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Physical and Dynamical Properties of the Atomic Spectrum and Squeezing of Information Entropy of Su(1,1) Interacting with Single Qubit

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Abstract: In this work, the interaction of an algebraic system of su(1,1) with qubit is investigated. Through Heisenberg's equations of motion, the constants of motion are calculated. By solving the Schröding equation, the wave function that defines the features of the proposed system is obtained. Some statistical quantities are studied to determine the properties of the interaction of the algebraic system su(1,1) with qubit. The mathematical formula for the atomic emission spectrum is calculated and its properties are studied for many parameter values. Moreover, the periods of squeezing are defined by the entropy squeezing. The results are also compared and statistical quantities are linked together.

Keywords: Atomic spectrum, Entropy squeezing, Su(1,1) Lie algebra, Variance squeezing.

1 Introduction

The spontaneous emission of radiation has been considered as a dominant source of the energy relaxation for a quantum system coupled to an environment. In this regard Purcell introduced the spontaneous emission probabilities at radio frequencies [1]. Suppressing of the decay by altering the electromagnetic environment over the system was obtained under the Purcell effect [1]. In this regard different effects have been examined. For example the effect two-level atom under the effect of the intensity-dependent coupling JCM [2] and Kerr medium [3] has been also investigated by Buzek and Jex (1991). Also, Agarwal and Puri have examined the effect of time-dependent spectra of the intermittent resonance fluorescence of single, laser-driven, three-level atoms due to electron shelving [4]. It was shown that the coherent peak, which is a steady state feature, is absent during the time evolution of the spectrum. Futhermore, control, fast reset and suppression of the spontaneous emission of a superconducting qubit have been studied [5,6]. Morevor, Moon and Noh have derived the analytical solutions for the line width and peak value of the absorption spectrum of the probe beam [7]. The study indicated that the analytic theory could provide accurate spectra at an arbitrary pump beam intensity and diameter.

Recently, Dorfmana and Mukamel (2018) have investigated the photon correlation spectroscopy of cavity polarizations in addition to the effect of different physical parameters of the spontaneous emission of the four-level atom coherently-driven [8]. The results illustrated that the adopted scheme is useful for the high-precision measurement of the center-of-mass wave functions of the nanolithography and atomic motion. More recently, the influence of time-dependence forms and intensity based on the physical properties of the transient spectrum has investigated [9]. The results revealed that the transient spectrum strongly depended on the time and intensity dependent. Moreover, there was a relation between the transient spectrum and field purity interacting with a two-level atom. Also, the effect of the deformed parameter emission spectrum and geometric phase of a qubit system that interacts with a field has been investigated [10]. It was shown that the variation of the geometric phase and emission spectrum were strongly influenced by the deformation parameter and initial condition of the system.

Fang et al, introduced an alternative definition of squeezing of the information entropy theory, which

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overcomes the disadvantages of the definition based on the Heisenberg uncertainty relation [11]. The squeezing of the information entropy has been applied on the two-level atom in the resonance fluorescence and (JCM) Jaynes-Cummings model. Based on the definition of Fang et al, for the entropy squeezing different investigations have been considered different systems such as two-level atom in a squeezed vacuum [12], two-level atom under the effect of a nonlinear medium [13], the effect of time dependent of single-mode JCM [14] and moving atoms in squeezed coherent field [15]. Moreover, the results of [16] have shown that the entropy squeezing and atomic inversion in the k-photon JCM were strongly affected by Stark shift and Kerr medium for different initial field states.

Recently, the influence of classical field effect on the dynamic behavior of entropy squeezing has been studied and compared with the entanglement [17]. It was shown that the appearance of squeezing in the information entropy can be controlled by the classical field. Also, the correlation between the entropy squeezing and entanglement of two qubits interacting with a two-mode field in the context of power low potentials has been explored [18]. The results showed that the temporal entropy squeezing and entanglement wre influenced by the qubit-qubit interaction and exponent parameter in the absence and presence of the atomic motion.

