A Study on Ricci Solitons in almost $C(\lambda)$ Manifolds

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Abstract: We show that $C(\lambda)$ manifold is cone if the Ricci Solitons $(g, V, \lambda_1)$, $n \geq 3$ is expanding and $\tau$-curvature tensor is zero where $\tau$ is a generalized curvature tensor and consists of Riemannian, Conformal, quasi-conformal, Conharmonic, Concircular, Pseudoprojctive, Projective and M-Projective etc., curvature tensors. Also it is shown that Ricci Solitons of $C(\lambda)$ manifolds are shrinking when $C$–Bochner curvature tensor is Zero.

Keywords: Almost $C(\lambda)$ manifolds, $\tau$-curvature tensor, C-Bochner curvature tensor, $\eta$-Einstein

1 Introduction

In 1982 Hamilton [4] introduced an a excellent tool for simplifying the structure of manifolds which smooth out the topology of the manifolds and to make them more symmetric.

$$\frac{dg}{dt} = -2Ricg \quad \text{(1)}$$

known as Hemalton Ricci flow equation and this is nothing but one type of heat equation.

A Ricci soliton is a natural generalization of an Einstein metric which moves under the Ricci flow simply by diffeomorphism of the initial metric [10]. A Ricci soliton is a triple $(g, V, \lambda_1)$ with $g$ a Riemannian metric, $V$ a vector field and $\lambda_1$ a real scalar such that

$$\mathcal{L}_V g + 2S + 2\lambda_1 g = 0, \quad \text{(2)}$$

where $S$ is a Ricci tensor of $M$ and $\mathcal{L}_V$ denotes the Lie derivative operator along the vector field $V$. The Ricci soliton is said to be shrinking, steady and expanding according as $\lambda_1$ is negative, zero and positive respectively.

In 1981, D. Janssen and L. Vanhecke [11] introduced the notion of almost $C(\lambda)$ manifolds and they have neatly explained the different types of the manifolds depending on the value $\lambda$. Further many authors Z. Olszak, R. Rosca [18] and S. V. Kharitonava [15] studied the flatness of curvature tensors in $C(\lambda)$ manifolds and Ali Akbar [2] has obtained results on Ricci tensor and quasi conformal curvature tensor of $C(\lambda)$ manifolds. Further G. Zhen, J. L. Cabrerizo, L. M. Fernandez and M. Fernandez [12] have studied $\xi$ conharmonic flat generalized Sasakian space forms on $C(\lambda)$ manifolds. In this paper we study the Ricci solitons in $C(\lambda)$ manifolds using the flatness condition on $\tau$-curvature tensor, C-Bochner curvature tensor, $W_2$-curvature tensor, $\tilde{P}$ Pseudo Quasi conformal curvature tensor.

2 Preliminaries

Let $M$ be a $n$-dimensional connected differentiable manifold endowed with an almost contact metric structure $(\phi, \xi, \eta, g)$, where $\phi$ is a tensor field of type $(1,1)$, $\xi$ is a vector field, $\eta$ an 1-form and $g$ is a Riemannian metric on $M$ such that [5].

$$\eta(\xi) = 1, \quad \text{(3)}$$

$$\phi^2 = -I + \eta \otimes \xi, \quad \text{(4)}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \quad \text{(5)}$$

$$g(X, \xi) = \eta(X), \quad \phi \xi = 0, \quad \eta(\phi X) = 0. \quad \text{(7)}$$

In [11] D. Janssen and L. Vanhecke introduced the almost $C(\lambda)$ manifolds, where $\lambda$ is a real number. Further Z. Olszak, R. Rosca [18] and others investigated such manifolds.

