Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/110136

(α,β)-Anti Fuzzy Filters of CI-Algebras

S. R. Barbhuiya[*]

Department of Mathematics, Srikishan Sarda College, Hailakandi, Hailakandi-788151, Assam, India

Received: 23 Mar. 2016, Revised: 3 Nov. 2016, Accepted: 7 Nov. 2016
Published online: 1 Jan. 2017

Abstract: The aim of this paper is to introduce the notion of (α,β)-anti fuzzy filters of CI-algebras and to investigate some of their basic properties. We also introduce the notion of doubt cartesian product of (α,β)-anti fuzzy filters and study their properties.

Keywords: CI-algebra, α-(anti)fuzzy filter, (α,β)-anti fuzzy filter, homomorphism.

1 Introduction

BCK and *BCI*-algebras are two important class of logical algebras introduced by Imai and Iseki [5,7] in 1966. It is known that the class of *BCK*-algebra is a proper subclass of the class of *BCI*-algebras. There exist several generalizations of BCK/BCI-algebras, as such BCH-algebras [5], dual BCK-algebras [10], d-algebras [13], etc. In [11] Meng introduced the notion of a CI-algebra as a generation of a BE-algebra and dual BCK/BCI/BCH-algebras. After the introduction of fuzzy set in 1965 by Zadeh [18] researcher are trying to fuzzify all the usual mathematical concept in almost every branch of mathematics. Jun [4,8] introduced the notion of doubt (anti) fuzzy ideals in BCK/BCI-algebras. The concept of doubt fuzzy ideals of BF-algebras was introduced by Barbhuiya in [2]. Biswas [1] introduced the concept of anti fuzzy subgroup. The concept of fuzzification of ideals in CI-algebra have introduced by Mostafa[12]. In [9] Kim considered ideal and filter of CI-algebra. In [3] Borzooei et al. discussed anti fuzzy filters of CI-algebras.

In [15,16,17] Sharma introduced the notion of (α,β)-anti fuzzy set and (α,β)-anti fuzzy subgroups of a group G. Modifying their idea, in this paper we apply the idea of (α,β)-anti fuzzy set to CI-algebras and introduce the notion of (α,β)-anti fuzzy filters of CI-algebras and establish some of their basic properties. We show that a fuzzy subset is (α,β)-AFF iff its complement $\mu^c_{\alpha,\beta}$ is FF. We also study union and intersection of two (α,β)-anti fuzzy filters of CI-algebras and image and pre image of (α,β)-anti fuzzy filters under homomorphic mapping.

2 Preliminaries

Definition 2.1 *An algebraic system $(X,*,1)$ of type (2,0) is called a CI-algebra if it satisfies the following axioms:*

(CI1)$x*x=1$,
(CI2)$1*x=x$,
(CI3)$x*(y*z)=y*(x*z)\ \forall\ x,y,z\in X$.

A CI-algebra X satisfying the condition $x*1=1$, is called a BE-algebra. In any CI-algebra X one can define a binary relation " $\leq$ " by $x\leq y$ if and only if $x*y=1$.
In any CI-algebra X has the following properties:

–$y*((y*x)*x)=1$,
–$(x*1)*(y*1)=(x*y)*1$,
–*if* $1\leq x$, *then* $x=1$, *for all* $x,y\in X$.

Example 2.2 *Let X = {1, a, b} with the following cayley table:*

Table 1: Example of CI-algebra.

*	1	a	b
1	1	a	b
a	1	1	1
b	1	1	1

Then $(X,*,1)$ is a CI-algebra.
In the rest of this paper, X is a CI-algebra, unless otherwise stated.

[*] Corresponding author e-mail: saidurbarbhuiya@gmail.com

A non-empty subset S of X is called a subalgebra of X if $x * y \in S$, for all $x, y \in S$. A fuzzy subset μ of X is called a fuzzy subalgebra of X if $\mu(x*y) \geq min\{\mu(x),\mu(y)\}$, for all $x, y \in S$. A nonempty subset F of X is called a filter of X if

(F1) $1 \in F$,
(F2) $x \in F,\ x*y \in F \Rightarrow y \in F\ \forall\, x,y \in X$.

A filter F of X is said to be closed if $x \in F$ implies $x*1 \in F$.

Definition 2.3 *A fuzzy set μ in X is called a fuzzy filter(FF) of X if it satisfies the following conditions:*

–(FF1) $\mu(1) \geq \mu(x)$,
–(FF2) $\mu(y) \geq \min\{\mu(x*y),\mu(x)\},\ \forall\, x,y \in X$.

Definition 2.4 *A fuzzy set μ in X is called a anti fuzzy filter(AFF) of X if it satisfies the following conditions:*

–(AFF1) $\mu(1) \leq \mu(x)$,
–(AFF2) $\mu(y) \leq \max\{\mu(x*y),\mu(x)\}\ \forall\, x,y \in X$.

Definition 2.5 *Let μ be a fuzzy subset of X and $\alpha \in [0,1]$ then the fuzzy set μ^{α} and μ_{α} of X are respectively called the α-fuzzy subset and α-doubt fuzzy subset of X with respect to fuzzy set μ and is defined as $\mu^{\alpha}(x) = min\{\mu(x),\alpha\}$ and $\mu_{\alpha}(x) = max\{\mu(x), 1-\alpha\}$, for all $x \in X$.*
Clearly $\mu^1 = \mu, \quad \mu^0 = \tilde{0}, \quad \mu_1 = \mu, \quad \mu_0 = \tilde{1}$.

Definition 2.6 *A fuzzy subset μ of X is called α-fuzzy filter (α-FF) of X if*

$(i)\mu^{\alpha}(1) \geq \mu^{\alpha}(x)$
$(ii)\mu^{\alpha}(y) \geq min\{\mu^{\alpha}(x*y),\mu^{\alpha}(x)\} \quad \forall x,y \in X, \alpha \in [0,1]$.

Example 2.7 *Let $X = \{1,a,b,c\}$ with the following cayley table:*

Table 2: Example of α-FF of X.

*	1	a	b	c
1	1	a	b	c
a	1	1	b	c
b	1	a	1	c
c	1	a	b	1

Define a fuzzy set μ by $\mu(1) = 0.8, \mu(a) = 0.4, \mu(b) = 0.4, \mu(c) = 0.3$, with $\alpha = 0.5$. Then μ is a 0.5-FF of X.

