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Unveiling of Geometric Generation of Composite Numbers`Exactly and Completely

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Abstract: We present a certain *geometrical* interpretation of the natural numbers, where these numbers appear as *joint products of 5- and 3-multiples* located at specified positions in a *revolving chamber*. Numbers without factors 2, 3 or 5 appear at one of *eight* such positions, after a specified amount of rotations of the chamber. Our approach determines the sets of rotations constituting *primes* at the respective eight positions, as the *complements* of the sets of rotations constituting *composite numbers* at the respective eight positions. These sets of rotations constituting composite numbers are exhibited in *strict rotation regularities* from a *basic 8×8-matrix* of the *mutual products* originating from the eight prime numbers located at the eight positions in the *original* chamber. These regularities are expressed in relation to the multiple 11^2 as an anchoring *reference point*. The *complete* set of composite numbers located at the eight positions is exposed as *eight* such sets of eight series. Each of the series is *completely* characterized by *four simple variables* when compared to a reference series anchored in 11^2. *Ad negativo* this also represents an *exact* and *complete* generation of all *prime numbers* as the union of the eight complement sets for these eight non-prime sets of eight series. By this an exact and complete *pattern* in composite numbers, as well as in prime numbers, are exhibited in the *maximum* sense of a pattern.

Keywords: Number theory, Composite numbers, Distribution of primes, Generation of primes

1. Introduction

The quest of finding patterns in order to generate and predict the occurrences of prime numbers has represented a mathematical riddle from ancient times and is still much cloaked in mystery. Leonhard Euler believed possible order in the sequence of prime numbers to remain ”a mystery into which the mind will never penetrate”. Paul Erdos commented that ”it will be another million years, at least, before we understand the primes”. Marcus du Sautoy has stated that ”if the Riemann Hypothesis is true, it explains why there are no strong patterns in the primes”. In some distinction to these more pessimistic opinions, Don Zagier in 1975 noted the paradox of prime numbers ”seeming to obey no other law than that of chance” while at the same time to ”exhibit stunning regularity, that there are laws governing their behavior, and that they obey these laws with almost military precision” (Havil 2003:171).

A significant contribution towards discovering a pattern in distribution of prime numbers was made in 1795 by Karl Friedrich Gauss who proved that the amount of primes p(n) until a natural number n followed the approximation law p(n)≈n/ln(n), reaching equality as limit from infinite increase in n. This implied a quite regular logarithmic distribution of prime numbers. Wolf (1997) found that the number of primes in successive intervals of length 2^{16} was distributed according to the self-similar 1/f behaviour known from complexity science. Some general discussion of fractal patterns in prime distributions was offered by Carlo (2010). Some elaboration of Gauss' discovery, congruent with global scaling theory, was presented by Müller (2000,2009). Luque and Lacasa (2009) further found that the statistical distribution of leading digits of prime numbers followed a first-digit Benfords law with striking precision.

Stretching beyond probability patterns of prime distributions Balog (1990) proved that there are infinitely many 3-by-3 squares of distinct primes where each row and each column forms an arithmetic progression, as well as analogous aritmethic progression for infinitely many 3-by-3-

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by-3 cubes of distinct primes where each row and each column and each vertical line forms an arithmetic progression. Green and Tao (2007,2008) expanded Balog's approach and found certain finite sequences of primes. They showed that there are infinitely many k-term arithmetic progressions of primes, i.e. that there exist infinitely many distinct pairs of nonzero integers and such that a, a + d,..., a + (k -1)d are all primes. Granville (2008) presented some generalizing contemplation of these results, and this was offered some further discussion by Sorensen (2008).

The presentation of the Ulam spiral (Stein et al. 1964,1967) indicated a curious tendency for prime numbers to distribute in a diagonal pattern when arranged in a rectangular grid spiraling out. From later inspection of different gaps between consecutive prime numbers, the most frequent gap was found to be of size 6, the so-called *jumping champion*, and an even more striking probability regularity was found for *multiples* of 6 (Wolf 1996; Odlyzko et al. 1999). Kumar et al. (2003) found from statistic-empirical examinations of prime numbers that gaps *between* gaps (by them called "increments") followed an exponential distribution with superposed periodic behavior of period three, similar to previously reported period six oscillations for first- order gaps, with the frequency of the increases significantly lower for multiples of 6 than for 6±k2. Szpiro (2004) investigated also gaps of higher order than second, and found related and striking statistical regularities at such orders as well.

