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Determination of Optimal Lot Size and Sampling Plan for Single-Producer Single-Retailer Supply Chain System

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Abstract: A supply chain of one producer and one retailer is considered in this paper. The production process produces items in lots with random fraction defective to satisfy the retailer with a deterministic demand. When a lot is completed, a random sample is drawn to sentence the produced lot. If the number of defective items in the sample is less than or equal to the specified acceptance number, the lot is accepted and sold at a regular price. On the other hand, if the lot is rejected, it is sold at a reduced price. The retailer is protected by a limit on the incoming quality. That is, the retailer dictates a certain Acceptable Quality Level, which must be respected by the producer. The objective of the model is to determine the optimal production run, sample size, and acceptance number simultaneously, to maximize the expected total profit made by the producer. The decision variables in this model are the production lot size, the sample size, and the acceptance number.

Keywords: Supply chain, lot size, quality control, acceptance sampling and sample size.

1 Introduction

The single-producer single-retailer supply chain has received a considerable focus in recent years because it is the building block for the wider supply chain. A supply chain can be very complex and link-by-link understanding can be very useful (Ben-Daya et al., 2008; Qin et al. 2013). In the single-producer single-retailer model, the producer manufactures an item in lots and delivers the produced lot to a retailer. Traditionally, the production process in single-producer single-retailer model is assumed to be perfect, that is all produced items are of acceptable quality. However, this assumption is not realistic in most situations. In this paper, the production process under consideration produces a lot with a random fraction of nonconforming items. In order to sentence the produced lot, a sample is drawn; and based on the sample, the lot is classified into one of two states: A high quality lot or a sub-standard lot. The state of the lot determines its sales price. The objective is to determine the optimal lot size and sampling plan parameters in order to maximize the profit achieved by the supplier. In the literature, there are many models that consider imperfect production processes. For example, Salameh and Jaber (2000) extended the classical EPQ/EOQ model by considering imperfect lots. They assume that low-quality items are sold as a single batch by the end of the screening process. The fraction of nonconforming items is assumed to be constant in this model. Yassine et al. (2012) considered EPQ model with a random fraction of produced inventory. They considered a 100% inspection policy to detect imperfect items at the end of the production cycle and derived optimal production quantity assuming that the imperfect items are scrapped at the end of production cycle. Then, they extended this base model to allow for consolidating the imperfect items during a single production run and over multiple production runs. In above models, sampling plan parameters are not included, instead, the authors considered 100% inspection. However, there are many models that integrate the sampling plan parameters with production related issues. For example, Boucher and Jafari (1991) developed a model that determines the optimal target value of the process when sampling plan parameters are imposed on the process-targeting problem. In other words, the plan is given and the process parameters are optimized using the plan as an input. Darwish and Duffuaa (2010) developed a mathematical model for determining the optimal process mean for a production process and the inspection

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plan parameters; the sample size, and the acceptance number. In this model, products are produced in lots and a single sampling plan is used to accept or reject the lots. However, they considered constant fraction of non-conforming items in the lot. Sheu et al. 2014 proposed a model with a variable sampling plan based on incapability index to accept or reject the lot. Their model distinguished among the products that are within the specification limits. Moreover, Chiu (2003) considered the effects of reworking of defective items on the economic production quantity where backorders are allowed. In this model, the author considered the defective rate as a random variable that follows a beta distribution. When the production run is complete, the reworking of defective items starts at a certain rate. However, sampling plan is not considered in this model. Another model is developed by Cheung and Leung (2000) who used (Q, S) policy to manage a two-item inventory control system. The orders for each item varies from lot to lot and accordingly the sampling plan will also be changed. Ben-Daya et al. (2006) considered a buyer who is subjected to deterministic demand and orders from a supplier. When a lot is received, the buyer uses some type of inspection policy, such as no inspection, sampling inspection, or 100% inspection. The fraction nonconforming items in the lot is assumed to be a random variable following a beta distribution. However, the sampling inspection is determined by imposing consumer and producer risk. Also, Ben-Daya and Noman (2008) extended Ben-Daya et al. (2006) by considering stochastic demand. Finally, Wu and Ouyang (2000) derived a stochastic inventory model with backorders and lost sales when the lot contains a fraction of nonconforming items that follows a beta distribution. In this model the order quantity, reorder point and lead time are decision variables. The purpose of this paper is to develop a model for a single-producer single-retailer supply chain where a producer manufactures a product in lots. It is assumed that the process is imperfect in a sense that it produces a random fraction of defective items which follows a beta probability distribution. The producer is responsible for replenishing the retailer?s inventory who observes a deterministic demand. When the producer completes the lot, a sample is drawn from the lot and based on the contents of the sample, the lot is classified as high-quality or sub-standard lot. The high-quality lot is sold at a premium price while the sub-standard lot is sold at a reduced price. Since the model optimizes the producer?s profit, the customer is protected by Acceptable Quality Level (AQL). The objective of the model is to determine the optimal production run, sample size, and acceptance number which maximize the expected total profit of the producer while the customer?s AQL is respected.

