Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/090540

Generating a Complex Form of Chaotic Pan System and its Behavior

Ahmed A. M. Farghaly[1,2,*]

[1] College of Science and Humanitarian Studies, Salman Bin Abdulaziz University, Al-Aflaj, Saudi Arabia
[2] Department of Mathematics, Faculty of Science, Assiut University, Assiut71516, Egypt

Received: 8 Feb. 2015, Revised: 10 May 2015, Accepted: 11 May 2015
Published online: 1 Sep. 2015

Abstract: In this article we introduce a new chaotic five dimensional (5D) continuous autonomous system involving complex variables,via 3D Pan system[Pan et al 2010]. The basic dynamical properties of the new system are analyzed such as equilibrium points, eigenvalues structures, and maximal Lyapunov exponent. We propose an approach for controlling chaotic attractor of this system by adding a complex periodic forcing. Computer simulations are calculated to study the behavior of this system.

Keywords: Chaotic, behavior, Complex, Pan system, control.

1 Introduction

For nearby 40 years, chaos theory was an interesting phenomenon of dynamical systems. It has been found to be very useful and has great potential in many disciplines such as the fields of communication, Laser, neural work, nonlinear circuits, and etc [1-3]. Famous examples of chaotic systems are Lorenze [4], Rössler [5], Chen [6], Lü [7], Henon map [8] and Pan [9]. Chaotic behavior have been widely studied on a great numbers of real variables. However, there also are many interesting cases of dynamical systems involving complex variables, like, Lorenz system which is used to describe and simulate the physics of detuned lasers and thermal convection of liquid flows [10]. In recent years, Mahmoud et al have been introduced and studied chaotic complex systems [11-16]. In 2010 Pan et al [9] proposed a new 3D chaotic system which is similar to the Lorenz chaotic attractor, but it is not topological equivalent. Here, similar to complex systems like Lorenz [10], Rössler [17], Chen and Lu [14], we wish to include complex variables in Pan system to get a higher-dimensional system. Some basic dynamical properties such as maximal Lyaponuv exponent, eigenvalues, chaotic behavior and chaos control of this new system are studied. The remainder of the paper is organized as follows: Section 2 explains the proposed new chaotic system and its dynamics. Section 3 discusses the controlling chaos of the new system and finally the concluding remark is given in section 4.

2 System description

The mathematical model of real Pan system is a system of non-linear ordinary differential equations as:

$$\left.\begin{aligned} \dot{x} &= \rho(y-x) \\ \dot{y} &= \sigma x - xz \\ \dot{z} &= xy - \beta z \end{aligned}\right\} \tag{1}$$

where ρ, σ and β are positive parameters [9]. Now, the Pan system with complex variables is:

$$\left.\begin{aligned} \dot{x} &= a(y-x) \\ \dot{y} &= cx - xz \\ \dot{z} &= -bz + \tfrac{1}{2}(\bar{x}y + \bar{y}x) \end{aligned}\right\} \tag{2}$$

where a, b and c are positive parameters, $x = u_1 + iu_2$ and $y = u_3 + iu_4$ are complex variables, $i = \sqrt{-1}$ and $z = u_5$ is a real variable, dots represent derivatives with respect to time and an over bar denotes complex conjugate variable. The real version of (2) which is a five dimensional chaotic

* Corresponding author e-mail: aafarghaly166@hotmail.com

autonomous system reads:

$$\left.\begin{aligned} \dot{u_1} &= a(u_3-u_1) \\ \dot{u_2} &= a(u_4-u_2) \\ \dot{u_3} &= cu_1-u_1u_5 \\ \dot{u_4} &= cu_2-u_2u_5 \\ \dot{u_5} &= -bu_5+u_1u_3+u_2u_4 \end{aligned}\right\} \tag{3}$$

2.1 Dissipativity and the existence of attractor

The divergence of the flow 3 is defined by:

$$\nabla F = \frac{\partial f_1}{\partial u_1}+\frac{\partial f_2}{\partial u_2}+\frac{\partial f_3}{\partial u_3}+\frac{\partial f_4}{\partial u_4}+\frac{\partial f_5}{\partial u_5} = -a-a+0+0+b = -2a+b$$