Boeyens (2004) arranged natural numbers along a spiral with period 24. All prime numbers larger than 3 are included in the form 6n±1, and thus can be connected by radial lines to form eight arms (the so-called *prime-number cross*) in the spiral. Boyens demonstrated that this strong tendency to symmetric periodicity in the prime distribution was remarkably similar to a hidden symmetry in the periodicities of nuclear synthesis, of chemical elements, and of DNA. This indicated that said pattern in prime number distribution expressed a "generalized, closed periodic law" of nature, which might suggest the existence of a still hidden prime number pattern of even stricter regularity closing the possibility space for occurrences of primes considered random.

The studies and results referred above proved or suggested different kinds of prime number patterns in certain combinations between probability distributions and non-random algorithmic determinations, with a tendency to also increase insights into the last aspect. In this perspective the study of Hibbs (2008) may be of special interest. Hibbs studied the development of second-order gaps between primes, and found this distribution to develop in a spiraling manner, as well as fractally and recursively, more specifically as indicating a certain double-helix pattern. On the one hand Hibbs' study thus was in agreement with some other crucial results from research into prime number patterns, while on the other hand his study did not have to apply probability mathematics. Therefore, Hibbs' study might be interpreted as a step towards the possible discovery of an exact pattern generating the complete set of prime numbers.

A step in the same direction was also provided by Khurana and Koul (2005) who proved, by a quite simple approach, that strictly regular patterns of prime numbers also exist absolutely for *small* intervals (including coverage of some large primes), and presented a novel deterministic (and quite computer efficient) primality test for such. In 2002 Agrawal et al. (2004) had established, as the first, a polynomial time deterministic algorithm to decide with certainty whether a number is prime. They also had proved that this *AKS primality* test vas universally valid. Johansen (2006) presented another deterministic primality test, related to Fibonacci reformulation of number theory (Johansen 2011), the so-called *Fibonacci neighbour primality test*, but without providing any proof for the universal validity of the test.

These developments indicated the possible existence of an exact and complete pattern of prime numbers exluding all uncertainties connected to probability treatments. A possible approach to achieve such a result was argued in Johansen (2006). A complete treatment and deductive proof establishing said achievement was presented in Johansen (2010), referred to in Rapoport (2011) as a "remarkable work". We will present some key points in this approach, followed by some discussion of the implied prime number pattern in relation to informatics.

2. Revolving generation of complete and exact pattern of composite numbers vs. prime numbers

We start out from a rewrite of natural numbers as combined multiples of the numbers 5 and 3:

(1) $N = m5 + n3; \quad m>0,\ n>0$

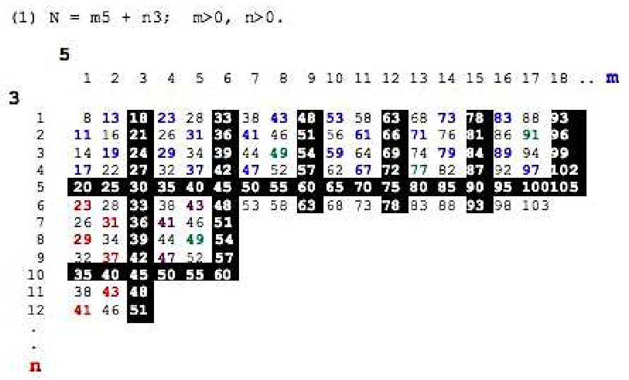
Obviously, this split code 5:3 can be performed to cover *any* sequence of whole numbers by simply lowering the bottom values of m and n.

The profound significance of the split code 5:3 in "Nature's code" is acknowledged and argued in the pioneering monumental work of Peter Rowlands (2007), and also with some stated connection (Rowlands 2007: 530, 550) to the contribution in Johansen (2006). Johansen (2011), with related references, including Strand (2011), presents further results and contemplations reinforcing and clarifying this significance.

From (1) we construct the following matrix:

There exist three possibilities to make a cut in the matrix in such a way that every number shows up only once. We denote these three bands of numbers by means of colour terms:

1. *The Blue Band*, corresponding to the five upper rows.



	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	.. m
1	8	13	18	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	
2	11	16	21	26	31	36	41	46	51	56	61	66	71	76	81	86	91	96	
3	14	19	24	29	34	39	44	49	54	59	64	69	74	79	84	89	94	99	
4	17	22	27	32	37	42	47	52	57	62	67	72	77	82	87	92	97	102	
5	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	
6	23	28	33	38	43	48	53	58	63	68	73	78	83	88	93	98	103		
7	26	31	36	41	46	51													
8	29	34	39	44	49	54													
9	32	37	42	47	52	57													
10	35	40	45	50	55	60													
11	38	43	48																
12	41	46	51																
.																			
.																			
n																			

Figure 1 The Revolving Chamber

2. *The Red Band*, corresponding to the three left columns.
3. *The Violet Band*, corresponding to a double diagonal field unfolding from first six columns of The Blue Band, or from first ten rows of The Red Band.

