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Dual to Ratio cum Product Type of Exponential Estimator for Population Mean in Stratified Random Sampling

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Abstract: In this article, we propose a novel dual to ratio cum product type of the exponential estimator in the stratified random sampling. The bias and the mean squared error equations of the proposed estimator, up to the first degree of approximation, have been obtained. The proposed estimator has been compared with the unbiased estimator in the stratified random sampling, combined ratio and product estimators, dual to combined ratio and product estimators. Our results show that the proposed estimator provides a great improvement in terms of relative efficiency. In addition, the results are supported by an empirical study.
Keywords: Ratio and product estimators, Population Mean, Duality, Efficiency.

1. Introduction

Bahl and Tuteja (1991) visualized the ratio and product estimators using the exponential function and these estimators were studied in the stratified random sampling by Singh et al. (2008). Then, Singh et al. (2009) visualized the ratio-cum-product type of the exponential estimators for the population mean. Using the Srivenkataramana (1980) transformation on the auxiliary variables, Tailor and Tailor (2012) obtained the dual estimators of the Singh et al. (2008) ratio and product types of the exponential estimator. Tailor and Chouhan (2013) and Ruiz Espejo, M. (2018) proposed the ratio-cum-product types of the exponential estimator for the population mean in the stratified random sampling.

Consider a finite population, $U = \{U_1, U_2, ..., U_N\}$ of N distinct and identifiable units, which is divided into L strata of sizes, N_h (h=1,2,…,L). Let Y be a study variable and X, Z be two auxiliary variables, taking the values of y_{hi}, x_{hi}, and z_{hi}, respectively, where i=1,2,3,…,N_h. Note that the auxiliary variable, X, is correlated with Z and negatively correlated with the study variable, Y. To estimate the population mean with some desirable properties on the basis of a stratified random sample, we define

$\bar{Y} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} y_{hi} = \frac{1}{N}\sum_{h=1}^{L} N_h\ \bar{Y}_h = \sum_{h=1}^{L} W_h\bar{Y}_h$: Population mean of the study variable, Y.

$\bar{X} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} x_{hi} = \sum_{h=1}^{L} W_h\bar{X}_h$: Population mean of the auxiliary variable, X.

$\bar{Z} = \frac{1}{N}\sum_{h=1}^{L}\sum_{i=1}^{N_h} z_{hi} = \sum_{h=1}^{L} W_h\bar{z}_h$: Population mean of the auxiliary variable, Z.

Here, our problem is to estimate the population mean of the study variable using the stratified random sampling.

Similarly, the unbiased estimators of the population mean of the auxiliary variables, X, and Z, are defined under the stratified random sampling, respectively, as:

$\bar{x}_{st} = \sum_{h=1}^{L} W_h\bar{x}_h$ and $\bar{z}_{st} = \sum_{h=1}^{L} W_h\bar{z}_h$,

where $\bar{x}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} x_{hi}$ and $\bar{z}_h = \frac{1}{n_h}\sum_{i=1}^{n_h} z_{hi}$ represent the sample means obtained from n_h values of the auxiliary variables X and Z, respectively, in the h^{th} stratum. Note that variances of the unbiased estimators, $\bar{y}_{st}$, $\bar{x}_{st}$, $\bar{z}_{st}$, are respectively given by

$$V(\bar{y}_{st}) = \sum_{h=1}^{L} W_h^2\gamma_h S_{yh}^2, \tag{1.1}$$

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$$V(\bar{x}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{xh}^2 , \tag{1.2}$$

$$V(\bar{z}_{st}) = \sum_{h=1}^{L} W_h^2 \gamma_h S_{zh}^2 \tag{1.3}$$

2. Estimators in Literature

In this section, we consider some estimators of the finite population mean in the literature. The variance and MSE equations of all these estimators are obtained up to the first order of approximation.

