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Stability Analysis of Traveling Wave Solutions for Generalized Coupled Nonlinear KdV Equations

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Abstract: In the present paper, an extended algebraic method is used for constructing exact traveling wave solutions for generalized coupled nonlinear KdV equations. By implementing the extended direct algebraic method, new exact solutions of the generalized coupled KdV equations are obtained. The present results are describing the generation and evolution of such waves, their interactions, and their stability. Moreover, the method can be applied to a wide class of coupled nonlinear evolution equations. The present traveling wave solutions have applications in physics.

Keywords: Extended direct algebraic method, Traveling wave solutions, Generalized coupled KdV equations

1 Introduction

The solitary and traveling wave solutions of coupled nonlinear partial differential equations which occur in many branches of physics have been subject of intense study as well in recent years. The coupled integrable systems, which come up in many mathematical and physical fields, have been studied extensively, and a lot of interesting results have been given from both the classification view and application fields [1-3]. Many coupled systems have been proposed since the soliton theory came into being in last century. Because of the rich structures of the soliton systems, both mathematicians and physicists have been paying more attention to them [4-6].

Since the first coupled KdV system was proposed by Hirota and Satsuma [7-8], it has been studied amply [9-10]. Some important coupled KdV models have been advanced, for example, the Fuchssteiner equation [10], the Itos system [11], the Drinfeld and Sokolov model [12], the Benjamin-Feir model [13], the Zharkov system [14], the Foursov model [15]. So far, some kinds of general coupled KdV equations have been applied in some fields such as in shallow stratified liquid [16], in fluid dynamical system [17-19], and in two-component Bose-Einstein condensates [20]. More recently, exact solutions of a coupled KdV system have been found with a formal variable separation approach [21]; some types of coupled KdV equations have been derived from a two-layer fluid system [22-28]; a new type of coupled KdV equation was found to be Painlevé-integrable; nonsingular solutions for a special coupled KdV system were discovered by means of the iterative Darboux transformations [29].

This paper is organized as follows: In Section one, An introduction is given. In section 2, the formulation of stability analysis solutions is presented. In section 3, we implemented the extended direct algebraic method method for finding the travelling wave solutions of the generalized KdV equations.

2 Stability of solutions

Hamiltonian system for which the momentum is given by

$$M=\frac{1}{2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}U_{ij}^{2}(t,x)\,dt\,dx,\qquad i=1,2\quad j=1,2,3,\quad (1)$$

where $U_1 = u(x,t)$ and $U_2 = v(x,t)$. The sufficient condition for discuss the stability of solution is

$$\frac{\partial M}{\partial \omega}>0,$$

where ω is the frequency.

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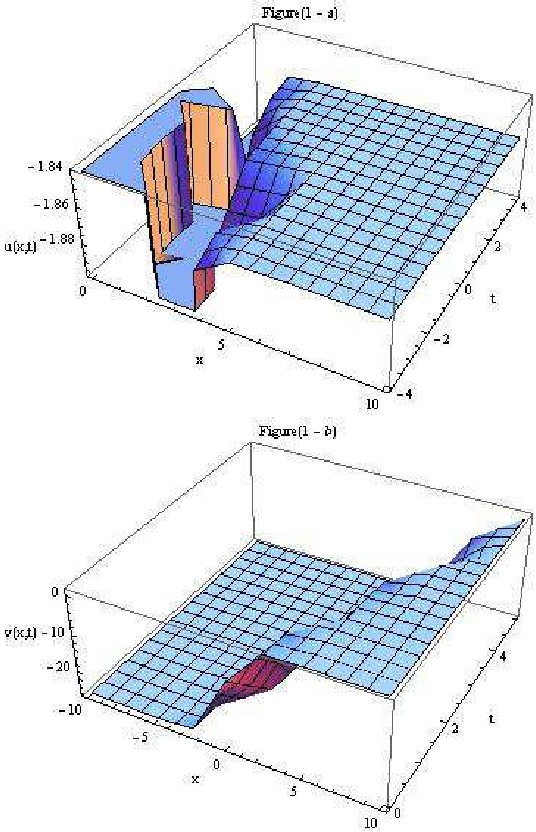

Fig. 1: (a-b). The solitary wave solutions for the quantity $u_1(x,t)$ and $v_1(x,t)$ by equations (9-10) for the generalized coupled system KdV equations (2).