There can not be any prime numbers in the row for n=5, nor in the columns that are multiples of m=3. Ignoring these rows and columns (illustrated by the black grid in fig. 1), prime candidates can only appear in the remaining "chambers" of the bands. Further, prime candidates can only appear at spots in the chambers where odd numbers are located (illustrated with the colours blue, red and violet, respectively). We notice that these spots are distributed in a zigzag pattern inside each chamber, and that this pattern alternates with its mirror pattern when progressing horizontally or vertically along a band. In the present context we will only study The Blue Band.

We apply the notion *original chamber* to denote the location of the first eight prime numbers in The Blue Band, not situated at black frames, at the (upper) left segment of fig. 1, i.e. the eight primes from 11 to 37. This original chamber is divided into its *left* (sub-)chamber, primes 11,13,17,19; and its *right* (sub-)chamber, primes 23,29,31,37. Then we imagine this left chamber *revolving in 3D* around the black vertical axis made up of the numbers 18,21,24,27,30. After *half* a rotation the four positions of the primes in the left chamber will cover the four positions of their respective enantimorphs in the right chamber, i.e. as 13 onto 23, 11 onto 31, 19 onto 29, and 17 onto 37. After a *whole* rotation, the four positions of the primes in the *left* chamber will cover the four positions of the corresponding numbers in the *left* (sub-)chamber of the *second* chamber in The Blue Band, the chamber to the *right* of the original chamber, i.e. as 13 onto 43, 11 onto 41, 19 onto 49, and 17 onto 47. After a whole rotation of the four positions of the primes in the *right* (original) (sub-)chamber, these positions will cover the four positions of the corresponding numbers in the *right* (sub-)chamber of the second chamber in The Blue Band, i.e. as 23 onto 53, 31 onto 61, 29 onto 59, and 37 onto 67. Hence, taken together, after a whole rotation of the *eight* positions of the primes in the original chamber, these eight positions will cover the eight positions of the corresponding numbers in the second chamber, and each of these last eight numbers is determined as the number at the corresponding position in the original chamber, *added with 30*. Obviously, after *multiple* rotations of the original chamber, the number in the arrival chamber is determined as the number at the corresponding position in the original chamber, added with the *same multiple of 30*. Also obviously, *any* odd number in The Blue Band is determined uniquely and can be written uniquely as the corresponding position in the original chamber, undergoing a *certain multiple of whole rotations*, which corresponds to the original number being added with the same multiple of 30. Hence, the eight positions of primes in the original chamber determines *uniquely and exhaustively all* odd numbers in chambers of The Blue Band when undergoing *all* possible integer multiples of whole rotations, which is equivalent to each of the original eight numbers being added with all corresponding integer multiples of 30.

To easily get a picture of the underlying prime number generator, we first imagine *all* remaining odd (blue) numbers in The Blue Band as being prime numbers. This is the case for the first two chambers of The Blue Band. However, in the third chamber, which can be imagined as constituted from the first (whole) *rotation* of the left, first chamber, the number of 49, i.e. 7×7, shows up as the first anomaly not being any prime number. Analogous anomalies will be the case for all powers of 7, as well as for all "clean multiples" of 7 (meaning those having a factor in a preceding chamber) located in chambers further to the right on The Blue Band. 7 is the only lower number *outside* and *before* our matrix, that acts as a "bullet" and "shoots out" odd numbers in The Blue Band, removing their prime number candidature. For example, the number of 77 is shot out (displayed by the colour green) from the prime number universe in chamber no. 5 after two rotations of chamber no. 1, being a multiple of the bullets 7 and 11. Prime numbers from the first chamber will deliver the same "ammunition" when exposed for sufficient rotations to manifest multiples made up as internal *products* of these prime numbers. Such multiples occur at corresponding "arrival spots" in upcoming chambers after further rotations. For example, the number of 143 is shot out from the prime number universe in chamber no. 10 after four rotations of chamber no. 2, being a multiple of the factor "bullets" 11 and 13. Quite obviously, *all* multiples of primes will expose the same pattern of shooting out corresponding prime number candidates occurring in proceeding chambers, without regard to the number of rotations of chamber no. 1 or no. 2 manifesting the prime factor bullets of the multiple. Hence, the over-all process of shooting out prime candidates can be imagined as successive out-shooting during consecutive rotation of chambers no. 1 and 2, due to more and more multiples from prime bullets, located in preceding chambers, becoming manifest along with further chamber rotations. This elimination process of prime candidates is obviously *exhaustive*. *All* prime candidates which is *not* shot out from the multiples of prime bullets occurring at preceding chambers *have* to be primes. Therefore, a complete mathematical description of this successive out-shooting of prime candidates will automatically *ad negativo* implicate also a complete, successive description of the generation of prime numbers. Here the prime numbers appear as the numbers *remaining* in chambers of The Blue Band *after* the shoot-out procedure has passed through the chamber where the prime candidate is located.