Hansen et al. (1946) envisaged a combined ratio estimator for the population mean $\bar{Y}$ as

$$\hat{\bar{Y}}_{RC} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right). \tag{2.1}$$

For negative correlation between the study and the auxiliary variables, the combined product estimator is defined as

$$\hat{\bar{Y}}_{PC} = \bar{y}_{st}\left(\frac{\bar{z}_{st}}{\bar{Z}}\right). \tag{2.2}$$

The biases and mean squared error equations of the combined ratio estimator $\hat{\bar{Y}}_{RC}$ in (2.1) and the combined product estimator $\hat{\bar{Y}}_{PC}$ in (2.2) are

$$B\left(\hat{\bar{Y}}_{RC}\right) = \frac{1}{\bar{X}}\sum_{h=1}^{L} W_h^2 \gamma_h (R_1 S_{xh}^2 - S_{yxh}), \tag{2.3}$$

$$B\left(\hat{\bar{Y}}_{PC}\right) = \frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h^2 \gamma_h (R_2^2 S_{zh}^2 + S_{yzh}) , \tag{2.4}$$

$$MSE\left(\hat{\bar{Y}}_{RC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R_1^2 S_{xh}^2 - 2R_1 S_{yxh}), \tag{2.5}$$

$$MSE\left(\hat{\bar{Y}}_{PC}\right) = \sum_{h=1}^{L} W_h^2 \gamma_h (S_{yh}^2 + R_2^2 S_{zh}^2 + 2R_2 S_{yzh}), \tag{2.6}$$

respectively, where $R_1 = \frac{\bar{Y}}{\bar{X}}$ and $R_2 = \frac{\bar{Y}}{\bar{Z}}$.

Bahl and Tuteja (1991) developed the ratio and product types of the exponential estimators for the population mean $\bar{Y}$ in the simple random sampling, respectively, as

$$\hat{\bar{Y}}_{Re} = \bar{y}\, exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right), \tag{2.7}$$

$$\hat{\bar{Y}}_{Pe} = \bar{y}\, exp\left(\frac{\bar{z} - \bar{Z}}{\bar{z} + \bar{Z}}\right), \tag{2.8}$$

Singh et al. (2008) adapted the estimators in (2.7) and (2.8) to the stratified random sampling as

$$\hat{\bar{Y}}_{Re}^{ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h - \bar{x}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h + \bar{x}_h)}\right], \tag{2.9}$$

$$\hat{\bar{Y}}_{Pe}^{ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}_h - \bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}_h + \bar{Z}_h)}\right]. \tag{2.10}$$

Tailor and Tailor (2012) used the Srivenkataramana (1980) transformation and obtained the dual of the estimators in (2.9) and (2.10) as

$$\hat{\bar{Y}}_{Re}^{*ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{x}^*_h - \bar{X}_h)}{\sum_{h=1}^{L} W_h(\bar{x}^*_h + \bar{X}_h)}\right], \tag{2.11}$$

$$\hat{\bar{Y}}_{Pe}^{*ST} = \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{Z}_h - \bar{z}^*_h)}{\sum_{h=1}^{L} W_h(\bar{Z}_h + \bar{z}^*_h)}\right], \tag{2.12}$$

where $\bar{x}^*_h = \frac{\bar{X}_h N_h - \bar{x}_h n_h}{N_h - n_h}$ and $\bar{z}^*_h = \frac{\bar{Z}_h N_h - \bar{z}_h n_h}{N_h - n_h}$ represent the dual estimators of the population means of $\bar{X}$ and $\bar{Z}$, respectively, in the h[th] stratum.

Singh (1967) utilized the information of the population mean of two auxiliary variables, $\bar{X}$ and $\bar{Z}$ and suggested the ratio-cum-product estimator for the population mean $\bar{Y}$ in the simple random sampling as

$$\hat{\bar{Y}}_{RP} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{z}}{\bar{Z}}\right). \tag{2.13}$$

Tailor and Tailor (2012) adapted the estimator in (2.13) to the stratified random sampling as

$$\hat{\bar{Y}}_{RP}^{ST} = \bar{y}_{st}\left(\frac{\bar{X}}{\bar{x}_{st}}\right)\left(\frac{\bar{z}_{st}}{\bar{Z}}\right). \tag{2.14}$$

Singh et al. (2009) defined the ratio-cum-product type of the exponential estimator in the simple random sampling as

$$\hat{\bar{Y}}_{RPe} = \bar{y}\, exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) exp\left(\frac{\bar{z}-\bar{Z}}{\bar{z}+\bar{Z}}\right). \tag{2.15}$$