Definition 2.8 *A fuzzy subset μ of X is called α-anti fuzzy filter (α-AFF) of X if*

$(i)\mu_{\alpha}(1) \leq \mu_{\alpha}(x)$
$(ii)\mu_{\alpha}(y) \leq max\{\mu_{\alpha}(x*y),\mu_{\alpha}(x)\} \quad \forall x,y \in X, \alpha \in [0,1]$.

Example 2.9 *Consider CI-algebra X as in Example 2.7. Define a fuzzy set μ in X by $\mu(1) = 0.3, \mu(a) = 0.6, \mu(b) = 0.6, \mu(c) = 0.8$ with $\alpha = 0.4$ Then μ is an 0.4-AFF of X.*

Proposition 2.10 *If μ is fuzzy filter of X, then μ is also α-FF as welL as α-AFF of X .*

Definition 2.11 *A fuzzy subset μ of X is called α-fuzzy subalgebra of X if $\mu^{\alpha}(x*y) \geq min\{\mu^{\alpha}(x),\mu^{\alpha}(y)\}, \quad \forall x,y \in X, \alpha \in [0,1]$.*

Definition 2.12 *A fuzzy subset μ of is called α-anti fuzzy subalgebra of X if $\mu_{\alpha}(x*y) \leq max\{\mu_{\alpha}(x),\mu_{\alpha}(y)\}, \quad \forall x,y \in X, \alpha \in [0,1]$.*

Definition 2.13 *Let X and Y be two non empty sets and $f : X \longrightarrow Y$ be a mapping. Let μ and ν be two fuzzy subsets of X and Y respectively. Then the image of μ under the map f is denoted by $f(\mu)$ and is defined by $f(\mu)(y)$, where $f(\mu)(y) = \begin{cases} \sup_{x\in f^{-1}(y)} \mu(x), & \text{if} \quad f^{-1}(y) = \{x \in X, f(x) = y\} \neq \phi \\ 0 & \text{otherwise} \end{cases}$*

also pre image of ν under f is denoted by $f^{-1}(\nu)$ and is defined as

$$f^{-1}(\nu)(x) = \nu(f(x)); \forall x \in X.$$

3 (α,β)-Anti fuzzy filters

Definition 3.1 *Let μ^{α} and μ_{β} denote respectively the α-fuzzy set and β-doubt fuzzy set of X (with respect to fuzzy set μ). Then the fuzzy set $\mu_{(\alpha,\beta)}$ defined by $\mu_{(\alpha,\beta)}(x) = max\{(\mu^{\alpha})^c(x),\mu_{\beta}(x)\} \quad \forall x \in X$ is called (α,β)-doubt fuzzy set of X (with respect to fuzzy set μ), where $\alpha,\beta \in [0,1]$ such that $\alpha+\beta \leq 1$.*

Remark 3.2 *(i)* $\mu_{(1,0)}(x) = max\{(\mu^1)^c(x),\mu_0(x)\} = max\{\mu^c(x),1\} = 1$.
(ii) $\mu_{(1,0)}(x) = max\{(\mu^0)^c(x),\mu_1(x)\} = max\{1,\mu(x)\} = 1$.

Definition 3.3 *Let (α,β)-anti fuzzy set $\mu_{(\alpha,\beta)}, t \in (01]$, and $\mu_{(\alpha,\beta)_t} = \{x \in X | \mu_{(\alpha,\beta)}(x) \geq t\}$ may be empty set. The set $\mu_{(\alpha,\beta)_t} \neq \phi$ is called the (α,β)-t-confidence set of μ.*
Again for (α,β)-anti fuzzy set $\mu_{(\alpha,\beta)}$ and $t \in (01]$, let $\mu^t_{(\alpha,\beta)} = \{x \in X | \mu_{(\alpha,\beta)}(x) \leq t\}$ may be empty set. The set $\mu^t_{(\alpha,\beta)} \neq \phi$ is called the (α,β)-t-doubt set of μ.

Example 3.4 *Consider CI-algebra X as in Example 2.7. Define a fuzzy set μ in X by $\mu(1) = 0.4, \mu(a) = 0.6, \mu(b) = 0.3, \mu(c) = 0.2$ take $\alpha = 0.4, \beta = 0.6$. Now we have $\mu_{\alpha,\beta}(x) = max\{(\mu^{\alpha})^c(x),\mu_{\beta}(x)\} = max\{1 - (\mu^{\alpha})(x),\mu_{\beta}(x)\} = max\{1 - min(\mu(x),\alpha), max(\mu(x),1-\beta)\}$, Therefore*
$\mu_{(0.4,0.6)}(1) = max\{1 - min(\mu(1),0.4), max(\mu(1),0.4)\} = max\{0.6,0.4\} = 0.6$
$\mu_{(0.4,0.6)}(a) = max\{1 - min(\mu(a),0.4), max(\mu(a),0.4)\} = max\{0.6,0.6\} = 0.6$

$\mu_{(0.4,0.6)}(b) = max\{1 - min(\mu(b),0.4), max(\mu(b),0.4)\} = max\{0.7,0.4\} = 0.7$
$\mu_{(0.4,0.6)}(c) = max\{1 - min(\mu(c),0.4), max(\mu(c),0.4)\} = max\{0.8,0.4\} = 0.8$
Then $\mu_{(0.4,0.6)_{6.5}} = \{b,c\}$, $\mu_{(0.4,0.6)_{0.8}} = \{c\}$, $\mu_{(0.4,0.6)_{0.9}} = \{\}$ *and* $\mu^{0.5}_{(0.4,0.6)} = \{\}, \mu^{0.7}_{(0.4,0.6)} = \{1,a,b\}$.

Remark 3.5 *If* $t \le \mu(x) \le max(\alpha,\beta) \le 1-t$, *then the set* $\mu_{(\alpha,\beta)_t} = \{x \in X | \mu_{(\alpha,\beta)}(x) \ge t\}$ *is a non empty and if* $1 - t \le \mu(x) \le 1 - min(\alpha,\beta)$, *then the set* $\mu^t_{(\alpha,\beta)} = \{x \in X | \mu_{(\alpha,\beta)}(x) \le t\}$ *is a non empty.*

Definition 3.6 *Let* μ *be a* (α,β)*-anti fuzzy set of* X *(with respect to fuzzy set* μ *), then* μ *is called* (α,β)*-anti fuzzy subalgebra of* X *if* $\mu_{(\alpha,\beta)}(x*y) \le max\{\mu_{(\alpha,\beta)}(x), \mu_{(\alpha,\beta)}(y)\}$, $\forall x,y \in X, \alpha,\beta \in [0,1]$.