The model of fig.1, as well as the general procedure of shooting out prime candidates, was presented in Johansen (2006: 127-9). The deduction of complete formulas to perform the out- shooting, according to this approach, in order to generate prime numbers exactly and completely was presented in Johansen (2010). Here we will recapitulate some crucial steps, notions and figures from this deduction.

We apply the following notation of the blue (odd) numbers' positions inside a chamber, using their positions inside the first two chambers as illustration:

Left chamber: a_1: position of 13; a_2: position of 11; a_3: position of 19; a_4: position of 17. Right chamber: b_1: position of 23; b_2: position of 31; b_3: position of 29; b_4: position of 37.

Then, all odd numbers in The Blue Band can be written as one of these positions combined with a specific number of rotations. As an example, 71 can be written as [2,a_2], meaning that 71 emerges at the position a_2 after 2 rotations of the original (left) chamber. Accordingly, 67 will be written as [1,b_4], etc.

Number position

	Left chamber				Right chamber			
	a_1	a_2	a_3	a_4	b_1	b_2	b_3	b_4
Start:	13	11	19	17	23	31	29	37
Step								
1			13- 5 (13) 17- 9 (17) 23-17 (23) 37-45 (37)			11- 3 (11) 19-11 (19) 29-27 (29) 31-31 (31)		
2		13- 7 (17)		19-14 (23) 37-50 (41)	11- 4 (13) 17-10 (19)		29-29 (31)	23-21 (29) 31-37 (37)
3		19-18 (29) 31-42 (41)			23-23 (31) 29-35 (37)	17-12 (23) 37-52 (43)		11- 5 (17) 13- 7 (19)
4	17-16 (29) 31-44 (43)	23-28 (37)	19-19 (31) 29-39 (41)				11- 6 (19) 13- 9 (23) 37-57 (47)	
5	11- 8 (23) 19-23 (37) 23-31 (48) 37-60 (49)			13-12 (29) 17-17 (31) 29-41 (43) 31-48 (47)				
6	13-13 (31) 29-45 (47)	37-65 (53)	11-10 (29) 31-50 (49)				17-20 (37) 19-25 (41) 23-32 (43)	
7		11-11 (31) 29-47 (49)			31-54 (53) 37-72 (59)	13-15 (37) 23-35 (47)		17-22 (41) 19-26 (43)
8		17-24 (43)		11-13 (37) 23-37 (49)	13-17 (41) 19-29 (47)		31-60 (59)	29-50 (53) 37-74 (61)

Figure 2 The basic 8×8-matrix of the non-primes generator in the revolving chamber

Fig. 2 describes the basic distributive structure of positions (illustrated as columns) in the chambers, manifesting from the specific numbers of rotations (illustrated in red) of the eight initial position numbers (illustrated in bold black) of the original chamber (i.e. chamber no.1, the left, and chamber no.2, the right, taken together), insofar as these rotations correspond to stepwise *multiplications* of the respective original position numbers with progressively larger multiplicators (illustrated in blue). The succession of multiplications, resulting in composite numbers (displayed in green if fig. 1), goes as follows, taking as example 11 as multiplicand:

Table 1: Self-referential generation of products with according positions and rotations

	multiplicator	*product - position - rotations*
1. row:	R_1.	11×11 at b_2 after 3 rotations
2. row:	R_2.	11×13 at b_1 after 4 rotations
3. row:	R_3.	11×17 at b_4 after 5 rotations
4. row:	the 3. “	11×19 at b_3 after 6 rotations
5. row:	the 4. “	11×23 at a_1 after 8 rotations
6. row:	the 5. “	11×29 at a_3 after 10 rotations
7. row:	the 6. “	11×31 at a_2 after 11 rotations
8. row:	the 7. “	11×37 at a_4 after 13 rotations
9. row:	the 8. “	M_1
10.row:	the 9. “	M_2
11.row:	the 10. “	M_3
...	...	...
...	...	...

where R_1 means ” the multiplicand number itself ”, R_2, means ” the closest blue number larger than itself” and R_3 means ” the 2. closest number larger than itself ”. Also, M_1 refers to ”11×(11+30) at b_2 after (3+11) rotations”, M_2, refers to ” 11×(13+30) at b_1 after (4+11) rotations” and M_3 means ”11×(17+30) at b_4 after (5+11) rotations ”.

As an example we can look at the number in the box [8,b_3] that manifests at position b_3, i.e. the same position as 29 in the original chamber, after the original number 31 is multiplied with the multiplicator 59 which is situated at the 8. row, i.e. 7 steps after the number 31 itself acts as multiplicator on itself. This box is reached after 60 rotations of the original chamber.