Tailor and Chouhan (2013) adapted the estimator in (2.15) to the stratified random sampling as

$$\hat{\bar{Y}}_{RPe}^{ST} = \bar{y}_{st}\, exp\left(\frac{\bar{X}-\bar{x}_{st}}{\bar{X}+\bar{x}_{st}}\right) exp\left(\frac{\bar{z}_{st}-\bar{Z}}{\bar{z}_{st}+\bar{Z}}\right)$$
$$= \bar{y}_{st} exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h-\bar{x}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h+\bar{x}_h)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}_h-\bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}_h+\bar{Z}_h)}\right]. \tag{2.16}$$

3. Suggested Estimator

Using the transformation of $x^* = \frac{N\bar{X}-n\bar{x}}{N-n}$, Srivenkataramana (1980) obtained the dual version of the classical ratio estimator as

$$\hat{\bar{Y}}_R^* = \bar{y}\left(\frac{\bar{x}^*}{\bar{X}}\right). \tag{3.1}$$

Similarly, we perform the duality to the estimator in (2.16) by using the following transformations on the auxiliary variables, X and Z,

$$\bar{x}^*{}_h = \frac{N_h\bar{X}_h - n_h\bar{x}_h}{N_h-n_h},$$
$$\bar{z}^*{}_h = \frac{N_h\bar{Z}_h - n_h\bar{z}_h}{N_h-n_h}$$

and we develop the new estimator as follows:

$$\hat{\bar{Y}}_{Pro}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{X}_h-\bar{x}^*{}_h)}{\sum_{h=1}^{L} W_h(\bar{X}_h+\bar{x}^*{}_h)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h(\bar{z}^*{}_h-\bar{Z}_h)}{\sum_{h=1}^{L} W_h(\bar{z}^*{}_h+\bar{Z}_h)}\right] \tag{3.2}$$
$$= \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\bar{X}_h-\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\bar{X}_h+\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)-\sum_{h=1}^{L} W_h\bar{Z}_h}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)+\sum_{h=1}^{L} W_h\bar{Z}_h}\right]$$
$$= \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h+n_h\bar{x}_h-N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h+N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h+n_h\bar{Z}_h-N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{Z}_h-n_h\bar{Z}_h+N_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)}\right]$$
$$= \left(\sum_{h=1}^{L} W_h\bar{y}_h\right) exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{x}_h-n_h\bar{X}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h+N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}\right] exp\left[\frac{\sum_{h=1}^{L} W_h\left(\frac{n_h\bar{Z}_h-n_h\bar{z}_h}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{N_h\bar{X}_h-n_h\bar{x}_h+N_h\bar{X}_h-n_h\bar{x}_h}{N_h-n_h}\right)}\right]$$

To obtain the bias and mean squared error of the proposed estimator, $\hat{\bar{Y}}_{Pro}^{*ST}$, we use the following notations:
$\bar{y}_h = \bar{Y}_h(1+\varepsilon_{0h}), \bar{x}_h = \bar{X}_h(1+\varepsilon_{1h}), \bar{z}_h = \bar{Z}_h(1+\varepsilon_{2h}),$
such that $E(\varepsilon_{0h}) = E(\varepsilon_{1h}) = E(\varepsilon_{2h}) = 0, E(\varepsilon_{0h}^2) = \lambda_h C_{yh}^2, E(\varepsilon_{1h}^2) = \lambda_h C_{xh}^2, E(\varepsilon_{2h}^2) = \lambda_h C_{zh}^2,$
$E(\varepsilon_{0h}\varepsilon_{1h}) = \lambda_h\rho_{yxh}C_{yh}C_{xh}, E(\varepsilon_{0h}\varepsilon_{2h}) = \lambda_h\rho_{yzh}C_{yh}C_{zh}, E(\varepsilon_{1h}\varepsilon_{2h}) = \lambda_h\rho_{xzh}C_{xh}C_{zh}.$

