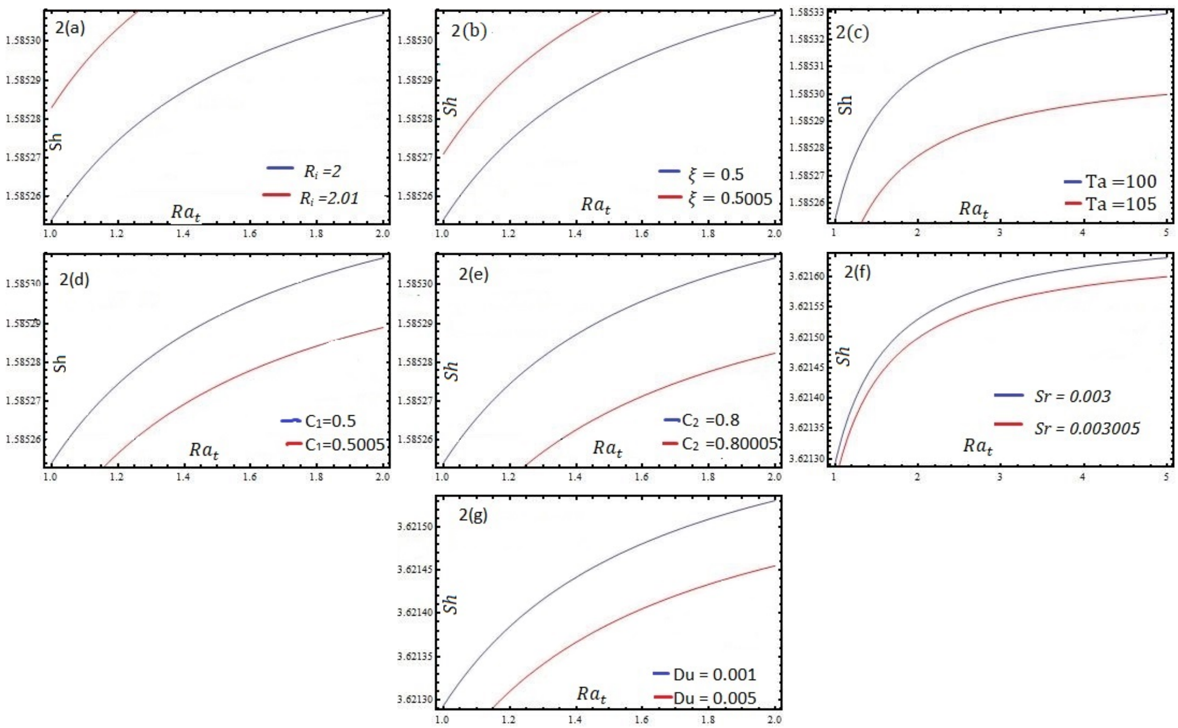


Fig. 2: Variability of Sh with Ra_t for different values of R_i, ξ, Ta, C_1, C_2, Sr, and Du respectively