For each of the eight different position numbers in the original chamber, the position of the multiplicands product in the 9. row (i.e. after 8 steps of the succession) is identical with the original position, the position of the multiplicands product in the 10. row (i.e. after 8+1 steps of the succession) is identical with the original position, etc.

This means that with respect to *position*, the 8 sequence of positions characteristic for the products progressing in steps from the original position for the smallest considered product of the respective multiplicands, just repeats in 8 steps cycles along with increasing additions of 30s to the multiplicator. (From now on we denote the number of such 30- additions with the symbol *m*.)

With regard to the number of *rotations*, we always have that after 8 steps the number of rotations added to the rotations in the product in row 1, required to manifest the product for the same multiplicand in row 9, is identical to the size of the multiplicand. Thus, as an example, for the multiplicand 11, the product in row 9 is reached as the 3 added rotations of its initial product in row 1, added with 11 new rotations, which gives 14 rotations. And the same must be the case with respect to the added numbers of rotations stepping from row 2 to 10, from row 3 to row 11, etc.

The same homology with respect to position and rotations occur for additions of 30s to the *multiplicand* (denoted with the symbol *n*).

Thus, all thinkable products (besides the trivial products of 2, 3 and 5, and the not so trivial products of 7) can be written uniquely as the square of one of the multiplicands located in the original chamber, successively added with increases in m and increases in n. Any of these products arrives in one of the 64 boxes of fig. 2, after a specified number of rotations, completely determined by the position and number of rotations of the initial squared product, and the sizes of m and n. If we, as an example, consider products arriving in box $[3, b_4]$, the non-primes entering this box from the total *11-path*, are given by the set:

(2) (11+n30) [(17+n30) +m30]
alternatively expressed as:
(2b) 37 + 30[5+ n(11+17) +m11 +n30(n+m)]

Non-primes entering this box from the total *13-path*, are given by the set:
(3) (13+n30) [(19+n30) +m30]
alternatively expressed as:
(3b) 37 + 30[7+ n(13+19) +m13 +n30(n+m)]

Products generated from the multiplicand 7 constitutes a special case that is covered by being represented by n=-1 in analogous expressions for boxes reached from the total 37- path. With regard to *positions* the path from 7 is identical to the path from 37; thus the two paths only differ with respect to the number of *rotations*. 37 is chosen in stead of 7 as original position number due to completing the original chamber in fig. 1 with a symmetrical structure between left and right chamber.

The expressions for the 64 boxes of products, developed in analogy to (2) and (2b), can be rewritten as *additives* of rotations compared to the rotations of products arriving in box $[1, b_1]$ as an anchoring box suitable as a general reference. We rewrite this reference box to box (11,11) which denotes all products arriving in the same position in fig. 2 from successive increases of m and n to the initial product 11×11 arriving in this box.

Horizontally, at the top of fig. 3, we list in succession the factors in the original chamber, acting as *multiplicands* in the 64 basic products represented in fig. 2. Vertically, to the left of fig. 3, we list in succession the numbers acting as *multiplicators* in the 64 basic products represented in fig. 2. Hence, all the 64 basic products, and all the clusters of non- primes generated from each of them, are also represented in fig. 3. The amount of rotations for the initial product in each box (i.e. for m=0 and n=0) is displayed in red in fig. 3, and the position number where products arrive (i.e. the columns of fig. 2) is displayed in black to the right of these numbers in red. Hence, fig. 3 displays the 64 boxes of products distributed among these 8 position numbers where the respective boxes arrive, as specified expressions of n- and m-additives of rotations compared to the reference box (11,11).

	11	13	17	19	23	29	31	37
11	0n 3-31							
13	2n 4-23	2n +2(m+n) 5-19						
17	6n 5-37	6n+2(m+n) 7-11	6n+6(m+n) 9-19					
19	8n 6-29	8n+2(m+n) 7-37	8n+6(m+n) 10-23	8n+8(m+n) 11-31				
23	12n 8-13	12n+2(m+n) 9-29	12n+6(m+n) 12-31	12n+8(m+n) 14-17	12n+12(m+n) 17-19			
29	18n 10-19	18n+2(m+n) 12-17	18n+6(m+n) 16-13	18n+8(m+n) 18-11	18n+12(m+n) 21-37	18n+18(m+n) 27-31		
31	20n 11-11	20n+2(m+n) 13-13	20n+6(m+n) 17-17	20n+8(m+n) 19-19	20n+12(m+n) 23-23	20n+18(m+n) 29-29	20n+20(m+n) 31-31	
37	26n 13-17	26n+2(m+n) 15-31	26n+6(m+n) 20-29	26n+8(m+n) 23-13	26n+12(m+n) 28-11	26n+18(m+n) 35-23	26n+20(m+n) 37-37	26n+26(m+n) 45-19
41		30n+2(m+n) 17-23	30n+6(m+n) 22-37	30n+8(m+n) 25-29	30n+12(m+n) 31-13	30n+18(m+n) 39-19	30n+20(m+n) 42-11	30n+26(m+n) 50-17
43			32n+6(m+n) 24-11	32n+8(m+n) 26-37	32n+12(m+n) 32-29	32n+18(m+n) 41-17	32n+20(m+n) 44-13	32n+26(m+n) 52-31
47				36n+8(m+n) 29-23	36n+12(m+n) 35-31	36n+18(m+n) 45-13	36n+20(m+n) 48-17	36n+26(m+n) 57-29
49					38n+12(m+n) 37-17	38n+18(m+n) 47-11	38n+20(m+n) 50-19	38n+26(m+n) 60-13
53						42n+18(m+n) 50-37	42n+20(m+n) 54-23	42n+26(m+n) 65-11
59							48n+20(m+n) 60-29	48n+26(m+n) 72-23
61								50n+26(m+n) 74-37