Where : $F=(f_1,f_2,f_3,f_4,f_5)=[10(u_3-u_1),10(u_4-u_2),16u_1-u_1u_5,16u_2-u_2u_5,-\frac{8}{3}u_5+u_1u_3+u_2]$. It means that system (3) is dissipative and its contraction rate is $\frac{dV}{dt}=-\frac{52}{3}V$ then $V=V_0e^{-\frac{52}{3}t}$ for the case $a=10$, $b=\frac{8}{3}$ and $c=16$. Therefore each volume containing the trajectory of this system decay to zero as $t\to\infty$ at an exponential rate $-\frac{52}{3}$. So, the asymptotic motion settles onto an attractor of (3). The attractors of system 3 are displayed in Fig.1. The parameters are chosen as $a=10$, $b=\frac{8}{3}$ and $c=16$ and the initial values are taken as : $u_1(0)=u_2(0)=1$, $u_3(0)=u_4(0)=-1$, $u_5(0)=10$. Fig.1 (a,b,c) shows the chaotic attractors in (u_1,u_3), (u_1,u_5) and (u_3,u_5) respectively. The waveforms of $u_1(t)$ and $u_5(t)$ in time domain are shown in Fig.1 (d and e). Other values of a, b and c can similarly studied such that $-2a+b<0$.

2.2 Maximal Lyapunov exponent (MLE)

The maximal Lyapunov exponent measures the exponent of the rate at which nearby trajectories diverge in state space. A positive maximal Lyapunov exponent is a strong indication of deterministic chaos. To calculate MLE we must put system 3 in the vector notation as follows:

$$\dot{U}(t)=F[U(t);\mu] \tag{4}$$

Where $U(t)=[u_1(t),...,u_5(t)]^T$ is the state vectors, $F=[f_1,...,f_5]^T$, μ is a set of parameters and $[...]^T$ denotes transpose. System (4) for small deviation δU from the solution $U(t)$ is:

$$\delta\dot{U}(t)=J_{ij}[U(t);\mu]\delta U, \qquad i,j=1,2,..,5 \tag{5}$$

Where $J_{ij}=\frac{\partial f_i}{\partial u_j}$ is the following Jacobian matrix:

$$J_{ij}=\begin{bmatrix} -a & 0 & a & 0 & 0 \\ 0 & -a & 0 & a & 0 \\ c-u_5 & 0 & 0 & 0 & -u_1 \\ 0 & c-u_5 & 0 & 0 & -u_2 \\ u_3 & u_4 & u_1 & u_2 & -b \end{bmatrix}. \tag{6}$$

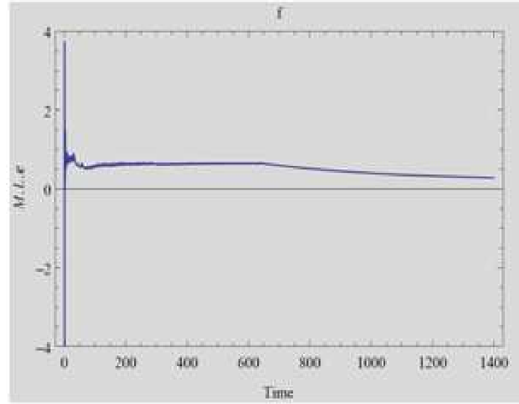

Fig. 2: The Maximal Lyapunov exponent of new system with the same initial conditions and parameter values of Fig.1.

The MLE of the system defined by:

$$\lambda_{max}=\lim_{t\to\infty}\frac{1}{t}\log\frac{\|\delta U(t)\|}{\|\delta U(0)\|} \tag{7}$$

To find λ_{max}, equations (5) and (6) must be numerically solved simultaneously. By using Mathematica software we calculate MLE with the same above parameters and initial values. The maximal Lyapunov exponent of the new system is obtained as ($\lambda_{max}\cong 0.63$). Fig.1(f) shows MLE.