2 Problem Statement

Consider a producer who produces an item in lots of size Q. At the beginning of the production cycle, the process is setup at a cost of A. It is assumed that the process is imperfect, that is, it produces a random fraction of non-conforming items (Y) which follows a beta probability distribution with parameters a and b as follows:

$$f_Y(y) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} y^{a-1}(1-y)^{b-1},\ y \in [0,1] \tag{1}$$

Where Γ is gamma function. Beta probability distribution is used in this paper to model the proportion of defective items in the lot because it is defined over real numbers between 0 and 1. Moreover, it contains two parameters which make it easy to control the shape of the distribution and consequently easy to fit real data. This makes it a better alternative than the uniform distribution that is defined between 0 and 1. The producer who can produce the item at a rate P is responsible for replenishing the retailer's inventory who is subjected to deterministic demand D. When a lot is completed, a sample of size n is inspected. On the basis of the information in this sample, a decision is made regarding lot disposition. If the number of non-conforming items in the sample, X, is less than or equal to acceptance number (d), then the lot is considered of high-quality and sold at a price v1. On the other hand, if X is greater than d, the lot is considered to be sub-standard and sold at a reduced price v_2 where $v_1 > v_2$. Moreover, the sample size is typically small in comparison to the lot size, thus, the probability of accepting a lot if the fraction of nonconforming items in the lot is y_0 is given by

$$P(X \le d | Y = y_0) = \sum_{x=0}^{d} \binom{n}{x} y_0^x (1-y_0)^{n-x} \tag{2}$$

Moreover, the retailer is protected by acceptable quality level, AQL, which is the poorest level of quality that is accepted by the retailer. It is important to indicate that n, d, and AQL are related as follows:

$$d \le \lfloor (AQL)n \rfloor \tag{3}$$

For example, if the retailer's AQL is 10% and the sample size chosen by the producer is 15 items, the acceptance number should be 1 or less to achieve the retailer's AQL. The notation used in developing the model is as follows:

P: Production rate

D: Demand rate,

T: A random variable represents inventory cycle length

ET: Expected value of inventory cycle length

v_1: Selling price per unit in the accepted lot,

v_2: Selling price per item in the sub-standard quality lot,

X: Number of non-conforming items in a sample,
Y: A random variable represents the fraction of nonconforming items in a lot
n: Sample size
P_a: Probability of accepting a lot,
C_i: Inspection cost per inspected item
AQL: Acceptable quality level.

3 Model Development

In this section, sample plan parameters and production decisions are integrated in one model with the objective of maximizing the producer?s expected total profit which is the difference between total revenue and total cost. Revenue is generated from selling a high-quality lot at a price v_1 and a sub-standard quality lot at a price v_2. Thus, the revenue generated from selling a lot (R) is a random variable and is given by:

$$R = \begin{cases} v_1 Q & X \leq d \\ v_2 Q & X > d \end{cases} \quad (4)$$

On the other hand, the costs that are included in the model are screening cost, setup cost and holding cost, and they are as follows:

1. Screening Cost: The producer incurs a variable cost C_i for each screened item. Therefore, the screening cost per cycle is nC_i.
2. Setup Cost: At the start of each production run, the producer incurs a setup cost A.
3. Holding Cost: There are two distinct periods in a production run. The first of which is the buildup period when the production starts until the whole lot is produced. The other period is consumption period whereas the producer begins satisfying the demand until the produced lot is consumed. This policy is known in the literature as lot-for-lot policy. During consumption period, the retailer may find some nonconforming items, which will be scrapped with no salvage value. Thus, the reduction in inventory is in fact more than the demand rate. Because the number of nonconforming items in a lot is very small compared to the lot size, we assume that the inventory at the end of the buildup period is decreased by Qy as shown in Figure 1. Hence, the inventory holding cost in one inventory cycle (HCC) is given by:

$$HC_C = h\left[\frac{1}{2}\frac{Q^2}{P} + \frac{1}{2}\frac{Q^2(1-Y)^2}{D}\right] \quad (5)$$