3 Coupled system of KdV equations

Gear and Grimshaw derived a system of coupled KdV equations to model interactions of long waves, for example in a stratified fluid. Specifically, their model is [3, 16, 25]

$$u_t + \left(u_{xx} + u^2 + \frac{\varepsilon}{2}v^2 + \varepsilon uv\right)_x = 0\,,$$

$$v_t + \left(v_{xx} + v^2 + \frac{\varepsilon}{2}u^2 + \varepsilon uv\right)_x = 0\,, \qquad (2)$$

where ε is constant. Consider the traveling wave solutions

$$u(x,t) = \sum_{i=0}^{m} a_i \varphi^i(\xi), \qquad v(x,t) = \sum_{j=0}^{n} b_j \varphi^j(\xi), \qquad (3)$$

and

$$\left(\frac{d\varphi}{d\xi}\right)^2 = \alpha^2\varphi^2 + \beta^2\varphi^3 + \lambda^2\varphi^4, \qquad (4)$$

where a_i, b_j, α ,β, λ, k and ω are arbitrary constants. Then equation (2) becomes

$$\omega u' + 2kuu' + k\varepsilon vv' + k\varepsilon uv' + k\varepsilon vu' + k^3u^{(3)} = 0\,,$$

$$\omega v' + 2kvv' + k\varepsilon uu' + k\varepsilon uv' + k\varepsilon vu' + k^3v^{(3)} = 0\,. \qquad (5)$$

Balancing the highest nonlinear terms and the highest order derivative terms in equation (5), we find $m_1 = 2$, $m_2 = 2$. Suppose the solution of equations (5) are in the form

$$u(\xi) = a_0 + a_1\varphi + a_2\varphi^2, \qquad v(\xi) = b_0 + b_1\varphi + b_2\varphi^2, \quad (6)$$

Substituting (6) into equation (5) yields a set of algebraic equations for a_0, a_1, a_2, ε, β, λ, α, b_0, b_1, b_2, k, ω. The solution of these system of equations can be found as:

The first set:

$$\varepsilon = -1\,, \quad a_0 = \frac{(-3+\sqrt{21})(k^3\beta^4 + 4\lambda^2\omega)}{72k\lambda^2}\,,$$

$$a_1 = \frac{(-3+\sqrt{21})(k^2\beta^2)}{3}\,, \quad a_2 = \frac{2(-3+\sqrt{21})(k^2\lambda^2)}{3},$$

$$b_0 = \frac{(-3-\sqrt{21})(k^3\beta^4+4\lambda^2\omega)}{72k\lambda^2}\,, \quad b_1 = \frac{(-3-\sqrt{21})(k^2\beta^2)}{3}\,,$$

$$b_2 = \frac{2(-3-\sqrt{21})(k^2\lambda^2)}{3}, \quad \alpha = -\frac{\beta^2}{2\lambda} \qquad (7)$$

The second set:

$$\varepsilon = -2, \quad a_0 = \frac{(-1+\sqrt{3})(k^3\alpha^2 + \omega)}{8k},$$

$$a_1 = -\frac{3}{2}(-1+\sqrt{3})\alpha\lambda k^2, \quad a_2 = -\frac{3}{2}(-1+\sqrt{3})k^2\lambda^2,$$

$$b_0 = -\frac{(1+\sqrt{3})(k^3\alpha^2+\omega)}{8k}, \quad b_1 = \frac{3}{2}(1+\sqrt{3})\alpha\lambda k^2,$$

$$b_2 = -\frac{3}{2}(1+\sqrt{3})k^2\lambda^2, \quad \beta = \sqrt{-2\alpha\lambda}, \qquad (8)$$