2.3 Eigenvalues test

System (3) has (11) terms, four quadratic nonlinearities $(u_1u_5,u_2u_5,u_1u_3$ and $u_2u_4)$ and three positive real constant parameters (a,b,c). The new system equation has two fixed points. The set of fixed points which satisfy this requirement are found by setting all the left hand side of equation (3) equal zero, and solving for u_1,u_2,u_3,u_4 and u_5:

$$\left.\begin{aligned} 0 &= a(u_3^*-u_1^*) \\ 0 &= a(u_4^*-u_2^*) \\ 0 &= cu_1^*-u_1^*u_5^* \\ 0 &= cu_2^*-u_2^*u_5^* \\ 0 &= -bu_5^*+u_1^*u_3^*+u_2^*u_4^* \end{aligned}\right\} \tag{8}$$

Two fixed points exist:

$$(u_1^*,u_2^*,u_3^*,u_4^*,u_5^*)=(9.39857\pm1.56278i,2.12572\mp6.90962i \\ ,9.39857\pm1.56278i,2.12572\mp6.90962i,16)$$

For the case when the fixed point is:

$$(u_1^*,u_2^*,u_3^*,u_4^*,u_5^*)=(9.39857+1.56278i,2.12572-6.90962i \\ ,9.39857+1.56278i,2.12572-6.90962i,16)$$

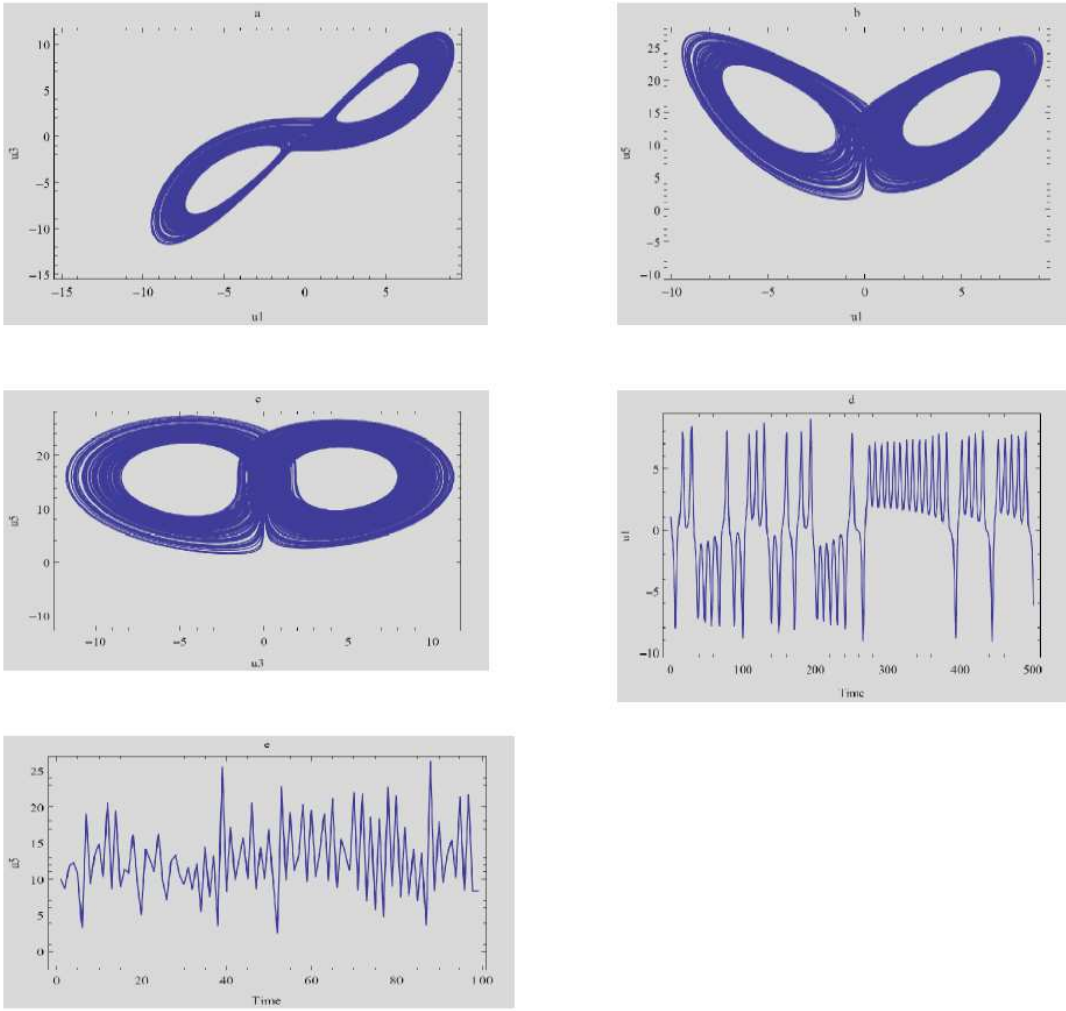

Fig. 1: The dynamical behavior of the new chaotic system : (a) $u_1 - u_3$ phase plane strange attractor, (b) $u_1 - u_5$ phase plane strange attractor, (c) $u_3 - u_5$ phase plane strange attractor, (d) $u_1(t)$ wave form, (e) $u_5(t)$ wave form, (f) Maximal lyapinove exponent versus time