Now, we present a model of the interactin of a bi-level atom with an electromagnetic field, as follows:

$$\frac{\hat{H}}{\hbar}=\frac{\omega_0}{2}\hat{\sigma}_z+\omega\hat{K}_z+\lambda\left(\hat{K}_+\hat{\sigma}_-+\hat{K}_-\hat{\sigma}_+\right) \tag{1}$$

where ω_0 is the atomic frequency, ω is the $\mathrm{su}(1,1)$ group frequency and λ represents the coupling factor. The $\mathrm{su}(2)$ Lie algebra operators $\hat{\sigma}_\pm$ and $\hat{\sigma}_z$ are governed by,

$$\hat{\sigma}_z\hat{\sigma}_\pm-\hat{\sigma}_\pm\hat{\sigma}_z=\pm2\hat{\sigma}_\pm,\qquad \hat{\sigma}_+\hat{\sigma}_--\hat{\sigma}_-\hat{\sigma}_+=\hat{\sigma}_z \tag{2}$$

while the quantum system subject to the operators $\hat{K}_-,\hat{K}_+$ and $\hat{K}_z$ obeyes

$$\hat{K}_z\hat{K}_\pm-\hat{K}_\pm\hat{K}_z=\pm\hat{K}_\pm,\qquad \hat{K}_-\hat{K}_+-\hat{K}_+\hat{K}_-=2\hat{K}_z \tag{3}$$

The present paper is organized, as follows. In the next section, the model is solved through the time evolution operator. Section Three addresses the atomic emission spectrum. In Section Four, periods of squeezing are defined. Conclusion is presented Section Five.

2 Physical system and wave function

The main procedure in this section is the calculation of the time dependent wave function. Heisenberg differential relationship is used to calculate the kinematic equations for the $\hat{K}_z$ and $\hat{\sigma}_z$ as follows:

$$i\dot{K}_z=\lambda(\hat{K}_+\hat{\sigma}_--\hat{K}_-\hat{\sigma}_+),\qquad i\dot{\sigma}_z=2\lambda(\hat{K}_-\hat{\sigma}_+-\hat{K}_+\hat{\sigma}_-), \tag{4}$$

Therefore, Based on the above-mentioned equation (4), we deduce the constant operator N, which is given by,

$$2\hat{C}=2\hat{K}_z+\hat{\sigma}_z, \tag{5}$$

After using the above equation (5), the Hamiltonian (1) becomes,

$$\frac{\hat{H}}{\hbar}=\omega\hat{C}+\delta\hat{\sigma}_z+\lambda\left(\hat{K}_+\hat{\sigma}_-+\hat{K}_-\hat{\sigma}_+\right) \tag{6}$$

where δ represents the detuning quantity and is given by

$$\delta=\frac{\omega_0-\omega}{2}. \tag{7}$$

We set the initial conditions, as follows: the atom starts movement from the next state

$$|\theta,\phi\rangle=\cos(\theta/2)|e\rangle+\sin(\theta/2)\exp(-i\phi)|g\rangle, \tag{8}$$

where θ denotes the coherence angle and ϕ represents phase. Suppose the atom of the algebraic system starts the interaction from the state

$$|\beta;k\rangle=\left(\frac{|\beta|^{2k-1}}{I_{2k-1}(2|\beta|)}\right)^{1/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!\Gamma(n+2k)}}|n;k\rangle, \tag{9}$$

where $I_n(x)$ is given by,

$$I_k(x)=\left(\frac{x}{2}\right)^k\sum_{n=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2n}}{n!\Gamma(n+k+1)}. \tag{10}$$

Where k is the Bargmann index. Therefore, using the initial conditions the general solution is given from the next relationship.