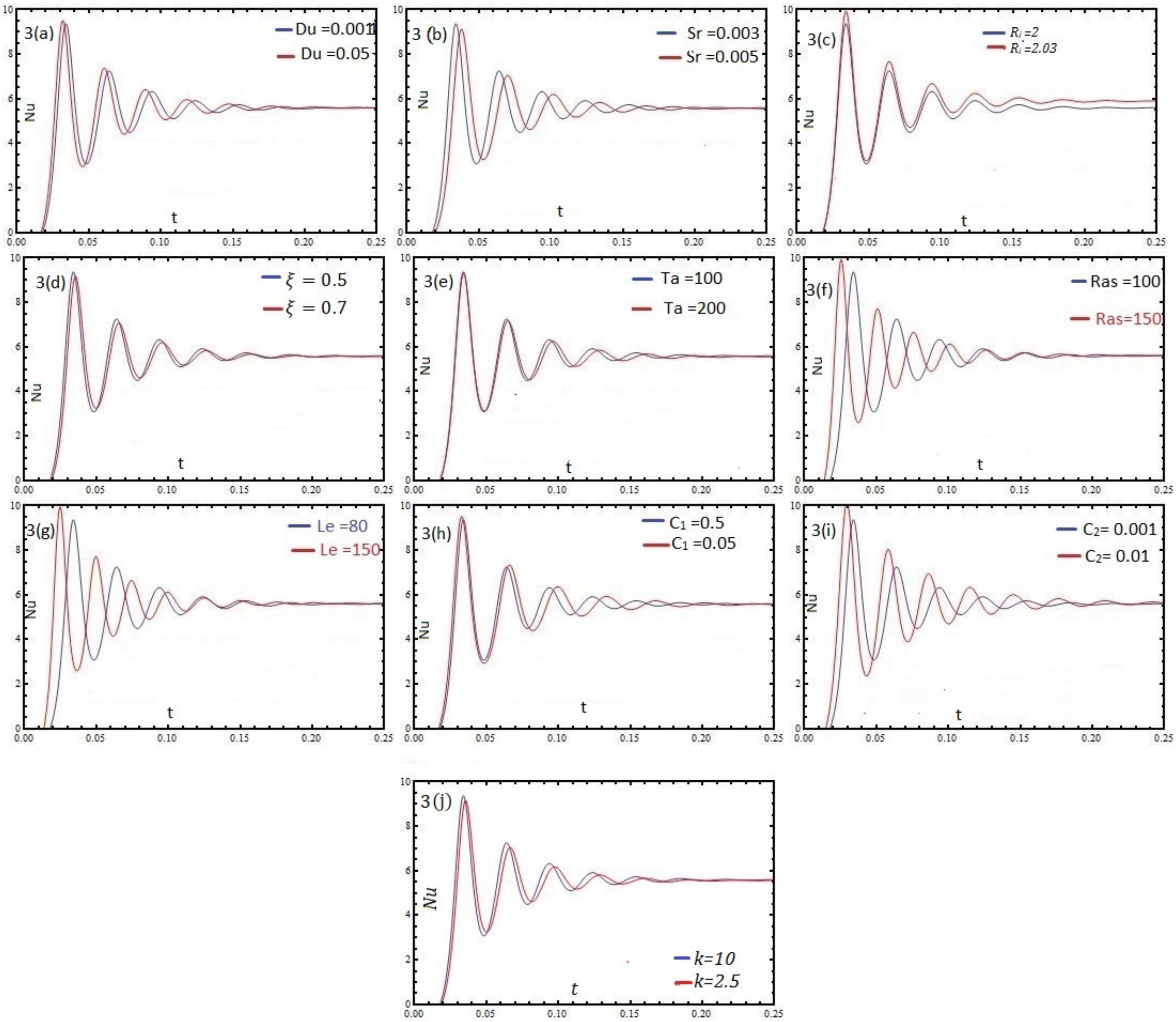


Fig. 3: Variability of Nu with t for different values of Du, Sr, R_i, ξ, Ta, Ra_S, Le, C_1, C_2,and k respectively.

7 conclusion

For both steady and unsteady scenarios, the couple stress fluid saturated rotating porous medium in the presence of an internal heat source with chemical reaction has been addressed to linear and non-linear analysis. Couple stress parameters, Dufour parameters Du, Soret parameters Sr, and reaction term k have been discovered to have a stabilising impact, as higher values of these factors result in reduced heat and mass transmission. Convection is destabilised by thermal and mechanical anisotropic parameters. In steady-state convection, the Lewis number stabilises the system, while in oscillatory convection, it destabilises it. In addition, η, ξ, Le, Ras, R_i increase heat transport and so destabilise the system. The convection is slowed by rotation, as well. Le, Ras, and ξ all have proportionate effects on mass transfer, but C and R_i have inverse effects.

Conflict of interest

The authors declare that there is no conflict regarding the publication of this paper.

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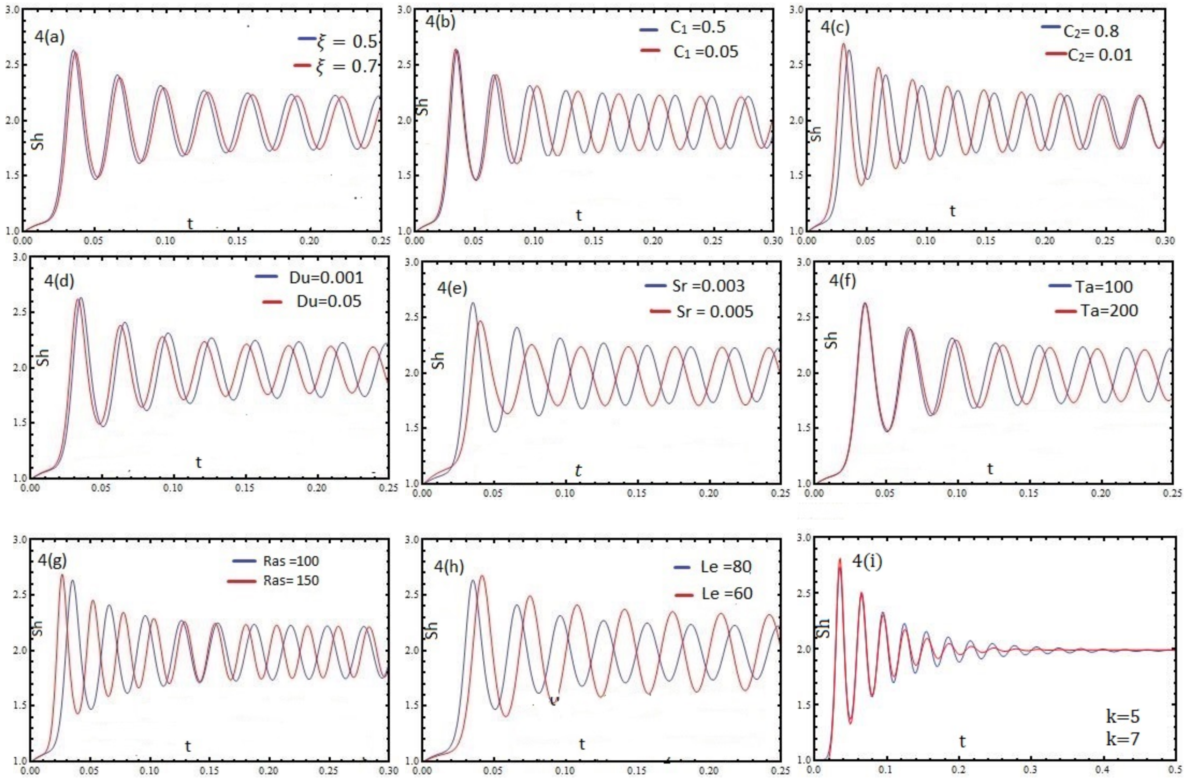