Example 3.7 *Consider CI-algebra* X *as in Example 2.7. Define a fuzzy set* μ *in* X *by* $\mu(1) = 0.4, \mu(a) = 0.6, \mu(b) = 0.3, \mu(c) = 0.2$ *take* $\alpha = 0.4, \beta = 0.6$. *Then the* $(0.4,0.6)$*-anti fuzzy set* $\mu_{(0.4,0.6)}(1) == 0.6, \mu_{(0.4,0.6)}(a) = 0.6, \mu_{(0.4,0.6)}(b) = 0.7, \mu_{(0.4,0.6)}(c) = 0.8$ *is an* $(0.4,0.6)$*-anti fuzzy subalgebra of* X.

Definition 3.8 *Let* μ *be a* (α,β)*-anti fuzzy set of* X *(with respect to fuzzy set* μ*,) then* μ *is called* (α,β)*-anti fuzzy filter ((*α,β*)-AFF) of* X *if the following condition hold:*

$(i)\, \mu_{(\alpha,\beta)}(1) \le \mu_{(\alpha,\beta)}(x)$,
$(ii)\, \mu_{(\alpha,\beta)}(y) \le max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\} \quad \forall x,y \in X, \alpha,\beta \in [0,1]$.

Example 3.9 *Consider CI-algebra* X *as in Example 2.7. Define a fuzzy set* μ *in* X *by* $\mu(1) = 0.4, \mu(a) = 0.6, \mu(b) = 0.3, \mu(c) = 0.2$ *take* $\alpha = 0.4, \beta = 0.6$. *Then the* $(0.4,0.6)$*-anti fuzzy set* $\mu_{(0.4,0.6)}(1) == 0.6, \mu_{(0.4,0.6)}(a) = 0.6, \mu_{(0.4,0.6)}(b) = 0.7, \mu_{(0.4,0.6)}(c) = 0.8$ *is an* $(0.4,0.6)$*-AFF of* X.

Theorem 3.10 *If* μ *is* α*-FF and* β*-AFF of* X*, then* μ *is also* (α,β)*-AFF of* X.

Proof Since μ is α-FF of X, we have

$(i)\, \mu^\alpha(1) \ge \mu^\alpha(x)$
$(ii)\, \mu^\alpha(y) \ge min\{\mu^\alpha(x*y), \mu^\alpha(x)\} \quad \forall x,y \in X, \alpha \in [0,1]$

Now

$$\begin{aligned}(i)\quad &\Rightarrow 1-\mu^\alpha(1) \le 1-\mu^\alpha(x)\\ &\Rightarrow (\mu^\alpha)^c(1) \le (\mu^\alpha)^c(x) \qquad (1)\\ & \qquad (2)\end{aligned}$$

and

$$\begin{aligned}(ii)\quad &\Rightarrow \mu^\alpha(y) \ge min\{\mu^\alpha(x*y), \mu^\alpha(x)\}\\ &\Rightarrow 1-\mu^\alpha(y) \le 1 - min\{\mu^\alpha(x*y), \mu^\alpha(x)\}\\ &\Rightarrow 1-\mu^\alpha(y) \le max\{1-\mu^\alpha(x*y), 1-\mu^\alpha(x)\}\\ &\Rightarrow (\mu^\alpha)^c(y) \le max\{(\mu^\alpha)^c(x*y), (\mu^\alpha)^c(x)\} \qquad (3)\end{aligned}$$

Also, since μ is β-AFF of X, we have

$$\begin{aligned}&(iii)\, \mu_\beta(1) \le \mu_\beta(x) \qquad (4)\\ &(iv)\, \mu_\beta(y) \le max\{\mu_\beta(x*y), \mu_\beta(x)\} \quad \forall x,y \in X, \alpha \in [0,1] \qquad (5)\end{aligned}$$

Now

$$\begin{aligned}(v)\, \mu_{(\alpha,\beta)}(1) &= max\{(\mu^\alpha)^c(1), \mu_\beta(1)\}\\ &\le max\{(\mu^\alpha)^c(x), \mu_\beta(x)\} \quad \text{By Eqn(1)and(4)}\\ &= \mu_{(\alpha,\beta)}(x)\\ (vi)\, \mu_{(\alpha,\beta)}(y) &= max\{(\mu^\alpha)^c(x), \mu_\beta(x)\}\\ &\le max\{max\{(\mu^\alpha)^c(x*y), (\mu^\alpha)^c(x)\}, max\{\mu_\beta(x*y), \mu_\beta(x)\}\} \quad \text{By Eqn(3)and(5)}\\ &= max\{max\{(\mu^\alpha)^c(x*y), \mu_\beta(x*y)\}, max\{(\mu^\alpha)^c(x), \mu_\beta(x)\}\}\\ &= max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}.\end{aligned}$$

Therefore from (v) and (vi) μ is (α,β)-AFF of X.

Theorem 3.11 *If* μ *be a FF of* X*, then* μ *is also* (α,β)*-AFF of* X.

Proof Since μ is FF of X, we can see that μ is α-FF and β-AFF of X by Proposition 2.10. Hence by above Theorem μ is (α,β)-AFF of X.

Remark 3.12 *The converse of above theorem need not be true i.e., a fuzzy set* μ *of* X *can be* (α,β)*-AFF without being FF of* X.

Example 3.13 *Consider a CI-algebra* $X = \{1,a,b\}$ *with the following cayley table:*

Table 3: Example of (α,β)-AFF of CI-algebra X.