Now, the proposed dual to ratio-cum-product type exponential estimator is expressed in terms of $e_i's$ as

$$\hat{\bar{Y}}_{Pro}^{*ST} = \left(\sum_{h=1}^{L} W_h\bar{Y}_h(1+\varepsilon_{0h})\right) exp\frac{\sum_{h=1}^{L} W_h\gamma_h(\bar{X}_h(1+\varepsilon_{1h})-\bar{X}_h))}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h-n_h(\bar{X}_h+\bar{X}_h(1+\varepsilon_{1h}))}{N_h-n_h}\right)} exp\frac{\sum_{h=1}^{L} W_h n_h\left(\frac{\bar{Z}_h-\bar{Z}_h(1+\varepsilon_{2h})}{N_h-n_h}\right)}{\sum_{h=1}^{L} W_h\left(\frac{2N_h\bar{X}_h-n_h(\bar{X}_h+\bar{X}_h(1+\varepsilon_{1h}))}{N_h-n_h}\right)}$$
$$= \bar{Y}\left(1+\frac{\sum_{h=1}^{L} W_h\bar{Y}_h\varepsilon_{0h}}{\bar{Y}}\right) exp\left[\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{2\bar{X}-\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}\right] exp\left[\frac{-\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{2\bar{Z}-\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}\right]$$
$$= \bar{Y}\left(1+\frac{\sum_{h=1}^{L} W_h\bar{Y}_h\varepsilon_{0h}}{\bar{Y}}\right) exp\left[\frac{\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{\bar{X}}}{2-\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\varepsilon_{1h}}{\bar{X}}}\right] exp\left[\frac{-\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{\bar{Z}}}{2-\frac{\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\varepsilon_{2h}}{\bar{Z}}}\right]$$

The proposed estimator in (3.2) can shortly be written in terms of $e_i's$ as

$$\hat{\bar{\bar{Y}}}^{*ST}_{Pro} = \bar{Y}(1+\varepsilon_0)exp\left(\frac{\varepsilon_1}{2-\varepsilon_1}\right)exp\left(\frac{-\varepsilon_2}{2-\varepsilon_2}\right), \quad (3.3)$$

where $\gamma_h = \frac{n_h}{N_h-n_h}$, $\varepsilon_0 = \frac{1}{\bar{Y}}\sum_{h=1}^{L} W_h\gamma_h\bar{Y}_h\,\varepsilon_{0h}$, $\varepsilon_1 = \frac{1}{\bar{X}}\sum_{h=1}^{L} W_h\gamma_h\bar{X}_h\,\varepsilon_{1h}$, $\varepsilon_2 = \frac{1}{\bar{Z}}\sum_{h=1}^{L} W_h\gamma_h\bar{Z}_h\,\varepsilon_{2h}$,

such that $E(\varepsilon_0) = E(\varepsilon_1) = E(\varepsilon_2) = 0, E(\varepsilon_0^2) = \frac{1}{\bar{Y}^2}\sum_{h=1}^{L}\lambda_h W_h S_{yh}^2$, $E(\varepsilon_1^2) = \frac{1}{\bar{X}^2}\sum_{h=1}^{L}\lambda_h W_h S_{xh}^2$,

$E(\varepsilon_2^2) = \frac{1}{\bar{Z}^2}\sum_{h=1}^{L}\lambda_h W_h S_{zh}^2$, $E(\varepsilon_0\varepsilon_1) = \frac{1}{\bar{Y}\bar{X}}\sum_{h=1}^{L}\lambda_h W_h S_{yxh}$, $E(\varepsilon_0\varepsilon_2) = \frac{1}{\bar{Y}\bar{Z}}\sum_{h=1}^{L}\lambda_h W_h S_{yzh}$,

$E(\varepsilon_1\varepsilon_2) = \frac{1}{\bar{Z}\bar{X}}\sum_{h=1}^{L}\lambda_h W_h S_{xzh}$.

Finally, using the exp function property, we can write (3.3) as

$$\hat{\bar{\bar{Y}}}^{*ST}_{Pro} = \bar{Y}(1+\varepsilon_0)\left(1+\frac{\varepsilon_1}{2}+\frac{\varepsilon_1^2}{4}+\frac{\varepsilon_1^2}{8}\right)\left(1-\frac{\varepsilon_2}{2}-\frac{\varepsilon_2^2}{4}+\frac{\varepsilon_2^2}{8}\right)$$

