MCDM based on Reciprocal Judgment Matrix: a Comparative Study of E-VIKOR and E-TOPSIS Algorithmic Methods with Interval Numbers

Yajie Dou∗, Pengle Zhang, Jiang Jiang, Kewei Yang and Yingwu Chen

College of Information System and Management, National University of Defense Technology, Changsha, Hunan, 410073, P. R. China

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Abstract: Interval number is a useful tool to handle the uncertainty brought by human factors in multi-criteria decision-making (MCDM) process. When faced with MCDM problems in real-world, the experts may be uncomfortable giving precise upper and lower bounds of the interval ratios on multi-criteria. Based on the reciprocal judgment matrices given by the experts through pairwise comparisons, the satisfaction degree of the multiple alternatives on single criterion is defined and the interval ratios was elicited by a linear programming model. The TOPSIS and VIKOR methods are extended with interval number and algorithmic E-VIKOR and E-TOPSIS methods are proposed. Finally, a numerical example of ranking indirect-fire weapon system alternative is given and a comparative experimental study is carried out based on the experts reciprocal judgment matrices generated by Monte Carlo simulation. The result illustrates the feasibilities and distinctive features of the two algorithmic methods.

Keywords: Multi-criteria decision-making (MCDM), reciprocal judgment matrix, E-VIORKR, E-TOPSIS, interval numbers

1 Introduction

Multi-Criteria Decision-Making (MCDM), also called as Multi-Criteria analysis, is often applied in the decision-making with multiple objectives in the field of Operations Research [1,2,3]. Especially, these objectives are conflicting with each other under the preference structure supplied by the decision-maker. The fundamentals of MCDM can be described as follows [1]: (1) construction of evaluation criteria related with decision-making goals for the alternatives; (2) generation of alternatives for achieving the decision-making goals; (3) computation of the alternatives value by the value functions for multiple criteria; (4) application of a normalized Multi-Criteria Analysis (MAC) methods; (5) searching the optimal alternative as the final decision-making result; (6) if the final alternative is unacceptable, multi-criteria optimization process is necessary to be carried out.

There are many literatures correlate with the MCDM problems and the techniques for these problems in various applications [1,2,3,4,5,6,7,8,9]. These methods were divided into four categories by Guitoni and Martel given as follows [4,5]: (1) elementary approaches, including Lexicographic method, weighted sum, Disjunctive method, Conjunctive methods and Maxi-min method; (2) the single synthesizing criterion methods, including multi-attribute value theory (MAVT),TOPSIS, simple multi-attribute rating technique (SMART),UTA (utility theory additive), MAUT (multi-attribute utility theory), EVAMIX,AHP (analytical hierarchy process), Fuzzy maxi-min and Fuzzy weighted sum; (3) the outranking synthesizing methods, including PROMETHEE, ELECTRE, ORESTE,MELCHIOR and REGIME; and (4) the mixed methods, including Fuzzy conjunctive method , Fuzzy disjunctive method, QUALIFLEX and Martel and Zaras method.

As a commonly used classical MCDM method with cardinal information, TOPSIS accounts for a ratio scale on the multiple criteria given by the experts as AHP matrix [6]. In TOPSIS, the importance weights of multiple criteria and the judgment ratios of alternatives under multiple criteria are given by crisp values, and both weights and ratios are normalized into indices without dimensions for the consequential aggregation [1]. The main principle of TOPSIS is that the optimal alternatives
should have the shortest distance from the positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS), and from the distances of which the preference order ranking of the alternatives was derived for the final decision-making [2]. In the recent research, TOPSIS method is widely used in various fields, including material selection [7], energy project [4], and supply chain management [8], and achieved a lot. However, an obvious drawback of TOPSIS is that it only focuses on the distances of the criteria values from the PIS and NIS without the relative importance of these distances. As one feasible and applicable method to implement within MCDM, the VIKOR approach was introduced [2] for multi-criteria optimization problem of complex alternatives and received a broad acceptance. Based on conflicting and different dimensions criteria, VIKOR method compares the closeness of all the alternatives with ideal alternative and performs a compromise ranking with mutual concessions.