Fig. 4: Variability of Sh with t for different values of ξ, C_1, C_2, Du, Sr, Ta, Ra_S, Le and k respectively.

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Appendix

Nomenclature

Latin Symbols	
c	Specific heat capacity
C	Concentration
d	Length of the porous layer
C_1	Couple stress viscosity parameter $\left(= \frac{\mu_c}{\mu d^2}\right)$
C_2	Couple stress parameter $\left(= \frac{\mu_c}{\mu d^4}\right)$
D_m	Solute diffusivity
D_T	Soret coefficient
Du	DuFour parameter, $Du = \frac{D_f \beta_T}{\varepsilon \kappa_{fz} \beta_C}$
Da	Darcy number $\left(= \frac{K_z}{d^2}\right)$
Pr	Prandtal number $\left(= \frac{\nu \varepsilon}{K_f}\right)$
Ta	Taylor number $\left(= \frac{2\Omega d^2 K_z}{\nu \varepsilon}\right)^2$
K_{Tx}	Effective thermal diffusivity in $x-$ direction
K_{Ty}	Effective thermal diffusivity in $y-$ direction
K_T	Thermal diffusivity $\left(= K_{Tz}(\hat{k}\hat{k}) + K_{Tx}(\hat{i}\hat{i} + \hat{j}\hat{j})\right)$
K	Permeability of the porous layer
K_x, K_y, K_z	Characteristic permeabilities in the x, y, and z directions
Le	Lewis number, $Le = \frac{\kappa_{fz}}{D_m}$
p	Reduced pressure

Ra_T	Darcy-Rayleigh number, $Ra_T = \frac{\rho_f g \beta_T \Delta T K_z d}{\varepsilon \mu \kappa_{fz}}$
Ra_S	Solutal Rayleigh number, $Ra_S = \frac{\rho_f g \beta_C \Delta C K_z d}{\varepsilon \mu \kappa_{fz}}$
k	Reactive term
a	Horizontal wavenumber
Sr	Soret parameter, $Sr = \frac{D_t \beta_C}{D_m \beta_T}$
T	Temperature
T_f	Temperature of fluid
T_s	Temperature of solid
Q	Internal heat source coefficient
R_i	Internal heat source parameter $\left(= \frac{Qd^2}{K_{Tz}}\right)$
ΔT	Temperature difference across the porous layer
T_l	Temperature of lower surface
T_u	Temperature of upper surface
t	Time
$\vec{q}$	Velocity
x, y, z	Space Co-ordinates
Greek symbols	
κ	Thermal diffusivity
κ_f	Thermal diffusivity for fluid
κ_s	Thermal diffusivity for solid
β_C	Coefficient of concentration expansion
β_T	Coefficient of thermal expansion
ε	Porosity
μ	Dynamic viscosity of the fluid
ν	Kinematic viscosity
ρ	Density of fluid
ρ_0	Reference density
χ	Diffusivity ratio, $\chi = \frac{(\rho_s c_s)\kappa_{fz}}{(\rho_f c_f)\kappa_{sz}}$
η	Thermal anisotropic parameter $\left(= \frac{K_{Tx}}{K_{Tz}}\right)$
Ψ	Stream function
V	Velocity inducted by rotation along y-component
Other symbols	
D	d/dz
δ^2	$a^2 + \pi^2$
δ_1^2	$a^2 + \frac{\pi^2}{\xi}$
δ_2^2	$\eta a^2 + \pi^2$
∇_1^2	$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$
Subscripts	
b	basic state
c	Critical
f	Fluid
s	Solid
superscripts	
$'$	Perturbed quantity
$*$	Dimensionless quantity