Thus, the total inventory cost per cycle, including the set up and inspection costs, is as follows:

$$TC_C = A + nC_i + \frac{hQ^2}{2}\left[\frac{1}{P} + \frac{(1-Y)^2}{D}\right] \quad (6)$$

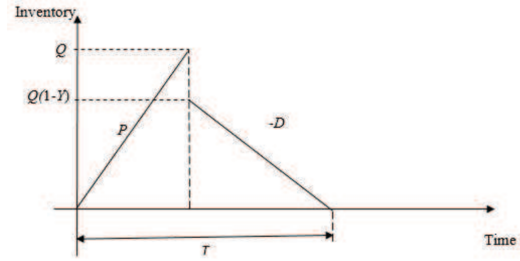

Fig. 1: Evolution of inventory over time.

We have to point out that TCC is a random variable and using the expectation operator on TCC in Equation (6) yields the expected total cost ETCC as follows.

$$ETC_C = A + nC_i + \frac{hQ^2}{2}\left[\frac{1}{P} + \frac{E((1-Y)^2)}{D}\right] \quad (7)$$

Proposition 1: Let Y be a random variable, representing the fraction of nonconforming items in a lot, which follows beta distribution with parameters a and b, then,

$$E((1-Y)^2) = \frac{b^2+b}{(a+b)^2+(a+b)}$$

The proof of the proposition is given in Appendix A. Therefore, expected total inventory cost in equation (7) becomes:

$$ETC_C = A + nC_i + \frac{hQ^2}{2D}\left[\frac{D}{P} + \frac{b^2+b}{(a+b)^2+(a+b)}\right] \quad (8)$$

Now, one can find the expected total profit per cycle (EPRC) based on two possible profit (PRC) values. It will be denoted as PRc1 for the case of X ? d, and as PRc2 for the case of X > d. Thus, the expected profit can be obtained from the following equations by using the conditional probabilities depending on fraction defective (X) found in the sample.

$$EPR_C = (PR_{C1})\, P(X \leq d) + (PR_{C2})\, P(X > d) \quad (9)$$

However, the depends on the random variable y, which takes 0?y?1. Therefore, the expected value of PR_{C1} has to be determined by the following equation:

$$(PR_{C1}|Y = y) = v_1 Q(1-y) - TC_C \quad (10)$$

And its expected value is as follows:

$$E(PR_{C1}) = \int_0^1 (v_1 Q(1-y) - TC_C) r(y) dy$$
$$== v_1 Q(1 - E(Y)) - ETC_C \quad (11)$$

In the case of PR_{C2}, the profit does not depend on y since the whole lot is sold at a cheaper price because of the fraction defective X > d. Thus, $(PR_{C2}|Y=y)=v_2Q-TC_C$, and therefore,

$$E(PR_{C2})=v_2Q-ETC_C \tag{12}$$

From Equations (9), (10), and (12), we obtain

$$EPR_C=\left(v_1\frac{b}{a+b}-v_2\right)Q\left[P(X\le d)\right]+v_2Q-ETC_C \tag{13}$$

Hence, the expected profit per cycle is:

$$EPR_C=(v_1\frac{b}{a+b}-v_2)QP_a+v_2Q \\ -A-nc-\frac{hQ^2}{2D}\left[\frac{D}{P}+\frac{b^2+b}{(a+b)^2+(a+b)}\right] \tag{14}$$