the Jacobian (6) becomes:

$$\begin{pmatrix} -10 & 0 & 10 & 0 & 0 \\ 0 & -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & -9.39857-1.56278i \\ 0 & 0 & 0 & 0 & -2.12572+6.90962i \\ 9.39857+1.56278i & 2.12572-6.90962i & 9.39857+1.56278i & 2.12572-6.90962i & -\frac{8}{3} \end{pmatrix} \quad (9)$$

The eigenvalues are found by solving the equation $|J-\lambda I| = 0$ yielding eigenvalues:

$$\lambda_1 = -12.557 - 5.06564 \times 10^{-15} i, \lambda_2 = -108.79911 \times 10^{-16} i,$$

$$\lambda_3 = -0.0548184 - 8.2434i, \lambda_4 = -0.0548184 - 8.2434,$$
$$\lambda_5 = 4.4408910^{-16} - 8.88178 \times 10^{-16} i$$

For the case when the fixed point is:

$$(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = (9.39857 - 1.56278i, 2.12572 + 6.90962i$$
$$, 9.39857 + 1.56278i, 2.12572 - 6.90962i, 16)$$

the Jacobian (6) becomes:

$$\begin{pmatrix} -10 & 0 & 10 & 0 & 0 \\ 0 & -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & -9.39857+1.56278i \\ 0 & 0 & 0 & 0 & -2.12572-6.90962i \\ 9.39857-1.56278i & 2.12572+6.90962i & 9.39857+1.56278i & 2.12572-6.90962i & -\frac{8}{3} \end{pmatrix} \quad (10)$$

The eigenvalues are found by solving the equation $|J-\lambda I| = 0$ yielding eigenvalues:

$$\lambda_1 = -12.557 - 5.06564 \times 10^{-15} i, \lambda_2 = -108.79911 \times 10^{-16} i,$$