$$= \bar{Y}\left(1-\frac{\varepsilon_1}{2}-\frac{\varepsilon_1^2}{8}+\frac{\varepsilon_2}{2}-\frac{\varepsilon_1\varepsilon_2}{2}+\frac{3\varepsilon_2^2}{8}+\varepsilon_0-\frac{\varepsilon_0\varepsilon_1}{2}+\frac{\varepsilon_0\varepsilon_2}{2}\right),$$

$$\hat{\bar{\bar{Y}}}^{*ST}_{Pro} - \bar{Y} = \bar{Y}\left(\varepsilon_0-\frac{\varepsilon_1}{2}+\frac{\varepsilon_2}{2}-\frac{\varepsilon_1^2}{8}+\frac{3\varepsilon_2^2}{8}-\frac{\varepsilon_1\varepsilon_2}{2}-\frac{\varepsilon_0\varepsilon_1}{2}+\frac{\varepsilon_0\varepsilon_2}{2}\right). \quad (3.4)$$

From (3.4), up to the first order approximation, the bias and MSE of the proposed estimator are obtained as

$$B\left(\hat{\bar{\bar{Y}}}^{*ST}_{Pro}\right) = \bar{Y}\sum_{h=1}^{L} W_h{}^2\lambda_h\left[-\frac{1}{8}R_1S_{xh}^2+\frac{3}{8}\gamma_h^2S_{xh}^2-\frac{1}{2}\gamma_hS_{yxh}+\frac{1}{2}\gamma_hS_{yzh}+\frac{1}{2}\frac{\gamma_hS_{xzh}}{\bar{Z}}\right] \quad (3.5)$$

$$MSE\left(\hat{\bar{\bar{Y}}}^{*ST}_{Pro}\right) = \sum_{h=1}^{L} W_h{}^2\lambda_h\left[S_{yh}^2+\frac{\gamma_h^2}{4}\left[(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh})-\gamma_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right]\right] \quad (3.6)$$

4. Efficiency Comparisons

In this section, the efficiency conditions of the proposed estimator, with respect to the usual unbiased estimator, combined ratio and product estimators, dual to combined ratio and product estimators, Singh et al. (2008) ratio and product types of the exponential estimator, dual to Singh et al. (2008) estimators given by Tailor and Tailor (2012) and Tailor and Chouhan (2013) estimator, are obtained and these conditions show that the proposed dual to ratio cum product type of the exponential estimator for the population mean, $\hat{\bar{\bar{Y}}}^{*ST}_{Pro}$, is more efficient than

(i) $\bar{y}_{st}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{\gamma_h^2}{4}\left(R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh}-\gamma_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right)\right]<0 \quad (4.1)$$

(ii) $\hat{\bar{\bar{Y}}}_{RC}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{1}{4}R_1^2S_{xh}^2(\gamma_h^2-4)+\frac{\gamma_h^2}{4}(R_2^2S_{zh}^2-2R_1R_2S_{xzh})-R_1S_{yxh}(\gamma_h-2)+\gamma_hR_2S_{yzh}\right]<0 \quad (4.2)$$

(iii) $\hat{\bar{\bar{Y}}}_{PC}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{1}{4}R_2^2S_{zh}^2(\gamma_h^2-4)+\frac{\gamma_h^2}{4}(R_1^2S_{xh}^2-2R_1R_2S_{xzh})-R_2S_{yzh}(\gamma_h-2)+\gamma_hR_1S_{yxh}\right]<0 \quad (4.3)$$

(iv) $\hat{\bar{\bar{Y}}}^{*}_{RC}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{\gamma_h^2}{4}(-3R_1^2S_{xh}^2+R_2^2S_{zh}^2-2R_1R_2S_{xzh})+\gamma_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right]<0 \quad (4.4)$$

(v) $\hat{\bar{\bar{Y}}}^{*}_{PC}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{\gamma_h^2}{4}(R_1^2S_{xh}^2-3S_{zh}^2-2R_1R_2S_{xzh})-\gamma_h\left(R_1S_{yxh}+R_2S_{yzh}\right)\right]<0 \quad (4.5)$$

(vi) $\hat{\bar{Y}}^{ST}_{Re}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{1}{4}R_1^2S_{xh}^2(\gamma_h^2-1)+\frac{\gamma_h^2}{4}(R_2^2S_{zh}^2-2R_1R_2S_{xzh})-R_1S_{yxh}(\gamma_h-2)+\gamma_hR_2S_{yzh}\right]<0 \quad (4.6)$$