Definition 21[11]: An almost $C(\lambda)$-Manifold $M$ is an almost contact manifold, if the Riemann curvature tensor...
satisfies the following property:
\[
R(X, Y, Z, W) = R(X, Y, φZ, φW) + λ[−g(X, Z)g(Y, W)] + g(X, W)g(Y, Z) + g(X, φZ)g(Y, φW) + g(X, φW)g(Y, φZ),
\]
(8)

\[
R(X, Y)Z = R(φX, φY)Z - λ[Xg(Y, Z) - g(X, Z)Y] - g(X, φW)g(Y, φZ),
\]
(10)

for a real number λ and X, Y, Z, W ∈ T(M).

**Definition 2[11]:** A normal almost C(λ)-manifold is called a C(λ) manifold. The authors [11] proved that cosymplectic, Sasakian, Kenmotsu manifolds are respectively C(0), C(1) and C(−1) manifolds. For Kenmotsu manifold the following holds
\[
(∇_X φ)Y = g(φX, Y)ξ - η(Y)φX.
\]
(13)

From (13), we have
\[
∇_X ξ = X - η(X)ξ.
\]
(14)

**Remark 1** Let (g, ξ, λ) be a Ricci soliton in an n-dimensional Kenmotsu manifold M. From (14) we have following identity
\[
(\mathcal{L}_ξ g)(X, Y) = 2[g(X, Y) - η(X)η(Y)].
\]
(15)

From (2) and (15), we get
\[
S(X, Y) = -(λ_1 + 1)g(X, Y) + η(X)η(Y).
\]
(16)

The above equation yields:
\[
QX = -(λ_1 + 1)X + η(X)ξ,
\]
(17)

\[
S(X, ξ) = -λ_1 η(X),
\]
(18)

\[
r = -λ_1 n - (n - 1),
\]
(19)

where S is the Ricci tensor, Q is the Ricci operator and r is the scalar curvature on M.

### 3 Ricci solitons on C(λ)-Manifolds with \(τ(X, Y)Z = 0\).

**Definition 3** The τ-curvature tensor [16] is given by
\[
τ(X, Y)Z = a_0R(X, Y, Z) - a_1S(X, Y, Z)X + a_2S(X, Y)Y + a_3S(X, Y)Z + a_4g(Y, Z)QX + a_5g(X, Z)QY + a_6g(X, Y)QZ + a_7[g(Y, Z)X - g(X, Z)Y],
\]
(20)

where \(a_0, … , a_7\) are some smooth functions on M. For different values of \(a_0, … , a_7\) the τ-curvature tensor reduces to the curvature tensor R, quasi-conformal curvature tensor, conformal curvature tensor, conharmonic curvature tensor, concircular curvature tensor, pseudo-projective curvature tensor, projective curvature tensor, \(M_i\)-projective curvature tensor, \(W_i\)-curvature tensors (\(i = 0, … , 9\), \(W_i\)-curvature tensors (\(j = 0, 1\)).

If τ curvature tensor vanishes identically then we say that the manifold is τ flat. Thus for a τ flat C(λ) manifold, we get
\[
a_0R(X, Y)Z = -a_1S(X, Y)X + a_2S(X, Y)Y + a_3S(X, Y)Z - a_4g(Y, Z)QX + a_5g(X, Z)QY + a_6g(X, Y)QZ - a_7[g(Y, Z)X - g(X, Z)Y],
\]
(21)

In view of (11) we have from (21)
\[
a_0R(φX, Y)Z = -a_0[g(Y, Z)X - g(X, Z)Y - g(φY, Z)φX + g(φX, Z)φY] - a_1S(Y, Z)X - a_2S(X, Z)Y - a_3S(X, Y)Z - a_4g(Y, Z)QX + a_5g(X, Z)QY - a_6g(X, Y)QZ - a_7[g(Y, Z)X - g(X, Z)Y],
\]
(22)

Take innerproduct with respect to ξ and Y = ξ in (22). By virtue of (7), (16), (17), (18) and on simplification, we get
\[
a_2S(X, Z) = [a_0 + λ_1 a_5 - a_7 r]g(X, Z)
\]
\[
+ [λ_1 a_1 - a_0 + λ_4 a_3 + λ_4 a_4 + λ_4 a_5]η(X)η(Z)
\]
(23)