$$\begin{aligned}|\psi(t)\rangle=&\left\{\left(\cos\hat{\mu}_1t-\frac{i\hat{\delta}}{\mu_1}\sin\hat{\mu}_1t\right)\cos\frac{\theta}{2}-ig\frac{\sin\hat{\mu}_1t}{\mu_1}\right.\\&\left.\hat{K}_-\exp\{-i\phi\}\sin\frac{\theta}{2}\right\}|\beta;k,e\rangle+\left\{\left[\cos\hat{\mu}_2t\right.\right.\\&\left.+\frac{i\hat{\delta}}{\mu_2}\sin\hat{\mu}_2t\right]\exp\{-i\phi\}\sin\frac{\theta}{2}\\&\left.-ig\frac{\sin\hat{\mu}_2t}{\mu_2}\hat{K}_+\cos\frac{\theta}{2}\right\}|\beta;k,g\rangle.\end{aligned} \tag{11}$$

where $\hat{\mu}_1=\delta^2+\lambda^2\hat{K}_-\hat{K}_+$ and $\hat{\mu}_2=\delta^2+\lambda^2\hat{K}_+\hat{K}_-$.

The general solution (wave function) is used to discuss some statistical quantities in the following sections.

3 The atomic emission spectrum

Recently, attention has been paid to addressing the properties of the atomic spectrum emitted from inside the cavity of the JCM. Many articles that explain the characteristics of the atomic emission spectrum [19]. The peak which has been obtained in the vacuum state is divided into two identical peaks in a coherent state [20]. The effect of Stark shift on the emission spectrum has also been studied. Hence, the previous symmetry is broken through the nonlinear interaction, either the Stark shift or the Kerr-like medium [21,22,23,24].

The atomic emission spectrum is given by,

$$S(\omega) = 2\Gamma \int_0^T dt_1 \int_0^T dt_2 \exp\Big[-(\Gamma - i\omega)(T - t_1) - (\Gamma + i\omega)(T - t_2)\Big] \Big\langle \psi | \hat{S}_{12}(t_1)\hat{S}_{21}(t_2) | \psi \Big\rangle^{ex} \quad (12)$$

where T denotes the time of interaction and $\frac{1}{\Gamma}$ represents the response time.

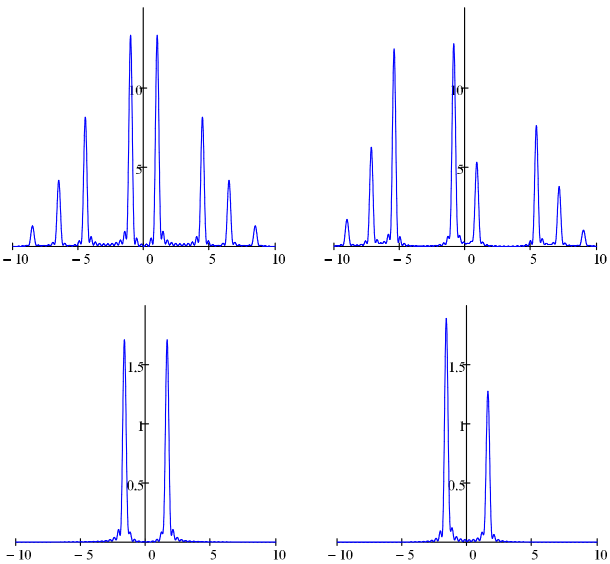

Fig. 1: $S(\omega)$ against ω with the intensity of the initial Barut-Girardello coherent state equal to, ($\beta = 2$), $T = 20$, $\Gamma = 0.05$, where (a) $k = \frac{1}{4}$, $\Delta = 0$, (b) $k = \frac{1}{4}$, $\Delta = 3\lambda$ (c) $k = 10$, $\Delta = 0$, (d) $k = 10$, $\Delta = 3\lambda$.

Now we begin analyzing and describing the properties of the atomic emission spectrum of the su(1,1) system interacting with a two-level atom for some k and Δ parameter values and fixed $|\beta|^2 = 4$, $\theta = 0$. In the absence of the detuning parameter $\Delta = 0$ and $k = \frac{1}{4}$, the result is displayed in figure (1a). It shows that the single-peaked in vacuum is divided into eight symmetrical peaks. The effect of the detuning factor on atomic emission has been considered in Fig. 1b. It is evident that the eight symmetric peaks originated in Fig. 1a. This symmetry is broken in the presence of the detuning factor, where the peaks decrease on the right side, while they increase on the other side as observed in Fig. 1b. On the other hand, we consider the effect of the Bargmann index on the evolution of atomic emission in the absence of the detuning factor. The increase of the Bargmann index ($k = 10$) occurs due to the values of the maximum values of the atomic spectrum as observed in Fig. 1c. After adding the detuning parameter to the interaction cavity, the symmetry is broken where the maximum values of the left peak is greater than those of the right peak as shown in Fig. 1d.