where $\alpha\lambda < 0$. Substituting equations (7-8) into (6), the following solutions of equation (2) can be obtained as:
Family I

$$u_1(x,t) = \frac{(-3+\sqrt{21})}{72k\beta^4\lambda^2}((k^3\beta^8 + 48k^3\alpha^4\lambda^4)$$

$$\left(1+\delta\tanh(\frac{\alpha}{2}(kx+\omega t))\right)^2 + .$$

$$4\beta^4\lambda^2(-6k^3\alpha^2 + \omega - 6k^3\alpha^2\delta\tanh(\frac{\alpha}{2}(kx+\omega t)))\Big)\,, \qquad (9)$$

$$v_1(x,t) = \frac{(3+\sqrt{21})}{72k\beta^4\lambda^2}((-k^3\beta^8 - 48k^3\alpha^4\lambda^4)$$

$$\left(1+\delta\tanh(\frac{\alpha}{2}(kx+\omega t))\right)^2 - .$$

$$4\beta^4\lambda^2(6k^3\alpha^2+\omega+6k^3\alpha^2\delta\tanh(\frac{\alpha}{2}(kx+\omega t)))\Big)\ , \tag{10}$$

$$u_2(x,t)=\frac{(-3+\sqrt{21})}{72k\lambda^2}\left(\frac{\delta\sinh(\alpha(kx+\omega t)))}{\mu+\cosh(\alpha(kx+\omega t))}-\right.$$
$$\frac{12k^3\alpha\beta^2\lambda(\mu+\cosh(\alpha(kx+\omega t))}{\mu+\cosh(\alpha(kx+\omega t))}+k^3\beta^4+4\lambda^2\omega+$$
$$\left.\frac{12k^3\alpha^2\lambda^2\,(\mu+\cosh(\alpha(kx+\omega t))+\delta\sinh(\alpha(kx+\omega t)))^2}{(\mu+\cosh(\alpha(kx+\omega t)))^2}\right)\ , \tag{11}$$

$$v_2(x,t)=\frac{(3+\sqrt{21})}{72k\lambda^2}\left(\frac{\delta\sinh(\alpha(kx+\omega t)))}{\mu+\cosh(\alpha(kx+\omega t))}-\right.$$
$$\frac{12k^3\alpha\beta^2\lambda(\mu+\cosh(\alpha(kx+\omega t))}{\mu+\cosh(\alpha(kx+\omega t))}-k^3\beta^4-4\lambda^2\omega$$
$$\left.-\frac{12k^3\alpha^2\lambda^2\,(\mu+\cosh(\alpha(kx+\omega t))+\delta\sinh(\alpha(kx+\omega t)))^2}{(\mu+\cosh(\alpha(kx+\omega t)))^2}\right)\ , \tag{12}$$

$$u_3(x,t)=\frac{-3+\sqrt{21}}{72k}\left(\frac{k^3\beta^4}{\lambda^2}+4\omega\right.$$
$$+\frac{24k^3\alpha^2(p+\delta\mu\sqrt{p^2+1}+\delta\cosh(\alpha(kx+\omega t))}{p+\sinh(\alpha(kx+\omega t))}$$
$$+\frac{\sinh(\alpha(kx+\omega t)))}{p+\sinh(\alpha(kx+\omega t))}$$
$$+\frac{48k^3\alpha^4\lambda^2\left(p+\delta\mu\sqrt{p^2+1}+\delta\cosh(\alpha(kx+\omega t))\right.}{\beta^4\,(p+\sinh(\alpha(kx+\omega t)))^2}$$
$$\left.+\frac{\sinh(\alpha(kx+\omega t)))^2}{\beta^4\,(p+\sinh(\alpha(kx+\omega t)))^2}\right) \tag{13}$$