Because of the different normalization methods and different aggregation functions used by the TOPSIS and VIKOR methods, a detailed and in-depth comparative analysis of the original TOPSIS and VIKOR was carried out by Opricovic and Tzeng [1]. Besides, TOPSIS and VIKOR are examined as two different MCDM methods by Reza Raei for some observation data samples from Tehran Stock Exchange to search for an appropriate alternative [9]. Besides, the E-VIKOR method with TOPSIS method was developed to help the decision-maker decide the optimal projects developmental strategy [10]. For TOPSIS and VIKOR method, the importance weights of multiple criteria and the judgment ratios of alternatives under multiple criteria are difficult to be given by crisp values for the experts when little information for judgment is available. Handling the uncertainty by interval number is receiving considerable attention by the recent researchers. The Extended VIKOR method for decision making problem with interval numbers by M. K. Sayadi [11] is compared with the extended TOPSIS method proposed by Jahanshahloo [3] in obtaining the compromise solution.

However, in fact, Camerer and Weber [12] suggest that an expert may be uncomfortable giving such precise upper and lower bounds of the interval ratios on multi-criteria. Yager and Kreinovich proposed [13] a formulation to obtain the upper and lower bounds of the interval ratios in a statistical method. Moreover, Guo [14] studied the Linear programming model for estimating and combining interval ratios based on pairwise subjective comparisons of the possibilities of the events.

The contributions of this paper are summarized as follows: based on the reciprocal judgment matrices given by the experts with regard to the satisfaction degree of the multiple alternatives on single criterion, the interval ratios are elicited by a linear programming model. The TOPSIS and VIKOR methods are extend for MCDM problems with interval number and algorithmic E-VIKOR and E-TOPSIS methods are examined by the experts reciprocal judgment matrices generated by Monte Carlo simulation.

The rest of the paper is organized as follows. Section 2 presents the elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons. The extended VIKOR with interval numbers for MCDM, E-VIKOR and its algorithmic process are given in Section 3. The extended TOPSIS with interval numbers for MCDM, E-TOPSIS and its algorithmic process are given in Section 4. An illustrative numerical example examines the two algorithmic methods and clarifies the main experimental results. Conclusions and future work are drawn in Section 6.

2 Elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons

In this section, Alternative Satisfaction Level (ASL) is proposed to indicate the comparative satisfaction level of the alternative with multi-criteria, which has two components: an alternative and a certain criterion, its expression is given as a function with two parameters.

**Definition 1.** Alternative Satisfaction Level (ASL), denoted as ASL(S_i, C_u), is an index indicating the judgment ratio of an alternative, S_i, on a criterion, C_u, when the alternative is checked and measured in the choosing process by the experts.

Because of the unavoidable uncertainty of the prediction with limited information, ASL(S_i, C_u) is an estimated value, based on the experiential knowledge of experts. It can be denoted as an interval number as follows:

\[
\text{ASL}(S_i, C_u) = [L_i^C_u, U_i^C_u]
\]

where the upper and lower bounds of ASL(S_i, C_u) are \(L_i^C_u\) and \(U_i^C_u\), restricted by the following inequalities:

\[
0 \leq L_i^C_u \leq U_i^C_u \leq 1
\]

It is clear that ASL(S_i, C_u) \(\in [0, 1]\). If \(L_i^C_u = U_i^C_u\), ASL(S_i, C_u) degenerates into a real number. And the center and width of the interval probability, ASL(S_i, C_u) are respectively defined as follows [15]:

\[
m(\text{ASL}(S_i, C_u)) = \frac{1}{2} (U_i^C_u + L_i^C_u)
\]

\[
w(\text{ASL}(S_i, C_u)) = U_i^C_u - L_i^C_u
\]

Considering that there are \(k (k \leq m)\) alternatives in a set of candidate alternatives \(S' = \{S_t, t = 1, 2, \dots, m\}\), having the corresponding functions to satisfy a certain criterion, C_u. For all the k alternatives, there are interval probability sets ASL(S, C_u)\(^1\) containing k elements.
The sufficient condition: If Definition 2 holds, that is,
\[ \forall k \in [L^C_{t-}, L^C_t], \]
where \( k \) represent all possible candidates of \( L^C_{t-}, L^C_t \) for the feasible solution. Therefore, Definition 2 is then given as follows [14, 16, 17]:

**Definition 2.** For \( \forall L^C_t \in [L^C_{t-}, L^C_t] \), there is an equation, \( \frac{L^C}{t} = 1 \).

**Theorem 1.** The interval set \( ASL(S, C_u)^* \) satisfies Definition 5 if and only if, the following conditions hold [14, 18]:

\[
\sum_{j=1}^{k} L^C_{t_j} \leq 1 \]

Then we have
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]

Since
\[
L^C_{t_1} \in [L^C_{t_1-}, L^C_{t_1+}] \]

It is easy to check and see that
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]
\[
L^C_{t_1} + L^C_{t_2} + \ldots + L^C_{t_k} = \leq 1, \forall t = 1, 2, \ldots, k, \]

This proves that Definition 2 is a sufficient condition of Theorem 1.