Figure 3 The 8×8 universal matrix of (11,11)-related additives of rotations for complete generation of non-primes.

Rotations for the platform for the additives, the reference box (11,11), are:
3 + 11n + 11(m+n) + n30(m+n)

The different amounts of rotations making up the complete set of products arriving in the reference box (11,11) can be represented as the series displayed in Fig. 4:

Colour coding:
30's
11's
1's

In fig. 4 the position of each number signify a unique product. As an example: The amount of rotations repre-

```
3+                          0+
3+                        1+
3+                      2+     1
3+                     3+    2
3+                   4+     3 + 1
3+                 5+     4 + 2
3+                6+     5 + 3 + 1
3+              7+     6 + 4 + 2
3+            8+     7 + 5 + 3 + 1
3+          9+     8 + 6 + 4 + 2
3+        10+     9 + 7 + 5 + 3 + 1
3+      11+     10+ 8 + 6 + 4 + 2
…       …      ……………………..
….     ….     ………………………
n       0       1   2   3   4   5
```

Figure 4 Make-up of the set of rotations for non-prime box (11,11) at position number 31 in the revolving chamber

```
3+4+                                        0+0
3+4+                                     1+1
3+4+                                  2+2  +   1+3
3+4+                              3+3  +   2+3
3+4+                           4+4  +   3+3 + 1+3
3+4+                       5+5  +   4+3 + 2+3
3+4+                    6+6  +   5+3 + 3+3 + 1+3
3+4+                7+7  +   6+3 + 4+3 + 2+3
3+4+             8+8  +   7+3 + 5+3 + 3+3 + 1+3
3+4+         9+9  +   8+3 + 6+3 + 4+3 + 2+3
3+4+     10+10  +   9+3 + 7+3 + 5+3 + 3+3 + 1+3
3+4+  11+11  +  10+3 + 8+3 +  6+3 + 4+3 + 2+3
……    ………………………………………………………..
……   ………………………………………………………...
n          0       1       2       3       4       5
```

Figure 5 Make-up of the set of rotations for non-prime box (19,13) at position number 37 in the revolving chamber.

sented by the black **4** at the row with blue 9 in the figure, is:

(5) $3 + 11 \times 9 + 30\ (8 + 6 + \mathbf{4}) = 642$

The natural number corresponding to this place in the revolving chamber after this amount of rotations:

(6) $642 \times 30 + 31 = 19291$

Hence, this black 4 in fig. 4, when interpreted in this manner, is just another way of writing the number 19291. Since this number is included in fig. 4, it is positioned in box (11,11) and with necessity a non-prime. Just for confirmation: This black 4 is located in fig. 5 at the position for the row indicated by the blue number, n+(m+n)=9, and the diagonal n=3. This gives the factor $(11+3\times30)$ from the value of n, and from the value of m the other factor $[(11+3\times30) + 3\times30]$, i.e. the product 101×191 which is 19291.

The pattern in fig. 4 generating all products arriving in box (11,11) is amazingly simple. The series generating all products arriving in the remaining 63 boxes show to be modified variations built on the same basic pattern. As an example:

Colour coding:
30's
11's
2's
2's
1's

For remaining boxes the modifications of the basic pattern exposed by fig. 4 appear only moderately more complex than the modification represented by fig. 5. Each of the 64 patterns, with corresponding series, can be *completely* characterized by *four simple variables* when compared to the reference series displayed in fig. 4 anchored in 11^2. The values of these four simple variables for the respective 64 boxes are calculated and listed in Johansen (2010:153,166-8). By this the expressions of rotations generating *all* composite numbers located at *same* position in the chamber is found as a set of *eight related series*. Hence, the *total* set of composite numbers located at all eight positions is exposed as *eight* such sets of eight series. This represents a *complete* exposition of composite numbers generated by a quite simple mathematical structure. *Ad negativo* this also represents a *complete* exposition of all *prime numbers* as the union of the eight complement sets for these eight non-prime sets of eight series.