It is worthwhile to indicate that this problem is a renewal reward stochastic process. Thus, the expected total profit per unit time (EPR) is simply the expected total profit per cycle (EPRC) divided by the expected cycle length (ET), that is

$$EPR=\frac{EPR_C}{ET} \tag{15}$$

From Figure 1, we can determine the inventory cycle length (T) as follows:

$$T=\frac{Q\left[D+(1-Y)P\right]}{DP} \tag{16}$$

Since $E(Y)=a/(a+b)$, expected cycle length (ET) will be given as follows:

$$ET=\frac{Q}{D}\left(\frac{D}{P}+\frac{b}{a+b}\right) \tag{17}$$

Using Equations (15), (16), and (17), we find the expected total profit per unit time as follows:

$$EPR=\frac{1}{\left(\frac{D}{P}+\frac{b}{a+b}\right)}\left[v_2D+(v_1\frac{b}{a+b}-v_2)DP(X\le d)- \right. \\ \left. \frac{D(A+nc)}{Q}-\frac{hQ}{2}\left[\frac{D}{P}+\frac{b^2+b}{(a+b)^2+(a+b)}\right]\right] \tag{18}$$

Proposition 2: Let X be the number of nonconforming items in a sample of size n with the acceptance number d, then the probability of accepting a lot is given by:

$$P(X\le d)=\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}\sum_{x=0}^{d}\binom{n}{x}\frac{\Gamma(x+a)\Gamma(n-x+b)}{\Gamma(n+a+b)}$$

The proof of proposition 2 can be found in Appendix B. The decision variables in the profit function are the lot size, sample size, and acceptance number. One important property of EPR function is its concavity, which is shown below by its second derivative.

$$\frac{\partial EPR}{\partial Q}=\frac{1}{(D/P+\frac{b}{a+b})} \\ \left[\frac{D(A+nc)}{Q^2}-\frac{h}{2}\left[\frac{D}{P}+\frac{b^2+b}{(a+b)^2+(a+b)}\right]\right] \tag{19}$$

$$\frac{\partial^2 EPR}{\partial Q^2}=\frac{1}{\left(\frac{D}{P}+\frac{b}{a+b}\right)}\left[\frac{2D(A+nc)}{Q^3}\right]>0 \tag{20}$$

Thus, EPR is concave in Q, consequently, the optimal lot size can be derived from Equation (19) as follows:

$$Q=\sqrt{\frac{2D(A+nc)}{h\left[\frac{D}{P}+\frac{b^2+b}{(a+b)^2+(a+b)}\right]}} \tag{21}$$

Using (18) and (21), EPR can be simplified (after eliminating Q) as follows:

$$EPR(n,d)=\frac{1}{\left(\frac{D}{P}+\frac{b}{a+b}\right)}\left[v_2D+(v_1\frac{b}{a+b}-v_2)DP(X\le d)- \right. \\ \left. \sqrt{2hD(A+nc)\left[D/P+\frac{b^2+b}{(a+b)^2+(a+b)}\right]}\right] \tag{22}$$

In order to solve this model, we suggest the following algorithmic steps:
Step 1: Let: n = 0, d = 0, EPR1 = -infinity
Step 2: $n=n+1,\quad j=d,\quad n_j=j/AQL$.
Step 3: Find Q from Equation (21) as: $Q=\sqrt{\frac{2D(A+nc)}{h\left[\frac{D}{P}+\frac{b^2+b}{(a+b)^2+(a+b)}\right]}}$
Step 4: Find $EPR_2=EPR$ (d, n, Q) from Equation (22)
Step 5: If $EPR_2>EPR_1$ then $EPR_1=EPR_2$, otherwise, go to step 7.
Step 6: if $d\ge n$, then d =0 and go to step 2, otherwise d = d +1 go to step 4.
Step 7: Stop, the previous solution is optimal.
The algorithm will be illustrated with a case example in the next section.

4 Sensitivity Analysis

In this section, we study the effects of the following parameters on the optimal solution:

1.Effect of shape parameter 1 of Beta distribution (a).

2.Effect of shape parameter 2 of Beta distribution (b).
3.Effect of acceptable quality level (AQL).
4.Effect of the selling price per unit in the high-quality lot (v1).
5.Effect of the selling price per unit in sub-standard quality lot (v2).
6.Effect of inspection cost (Ci).

It should be noted that the parameters a and b of the beta distribution are important since they affect the probability of the fraction non-conforming in a lot. Therefore, we have included the effects of changes in these parameters on various performance measures. A typical case example is used to illustrate the effects of several parameters on optimal lot size, expected profit, and other measures. The values of the parameters for the selected case example are given in table 1.

Table 1: Basic data for sensitivity analysis.