$$v_3(x,t)=\frac{3+\sqrt{21}}{72k}\left(-\frac{k^3\beta^4}{\lambda^2}-4\omega\right.$$
$$-\frac{24k^3\alpha^2(p+\delta\mu\sqrt{p^2+1}+\delta\cosh(\alpha(kx+\omega t))}{p+\sinh(\alpha(kx+\omega t))}$$
$$+\frac{\sinh(\alpha(kx+\omega t)))}{p+\sinh(\alpha(kx+\omega t))}$$
$$-\frac{48k^3\alpha^4\lambda^2\left(p+\delta\mu\sqrt{p^2+1}+\delta\cosh(\alpha(kx+\omega t))\right)}{\beta^4\,(p+\sinh(\alpha(kx+\omega t)))^2}$$
$$\left.-\frac{\sinh(\alpha(kx+\omega t)))^2}{\beta^4\,(p+\sinh(\alpha(kx+\omega t)))^2}\right) \tag{14}$$

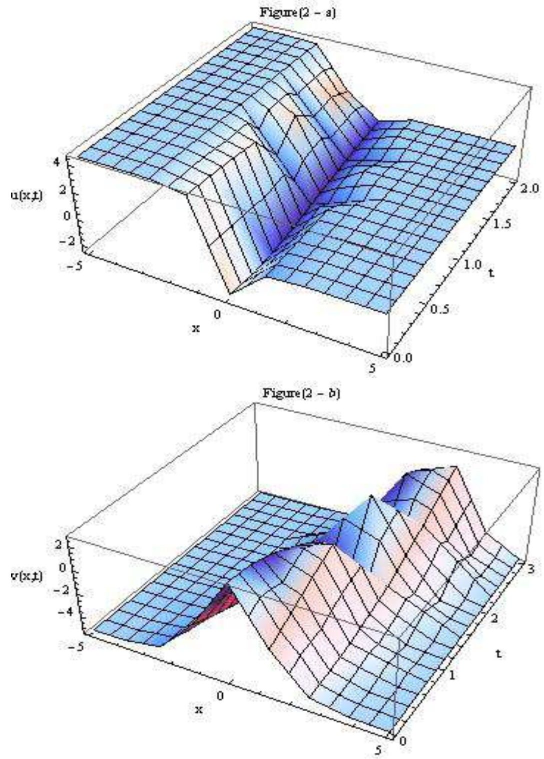

Fig. 2: (a-b). The solitary wave solutions for the quantity $u_2(x,t)$ and $v_2(x,t)$ by equations (11-12) for the generalized coupled system KdV equations (2).

Family II

$$u_1(x,t)=\frac{(-1+\sqrt{3})}{8k}\left(k^3\alpha^2+\omega-\right.$$
$$\frac{12k^3\alpha^3\lambda}{\beta^2}\left(1+\delta\coth(\frac{\alpha}{2}(kx+\omega t))\right).$$
$$\left.+\frac{12k^3\alpha^4}{\beta^4}\left(\lambda+\lambda\delta\coth(\frac{\alpha}{2}(kx+\omega t))\right)^2\right)\ , \tag{15}$$

$$v_1(x,t)=\frac{(1+\sqrt{3})}{8k}\left(-k^3\alpha^2-\omega+\right.$$
$$\frac{12k^3\alpha^3\lambda}{\beta^2}\left(1+\delta\coth(\frac{\alpha}{2}(kx+\omega t))\right).$$
$$\left.-\frac{12k^3\alpha^4}{\beta^4}\left(\lambda+\lambda\delta\coth(\frac{\alpha}{2}(kx+\omega t))\right)^2\right)\ , \tag{16}$$

$$u_2(x,t)=\frac{(-1+\sqrt{3})}{8k(\mu+\cosh(\alpha(kx+\omega t)))^2}.$$
$$\left(3k^3\alpha^2\delta^2\sinh(\alpha(kx+\omega t)^2)\right.$$
$$\left.+(-2k^3\alpha^2+\omega)(\mu+\cosh(\alpha(kx+\omega t)))^2\right)\ , \tag{17}$$