The necessary condition: If Theorem 1 holds, According to Eq. (1)
\[
L^C_{t_1} \leq L^C_{t_2} \leq L^C_{t_3} \]

Then we have [14]
\[
\forall L^C_t \in [L^C_{t-}, L^C_t], (10) \]

Clearly, there exists \( L^C_{t_i} \leq L^C_{t_j} \leq L^C_{t_k} \), \( h \in [1, 2, \ldots, k] \), \( h \neq t \), which satisfies the above conditions. So,
\[
\sum_{i=1}^{k} L^C_t \leq (k-1)(L^C_{t_i} + L^C_{t_j} + \ldots + L^C_{t_k}) \leq k \]

Which can be translated into the following:
\[
\sum_{i=1}^{k} L^C_t (k-1) = k - \sum_{i=1}^{k} L^C_t \leq k \]

This proves that Definition 2 is a sufficient condition of Theorem 1.

**Definition 3.** The First-Ignorance of \( ASL(S, C_u)^* \) denoted as \( I^1(ASL(S, C_u)^*) \), is defined by the sum of the width of the intervals as follows [14, 18]:
\[
I^1(ASL(S, C_u)^*) = \frac{1}{k} \sum_{i=1}^{k} w(ASL(S, C_u)) \]

Similar definitions and theorems have been used in the literature [16, 17] as the constraints and operations of the interval probability. When there is little information available for the experts to predict \( SSL(S, C_u) \), a precise estimation of \( L^C_{t_i} \) and \( L^C_{t_j} \) is difficult to achieve. In fact, an expert is more comfortable stating a personal preference towards a set of alternatives by means of pairwise comparison and determining which one has more possibility to satisfy a certain criterion. Wang [19] introduces a goal programming model to obtain interval weights from imprecise preference in MCDA. Guo [14] elicits the interval-valued probabilities, based on a linear and...
quadratic programming model from subjective pairwise comparisons for the likelihood among several events. The First ignorance model based on linear programming approach is employed in this study to minimize the imprecision of pairwise comparisons.

Considering the pairwise comparison process for each pair of candidate alternatives in a finite set \( S = \{ S_t, t = 1, 2, \ldots, m \} \), the possible judgment score from experts on alternative \( S_t \) and \( S_h \) (\( t, h \in N^+, t \neq h \)) is denoted as \( a_{th} \), which is an integer number \( [1, 9] \).

Next, we have a \( k \times k \) comparison matrix \( A(C_u) \) with regard to \( k \) alternatives, which can satisfy a certain criterion, \( C_u \), requirement as follows \([14, 18]\):

\[
A(c_{ij})_{k \times k} = \begin{bmatrix}
1 & a_{12} & \cdots & a_{1k} \\
a_{21} & 1 & \cdots & a_{2k} \\
\vdots & \vdots & \ddots & \vdots \\
a_{k1} & a_{k2} & \cdots & 1
\end{bmatrix}
\]  

(16)

Assumption 2. Even if there are multiple experts, there is only one \( k \times k \) comparison matrix \( A(C_u) \) for a certain criterion, \( C_u \), which represents the integrated preference information after a conference discussion.

The interval ratio \( ASL(S_t, C_u)/ASL(S_h, C_u) \) can be calculated by interval arithmetic as follows \([14, 20]\):

\[
\text{ASL}(S_t, C_u)/\text{ASL}(S_h, C_u) = [L_t^{C_u}/L_h^{C_u}, L_t^{C_u} + L_h^{C_u}]/L_h^{C_u}
\]  

(17)

Assumption 3. \([14, 20, 21]\) the given pairwise comparison \( a_{th} \) should belong to the estimated interval ratio \( ASL(S_t, C_u)/ASL(S_h, C_u) \), that is

\[
a_{th} \in [L_t^{C_u}/L_h^{C_u}, L_t^{C_u} + L_h^{C_u}]/L_h^{C_u}
\]  

(18)

\[
\Leftrightarrow L_t^{C_u} - a_{th}L_h^{C_u} \leq 0 \quad \forall h \neq t \quad \text{(19)}
\]

\[
\Leftrightarrow \begin{cases} 
L_t^{C_u} - a_{th}L_h^{C_u} \leq 0, \\
L_t^{C_u} + a_{th}L_h^{C_u} \geq 0, \\
L_t^{C_u} \geq \epsilon
\end{cases}
\]  

(20)

where \( \epsilon \) is a very small positive real number.