*	1	a	b
1	1	a	b
a	1	1	1
b	1	1	1

Define a fuzzy set μ *by* $\mu(1) = 0.5, \mu(a) = 0.6, \mu(b) = 0.4$ *take* $\alpha = 0.5, \beta = 0.6$. *Now we have* $\mu_{\alpha,\beta}(x) = max\{(\mu^\alpha)^c(x), \mu_\beta(x)\} = max\{1 - (\mu^\alpha)(x), \mu_\beta(x)\} = max\{1 - min(\mu(x),\alpha), max(\mu(x), 1-\beta)\}$, *Therefore*
$\mu_{(0.5,0.6)}(1) = max\{1 - min(\mu(1),0.5), max(\mu(1),0.4)\} = max\{0.5,0.5\} = 0.5$
$\mu_{(0.5,0.6)}(a) = max\{1 - min(\mu(a),0.5), max(\mu(a),0.4)\} = max\{0.5,0.6\} = 0.6$
$\mu_{(0.5,0.6)}(b) = max\{1 - min(\mu(b),0.5), max(\mu(b),0.4)\} = max\{0.6,0.4\} = 0.6$
one can be easily verify that μ *is* $(0.5,0.6)$*-AFF of* X *but* μ *is not a FF of* X.

Proposition 3.14 *Let* μ *be a fuzzy subset of* X*,* $\alpha \le p$ *and* $\beta \le 1-q$ *where* $p = inf\{\mu(x) : \forall x \in X\}$ *and* $q = sup\{\mu(x) : \forall x \in X.\}$ *Then* μ *is an* (α,β)*-AFF of* X.

Proof Here $\alpha \leq p$ and $\beta \leq 1-q, \alpha+\beta \leq p+1-q \leq q+1-q=1$. Also, $p = inf\{\mu(x) : \forall x \in X\} \geq \alpha \Rightarrow \mu(x) \geq \alpha \quad \forall x \in X$.
Therefore $\mu^{\alpha}(x) = min\{\mu(x), \alpha\} = \alpha$
Similarly, we can show $\mu_{\beta}(x) = 1-\beta$,
Now
$\mu_{(\alpha,\beta)}(x) = max\{(\mu^{\alpha})^c(x), \mu_{\beta}(x)\} = max\{1-\alpha, 1-\beta\}$
Hence both $\mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x)$ and $\mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\} \quad \forall x,y \in X, \alpha \in [0,1]$ satisfied.
Hence μ is (α,β)-AFF of X.

Proposition 3.15 *Let μ be an (α,β)-AFF of X, then the following hold*
(i) If $x \leq y$, then $\mu_{(\alpha,\beta)}(y) \leq \mu_{(\alpha,\beta)}(x)$
(ii) If $x * y \geq z$, then $\mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x), \mu_{(\alpha,\beta)}(z)\}$.

Proof
(i) Let $x \leq y$. Then $(x*y) = 1$

$$\begin{aligned}\mu_{(\alpha,\beta)}(y) &\leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}\\ &= max\{\mu_{(\alpha,\beta)}(1), \mu_{(\alpha,\beta)}(x)\}\\ &= max\{\mu_{(\alpha,\beta)}(x)\} \qquad [\text{ Since } \mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x), \quad \forall x \in X.]\end{aligned}$$

(ii) Since $x*y \geq z$, we have $z*(x*y) = 1$, Now

$$\begin{aligned}\mu_{(\alpha,\beta)}(x*y) &\leq max\{\mu_{(\alpha,\beta)}(z*(x*y)), \mu_{(\alpha,\beta)}(z)\}\\ &= max\{\mu_{(\alpha,\beta)}(1), \mu_{(\alpha,\beta)}(z)\}\\ &= max\{\mu_{(\alpha,\beta)}(z) \qquad [\text{ Since } \mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x), \quad \forall x \in X.]\\ \text{Now} \quad \mu_{(\alpha,\beta)}(y) &\leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}\\ &= max\{\mu_{(\alpha,\beta)}(z), \mu_{(\alpha,\beta)}(x)\}.\end{aligned}$$

Theorem 3.16 *If μ is (α,β)-AFF of X, then the set $X_{\mu} = \{x \in X \mid \mu_{(\alpha,\beta)}(x) = \mu_{(\alpha,\beta)}(1)\}$ is a filter.*

Proof Clearly $1 \in X_{\mu}$. Let $x*y, x \in X_{\mu}$. Then
$\Rightarrow \mu_{(\alpha,\beta)}(x*y) = \mu_{(\alpha,\beta)}(x) = \mu_{(\alpha,\beta)}(1)$
Now

$$\begin{aligned}\mu_{(\alpha,\beta)}(y) &\leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}\\ &= max\{\mu_{(\alpha,\beta)}(1), \mu_{(\alpha,\beta)}(1)\}\\ &= \mu_{(\alpha,\beta)}(1)\\ \text{Therefore} \Rightarrow \mu_{(\alpha,\beta)}(y) &\leq \mu_{(\alpha,\beta)}(1)\\ \text{Also, } \mu_{(\alpha,\beta)}(1) &\leq \mu_{(\alpha,\beta)}(y)\\ \text{Hence, } \mu_{(\alpha,\beta)}(y) &= \mu_{(\alpha,\beta)}(1) \Rightarrow y \in X_{\mu}\end{aligned}$$

Therefore X_{μ} is a filter.

Definition 3.17 *Let μ be a (α,β)-anti fuzzy set of X (with respect to fuzzy set μ), then μ is called (α,β)-fuzzy filter ((α,β)-FF) of X if the following condition holds:*

$(i) \quad \mu_{(\alpha,\beta)}(1) \geq \mu_{(\alpha,\beta)}(x)$
$(ii) \quad \mu_{(\alpha,\beta)}(y) \geq min\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\} \quad \forall x,y \in X, \alpha \in [0,1].$

Theorem 3.18 *A fuzzy set μ is (α,β)-AFF of X, iff its complement $\mu^c_{\alpha,\beta}$ is a fuzzy filter of X.*

Proof Sinceμ is (α,β)-AFF of X, we have

$(i)\, \mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x)$
$(ii)\, \mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\} \quad \forall x,y \in X, \alpha \in [0,1]$

Now (i) implies

$$\begin{aligned}\mu_{(\alpha,\beta)}(1) &\leq \mu_{(\alpha,\beta)}(x),\\ \text{Hence, } 1-\mu_{(\alpha,\beta)}(1) &\geq 1-\mu_{(\alpha,\beta)}(x).\\ \text{Therefore, } \mu^c_{(\alpha,\beta)}(1) &\geq \mu^c_{(\alpha,\beta)}(x)\end{aligned}$$