$$\lambda_3 = -0.0548184 - 8.2434i, \lambda_4 = -0.0548184 - 8.2434,$$

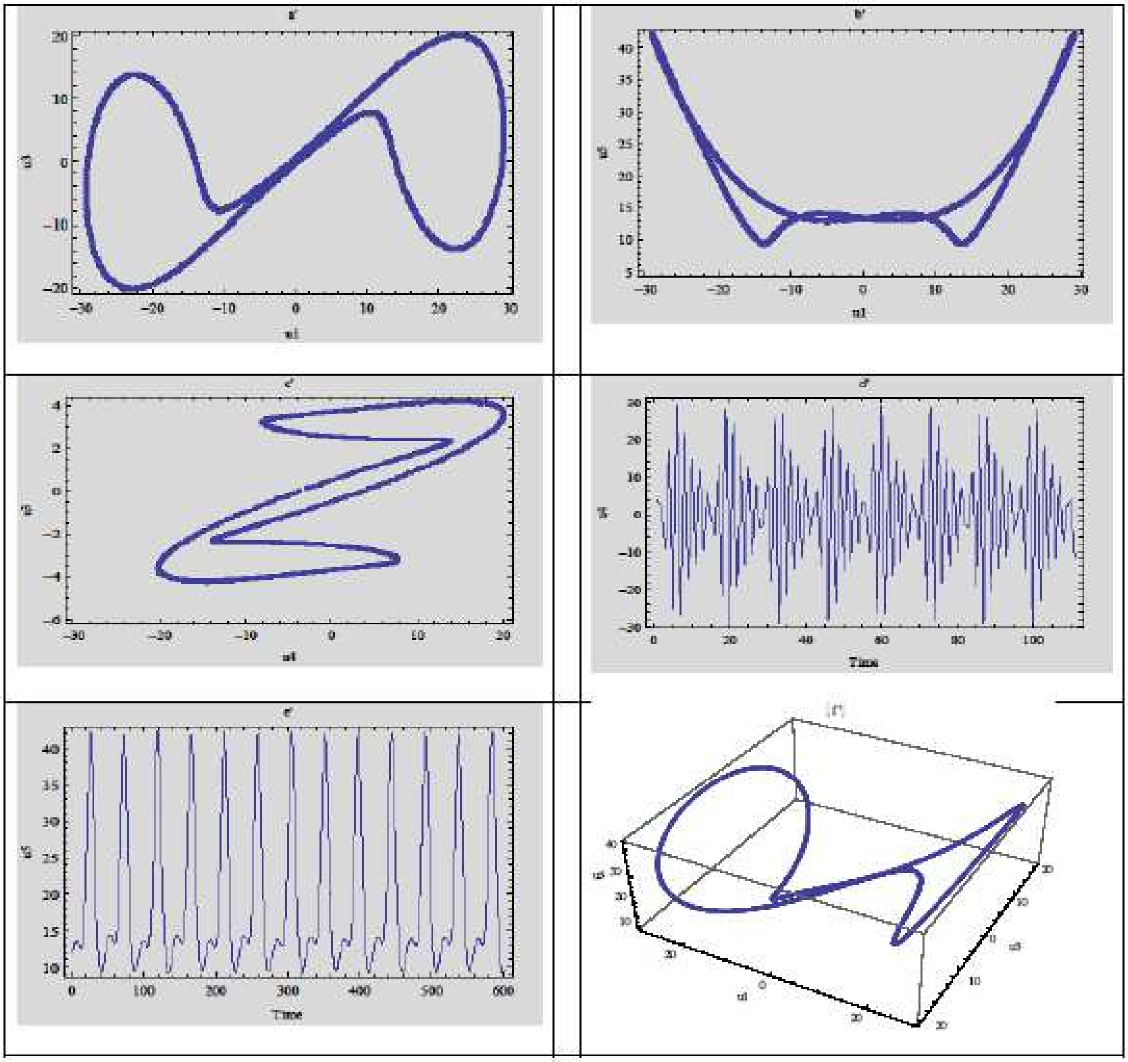

Fig. 3: The dynamical behavior of the new chaotic system after control: (a) $u_1 - u_3$ phase plane,(b) $u_1 - u_5$ phase plane, (c) $u_3 - u_5$ phase plane, (d) $u_4(t)$ wave form (e) $u_5(t)$ wave form (f) chaotic attractor in (u_1, u_3, u_5) after control.

$$\lambda_5 = 4.4408910^{-16} + 8.88178 \times 10^{-16} i$$

For the case when the fixed point is:

$$(u_1^*, u_2^*, u_3^*, u_4^*, u_5^*) = (9.39857 - 1.56278i, 2.12572 + 6.90962i \\ , 9.39857 + 1.56278i, 2.12572 - 6.90962i, 16)$$

Note that for all fixed points there is one λ has positive real part. Consequently the fixed points are unstable and this implies chaos as mentioned above in Fig. 1 .

3 Controlling chaos of a new system

In this section, the main results with a new and simple control low will be discussed. Based on the addition of complex periodic forcing to the first equation of system (2), so system (2) becomes:

$$\left.\begin{array}{rl} \dot{x} = & a(y-x) + (1+i)k\cos\omega t \\ \dot{y} = & cx - xz \\ \dot{z} = & -bz + \frac{1}{2}(\bar{x}y + \bar{y}x), \end{array}\right\} \tag{11}$$

where ω and k are positive parameters. The controlled system (11) in the real version with $u_6 = \omega t$ is:

$$\left.\begin{array}{rl} \dot{u_1} = & a(u_3 - u_1) + k\cos u_6, \\ \dot{u_2} = & a(u_4 - u_2), \\ \dot{u_3} = & cu_1 - u_1 u_5 + k\cos u_6, \\ \dot{u_4} = & cu_2 - u_2 u_5, \\ \dot{u_5} = & -bu_5 + u_1 u_3 + u_2 u_4, \\ \dot{u_6} = & \omega. \end{array}\right\} \tag{12}$$

Numerical simulations are used to investigate the controlled chaotic system (12) using Mathematica software version 9. When the parameters are $a = 10, b = \frac{8}{3}$ and $c = 16$ and the additional parameters ($\omega = 6.75$ and $k = 250$). The initial values are taken as: $u_1(0) = u_2(0) = 1$, $u_3(0) = u_4(0) = -1$ and $u_5(0) = 10$. In the controller time t, one can see when is greater than or equal to 44 , chaos attractor disappear. The behavior of the controlled chaotic system (12) are displayed in Fig.3.(a, b, c, d, e and f).