3. Informational discussion of the enclosed pattern of composite numbers vs. prime numbers

```
2           x x x x x x x x x x x x x x x x x x x x x x x x x x x x x
3            x  x  x  x  x  x  x  x  x  x  x  x  x  x  x  x  x  x  x
5              x    x    x    x    x    x    x    x    x    x    x
Total       xxxxx0xxx0x0xxx0x0xxx0xxxxx0x0xxxxx0xxx0x0xxx0x0xxx0xxxxx
Gap numbers 7  11 13 17 19  23   29 31   37  41 43 47 49  53
```

Figure 6 What is a pattern?

When performing a cut in a constrained appearance of information, the criterion for a pattern is that it is possible from the information residing at one side of the cut to predict *some* information residing at the other side of the cut with *more than random* probability (cf. the definition by Bateson 1972:131). This constitutes the criterion for a *minimal* pattern. The criterion for a *maximal* pattern is to predict with *zero* randomness *all* information residing at the other side. Trivially, the exact and complete occurrences of the multiples of 2 can be predicted from the information before a cut performed after 2×2, and the same holds for the multiples of 3 as predicted from the information before a cut performed after 3×2, and for the multiples of 5 as predicted from the information before a cut performed after 5×2. Hence, all these three cases unveil *maximal* patterns. Also quite trivially, the exact and complete occurrences of multiples having 2, 3 or 5 as a factor, can be predicted from the information before the last of said cuts, repeating with periodicity 30 (= 2×3×5). Hence, also this case unveils a maximal pattern. Obviously, as illustrated in fig. 6, such a maximal pattern automatically implies a corresponding prediction of the exact and complete occurrences of the *gaps*, i.e. the *absent* information, in the positive over-all pattern, by simply performing the switch from the positive pattern to its *negative gestalt*. Also evident, the same must be the case whatever the mathematical complexity of the positive pattern. Since our deductive exhibition in Johansen (2010) represents a prediction of the exact and complete occurrences of composite numbers after an initial cut, performance of the gestalt switch from this positive pattern to the prediction of the exact and complete occurrences of *gaps* between composite numbers, i.e. of the residences of the prime numbers, must also represent a maximum pattern.

The complementarity between the two classes of gaps, representing the two sides of the same coin, is implied also in for example the treatment by Szpiro (2004). Szpiro noted correctly that his ”findings point to the presence of some structure in the prime number sequence that has hitherto not been identified”, but made ”no attempt to explain the phenomenon”. In distinction to this our treatment firstly and independently focuses and deduces the exact and complete residences of the consecutive composite numbers that *make up* the gaps between consecutive prime numbers in the exposed strict and exact regularity.

We may illustrate our approach in some analogy to the keys on a piano:

The cut for contemplating the pattern is represented by the eight prime numbers in our original chamber. From these eight number positions, the occurrences of the eight basic multiplicators for the respective eight multiplicands are predicted with zero randomness. This constitutes the self-referential basic matrix of products displayed in fig. 2, from which all other composite numbers are deduced and predicted with zero randomness. More precisely, that is when already presupposing the trivial maximum pattern of composite numbers with factors 2, 3 or 5 (easily predicted with zero randomness from the original chamber), and treating the multiplicand 7 as one negative rotation of 37 (which in our analogy may be thought somewhat similar to the left little finger hitting the key one octave lower from the black key 37). Thus, the exact and complete occurrences of all composite numbers are predicted with zero randomness from the cut after the first chamber, and the exact and complete occurrences of non-trivial composite numbers are predicted with zero randomness from the

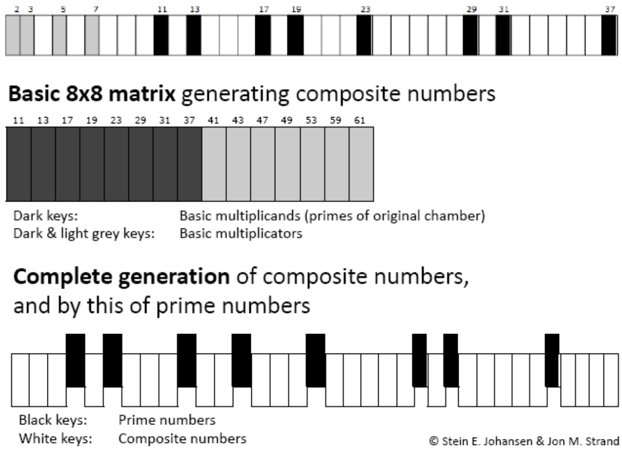


Figure 7 The Piano Analogy

eight prime numbers in the original chamber. This may be compared to a pianist touching eight black keys at the left of the piano with his hands and from there touching all remaining white keys in succession in one swiping movement. The keys he does not touch, is then the totality of black keys after the first eight ones, corresponding to the gaps representing the prime numbers. By simply performing the gestalt switch, our deduction of a maximal pattern also represents an exact and complete prediction of all non-trivial prime numbers, i.e. a deduction of a *maximal* pattern of *prime numbers* from the cut after the eight prime numbers in the original chamber.