Now (ii) implies

$$\begin{aligned}&\mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}.\\ \text{Hence, } &\Rightarrow 1-\mu_{(\alpha,\beta)}(y) \geq 1-max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\} \quad \text{and so,}\\ &1-\mu_{(\alpha,\beta)}(y) \geq min\{1-\mu_{(\alpha,\beta)}(x*y), 1-\mu_{(\alpha,\beta)}(x)\}\\ \text{Therefore, } &\mu^c_{(\alpha,\beta)}(y) \geq min\{\mu^c_{(\alpha,\beta)}(x*y), \mu^c_{(\alpha,\beta)}(x)\}.\end{aligned}$$

Remark 3.19 *Note that $\mu^c_{(\alpha,\beta)}(x) = min\{\mu_{\alpha}(x), \mu^c_{\beta}(x)\}$.*

Theorem 3.20 *If μ and ν are two (α,β)-AFFs of X, then $\mu \cup \nu, \mu \cap \nu$ are also (α,β)-AFF of X.*

Proof Let $x,y \in X$. Now we have

$$\begin{aligned}(i) \quad (\mu\cup\nu)_{(\alpha,\beta)}(1) &= max\{\mu_{(\alpha,\beta)}(1), \nu_{(\alpha,\beta)}(1)\}\\ &\leq max\{\mu_{(\alpha,\beta)}(x), \nu_{(\alpha,\beta)}(x)\}\\ &= (\mu\cup\nu)_{(\alpha,\beta)}(x)\\ \text{Hence,} \quad (\mu\cup\nu)_{(\alpha,\beta)}(1) &\leq (\mu\cup\nu)_{(\alpha,\beta)}(x)\\ (ii) \quad (\mu\cup\nu)_{(\alpha,\beta)}(y) &= max\{\mu_{(\alpha,\beta)}(y), \nu_{(\alpha,\beta)}(y)\}\\ &\leq max\{max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}, max\{\nu_{(\alpha,\beta)}(x*y), \nu_{(\alpha,\beta)}(x)\}\}\\ &= max\{max\{\mu_{(\alpha,\beta)}(x*y), \nu_{(\alpha,\beta)}(x*y)\}, max\{\mu_{(\alpha,\beta)}(x), \nu_{(\alpha,\beta)}(x)\}\}\\ &= max\{(\mu\cup\nu)_{(\alpha,\beta)}(x*y), (\mu\cup\nu)_{(\alpha,\beta)}(y)\}\\ \text{Hence,} \quad (\mu\cup\nu)_{(\alpha,\beta)}(y) &\leq max\{(\mu\cup\nu)_{(\alpha,\beta)}(x*y), (\mu\cup\nu)_{(\alpha,\beta)}(x)\}.\end{aligned}$$

Therefore $\mu \cup \nu$ is (α,β)-AFF of X.
Again let $x,y \in X$. Now we have

$$\begin{aligned}(i) \quad (\mu\cap\nu)_{(\alpha,\beta)}(1) &= min\{\mu_{(\alpha,\beta)}(1), \nu_{(\alpha,\beta)}(1)\}\\ &\leq min\{\mu_{(\alpha,\beta)}(x), \nu_{(\alpha,\beta)}(x)\}\\ &= (\mu\cap\nu)_{(\alpha,\beta)}(x)\\ \text{Hence,} \quad (\mu\cap\nu)_{(\alpha,\beta)}(1) &\leq (\mu\cap\nu)_{(\alpha,\beta)}(x)\\ (ii) \quad (\mu\cap\nu)_{(\alpha,\beta)}(y) &= min\{\mu_{(\alpha,\beta)}(y), \nu_{(\alpha,\beta)}(y)\}\\ &\leq min\{max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}, max\{\nu_{(\alpha,\beta)}(x*y), \nu_{(\alpha,\beta)}(x)\}\}\\ &= min\{max\{\mu_{(\alpha,\beta)}(x*y), \nu_{(\alpha,\beta)}(x*y)\}, max\{\mu_{(\alpha,\beta)}(x), \nu_{(\alpha,\beta)}(x)\}\}\\ &= min\{(\mu\cap\nu)_{(\alpha,\beta)}(x*y), (\mu\cap\nu)_{(\alpha,\beta)}(y)\}\\ \text{Hence,} \quad (\mu\cap\nu)_{(\alpha,\beta)}(y) &\leq max\{(\mu\cap\nu)_{(\alpha,\beta)}(x*y), (\mu\cap\nu)_{(\alpha,\beta)}(x)\}.\end{aligned}$$

Therefore $\mu \cap \nu$ is (α,β)-AFF of X.

Theorem 3.21 *A fuzzy subset μ of X is a (α,β)-AFF iff for every $t \in (0 1]$,$\mu^t_{(\alpha,\beta)}$ is a filter of X, when $\mu^t_{(\alpha,\beta)} \neq \phi$.*

Proof Let $x,y \in X, \alpha \in [0,1]$ and μ be a (α,β)-AFF. Therefore we have $\mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x)$ and $\mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}$. Also let $x \in \mu^t_{(\alpha,\beta)}$ hence $\mu_{(\alpha,\beta)}(x) \leq t$. Now $\mu_{(\alpha,\beta)}(1) \leq \mu_{(\alpha,\beta)}(x) \leq t$ which implies $\mu_{(\alpha,\beta)}(1) \leq t$ and so $1 \in \mu^t_{(\alpha,\beta)}$. Again let $x*y, x \in \mu^t_{(\alpha,\beta)}$. Hence $\mu_{(\alpha,\beta)}(x*y) \leq t$ and $\mu_{(\alpha,\beta)}(x) \leq t$. Now $\mu_{(\alpha,\beta)}(y) \leq max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}$ implies $\mu_{(\alpha,\beta)}(y) \leq t$. Which implies $y \in \mu^t_{(\alpha,\beta)}$. Therefore $\mu^t_{(\alpha,\beta)}$

is a filter of X.
Conversely,
Assume $\mu^t_{(\alpha,\beta)}$ is a filter of X, to prove μ is (α,β)-AFF, if μ is not a (α,β)-AFF, then at least one of $\mu_{(\alpha,\beta)}(1) > \mu_{(\alpha,\beta)}(x)$ and $\mu_{(\alpha,\beta)}(y) > max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}$ must hold for at least some $x,y \in X$. Suppose $\mu_{(\alpha,\beta)}(1) > \mu_{(\alpha,\beta)}(x)$ holds for $x = x'$. Then choose $t = \{\mu_{(\alpha,\beta)}(1) + \mu_{(\alpha,\beta)}(x')\}/2 \in (0,1]$.