Parameter	Value	Parameter	Value
P	2000	v_1	15
D	1000	v_2	10
h	1	α	3
A	100	β	3
C_i	0.5	AQL	0.1

4.1 Effects of Shape Parameter 1 of Beta Distribution (a)

For a fixed value of b, the probability that fraction of non-conforming items in the lot is less than y0, that is P(Y < y0), is lower when the shape parameter (a) is large. Thus, the expected profit of the producer is expected to increase as shown in Figure 2. However, the sample size is expected to decrease because the quality of the lot is higher for high a and the need to take a larger sample size is not strong. This relation between a and n is depicted in Figure 3. It is to be noted that the step-like shape of optimal n is due to the constraint that the producer should respect AQL of the retailer as shown by inequality (3). Since Q is the number of non-conforming items in the lot, it is anticipated that it increases. This is true because the cost of defective items will be reduced when the quality of the lot is high.

Figure 4 shows unexpected trend when a = 10 where the lot size decreases, this is because at this particular point the sample size optimal.

4.2 Effects of Shape Parameter 2 of Beta Distribution (b)

Generally speaking, for Beta distribution, high values of b correspond to high probability of fraction of

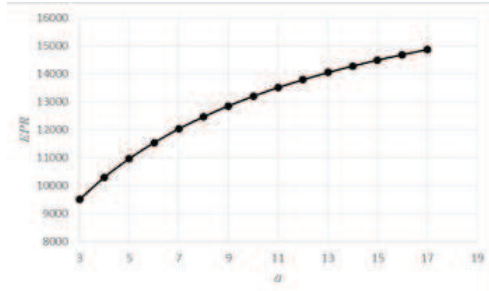

Fig. 2: Effect of shape parameter of beta distribution parameter (a) on EPR.

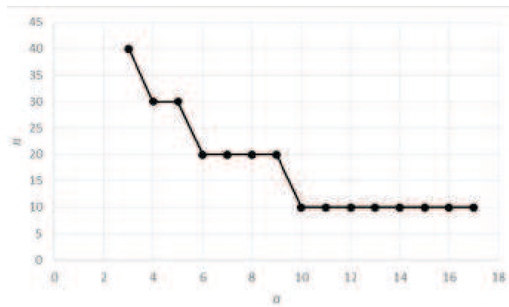

Fig. 3: Effect of shape parameter 1 (a) of beta distribution on sample-size.

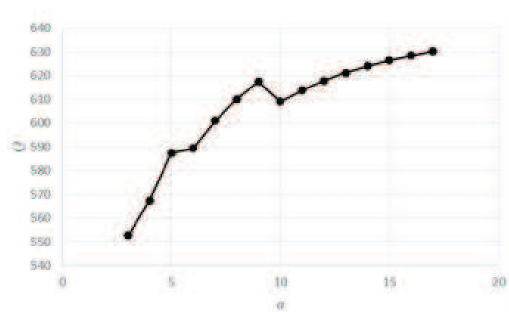

Fig. 4: Effect of shape parameter 1 (a) of beta distribution on lot-size.

non-conforming items in the lot. Thus, the expected total profit should decrease with b as shown by Figure 5. However, the expected total profit increases for b > 10. This trend is due to fact that AQL = 0.1 which must be observed by the producer, and optimal n = 10 (shown in Figure 5), consequently, optimal d = 1 (obtained from inequality 3). In other words, lots with high percentage of non-conforming items can be considered as acceptable while the retailer?s AQL is still satisfied. Moreover, optimal n is reduced when b is higher (see Figure 6). This is due to the probability of high percentage of non-conforming items in the lot, that is, it is easy to detect a low quality lot even with small sample size. Also, Figure 6 shows that the lot size is low for high values of b because it is not optimal to increase the production lot when the process produces high percentage of defective items.

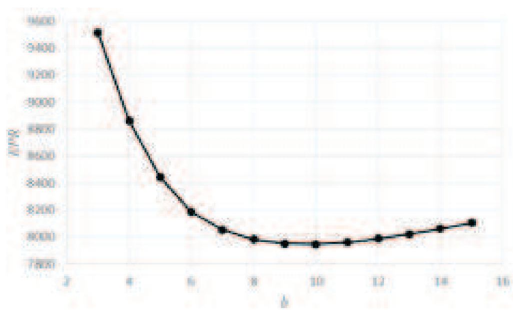

Fig. 5: Effect of shape parameter 2 (b) of beta distribution on EPR.