To determine the interval set \( ASL(S,C_u) \) with the smallest First ignorance (Definition 3), obtaining the interval probabilities from the expert opinion can be derived by the following optimization model \([14, 18]\):

\[
\min \ L_t^{C_u} \text{ I}^1(ASL(S, C_u^*)) = \frac{k}{k} \text{ } \sum_{i=1}^{k} (L_t^{C_u^*} - L_h^{C_u^*})
\]  

s.t. \( L_t^{C_u^*} + L_t^{C_u^*} + \ldots + L_t^{C_u^*} + L_t^{C_u^*} + \ldots + L_t^{C_u^*} \leq 1 \)

\forall t = 1, 2, \ldots, k,

\( L_t^{C_u^*} + L_t^{C_u^*} + \ldots + L_t^{C_u^*} + L_t^{C_u^*} + \ldots + L_t^{C_u^*} \geq 1 \)

\forall t = 1, 2, \ldots, k,

\( L_t^{C_u^*} - a_{th}L_h^{C_u^*} \leq 0 \quad \forall (t, h = 1, 2, \ldots, k, h > t),

\( L_t^{C_u^*} - a_{th}L_h^{C_u^*} \geq 0 \quad \forall (t, h = 1, 2, \ldots, k, h > t),

\( L_t^{C_u^*} \geq \epsilon \quad \forall t = 1, 2, \ldots, k,

\( L_t^{C_u^*} - L_h^{C_u^*} \geq 0 \quad \forall t = 1, 2, \ldots, k.

(21)

3 The algorithmic E-VIKOR method with Interval numbers for MCDM

The interval numbers are often considered as a useful tool when determining the precise values of the criteria is of

Fig. 1: The possible judgment score axes and the explanations of the scores
difficulty or impossibility. Therefore, the fundamental of the extended VIKOR with interval numbers for solving the MCDM problems has been studied by some researchers [2, 10, 11]. We extend the recent methods to E-VIKOR with the possibility degree of interval numbers. At first, we assume that a ratios matrix on multiple criteria with interval numbers is formulated as:

\[ S_1 \begin{bmatrix} f^U_{11} & f^L_{11} \\ f^U_{12} & f^L_{12} \\ \vdots & \vdots \\ f^U_{1n} & f^L_{1n} \end{bmatrix} \quad \ldots \quad S_m \begin{bmatrix} f^U_{m1} & f^L_{m1} \\ f^U_{m2} & f^L_{m2} \\ \vdots & \vdots \\ f^U_{mn} & f^L_{mn} \end{bmatrix} \]

(22)

where \( S_1, S_2, \ldots, S_m \) are candidate alternatives for decision makers to choose, \( C_1, C_2, \ldots, C_n \) are multiple criteria with which all the alternatives performance can be measured, \( f_{ij}^L \) is the ratio of an alternative \( S_i \) with respect to criterion \( C_j \) and its upper and lower bounds are \( f_{ij}^U \) and \( f_{ij}^L \). \( w_j \) is the weight of criterion \( C_j \).

The algorithmic E-VIKOR method with interval numbers is comprised of the following steps [2, 10]:

**Step 1:** Identification of the PIS and NIS.

\[ S^* = \{ f^*_i, \ldots, f^*_n \} = \{ (\max_i f^U_{ij} | j \in I) \text{ or } (\min_i f^L_{ij} | j \in J) \} \]

(23a)

\[ S^- = \{ f^-_i, \ldots, f^-_n \} = \{ (\min_i f^U_{ij} | j \in I) \text{ or } (\max_i f^L_{ij} | j \in J) \} \]

(23b)

where the criteria are divided into two types, benefit criteria and cost criteria, which are indicated by index \( I \) and \( J \), respectively. It is clearly that \( S^* \) is the PIS and \( S^- \) represents NIS.