$$\text{Hence}\mu_{(\alpha,\beta)}(1) > t > \mu_{(\alpha,\beta)}(x') \tag{6}$$

Now (6)$\Rightarrow$ $\mu_{(\alpha,\beta)}(x') < t$ which implies $x' \in \mu^t_{(\alpha,\beta)}$, Since $\mu^t_{(\alpha,\beta)}$ is a filter of X, we have $1 \in \mu^t_{(\alpha,\beta)}$. Hence $\mu_{(\alpha,\beta)}(1) < t$ which contradicts (6). Again if $\mu_{(\alpha,\beta)}(y) > max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}$ for some x', y', then choose
$t = \{\mu_{(\alpha,\beta)}(y') + max\{\mu_{(\alpha,\beta)}(x'*y'), \mu_{(\alpha,\beta)}(x')\}\}/2 \in (0,1]$. Hence

$$\mu_{(\alpha,\beta)}(y') > t > max\{\mu_{(\alpha,\beta)}(x'*y'), \mu_{(\alpha,\beta)}(x')\} \tag{7}$$

Now (7)$\Rightarrow$ $\mu_{(\alpha,\beta)}(x'*y'), \mu_{(\alpha,\beta)}(x') < t$ which implies $x'*y', x' \in \mu^t_{(\alpha,\beta)}$. Since $\mu^t_{(\alpha,\beta)}$ is a filter of X, we have $y' \in \mu^t_{(\alpha,\beta)}$. Then $\mu_{(\alpha,\beta)}(y') < t$ which contradicts (7). Therefore μ is a (α,β)-AFF of X.

4 Homomorphism of (α,β)-anti Fuzzy Filters

In this section, we investigate the image and the pre-image of (α,β)-AFF of a *CI*-algebra under homomorphism.

Definition 4.1 *Let f be a mapping defined on a set X. If μ is a fuzzy set in X, then the fuzzy set ν in $f(X)$ defined by*

$$\nu(y) = \inf_{x \in f^{-1}(y)} \mu(x)$$

for all $y \in f(X)$ is called the doubt image of μ under f. If ν is a fuzzy set in $f(X)$ then the fuzzy set $\mu = \nu o f$ in X (i.e., the fuzzy set defined by $\mu(x) = \nu(f(x))$, for all $x \in X$) is called pre image of ν under f.

Lemma 4.2[13] *Let $f: X \longrightarrow Y$ be a mapping and μ, ν be two fuzzy subsets of X and Y repectively, then*
$(i) f^{-1}(\nu_{(\alpha,\beta)}) = (f^{-1}(\nu))_{(\alpha,\beta)}$
$(ii) f(\mu_{(\alpha,\beta)}) = (f(\mu))_{(\alpha,\beta)}$

Definition 4.3 *A mapping $f: X \longrightarrow Y$ is said to be homomorphism if $f(x*y) = f(x)*f(y)$, $\forall x,y \in X$.*

Theorem 4.4 *If $f: X \longrightarrow Y$ be a homomorphism, then $f(1) = 1'$*

Proof We have $f(1) = f(x*x) = f(x)*f(x) = 1'$.

Theorem 4.5 *Let $f: X \longrightarrow Y$ be an onto homomorphism. If ν be an (α,β)-AFF of Y, then the pre image of ν under f is also a AFF of X.*

Proof Let μ be the pre image of ν under f. Then $\mu(x) = \nu(f(x))$, for all $x \in X$. Since ν is an (α,β)-AFF of Y. We have $\nu_{(\alpha,\beta)}(f(1)) \le \nu_{(\alpha,\beta)}(f(x)) = \mu_{(\alpha,\beta)}(x)$. Also $\nu_{(\alpha,\beta)}(f(1)) = \nu_{(\alpha,\beta)}(f(1)) = \mu_{(\alpha,\beta)}(1)$ and so $\mu_{(\alpha,\beta)}(1) \le \mu_{(\alpha,\beta)}(x)$ for all $x \in X$. Also

$$\begin{aligned}
\mu_{(\alpha,\beta)}(y) &= f^{-1}(\nu_{(\alpha,\beta)})(y) \\
&= \nu_{(\alpha,\beta)}(f(y)) \\
&\le max\{\nu_{(\alpha,\beta)}((f(x)*f(y)), \nu_{(\alpha,\beta)}(f(x)\} \\
&= max\{\nu_{(\alpha,\beta)}(f(x*y)), \nu_{(\alpha,\beta)}(f(x)\} \\
&= max\{f^{-1}(\nu_{(\alpha,\beta)})(x*y), f^{-1}(\nu_{(\alpha,\beta)})(x)\} \\
&= max\{(f^{-1}(\nu))_{(\alpha,\beta)}(x*y), (f^{-1}(\nu))_{(\alpha,\beta)}(x)\} \\
&= max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}. \\
\text{Therefore} \quad \mu_{(\alpha,\beta)}(y) &\le max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}.
\end{aligned}$$

Since $f(y) \in Y$ is arbitrary and f is onto, $y \in X$ is also, therefore
$\mu_{(\alpha,\beta)}(y) \le max\{\mu_{(\alpha,\beta)}(x*y), \mu_{(\alpha,\beta)}(x)\}$ is true for all $x,y \in X$.
Hence $\mu = f^{-1}(\nu)$ is (α,β)-AFF of X.

Definition 4.6 *A fuzzy set μ of X has inf property if for any subset T of X, there exists $t_0 \in T$ such that*

$$\mu(t_0) = \inf_{t \in T} \mu(t)$$

Theorem 4.7 *Let $f: X \longrightarrow Y$ be an onto homomorphism. If μ be an (α,β)-AFF of X with inf property, then $f(\mu)$ is an (α,β)-AFF of Y.*

Proof Let μ be an (α,β)-AFF of X with *inf property* and $y \in Y$. Since f is onto, there exists $x \in X$ such that $y = f(x)$, now

$$\begin{aligned}
(f(\mu))_{(\alpha,\beta)}(f(1)) &= \inf_{t \in f^{-1}(1')} \mu_{(\alpha,\beta)}(t) \\
&= \mu_{(\alpha,\beta)}(1) \\
&\le \mu_{(\alpha,\beta)}(x) \\
&= \inf_{t \in f^{-1}(x)} \mu_{(\alpha,\beta)}(t) \\
&= f(\mu_{(\alpha,\beta)})(f(x)) \\
\text{Therefore}(f(\mu))_{(\alpha,\beta)}(f(1)) &\le f(\mu_{(\alpha,\beta)})(f(x))
\end{aligned}$$