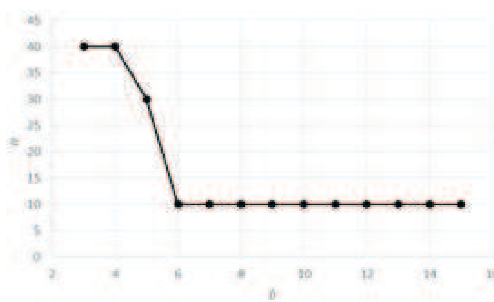

Fig. 6: Effect of shape parameter 2 (b) of beta distribution on sample-size.

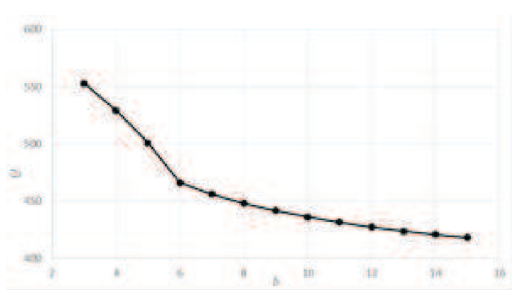

Fig. 7: Effect of shape parameter of beta distribution parameter (b) on lot-size.

4.3 Effects of Acceptable Quality Level (AQL)

As shown in Figure 8, the expected total profit will decrease as the AQL increases due to the increase of accepted lots with more defective items. Moreover, as the AQL increases, the sample size and lot size decreases in order to decrease the effect of defective items (see Figures 9 and 10).

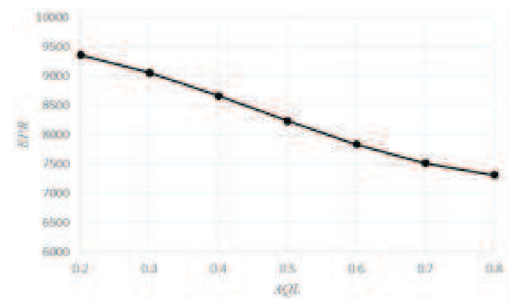

Fig. 8: Effect of Accepted Quality Level (AQL) on EPR.

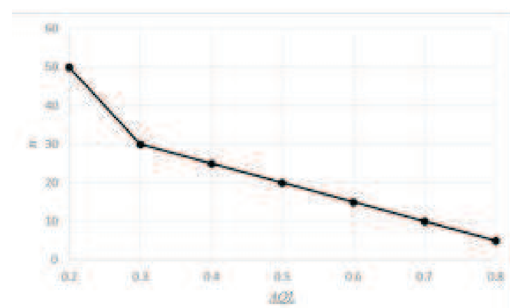

Fig. 9: Effect of Accepted Quality Level (AQL) on sample-size.

4.4 Effects of the Unit Selling Price for the High-Quality Lots

As v_1 increases, the expected profit will increase (see Figure 11). Moreover, increasing v_1 will increase the probability of accepting the lot, and thus, forcing the sample size and the lot size to decrease (see Figures 12 and 13).

4.5 Effects of the Unit Selling Price in Sub-Standard Quality Lots

As v_2 increases, the expected profit will increase linearly as shown in Figure 14. Also, increasing v_2 will increase

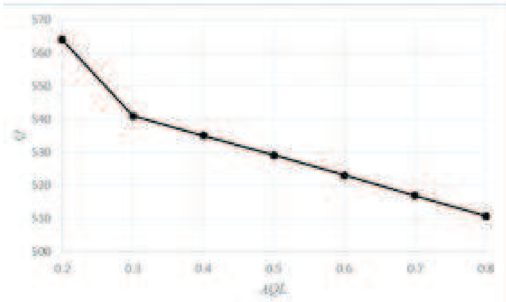

Fig. 10: Effect of Accepted Quality Level (AQL) on lot-size.

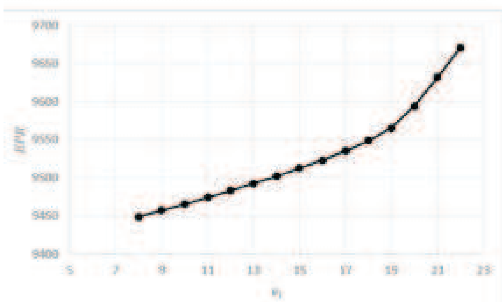

Fig. 11: Effect of the unit selling price in high-quality lot on EPR.