**Step 2:** Calculation of intervals \([R^U_i, R^L_i]\) and \([x^U_i, x^L_i]\), \( i = 1, 2, \ldots, m \).

\[ R^U_i = \max \left\{ w_j \left( f^U_{ij} - f^L_{ij} \right) \right\} \quad | j \in I, w_j \left( f^U_{ij} - f^L_{ij} \right) \right\} \quad | j \in J \]

(24a)

\[ R^L_i = \max \left\{ w_j \left( f^U_{ij} - f^L_{ij} \right) \right\} \quad | j \in I, w_j \left( f^U_{ij} - f^L_{ij} \right) \right\} \quad | j \in J \]

(24b)

\[ x^U_i = \sum_{j \in J} w_j \left( f^U_{ij} - f^L_{ij} \right) + \sum_{j \in J} w_j \left( f^L_{ij} - f^U_{ij} \right) \]

(25a)

\[ x^L_i = \sum_{j \in J} w_j \left( f^L_{ij} - f^U_{ij} \right) + \sum_{j \in J} w_j \left( f^U_{ij} - f^L_{ij} \right) \]

(25b)

**Step 3:** Calculation of interval \( Q_i = [Q^U_i, Q^L_i] \) in the following formulation:

\[ Q^U_i = v \left( \frac{x^U_i - x^*}{x^- - x^*} \right) + (1 - \lambda) \left( \frac{R^L_i - R^*}{R^* - R^L_i} \right) \]

(26a)

\[ Q^L_i = v \left( \frac{x^L_i - x^*}{x^- - x^*} \right) + (1 - \lambda) \left( \frac{R^L_i - R^*}{R^* - R^L_i} \right) \]

(26b)

where

\[ x^* = \min x^U_i, x^- = \max x^L_i \]

\[ R^* = \min R^U_i, R^- = \max R^L_i \]

In general, it is supposed that \( \lambda = 0.5 \) and it represents the strategy weight of "the majority of criteria".

**Step 4:** Selection of the best alternative that has minimum \( Q_i \), a new method for comparison of interval numbers as follows [10, 11]:

Let \( Q_i = [Q^U_i, Q^L_i] \) and \( Q_j = [Q^U_j, Q^L_j] \) be two interval numbers that the decision-makers have to choose minimum one between them.

When \( Q^U_i = Q^U_j \) and \( Q^L_i \neq Q^L_j \), that is, both interval numbers \( Q_i \) and \( Q_j \) are exact real numbers, then we can have [22]:

\[ p(Q_i \geq Q_j) = \begin{cases} 1 & \text{if } Q_i > Q_j \\ 1/2 & \text{if } Q_i = Q_j \\ 0 & \text{if } Q_i < Q_j \end{cases} \]

(27)

When \( Q^L_i = Q^L_j \) and \( Q^U_i \neq Q^U_j \), we can have

\[ p(Q_i \geq Q_j) = \begin{cases} 1 & \text{if } Q_i > Q_j \\ \frac{Q^U_i - Q^U_j}{Q^L_j - Q^L_i} & \text{if } Q_i \leq Q_j \leq Q^U_i \\ 0 & \text{if } Q_i < Q^L_j \end{cases} \]

(28)

When \( Q^U_i \neq Q^U_j \) and \( Q^L_i = Q^L_j = Q^U_i \), \( p(Q_i \geq Q_j) \) is formulated as

\[ p(Q_i \geq Q_j) = \begin{cases} 1 & \text{if } Q^U_i > Q_i \\ \frac{Q^U_i - Q_j}{Q^L_j - Q^U_i} & \text{if } Q^U_i \leq Q_i \leq Q^L_j \\ 0 & \text{if } Q^U_i < Q_i \end{cases} \]

(29)

Generally, the most common case is that \( Q^U_i \neq Q^U_j \) and \( Q^L_i \neq Q^L_j \), then, the two interval numbers, \( Q_i \) and \( Q_j \), are shown in Fig. 2 [22]. It is easily to see that a shadowed rectangle with two parts divided by straight line \( y = x \), marked as \( s^- \) and \( s^+ \), respectively, in different
colors. It is formed by four peaks \((Q^1_i, Q^1_i), (Q^m_i, Q^m_i), (Q^1_j, Q^1_j), (Q^m_j, Q^m_j)\). \(s\), \(s'\) and \(s''\) represent the corresponding area in the Fig. 2, respectively. The area of the whole rectangle is \(s\). The area that \(y > x\) belongs to the area \(s''\) and the rest area is \(s'\).

\[\text{Fig. 2: The relationship between two general interval numbers}\]

**Definition 4.** Let \(Q_i = [Q^1_i, Q^m_i]\) and \(Q_t = [Q^1_t, Q^m_t]\) and \(Q^1_i \neq Q^1_t\) and \(Q^m_i \neq Q^m_t\), then the definition of the possibility degree of \(Q_i \geq Q_t\) can be given as follows:

\[p(Q_i \geq Q_t) = \frac{s'}{s}\]  

(30)

where \(s = (Q^1_i - Q^1_t)(Q^m_i - Q^m_t)\).