Again let $x', y' \in Y$, let $x_0 \in f^{-1}(x'), y_0 \in f^{-1}(y')$ be such that

$$(\mu_{(\alpha,\beta)})(x_0) = \inf_{t \in f^{-1}(x')} (\mu_{(\alpha,\beta)})(t), \quad (\mu_{(\alpha,\beta)})(y_0) = \inf_{t \in f^{-1}(y')} (\mu_{(\alpha,\beta)})(t)$$

$$(\mu_{(\alpha,\beta)})(x_0 * y_0) = \inf_{t \in f^{-1}(x'*y')} (\mu_{(\alpha,\beta)})(t)$$

Now

$$\begin{aligned}
(f(\mu))_{(\alpha,\beta)}(y') &= \inf_{t \in f^{-1}(x')} (\mu_{(\alpha,\beta)})(t) \\
&= (\mu_{(\alpha,\beta)})(x_0) \\
&\le max\{(\mu_{(\alpha,\beta)})(x_0 * y_0), (\mu_{(\alpha,\beta)})(x_0)\} \\
&= max\{\inf_{t \in f^{-1}(x'*y')} (\mu_{(\alpha,\beta)})(t), \inf_{t \in f^{-1}(x')} (\mu_{(\alpha,\beta)})(t)\} \\
&= max\{(f(\mu_{(\alpha,\beta)}))(x'*y'), \mu_{(\alpha,\beta)}(x')\}
\end{aligned}$$

Therefore $f(\mu)$ is an (α,β)-AFF of Y.

5 Doubt Cartesian Product of (α,β)-anti Fuzzy Filters

Definition 5.1 *Let μ and ν be two (α,β)-anti fuzzy sets in a set X. Then their doubt cartesian product $\mu\times\nu : X\times X \longrightarrow [0,1]$ is defined by $(\mu\times\nu)(x,y) = max\{\mu(x),\nu(x)\}$, for all $x,y\in X$.*

Theorem 5.2 *If μ and ν are two (α,β)-anti fuzzy filters of X, then $\mu\times\nu$ is also an (α,β)-anti fuzzy filters of $X\times X$.*

Proof Let $x,y\in X$. Now we have

$$\begin{aligned}
(i)\quad (\mu\times\nu)_{(\alpha,\beta)}(1) &= max\{\mu_{(\alpha,\beta)}(1),\nu_{(\alpha,\beta)}(1)\}\\
&\le max\{\mu_{(\alpha,\beta)}(x),\nu_{(\alpha,\beta)}(x)\}\\
&= (\mu\times\nu)_{(\alpha,\beta)}(x).\\
\text{Hence}\quad (\mu\times\nu)_{(\alpha,\beta)}(1) &\le (\mu\times\nu)_{(\alpha,\beta)}(x).\\
(ii)\quad (\mu\times\nu)_{(\alpha,\beta)}(y) &= max\{\mu_{(\alpha,\beta)}(y),\nu_{(\alpha,\beta)}(y)\}\\
&\le max\{max\{\mu_{(\alpha,\beta)}(x*y),\mu_{(\alpha,\beta)}(x)\},max\{\nu_{(\alpha,\beta)}(x*y),\nu_{(\alpha,\beta)}(x)\}\}\\
&= max\{max\{\mu_{(\alpha,\beta)}(x*y),\nu_{(\alpha,\beta)}(x*y)\},max\{\mu_{(\alpha,\beta)}(x),\nu_{(\alpha,\beta)}(x)\}\}\\
&= max\{(\mu\times\nu)_{(\alpha,\beta)}(x*y),(\mu\times\nu)_{(\alpha,\beta)}(x)\}.\\
\text{Hence}\quad (\mu\times\nu)_{(\alpha,\beta)}(y) &\le max\{(\mu\times\nu)_{(\alpha,\beta)}(x*y),(\mu\times\nu)_{(\alpha,\beta)}(x)\}.
\end{aligned}$$

Therefore $(\mu\times\nu)$ is a (α,β)-AFF of $X\times X$.

Theorem 5.3 *If μ and ν be two (α,β)-anti fuzzy subsets of X such that $\mu\times\nu$ is a (α,β)-anti fuzzy filters of $X\times X$, then*
(i) either $\mu_{(\alpha,\beta)}(x)\ge\mu_{(\alpha,\beta)}(1)$ or $\nu_{(\alpha,\beta)}(x)\ge\nu_{(\alpha,\beta)}(1)$, for all $x\in X$.
(ii) if $\mu_{(\alpha,\beta)}(x)\ge\mu_{(\alpha,\beta)}(1)$ for all $x\in X$, then either $\mu_{(\alpha,\beta)}(x)\ge\nu_{(\alpha,\beta)}(1)$ or $\nu_{(\alpha,\beta)}(x)\ge\nu_{(\alpha,\beta)}(1)$.
(iii) if $\nu_{(\alpha,\beta)}(x)\ge\nu_{(\alpha,\beta)}(1)$ for all $x\in X$, then either $\mu_{(\alpha,\beta)}(x)\ge\mu_{(\alpha,\beta)}(1)$ or $\nu_{(\alpha,\beta)}(x)\ge\mu_{(\alpha,\beta)}(1)$.