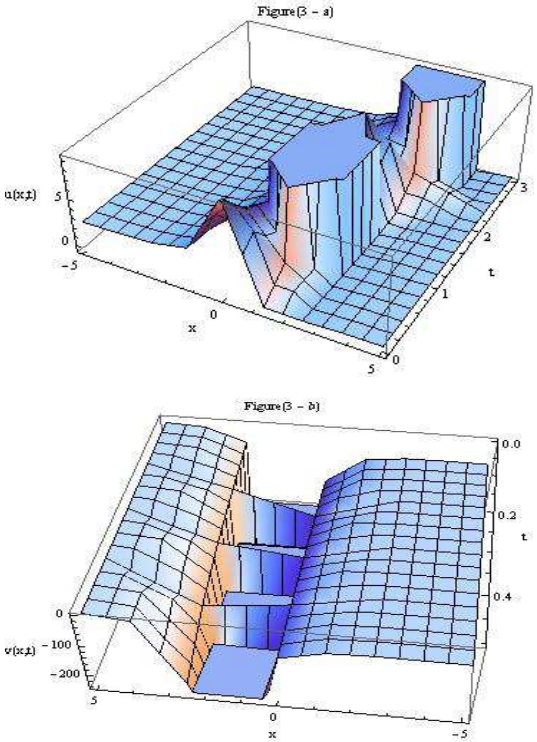

Fig. 3: (a-b). The solitary wave solutions for the quantity $u_3(x,t)$ and $v_3(x,t)$ by equations (13-14) for the generalized coupled system KdV equations (2).

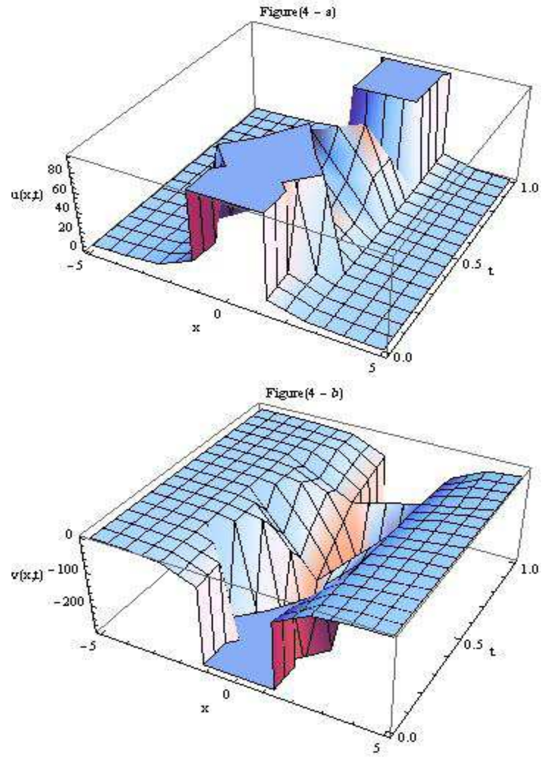

Fig. 4: (a-b). The solitary wave solutions for the quantity $u_1(x,t)$ and $v_1(x,t)$ by equations (15-16) for the generalized coupled system KdV equations (2).

$$v_2(x,t)=\frac{(1+\sqrt{3})}{8k(\mu+\cosh(\alpha(kx+\omega t)))^2}\cdot\left(-3k^3\alpha^2\delta^2\sinh(\alpha(kx+\omega t)^2)+(2k^3\alpha^2-\omega)(\mu+\cosh(\alpha(kx+\omega t)))^2\right), \quad (18)$$

$$u_3(x,t)=\frac{-1+\sqrt{3}}{8k\beta^4}\Bigg(k^3\beta^4\lambda^2+\omega\beta^4-12k^3\beta^2\lambda^3\lambda\left(1+\delta\frac{p+\sinh(\alpha(kx+\omega t))}{\mu\sqrt{p^2+1}+\cosh(\alpha(kx+\omega t))}\right).+12k^3\lambda^2\lambda^4\left(1+\delta\frac{p+\sinh(\alpha(kx+\omega t))}{\mu\sqrt{p^2+1}+\cosh(\alpha(kx+\omega t))}\right)^2\Bigg) \quad (19)$$