Consequentially, the possibility degree of \(Q_i \geq Q_t\) is derived by the formulation [11, 22]:

\[p(Q_i \geq Q_t) = \frac{s''}{s}\]  

(31)

**Step 5:** According to the degree of possibility of all the \(Q_i\), we assume that the acceptable degree is above 0.5 and a ranking of all the alternatives.

**4 The algorithmic E-TOPSIS method with Interval numbers for MCDM**

The algorithmic E-TOPSIS method with Interval numbers for MCDM is proposed in this section [1, 3, 7, 8, 9].

**Step 1:** Identification and construction of evaluation criteria for all the alternatives according to the decision-making Goals.

**Step 2:** Generation of alternatives for achieving the decision-making goals;

**Step 3:** Computation of the alternatives ratios in interval number by the value functions on multiple criteria;

**Step 4:** Identification of the weights of multiple criteria.

**Step 5:** Construction of the interval judgment matrix and the interval normalized judgment matrix.

Based on the ratios matrix on multiple criteria with interval numbers given by Eq. (14), the normalized values \(\bar{\Delta}^T_{ij}\) and \(\bar{\Delta}^U_{ij}\) can be calculated as follows [3]:

\[\bar{\Delta}^T_{ij} = f^T_{ij} \sqrt{\sum_{j=1}^{m} (f^T_{ij})^2 + (f^T_{ij})^2}, \quad j = 1, \ldots, m, i = 1, \ldots, n\]  

(32a)

\[\bar{\Delta}^U_{ij} = f^U_{ij} \sqrt{\sum_{j=1}^{m} (f^U_{ij})^2 + (f^U_{ij})^2}, \quad j = 1, \ldots, m, i = 1, \ldots, n\]  

(32b)

It clearly that the normalized interval number \([\bar{\Delta}^T_{ij}, \bar{\Delta}^U_{ij}]\) is originated from interval number \([f^T_{ij}, f^U_{ij}]\). Beyond all doubt, the normalized interval number \([\bar{\Delta}^T_{ij}, \bar{\Delta}^U_{ij}]\) is belonging to range \([0, 1]\).

**Step 6:** Construction of the interval weighted normalized judgment matrix.

Let us take the different weight of each criterion into consideration. The elements of a weighted normalized interval judgment matrix can be given as follows [7, 8]:

\[v^T_{ij} = w_j \bar{\Delta}^T_{ij}, \quad j = 1, \ldots, n, i = 1, \ldots, m\]  

(33a)

\[v^U_{ij} = w_j \bar{\Delta}^U_{ij}, \quad j = 1, \ldots, n, i = 1, \ldots, m\]  

(33b)

where \(w_j\) is the weight of the criterion \(j\) and \(\sum_{j=1}^{n} w_j = 1\).

**Step 7:** Identification of negative ideal solution and positive ideal solution.

As a consequence, the negative ideal solution and positive ideal solution can be identified as [9]

\[\bar{s}^+ = \{\bar{v}^+_1, \ldots, \bar{v}^+_m\} = \{(\max_{i} v^T_{ij}, j \in I), (\min_{i} v^T_{ij}, j \in J)\}, \quad (34a)\]

\[\bar{s}^- = \{\bar{v}^-_1, \ldots, \bar{v}^-_m\} = \{(\min_{i} v^T_{ij}, j \in I), (\max_{i} v^T_{ij}, j \in J)\}, \quad (34b)\]

where the criteria are divided into two types, benefit criteria and cost criteria, which are indicated by index \(I\) and \(J\), respectively.

**Step 8:** Calculation of the distance of each alternative from negative ideal solution and positive ideal solution, respectively.