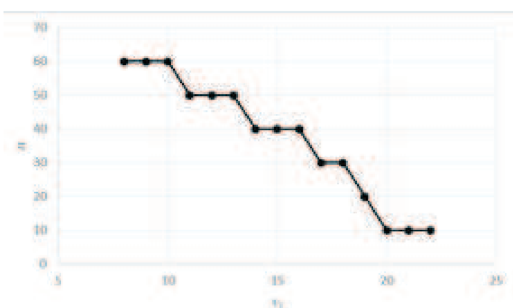

Fig. 12: Effect of the unit selling price in high-quality lot on sample-size.

the portion of units that will be sold at the secondary market. This will decrease the probability of acceptance forcing the sample size as well as the lot size to increase (see Figures 15 and 16).

4.6 Effects of Inspection Cost

Figure 17 illustrates the relationship between the inspection cost and the expected profit. It shows that as the inspection cost increases, the expected profit

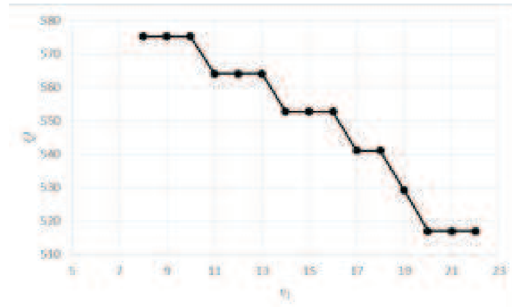

Fig. 13: Effect of the unit selling price in high-quality lot on lot-size.

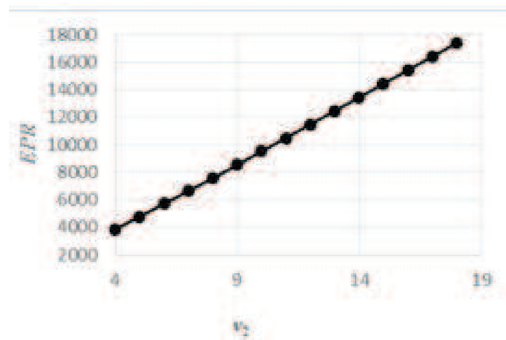

Fig. 14: Effect of unit selling price for the sub-standard quality lot on EPR.

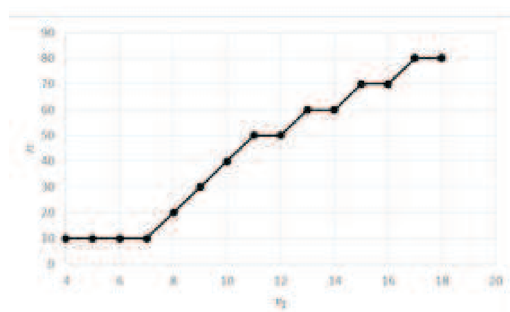

Fig. 15: Effect of unit selling price for the sub-standard quality lot on sample-size.

decreases. However, the rate of decrease in the expected profit decreases as inspection cost increases. This is due to decrease of the sample size and lot size (see Figures 18 and 19).

5 Conclusion

A single-producer single-retailer supply chain is considered in this paper. The producer produces a single

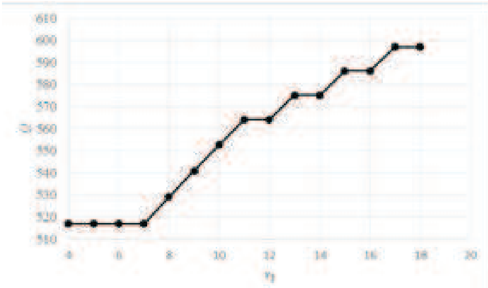

Fig. 16: Effect of unit selling price for the sub-standard quality lot on lot-size.

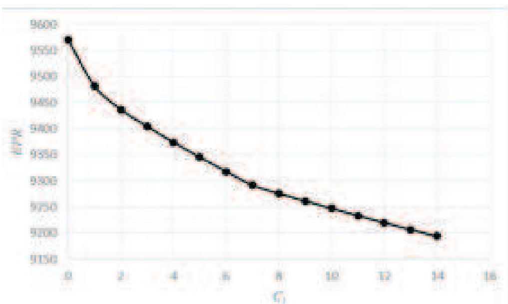

Fig. 17: Effect of the unit selling price for the high-quality lot on EPR.