Taking X = Z = \(e_i\) in (23) and summing over \{\(e_i\) : \(i = 1, 2, … , n\}). Then we get on simplification
\[
λ_1 = \frac{a_7 r(n + 1) - a_0(n + 1) + a_2 r}{a_5 n + a_1 + a_3 + a_4 + a_6},
\]
(24)

From the definition (31) we have the following:

The quasi conformal curvature tensor \(C\) if
\[
a_1 = -a_2 = a_4 = -a_5 = a_6 = 0,
\]
\[
a_7 = \frac{1}{(n - 1)(n - 2)},
\]

The conformal curvature tensor C if
\[
a_0 = 1, a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n - 2},
\]
\[
a_3 = a_6 = 0, a_7 = \frac{1}{(n - 1)(n - 2)},
\]

The conharmonic curvature tensor N if
\[
a_0 = 1, a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{n(n - 1)},
\]
\[
a_3 = a_6 = 0 = a_7 = 0,
\]

The concircular curvature tensor if
\[
a_0 = 1, a_1 = a_2 = a_3 = a_4 = a_5 = a_6 = 0,
\]
\[
a_7 = -\frac{1}{n(n - 1)},
\]

The pseudo-projective curvature tensor \(\tilde{P}\) if
\[
a_1 = -a_2, a_3 = a_4 = a_5 = a_6 = 0,
\]
\[
\alpha_7 = -\frac{1}{n} \left( \frac{a_0}{n-1} + a_1 \right),
\]
The projective curvature tensor \( P \) if
\[
a_0 = 1, a_1 = -a_2 = -\frac{1}{(n-1)},
\]
\[
a_3 = a_4 = a_5 = a_6 = a_7 = 0,
\]
The \( M \)-projective curvature tensor if
\[
a_0 = 1, a_3 = a_6 = a_7 = 0,
\]
\[
a_1 = -a_2 = a_4 = -a_5 = -\frac{1}{2(n-1)},
\]
The \( W_0 \)-curvature tensor if
\[
a_0 = 1, a_2 = a_3 = a_4 = a_6 = 0 = a_7 = 0,
\]
\[
a_1 = -a_5 = -\frac{1}{n-1},
\]
The \( W'_0 \)-curvature tensor if
\[
a_0 = 1, a_2 = a_3 = a_4 = a_5 = a_6 = 0 = a_7 = 0,
\]
\[
a_1 = -a_5 = \frac{1}{n-1},
\]
The \( W_1 \)-curvature tensor if
\[
a_0 = 1, a_3 = a_4 = a_5 = a_6 = a_7 = 0,
\]
\[
a_1 = -a_2 = \frac{1}{n-1},
\]
The \( W'_1 \)-curvature tensor if
\[
a_0 = 1, a_3 = a_4 = a_5 = a_6 = 0 = a_7 = 0,
\]
\[
a_1 = -a_2 = -\frac{1}{n-1},
\]
The \( W_2 \)-curvature tensor if
\[
a_0 = 1, a_1 = a_2 = a_3 = a_6 = a_7 = 0,
\]
\[
a_4 = -a_5 = -\frac{1}{n-1},
\]
The \( W_3 \)-curvature tensor if
\[
a_0 = 1, a_1 = a_3 = a_5 = a_6 = a_7 = 0,
\]
\[
a_2 = -a_4 = -\frac{1}{n-1},
\]
The \( W_4 \)-curvature tensor if
\[
a_0 = 1, a_1 = a_2 = a_3 = a_4 = a_7 = 0,
\]
\[
a_5 = -a_6 = \frac{1}{n-1},
\]
The \( W_5 \)-curvature tensor if
\[
a_0 = 1, a_1 = a_3 = a_4 = a_6 = a_7 = 0,
\]
\[
a_2 = -a_5 = -\frac{1}{n-1},
\]
The \( W_6 \)-curvature tensor if
\[
a_0 = 1, a_2 = a_3 = a_4 = a_5 = a_6 = a_7 = 0,
\]
\[
a_1 = -a_6 = -\frac{1}{n-1},
\]
The \( W_7 \)-curvature tensor if
\[
a_0 = 1, a_2 = a_3 = a_5 = a_6 = a_7 = 0,
\]
\[
a_1 = -a_4 = -\frac{1}{n-1},
\]
The \( W_8 \)-curvature tensor if
\[
a_0 = 1, a_1 = a_2 = a_5 = a_6 = a_7 = 0,
\]
\[
a_3 = -a_4 = \frac{1}{n-1},
\]
Also we have the following table by virtue of (24) and flat curvature tensors:

<table>
<thead>
<tr>
<th>Curvature Tensor</th>
<th>( \lambda_3 )</th>
<th>Ricci soliton</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{C} = 0 )</td>
<td>( \lambda_3 = -\frac{\left(n+1\right)\left(n^2+1\right)}{2n(n-1)} )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( C = 0 )</td>
<td>( \lambda_3 = \frac{\left(n^2+1\right)}{2n(n-1)} )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( N = 0 )</td>
<td>( \lambda_3 = \frac{1}{n-1} )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( F = 0 )</td>
<td>( \lambda_3 = \frac{-\left(n^2+1\right)}{2n(n-1)} )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W = 0 )</td>
<td>( \lambda_3 = -\frac{1}{n-1} )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_1 = 0 )</td>
<td>( \lambda_3 = \frac{n-1}{n-2} )</td>
<td>Expanding</td>
</tr>
<tr>
<td>( W_2 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_3 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_4 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_5 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_6 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
<tr>
<td>( W_7 = 0 )</td>
<td>( \lambda_3 = (n-1) )</td>
<td>Shrinking</td>
</tr>
</tbody>
</table>

We use the following results:

**Definition 3.1.** [8] **Asymptotic curvature ratio:**

The asymptotic curvature ratio of a complete noncompact Riemannian manifold \( (M^n, g) \) is defined by

\[
A(g) := \limsup_{r \to +\infty} r_p(x)^2 |Rm(g)(x)|.
\]
Noted that it is well-defined since it does not depend on the reference point \( p \in M^n \). Moreover, it is invariant under scalings. This geometric invariant has generated a lot of interest: See for example for a static study of the asymptotic curvature ratio and linking this invariant with the Ricci flow. Note also that Gromov and Lott–Shen have shown that any paracompact manifolds can support a complete metric \( g \) with finite \( A(g) \). Therefore, the only geometric constraint is the Ricci solitons structure.

**Theorem 3.1.** [Cone structure at infinity] Let \((M^n, g, \nabla_f)\), \(n \geq 3\), be a complete expanding gradient Ricci soliton with finite \( A(g) \).

For \( p \in M^n \), \( (M^n, t^{-2}g, p) \), Gromov-Hausdorff converges to a metric cone \((C(S_m), d_m, x_m)\) over a compact length space \( S_m \). Moreover,

1. \( C(S_m) \setminus \{x_m\} \) is a smooth manifold with a \( C^1, \alpha \) metric \( g_m \) compatible with \( d_m \) and the convergence is \( C^1, \alpha \) outside the apex \( x_m \).
2. \((S_m, g_{s_m})\) where \( g_{s_m} \) is the metric induced by \( g_m \) on \( S_m \), is the \( C^1, \alpha \) limit of the rescaled levels of the potential function \( f \).

\[
(f^{-1}(t^2), t^{-2}g_{r/4}) \quad \text{where} \quad g_{r/4} \quad \text{is the metric induced by} \quad g \quad \text{on} \quad f^{-1}(r^2/4).
\]