Based on the \(n\)-dimensional Euclidean distance, the distance of each alternative from the negative ideal solution is formulated as [1, 3]:

\[d^-_i = \left\{ \sum_{j \in J} (v^T_{ij} - v^-_j)^2 + \sum_{j \in J} (v^U_{ij} - v^-_j)^2 \right\}^{\frac{1}{2}}, i = 1, \ldots, m.\]

(35a)
Meanwhile, the distance of each alternative from the positive ideal solution is formulated as:

$$d_i^+ = \left\{ \sum_{j \in J} (v_{ij}^+ - v_{ij}^-)^2 + \sum_{j \in J} (v_{ij}^-)^2 \right\}^{1/2}, i = 1, \ldots, m.$$  

(35b)

**Step 9:** Calculation of the closeness coefficient of each alternative to positive ideal solution.

Once the value of $d_i^+$ and $d_i^-$ is obtained for each alternative, a closeness coefficient can be calculated from them to help the decision-makers rank all the alternatives. The closeness coefficient of the alternative $S_i$ with respect to $S^+$ is defined as [3]

$$R_i = d_i^- / (d_i^+ + d_i^-), i = 1, \ldots, m$$  

(36)

**Step 10:** Ranking all the alternatives in a preference order according to the value of closeness coefficient.

It can be seen that with the value of $R_j$ approaching to 1, the alternative $S_i$ is becoming to be closer to $S^+$ and farther from $S^-$. Therefore, we can use the closeness coefficient $R_j$ to rank all the alternatives and determine which one is the optimal alternative for the decision-making goals.

### 5 Numerical Experiment and Results

#### 5.1 Experiment description

Based on the scenario carried out by Jussi [23], the illustrative example, demonstrates the application of the
Table 4: Reciprocal judgment matrix on $C_4$

<table>
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Table 5: The Interval normalized decision matrix

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<th>C2R</th>
<th>C3L</th>
<th>C3R</th>
<th>C4L</th>
<th>C4R</th>
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</table>

The experts are requested to give four reciprocal judgment matrices by pairwise comparison of all the alternative with regard to four capability, respectively. Monte Carlo method is employed to generate acceptable consistency reciprocal judgment matrix ($CR < 0.1$) to simulate the judgments from the experts. (See Table 1, Table 2, Table 3 and Table 4). The weights of four criteria are given as the same value, $w_1 = w_2 = w_3 = w_4 = 0.25$

5.2 Results analysis and discussion

The interval normalized decision matrix is shown in Table 5, however, some ratios are precise point values. For example, the upper and lower bounds of the ratio of alternative $S_1$ with respect to criterion $C_1 \sim C_4$ are equal to each other. PIS and NIS computed by E-VIKOR is given in Table 6 and Table 7 shows shows the $x_i$ and $R_i$ interval numbers. $Q_i$ interval numbers is shown in Table 8 and Table 9 shows The degree of possibility of all the $Q_i$ by comparison. The final result of the alternatives ranking by E-VIKOR is listed in Table 10. $S_5$ and $S_3$ both rank the top and the worst three alternatives are $S_8$, $S_9$ and $S_{10}$. The Interval weighted normalized judgment matrix by E-TOPSIS is shown in Table 11. Distance of each alternative from the positive idea solution and negative idea solution by E-TOPSIS are given in Table. 12 and Table 13. Closeness coefficient and ranking by E-TOPSIS are shown in Table 14. The final ranking order of the alternatives determined by E-TOPSIS is clear and distinctive. $S_7$ ranks the top and in the bottom lays $S_5$, $S_4$ and $S_2$ rank near to the top and the worst three alternatives are $S_7$, $S_9$ and $S_5$.  

---

<table>
<thead>
<tr>
<th>$S_1$</th>
<th>$S_2$</th>
<th>$S_3$</th>
<th>$S_4$</th>
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<th>$S_7$</th>
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<td>1.1</td>
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<td>1</td>
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</tr>
<tr>
<td>$S_9$</td>
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Table 6: PIS and NIS computed by E-VIKOR

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Table 7: $x_i$ and $R_i$ interval numbers

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Table 8: $Q_i$ interval numbers

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<th>$Q^U_i$</th>
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Table 9: The degree of possibility of all the $Q_i$ by comparison

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<th>$S_{10}$</th>
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Table 10: Ranking of all the alternatives by E-VIKOR

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</table>

Table 11: The Interval weighted normalized judgment matrix by E-TOPSIS

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<th>C1R</th>
<th>C2L</th>
<th>C2R</th>
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<th>C3R</th>
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<td>0.0245</td>
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<td>0.0215</td>
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<td>0.0233</td>
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<td>0.0215</td>
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<td>0.0233</td>
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<td>$S_8$</td>
<td>0.0035</td>
<td>0.0036</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0029</td>
<td>0.0029</td>
<td>0.0031</td>
<td>0.0032</td>
</tr>
<tr>
<td>$S_9$</td>
<td>0.0035</td>
<td>0.0036</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0029</td>
<td>0.0029</td>
<td>0.0031</td>
<td>0.0032</td>
</tr>
<tr>
<td>$S_{10}$</td>
<td>0.0035</td>
<td>0.0036</td>
<td>0.0024</td>
<td>0.0026</td>
<td>0.0029</td>
<td>0.0029</td>
<td>0.0031</td>
<td>0.0032</td>
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6 Conclusion

In this paper, we studied the extended VIKOR and TOPSIS algorithmic methods with interval number for MCDM problem based on reciprocal judgment matrix and carried out a comparative experiment.