4 Phenomena of squeezing

As known that all previous investigations are based on Heisenberg uncertainty inequality which is the standard limit for measurements of quantitative fluctuations for checking squeezing [25,26,27,11]. The formula was depended on standard deviations of physical factors within the proposed quantum system. These physical quantities are the most natural quantifying of fundamental uncertainty related to the quantum fluctuations [11,12]. Squeezing of information entropy is identified through the formula,

$$E(\hat{\sigma}_i) = \delta H(\hat{\sigma}_i) - \frac{2}{\sqrt{\delta H(\hat{\sigma}_z)}}, \quad (13)$$

where $H(\hat{\sigma}_i)$ is the Shannon entropies for $i = x$ or y.

Therefore, squeezing on the information entropy is obtained if $\delta H(\hat{\sigma}_i) < \frac{2}{\sqrt{\delta H(\hat{\sigma}_z)}}$.

To visualize the influence of the Bargmann index and detuning parameter on the dynamics of the squeezing of information entropy in the two components we have plotted in Figs. 2 the time variation of the entropy squeezing comments when the system initially in the Barut-Girardello coherent state of su$(1,1)$. In the standard case where the effect of detuning parameter is ignored, squeezing occurs in a periodic manner on the component $\hat{\sigma}_y$ where no squeezing at all on the component $\hat{\sigma}_x$. As the detuning parameter is considered. We observe that the squeezing on $\hat{\sigma}_y$ decreases gradually compared with the absence of detuning parameter (see 2b). Figures 2(c,d) show the effect of high value of the Bargmann index on the evolution of the entropy squeezing components. The main comparison between 2(a,b) and 2(c,d) shows that the entropy squeezing behavior is more chaotic and less of squeezing appears on the component $\hat{\sigma}_y$. The results of Fig. (2) emphasized that the physical and dynamical properties of the entropy squeezing are very sensitive to the physical system parameters.

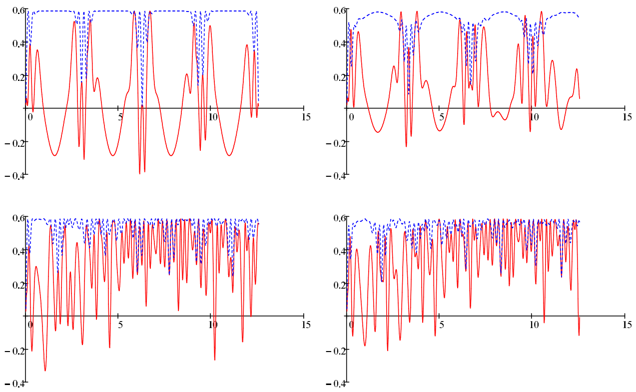

Fig. 2: The entropy squeezing as function of λt with the intensity of the initial Barut-Girardello coherent state equal to, $\beta = 5$ where (a) $k = \frac{1}{4}, \Delta = 0$, (b) $k = \frac{1}{4}, \Delta = 4\lambda$ (c) $k = 10, \Delta = 0$, (d) $k = 10, \Delta = 4\lambda$.

5 Conclusion

The present paper addressed the dynamics of the atomic emission spectrum and the entropy squeezing. Also, we examined the influence of detuning parameter on the dynamical behavior of the atomic spectrum and the squeezing of the information entropy components. On the other hand the influence of the initial state based on the Bargmann index on the dynamical properties of spectrum and entropy squeezing of the atom was explored. Moreover, we have explored the similarity between the entropy squeezing and atomic spectrum. The findings demonstrated that the dynamical and physical properties of the atomic spectrum and entropy squeezing were strongly affected by the detuning parameter and the Bargmann index.

Conflict of Interest

The authors declare that they have no conflict of interest.

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