$$v_3(x,t)=\frac{1+\sqrt{3}}{8k\beta^4}\Bigg(-k^3\beta^4\lambda^2-\omega\beta^4+12k^3\beta^2\lambda^3\lambda\left(1+\delta\frac{p+\sinh(\alpha(kx+\omega t))}{\mu\sqrt{p^2+1}+\cosh(\alpha(kx+\omega t))}\right).-12k^3\lambda^2\lambda^4\left(1+\delta\frac{p+\sinh(\alpha(kx+\omega t))}{\mu\sqrt{p^2+1}+\cosh(\alpha(kx+\omega t))}\right)^2\Bigg) \quad (20)$$

4 Results and discussion

Figures (1a-1b), represent the evolution of the bright and dark solitary wave solutions (9-10) of the generalized coupled system KdV equations (2), with $k=2$, $\omega=0.8$, $\alpha=1.4$, $\beta=1.5$, $\lambda=0.6$, $\delta=1.5$ and $k=1.2$, $\omega=-1.8$, $\alpha=1.3$, $\beta=1.5$, $\lambda=0.6$, $\delta=-1.5$. The bright solitary wave solution is stable with respect to equation (1) in the interval $[0,10]$ and $[-4,4]$, the dark solitary wave solution is stable in the interval $[-10,10]$ and $[0,5]$.

The evolution of the Dark-in-the-Bright solitary wave solutions (11-12) are represented in figures (2a-2b) of the generalized coupled system KdV equations (2), with $k=2$, $\omega=0.8$, $\alpha=1.4$, $\beta=1.5$, $\lambda=0.6$, $\delta=1.5$, $\mu=0.5$ and $k=1.2$, $\omega=-0.8$, $\alpha=-1.4$, $\beta=1.5$, $\lambda=0.8$, $\delta=1.5$, $\mu=1.3$. The Dark-in-the-Bright solitary wave solution is stable with respect to equation (1) in the interval $[-5,5]$ and $[0,2]$, the Dark-in-the-Bright solitary wave solution is stable in the interval $[-5,5]$ and $[0,3]$.

Figures (3a-3b). Evolution of the Dark-in-the-Bright solitary wave solutions (13-14) of the generalized coupled system KdV equations (2), with $k=1.2$, $\omega=-0.8$, $\alpha=-0.9$, $\beta=1.5$, $\lambda=0.8$, $\delta=1.5$, $\mu=1.5$, $p=0.7$ and $k=-0.5$, $\omega=0.6$, $\alpha=$

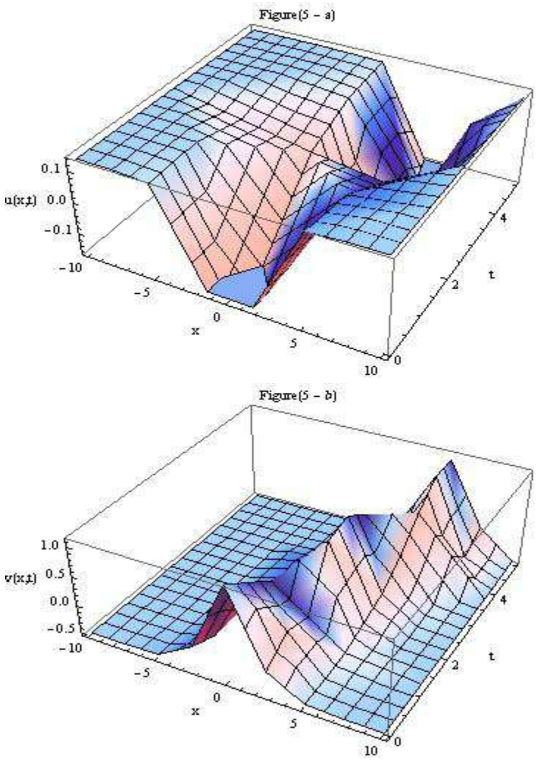

Fig. 5: (a-b). The solitary wave solutions for the quantity $u_2(x,t)$ and $v_2(x,t)$ by equations (17-18) for the generalized coupled system KdV equations (2).