When faced with MCDM problems in real-world, the experts may be uncomfortable giving crisp ratios or precise upper and lower bounds of the interval ratios on multi-criteria. Based on the reciprocal judgment matrices given by the experts with regard to the satisfaction degree of the multiple alternatives on single criterion, the interval ratios was elicited by a linear programming model. The TOPSIS and VIKOR methods are extend for MCDM problems with interval number and algorithmic E-VIKOR and E-TOPSIS methods are examined by the experts reciprocal judgment matrices generated by Monte Carlo simulation. A numerical example of ranking indirect-fire weapon system alternative is given and illustrates the feasibilities and distinctive features of the VIKOR and TOPSIS algorithmic methods with interval number.

Improvements can be made for future studies in the following ways: The programming model of elicitation of interval probability from reciprocal judgment matrix by pairwise comparisons can be improved. Furthermore, the TOPSIS and VIKOR can be extended with other classical methods (Fuzzy triangle number and ANP methods, etc.) for MCDM problems.

Acknowledgement

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References


Table 12: Distance of each alternative from the positive idea solution

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>DU1</th>
<th>DU2</th>
<th>DU3</th>
<th>DU4</th>
<th>DU5</th>
<th>DU6</th>
<th>DU7</th>
<th>DU8</th>
<th>DU9</th>
<th>DU10</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.0446</td>
<td>0.0279</td>
<td>0.0287</td>
<td>0.0287</td>
<td>0.0538</td>
<td>0.0535</td>
<td>0.0535</td>
<td>0.0635</td>
<td>0.0635</td>
<td>0.0635</td>
</tr>
</tbody>
</table>

Table 13: Distance of each alternative from the negative idea solution

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>DU1</th>
<th>DU2</th>
<th>DU3</th>
<th>DU4</th>
<th>DU5</th>
<th>DU6</th>
<th>DU7</th>
<th>DU8</th>
<th>DU9</th>
<th>DU10</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.0637</td>
<td>0.0575</td>
<td>0.0562</td>
<td>0.0562</td>
<td>0.0301</td>
<td>0.031</td>
<td>0.031</td>
<td>0.0448</td>
<td>0.0448</td>
<td>0.0448</td>
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</table>

Table 14: Closeness coefficient and ranking by E-TOPSIS

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
<th>S5</th>
<th>S6</th>
<th>S7</th>
<th>S8</th>
<th>S9</th>
<th>S10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank</td>
<td>4</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>10</td>
<td>9</td>
<td>8</td>
<td>5</td>
<td>7</td>
<td>6</td>
</tr>
</tbody>
</table>


Yajie Dou received the M.Sc. degree in engineering from the National University of Defense Technology (NUDT), Changsha, P. R.China, in 2011. He is pursuing the Ph.D. degree in the School of Information System and Management Science at NUDT. His main research interests include weapon system portfolio decision and effectiveness evaluation.

Pengle Zhang received the B.Sc. degree in Management from NUDT in 2013. He is currently a Master candidate in the School of Information System and Management Science at NUDT. His main research interests include system of systems requirement planning, and effectiveness evaluation.

Jiang Jiang received the B.Sc. degree and M.Sc. degree in system engineering from NUDT, PRC in 2004 and 2006, respectively. He is currently a lecture in School of Information System and Management, NUDT. His research interests include multiple criteria decision making and risk analysis.

Kewei Yang received the Ph.D. degree in Management from NUDT in 2004. He is an Associate Professor at the College of Information System and Management in NUDT. His research interests focus on Intelligent Agent Simulation, Defense Acquisition and System of Systems requirement modeling.

Yingwu Chen received the M.Sc. degree in system engineering, and the Ph.D. degree in engineering from NUDT in 1987 and 1994, respectively. He is a Professor in NUDT. His current research interests include assistant decision making systems for planning, decision-making systems for project evaluation, management decisions and system of systems engineering.