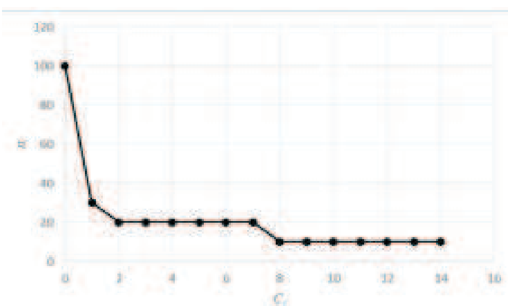

Fig. 18: Effect of the unit selling price for the high-quality lot on sample-size.

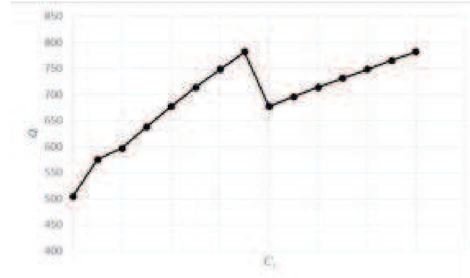

Fig. 19: Effect of the unit selling price for the high-quality lot on lot-size.

item to satisfy a retailer who observes a deterministic demand. The production process under consideration is imperfect and produces a random proportion of defective items which follow a beta distribution. A sampling inspection is used to classify the lot in one of two categories; high-quality or sub-standard quality. In the first case, the lot is sold at a regular price, while in the second case it is sold at a reduced price. The retailer is protected by a limit on the incoming quality, called acceptable quality level, AQL. In other words, the producer must respect the AQL of the retailer. The objective of the model is to determine the optimal production run, sample size, and acceptance number which maximize the expected total profit of the producer. The decision variables in this model are the production lot size, the sample size, and the acceptance number. The concavity of the expected profit with respect to the lot size is established and an algorithmic method is proposed to obtain a solution for the problem. In addition to the solution, parametric sensitivity analysis are presented and discussed with respect to a selected case example. Sensitivity analysis on the model's key parameters reveal that the optimal expected profit and optimal lot size are greatly affected by the distribution of defective items as well as the inspection cost. Therefore, the integrated system leads to significant cost saving. The model in this paper can be extended in many directions. For example, a single-producer and a multi-retailer case can be investigated. Another possible extension is considering a stochastic demand instead of the deterministic demand used in this paper.

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APPENDIX A: Proof of Proposition 1.

$E((1-Y)^2) = \int_0^1 (1-y)^2 f_Y(y)dy$ Where $f_Y(y) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} y^{a-1}(1-y)^{b-1},\ y \in [0,1]$ Thus,
$E((1-Y)^2) = 1 - 2E(Y) + E(Y^2)$
$E((1-Y)^2) = 1 - 2\frac{a}{a+b} + E(Y^2)$
$E(Y^2) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^1 y^{a+1}(1-y)^{b-1}dy$ using $\int_0^a x^m(a^n - x^n)^p dx = \frac{a^{m+1+np}\Gamma[(m+1)/n]\Gamma[p+1]}{n\Gamma[(m+1)/n+p+1]}$ We find,
$E(Y^2) = \frac{a}{(a+b)} \frac{(a+1)}{(a+b+1)}$

Thus $E((1-Y)^2) = \frac{b^2+b}{(a+b)^2+(a+b)}$,and the proof is complete.

APPENDIX B: Proof of Proposition 2.

As we have mentioned earlier, for a given fraction of defective items, in the lot, the number of nonconforming items in a sample, X, follows a binomial distribution that is:
$P(X=x|y) = \binom{n}{x} y^x (1-y)^{n-x}$ Thus,

$$P(X = x) = \int_0^1 \binom{n}{x} q^x (1-q)^{n-x} f(q)dq = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \binom{n}{x} \int_0^1 y^{x+a-1}(1-y)^{n-x+b-1}dy$$

Which can be simplified as follows:
$P(X=x) = \binom{n}{x} \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{\Gamma(x+a)\Gamma(n-x+b)}{\Gamma(a+b+n)}$ therefore,
$P(X \leq d) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \sum_{x=0}^{d} \binom{n}{x} \frac{\Gamma(x+a)\Gamma(n-x+b)}{\Gamma(a+b+n)}$, this completes the proof.

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