4 Conclusions

In this paper, we introduced a new 5 dimensional system, which is called a chaotic complex Pan system. The new system is generated from a famous Pan system after replacing the real variables of the first and second equations of (1) with complex ones. The maximal Lyapunov exponent of this system is positive as shown in Fig. 2 which means that our system is chaotic. System (2) is controlled by adding the complex forcing to the first equation and the control time starts from $t \geq 44$. Since the new system has more complex dynamical behavior , it is believed that the system will have a broad applications in various information systems.

Acknowledgment

The author would like to thank the deanship of scientific research at Salman bin Abdulaziz University for supporting this paper.

References

[1] G.Wang, X.Zhang, Y.Zheng, Li YA, New Modified Hyperchaotic Lü System, Physica A No. 371, (2006), pp. 260-272.
[2] X. Wang, M. Wang, A Hyperchaos Generated Form Lorenz System, Physica A, No. 387,(2008), pp. 3751-3758.
[3] J. Xu, G. Cai, S. Zheng, A Novel Hyperchaotic System and its Control, Journal of Uncertain system, Vol. 3, No. 2, (2009), pp. 137-144.
[4] E.N. Lorenz,Deterministic non-periodic flow, J. Atmospheric Sci., 20 (1963), pp. 130-141.
[5] O.E. Rössle, An equation for hyperchaos, Phys. Lett. A, 71 (2-3) (1979), pp. 155-157.
[6] G. Chen, T. Ueta, Yet another chaotic attractor, Int. J. Bifurc. Chaos, 9 (1999), pp. 1465-1466.
[7] LÜ, J. G. Chen, A new chaotic attractor coined, Int. J. Bifurc. Chaos, 12(3), 6590661 (2002).
[8] M. Henön, A two-dimensional mapping with a strange attractor, Commun. Math. Phys. 50 (1976),p. 69.
[9] L. Pan, D. Xu, W. Zhou, Controlling a novel chaotic attractor using linear feedback, J. Inform. Comput. Sci., 5 (2010), pp. 117-124.
[10] A. Rauh, HanibalL and Abraham N B, Global stability properties of the complex Lorenz model, Physica D 99 (1996), pp 45-58.
[11] G. M. Mahmoud, T. Bountis, G. M. AbdEl-Latif, and E. E.Mahmoud, Chaos synchronization of two different chaotic complex Chen and L systems, Nonlinear Dynamics Vol.55, No.1-2, PP.43-53 (2009)
[12] G. M. Mahmoud and A. A. M. Farghaly, Chaos control of chaotic limit cycles of real and complex van der Pol oscillators, Chaos, Solitons & Fractals, 21(2004), PP. 915-924.
[13] G. M. Mahmoud and A. A. M. Farghaly, On Stabilization of coupled complex nonlinear Schrödinger equations, Int. J. Modern Physics C15, 6 (2004),PP.845-866
[14] G. M. Mahmoud, S.A. Aly, and A.A. Farghaly, On chaos synchronization of a complex two coupled dynamos system, Chaos, Solitons & Fractals Vol.33,No. 1, PP.178-187 (2007).
[15] G. M. Mahmoud, T. Bountis and E. E. Mahmoud ,"Active control and global synchronization for complex Chen and L systems, , Int. J. of Bifurcation and Chaos, Vol.17, No.12(2007),PP4295-4308.
[16] G.M.Mahmoud, M. A. Al-Kashif , Basic properties and chaotic synchronization of complex Lorenz system, , and S.A. Aly, Int. J. Mod. Phys. C, Vol. 18,No.2(2007), PP. 253-265.
[17] Guo-Cheng Wua, Dumitru Baleanuc, Chaos synchronization of the discrete fractional logistic map, Signal Processing, **102**, 96-99 (2014).

Ahmed A. M. Farghaly received the PhD degree in Mathematics for Faculty of Science at Assuit university and Oidenbury university in Germany. His research interests are in the areas of applied mathematics and mathematical physics including the dynamical system non-linear contra theory-chaotic behavior in complex dynamical systems-chaotic synchronization dynamical system. He has published research articles in reputed international journals of mathematical and engineering sciences. He is referee of mathematical journals