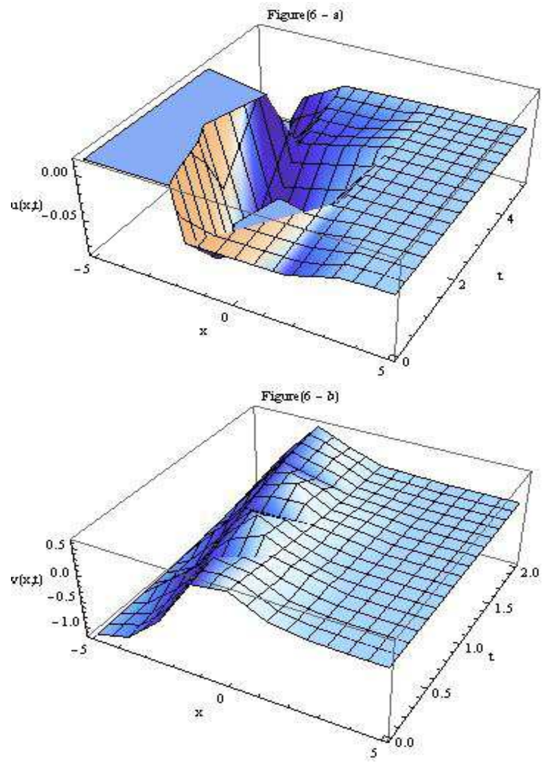

Fig. 6: (a-b). The solitary wave solutions for the quantity $u_3(x,t)$ and $v_3(x,t)$ by equations (19-20) for the generalized coupled system KdV equations (2).

1.2 , $\beta = -0.5$, $\lambda = 0.5$, $\delta = 1.5$, $\mu = 0.4$, $p = 0.8$ are stable in the interval $[-5,5]$ and $[0,3]$; $[-5,5]$ and $[0,5]$.

Figures (4a-4b), represent the evolution of the Dark-in-the-Bright solitary wave solutions (15-16) of the generalized coupled system KdV equations (2), with $k = 0.5$, $\omega = -0.6$, $\alpha = -0.8$, $\beta = 0.5$, $\lambda = 0.5$, $\delta = 1.5$ and $k = 0.5$, $\omega = -0.6$, $\alpha = 0.8$, $\beta = -0.5$, $\lambda = 0.5$, $\delta = 1.5$. The Dark-in-the-Bright solitary wave solution are stable with the condition (1) in the interval $[-5,5]$ and $[0,1]$.

The evolution of the Dark-in-the-Bright solitary wave solutions (17-18) are represented in figures (5a-5b) of the generalized coupled system KdV equations (2), with $k = -1.5$, $\omega = 1.6$, $\alpha = -0.7$, $\beta = 1.4$, $\lambda = 0.8$, $\delta = 1.2$, $\mu = 0.9$. The Dark-in-the-Bright solitary wave solutions are stable in the interval $[-10,10]$ and $[0,5]$.

Figures (6a-6b). Evolution of the Dark-in-the-Bright solitary wave solutions (19-20) of the generalized coupled system KdV equations (2), with $k = 1.3$, $\omega = 1.2$, $\alpha = 0.9$, $\beta = -1.4$, $\lambda = 0.8$, $\delta = 1.2$, $\mu = 0.9$, $p = 1$. are stable in the interval $[-5,5]$ and $[0,5]$; $[-5,5]$ and $[0,2]$.

5 Conclusion

An analytic study was conducted on coupled partial differential equations. We formally derived one soliton solutions for generalized coupled system of KdV equations. However, using another distinct approach, we derived one traveling wave solutions for each generalized coupled system of KdV equations. The structures of the obtained solutions are distinct and stable.

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