To our knowledge such a maximal pattern of prime numbers has never previously been discovered, not to say: deduced, in mathematics. There exist many *computational* methods to find prime numbers, but these moves to and from and forwards and backwards between prime numbers and composite numbers. Hence, they do not establish any cut where the class of prime numbers (or of composite numbers) is predicted independently (of the complementary class of natural numbers), exactly, completely and irreversibly from one side of the cut to the other. Thus, such methods, as for example the ancient sieve of Eratosthenes, are of course able to find the primes, but without knowledge or claim of any *pattern* existing in the primes.

Our exhibition has deduced such a pattern exactly and completely in the maximal sense of a pattern, and this was achieved from also unveiling the *generator* of said pattern. Thus, as far as we can see, the quest indicated by the initial quote by John T. Tate appears completed. For upcoming references we denote this discovery as *Johansen Revolving Prime Number Code*, abbreviated to *Johansen Revolver*, further abbreviated to *JR*. We denote the initial software expression of JR (cf. appendix) as *Johansen Revolver - Strand Algorithm*, abbreviated to *JR-SA*. A further software expression, showing able to pick the prime numbers (and only the prime numbers) in correct succession for freely chosen intervals of natural numbers, has been denoted as *Johansen Revolver - Strand Longrange Algorithm*, abbreviated to *JR-SLA*.

Inspection of the JR-SA program of the appendix may be fruitful to indicate that successful and good programming of such mathematical results is more than a straight forward 1:1 translation between mathematics and software, and also require an element of creative skills, somewhat similar to translation between different natural languages. The author was surprised that the consecutive complement of the whole 88 sets of series proved possible to become programmed in basically merely 29 lines of code (the main ”bow” part of the program). As in the case of the Mandelbrot fractal, this may also serve as some indication of nature’s tendency to prefer quite simple algorithms, presenting as the foremost quest for science to seek and reconstruct *the* simplicity. It may also be the case that further contemplation of such software formulations will show fruitful for mathematical ideas and approaches, with the possibility for some co-evolution between upcoming advances in prime number mathematics vs. related informatics. It seems of special interest to investigate from further computational studies from JR-SA whether there also can be discovered a maximum pattern in the prime numbers from a direct, positive formulation of the gestalt, i.e. *without* anymore having to perform the gestalt switch.

As e.g. noted by Khurana and Koul (2005) probabilistic primality tests, like Solovay- Strassen and Miller-Rabin, have the disadvantage compared to deterministic primality tests that the algorithm must run many times for satisfactory ruling out of composite numbers. Other things equal, deterministic primality tests should therefore be sought developed. However, the efficiency of deterministic primality tests will depend on plural factors, such as choice of software language, degree of algorithmic optimation, organization of parallel software distrubution, degree of convertion from software to hardware, processor capacity etc. Therefore, improvement in computational efficiency stemming from a novel deterministic pattern in prime numbers discovered by means of mathematics, whether by deduction or by extensive successful number tests, may be hard to predict in the early days after the discovery, and may also depend much on coordinated efforts. On the other hand it seems reasonable to assume that enclosure of a maximum pattern in the prime numbers, which at the same time displays a quite simple mathematical structure, as in the case of JR, may yield future improvements in computational efficiency not possible to achieve *without* knowing the exact, hidden structure.

Plural recent achievements in mathematics and science indicate a strong and intimate relation between hidden patterns in prime numbers and important patterns in natural systems, such as argued in the referenced works by Kumar et al. (2003), Higgs (2008), Müller (2009), Boeyens (2003), Rapoport (2011) and Johansen (2011). This may indicate that prime numbers should be reconsidered as abstracted features of quite general informational relations, patterns and laws. Hibbs (2008) advocates a ”new prime number mindset” and writes:

Instead of thinking of the incremental growth of prime numbers as a value on a number scale, we need to think of them more in terms of an information container in a relational structure. The real applied value for that type of information container could be virus, cancer, density, mass, molecular, momentum, torque, etc.

In this perspective one might also expect the likelihood of further co-evolution between prime number patterning and information science in the very foundations of number theory vs. informatics.

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The software can be sent under request.