(vii) $\hat{\bar{Y}}^{ST}_{Pe}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{1}{4}R_2^2S_{zh}^2(\gamma_h^2-1)+\frac{\gamma_h^2}{4}(R_1^2S_{xh}^2-2R_1R_2S_{xzh})+R_2S_{yzh}(\gamma_h-2)+\gamma_{hR_1S_{yxh}}\right]<0 \quad (4.7)$$

(viii) $\hat{\bar{Y}}^{*ST}_{Re}$ if

$$\sum_{h=1}^{L} W_h{}^2\lambda_h\left[\frac{\gamma_h^2}{4}(R_2^2S_{zh}^2-2R_1R_2S_{xzh})-\gamma_hR_2S_{yzh}\right]<0 \quad (4.8)$$

(ix) $\hat{\bar{Y}}^{*ST}_{Pe}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\frac{\gamma_h^2}{4}\left(R_1^2 S_{xh}^2 - 2R_1 R_2 S_{xzh}\right) - \gamma_h R_1 S_{yxh}\right] < 0 \qquad (4.9)$$

(x) $\hat{\bar{Y}}_{PRe}^{*ST}$ if

$$\sum_{h=1}^{L} {W_h}^2 \lambda_h \left[\frac{1}{4}\left(R_1^2 S_{xh}^2 - 2R_1 R_2 S_{xzh}\right)(\gamma_h^2 - 1) - (\gamma_h - 1)\left(R_1 S_{yxh} - R_2 S_{yzh}\right)\right] < 0 \qquad (4.10)$$

5. Empirical Study

A data set is considered to see the efficiency of the proposed estimator with respect to the estimators in literature. Descriptive statistics of the population are given in Table 1.

Population [Source: Murthy (1967), p. 228]

Y : Output

X : Fixed capital

Z : Number of workers

Table 1. Descriptive Statistics of the Population

	Stratum I	Stratum II
N_h	5	5
n_h	2	2
$\bar{Y}_h$	1925	3115.60
$\bar{X}_h$	214.40	333.80
$\bar{Z}_h$	51.80	60.60
S_{yh}	615.92	340.38
S_{xh}	74.87	66.35
S_{zh}	0.75	4.84
S_{yxh}	39360.68	22356.50
S_{yzh}	-411.20	-1536.00
S_{xzh}	38.08	287.92

Table 2. PRE and MSE Values of the Estimators

Estimators	PRE	MSE
$\bar{y}_{st}$	100	74282.40
$\hat{\bar{Y}}_{RC}$	313.75	23675.66
$\hat{\bar{Y}}_{PC}$	116.32	63863.01
$\hat{\bar{Y}}_{RC}^{*}$	432.03	17193.81
$\hat{\bar{Y}}_{PC}^{*}$	123.74	60031.03
$\hat{\bar{Y}}_{Re}^{ST}$	173.94	42705.76
$\hat{\bar{Y}}_{Pe}^{ST}$	107.94	68818.23
$\hat{\bar{Y}}_{Re}^{*ST}$	234.96	31614.91
$\hat{\bar{Y}}_{Pe}^{*ST}$	111.95	66353.19
$\hat{\bar{Y}}_{RPe}^{*ST}$	244.86	30336.68
$\hat{\bar{Y}}_{Pro}^{*ST}$	**454.56**	**16341.60**

From Table 2, we clearly observe that the proposed estimator is the most efficient estimator for this data set. It is interesting that the classical combined ratio estimator is more efficient than the recent estimators in literature. However, the dual of the combined ratio estimator is more efficient than the classical combined ratio estimator.

6. Conclusion

We propose a dual ratio cum product type of the exponential estimator for the population mean under the Stratified Random Sampling design. It is shown that the proposed estimator can perform better than the existing estimator in theory and in application. It is surprisingly observed from Table 2 that the proposed estimator, which is the dual of $\hat{\bar{Y}}_{RPe}^{*ST}$ given by Tailor and Chouhan (2013), is more efficient than the dual of the combined ratio estimator although the combined ratio estimator is more efficient than the estimator of Tailor and Chouhan (2013). Then, we can say that the dual operation may enormously improve the efficiency to some estimators.

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