Finally we can ensure that

\[
|K_{g_{s_m}}| \leq A(g) \text{ in Alexandrov sense} \quad (26)
\]

\[
\frac{\text{Vol}(A_m, g_{s_m})}{n} = \lim_{r \to +\infty} \frac{\text{Vol}(q, r)}{r^n}, \quad q \in M^n \quad (27)
\]

As direct consequence of Theorem 3, in case of vanishing asymptotic curvature ratio, we get the following:

**Corollary 3.2.** [Asymptotically flatness]. Let \((M^n, g, \nabla_f)\), \(n \geq 3\), be a complete expanding gradient Ricci soliton. Assume

\[
A(g) = 0.
\]

Then, with the notations of Theorem 3, \((S_m, g_{s_m}) = \cap_i I(S^n - 1/G_i, g_{s_i})\) and \((C(S_m), d_m, x_m) = (C(S_m), \text{eucl}, 0)\) where \( G_i \) are finite groups of Euclidean isometries and \( |I| \) is the (finite) number of ends of \( M^n \).

Moreover, for \( p \in M^n \),

\[
\sum \frac{\omega_n}{|G_i|} = \lim_{r \to +\infty} \frac{\text{Vol}(p, r)}{r^n} \quad (28)
\]

where \( \omega_n \) is the volume of the unit Euclidean ball.

From (24), (25), Theorem 3 and corollary 3 we have

**Theorem 3.3.** If the Ricci soliton \((g, V, \lambda_1)\), \(n \geq 3\) is for zero \( \tau \)-curvature expanding at \( \infty \) then it has cone structure at \( \infty \), provided asymptotic curvature \( A(G) \) is finite or otherwise it is asymptotically flat.

**Remark 3.4.** The independent calculations for different curvature tensors which are particular cases of \( \tau \)-curvature tensor will yield the same results of Theorem 3.

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**4 Ricci solitons on \( C(\lambda) \)-Manifolds with \( B(X, Y)Z = 0 \).**

S. Bochner introduced a Kähler analogue of the Weyl conformal curvature tensor by purely formal considerations, which is now well known as the Bochner curvature tensor [7]. A geometric meaning of the Bochner curvature tensor is given by D.E. Blair in [6] by using the Boothby-Wang’s fibration. In 1969, Matsumoto and Chuman [14] constructed the notion of C-Bochner curvature tensor in a Sasakian manifold and studied its several properties.

The C-Bochner curvature tensor [13] \( B \) in \( M \) is defined by

\[
\begin{align*}
B(X, Y)Z &= R(X, Y)Z + \frac{1}{n+3} g(X, Z)QY - S(Y, Z)X \\
& \quad - g(Y, Z)QX + S(X, Z)Y + g(\phi X, Z)\phi Y \\
& \quad - S(\phi Y, Z)\phi X - g(\phi Y, Z)Q\phi X + S(\phi X, Z)\phi Y \\
& \quad + 2S(\phi X, Y)\phi Z + 2g(\phi X, Y)Q\phi Z + \eta(Y)\eta(Z)QX \\
& \quad - \eta(Y)S(X, Z)\xi - \eta(X)S(Y, Z)\xi - \eta(X)\eta(Z)QY \\
& \quad - \frac{D}{n+3} g(X, Z)\phi Y - (\eta(Y)\eta(Z)QX) \\
& \quad + 2\eta(Y)g(X, Z)\xi - \eta(Y)\eta(Z)X \\
& \quad + \eta(X)\eta(Z)Y - \eta(X)g(Y, Z)\xi \\
& \quad - \frac{D-4}{n+3} g(X, Z)Y - g(Y, Z)X,
\end{align*}
\]

where \( D = \frac{n+3}{n+1} \).