Proof (i) Suppose that $\mu_{(\alpha,\beta)}(x)<\mu_{(\alpha,\beta)}(1)$ and $\nu_{(\alpha,\beta)}(x)<\nu_{(\alpha,\beta)}(1)$ for some $x,y\in X$. Then

$$\begin{aligned}
(\mu\times\nu)_{(\alpha,\beta)}(1) &= max\{\mu_{(\alpha,\beta)}(1),\nu_{(\alpha,\beta)}(1)\}\\
&\ge max\{\mu_{(\alpha,\beta)}(x),\nu_{(\alpha,\beta)}(x)\}\\
&= (\mu\times\nu)_{(\alpha,\beta)}(x).
\end{aligned}$$

Which contradicts the fact that $\mu\times\nu$ is a (α,β)-anti fuzzy filter of $X\times X$.
(ii) Assume that there exists $x,y\in X$ such that $\mu_{(\alpha,\beta)}(x)<\nu_{(\alpha,\beta)}(1)$ and $\nu_{(\alpha,\beta)}(x)<\nu_{(\alpha,\beta)}(1)$. Then

$$\begin{aligned}
(\mu\times\nu)_{(\alpha,\beta)}(1) &= max\{\mu_{(\alpha,\beta)}(1),\nu_{(\alpha,\beta)}(1)\} = \nu_{(\alpha,\beta)}(1).And\\
(\mu\times\nu)_{(\alpha,\beta)}(x) &= max\{\mu_{(\alpha,\beta)}(x),\nu_{(\alpha,\beta)}(x)\}\\
&< max\{\nu_{(\alpha,\beta)}(1),\nu_{(\alpha,\beta)}(1)\}\\
&= \nu_{(\alpha,\beta)}(1) = (\mu\times\nu)_{(\alpha,\beta)}(1).\\
\text{Therefore}\quad (\mu\times\nu)_{(\alpha,\beta)}(x) &< (\mu\times\nu)_{(\alpha,\beta)}(1).
\end{aligned}$$

Which contradicts the fact that $\mu\times\nu$ is a (α,β)-anti fuzzy filter of $X\times X$. Hence (ii) holds.
(iii) Similar to proof of (ii).

Theorem 5.4 *If μ and ν be two (α,β)-anti fuzzy subsets of X such that $\mu\times\nu$ is a (α,β)-anti fuzzy filter of $X\times X$, then either μ or ν is an (α,β)-anti fuzzy filter of X.*

Proof Straightforward.

6 Application

We can apply (α,β)-anti fuzzy filters of *CI*-algebras in artificial intelligence, computer science, medicine, control engineering, decision theory, expert systems, operations research, pattern recognition, and robotics and many more fields.

7 Conclusions and future work

In this paper, we have introduced the concept of (α,β)-anti fuzzy filters of *CI*-algebras and investigated some of their useful properties. In my opinion, these definitions and results can be extended to other algebraic systems like BCK/BCI/BF/BH/BCH-algebras. In [4,6,8] Huang et al. defined fuzzy ideal of BCK-algebras in some different way, now this ideal some author called as doubt fuzzy ideal and some other called as anti fuzzy ideal i.e., doubt fuzzy and anti fuzzy are same meaning. But the (α,β)-anti fuzzy ideal is little different from anti fuzzy ideal. In the future, the following studies may be carried out:
(1) (α,β)-anti fuzzy BCI-positive implicative ideals of BCI-algebra
(2) (α,β)-anti fuzzy maximal ideals of BCK-algebras.
(3) (α,β)-intuitionistic anti fuzzy ideals of BCK-algebra.
(4) An application of (α,β)-anti fuzzy filters of *CI*-algebras in medical diagnosis.

References

[1] R. Biswas, *Fuzzy subgroups and anti fuzzy subgroups*, Fuzzy sets and systems **35,** 121-124 (1990).
[2] S. R. Barbhuiya, *Doubt fuzzy ideals of BF-algebra*, IOSR Journal of Mathematics (IOSR-JM), **10(2)** , Ver. VII (Mar-April), 65-70 (2014).
[3] R. A. Borzooei, A. B. Saeid, A. Rezaei and R. Ameri, *Anti fuzzy filters of CI-algebras*, Afr. Mat. **25(4)**, 1197-1210 (2014).
[4] S. M. Hong and Y. B. Jun, *Anti fuzzy ideals in BCK-algebras*, Kyungpook Math. **38**,145-150 (1998).
[5] Q. P. Hu and X. Li, *On BCH-algebras, Math. Seminar Notes*, 11, No.2, part 2, 313-320 (1983).
[6] F. Y. Huang, *Another definition of fuzzy BCI-algebras-in Chinese*, Selected papers on BCK and BCI-algebras (P. R. China). **1** ,91-92 (1992).
[7] Y. Imai and K. Iseki, *On Axiom systems of Propositional calculi XIV*, Proc, Japan Academy, **42**, 19-22 (1966).
[8] Y. B. Jun, *Doubt fuzzy BCK/BCI-algebras*, Soochow Journal of Mathematics **20(3)**, 351-358 (1991).
[9] K. H. Kim, *A Note on CI-algebras*, International Mathematical Forum, **6 (1)**,1-5 (2011).
[10] K. H. Kim and Y. H. Yon, *Dual BCK-algebra and MV-algebra*, Sci. Math. Jpn., **66**, 247-253 (2007).
[11] B. L. Meng, *CI-algebras*, Sci. Math. Japo. Online, 695-701 e-2009.

[12] S. M. Mostafa, M. A.Abdel Naby and O. R.Elgendy, *Fuzzy ideals in CI-algebras*, Journal of American Science, **7(8)**, 485-488 (2011).

[13] J. Negger and H. S. Kim, *On d-algebras*, Math Slovaca **40(1)**, 19-26 (1999).

[14] A. B. Saeid, *Some results in Doubt fuzzy BF- algebras*, Analele Universitatii Vest Timisoara seria Mathematica-Informatica, **XLIX**,125-134 (2011).

[15] P. K. Sharma, *α-Anti Fuzzy Subgroups*, International Review of Fuzzy Mathematics (July-Dec 2012).

[16] P. K. Sharma, *(α,β)-Fuzzy Subgroups*, Fuzzy Sets Rough Sets and Multivalued Operations and Applications (July-Dec 2012).

[17] P. K. Sharma, *(α,β)-Anti Fuzzy Subgroups* , International Journal of Fuzzy Mathematics and Systems **3(1)**, 61-74 (2013).

[18] L. A. Zadeh, *Fuzzy sets*, Information and Control **8**, 338-353 (1965).

Saidur Rahman Barbhuiya is an Assistant Professor and Head Department of Mathematics , Srikishan Sarda College, Hailakandi, Assam (India). He received his M. Sc. and Ph. D. degree from Assam University, Silchar, Assam (India) . He is referee and Editorial board member of several international journals. His research interests include Fuzzy sets, Fuzzy algebraic structures and their applications. In his credit, he has a number of papers published in various national and international journals and conferences. Mr. Barbhuiya is a life member of Association of Mathematics Teachers of India (AMTI), Chennai, life member of Assam Science Society and life member of Barak Education Society, Silchar and a member of Board of Undergraduate Studies (BUGS) in Mathematics, Assam University, Silchar.