If \( B \) vanishes identically then we say that the manifold is C-Bochnerly flat. Thus for a C-Bochnerly flat \( C(\lambda) \) manifold, we get

\[
\begin{align*}
R(X, Y)Z &= -\frac{1}{n+3} [g(X, Z)QY - S(Y, Z)X - g(Y, Z)QX \\
& \quad + S(X, Z)Y + g(\phi X, Z)\phi Y - S(\phi Y, Z)\phi X \\
& \quad - g(\phi Y, Z)Q\phi X + S(\phi X, Z)\phi Y + 2S(\phi X, Y)\phi Z \\
& \quad + 2g(\phi X, Y)Q\phi Z + \eta(Y)\eta(Z)QX - \eta(Y)S(X, Z)\xi \\
& \quad + \eta(X)S(Y, Z)\xi - \eta(X)\eta(Z)QY \\
& \quad + \frac{D}{n+3} [g(X, Z)\phi Y - (\eta(Y)\eta(Z)QX) \\
& \quad + 2\eta(Y)g(X, Z)\xi - \eta(Y)\eta(Z)X \\
& \quad + \eta(X)\eta(Z)Y - \eta(X)g(Y, Z)\xi \\
& \quad + \frac{D-4}{n+3} g(X, Z)Y - g(Y, Z)X],
\end{align*}
\]
In view of (11) we get from (30)\[R(\phi X, \phi Y)Z = g(X, Z)Y - Xg(Y, Z) + \phi X g(\phi Y, Z)\]
\[- g(\phi X, Z)\phi Y - \frac{1}{n+3}g(X, Z)QY\]
\[- S(Y, Z)X - g(Y, Z)QX + S(X, Z)Y\]
\[+ g(\phi X, Z)Q\phi Y - S(\phi Y, Z)\phi X - g(\phi Y, Z)Q\phi X\]
\[+ S(\phi X, Z)\phi Y + 2S(\phi X, Y)\phi Z + 2g(\phi X, Y)\phi Z\]
\[+ \eta(Y)\eta(X)QX - \eta(Y)S(X, Z)\xi + \eta(X)S(Y, Z)\xi\]
\[+ \eta(X)\eta(Z)QY\]
\[+ \frac{D+n-1}{n+3}[g(\phi X, Z)\phi Y - g(\phi Y, Z)\phi X\]
\[+ 2 g(\phi X, Y)\phi Z\]
\[= \frac{D}{n+3}[\eta(Y)g(X, Z)\xi - \eta(Y)\eta(Z)X\]
\[+ \eta(X)\eta(Z)Y - \eta(X)g(Y, Z)\xi]\]
\[+ \frac{D-4}{n+3}[g(X, Z)Y - g(Y, Z)X] \tag{31}\]

Taking inner product with respect to $\xi$ and $Y = \xi$ in (31). By virtue of (7) (16), (17), (18) and on simplification, we get
\[\left[1 + \frac{D-4}{n+3} - \frac{D}{n+3} + \frac{1}{n+3}\right] g(X, Z) - \eta(X)\eta(Z) = 0 \tag{32}\]

Taking $X = Z = e_i$ in (32) and summing over $i = 1, 2, \ldots, n$. Then we get
\[\left[1 + \frac{D-4}{n+3} - \frac{D}{n+3} + \frac{1}{n+3}\right] (n-1) = 0 \tag{33}\]

On simplification, we get
\[\lambda_1 = -(n-1) \tag{34}\]

Thus we can state the following:

**Theorem 4.1.** A Ricci soliton in $C(\lambda)$-manifolds satisfying $B = 0$ is shrinking.

### 5 Conclusion

We use concept of asymptotic curvature $A(G)$ and results on cone structure at $\infty$ of an expanding gradient Ricci Soliton of [8]. It is shown that $C(\lambda)$-manifold looks like cone at $\infty$ provided asymptotic curvature $A(G)$ is finite and $\tau$-curvature is zero. If $A(G)$ is not finite at $\infty$ then $C(\lambda)$ is asymptotically flat. Further it is shown that Ricci Soliton of $C(\lambda)$ manifolds is shrinking, when $B = 0$.

### References


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