Applied Mathematics & Information Sciences Letters
An International Journal

http://dx.doi.org/10.18576/amisl/070101

The Concentration Distribution Surrounded the Growing of Gas Bubbles in the Bio Tissues of Divers

A. K. Abu-Nab[1,*] *and S. A. Mohammadein*[2]

[1] Department of Mathematics and Computer Science, Faculty of Science, Menoufia University, Shebin El-Koom, 32511, Egypt
[2] Department of Mathematics, Faculty of Science, Tanta University, Tanta, 31511, Egypt

Received: 1 Jun. 2018, Revised: 7 Nov. 2018, Accepted: 11 Nov. 2018
Published online: 1 Jan. 2019

Abstract: The concentration distribution around growing gas bubble in the blood and bio tissues of divers who ascend to surface too quickly is obtained by Mohammadein and Mohamed model [12] for variant and constant ambient pressure through the decompression process. The mathematical model describing this problem consists of four main equations: mass, convective diffusion, Fick's and Laplaces equations. The mathematical model is solved analytically to obtain the concentration distribution around a growing gas bubble in biotissues. The growth of gas bubble is affected by initial concentration difference ΔC_0, diffusivity of gas in tissue D_T , the constant K_d at decompression, surface tension σ , initial void fraction ϕ_0. The relation between the growth of gas bubble$R(t)$ and time t is obtained from the definition of the concentration distribution around a growing gas bubble in biotissues. The relation between the growth of gas bubble and time is studied under the effect of two different values of initial void fractionϕ_0,critical bubble radius R_c. The present model is compared with Mohammadein and Mohammed model [12]

Keywords: bubble growth, biotissues, concentration distribution, decompression sickness (DCS)

1 Introduction

The dissolution of gas bubbles in a liquid gas solution with simultaneous chemical reaction between the dissolved gas and the liquid is of concern in several problems of biological and physical interest. Among them may be mentioned the dissolution of oxygen bubbles in blood in extra-corporeal circulation during open heart surgery [1,2], and that of carbon dioxide bubbles in water. When an oxygen bubble in whole blood the oxygen gas diffuses across the bubble surface into the blood and immediately combines with the under saturated hemoglobin is situated. Only a small fraction of the oxygen gas is dissolved in the plasma. The chemical reaction between the oxygen and the hemoglobin takes place in the concentration boundary layer. More than sixty years ago, Epstein and Plesset published a seminal article in which they showed how to semi-quantitatively estimate the rate of gas bubble growth or dissolution, for a bubble embedded in a liquid medium containing the dissolved gas of which the bubble is comprised. They applied their expressions to air bubbles suspended in water, containing dissolved air. Their rate expressions have been experimentally found to be largely correct, and the precise degree of validity of their model remains the subject of active research. For small dissolving bubbles, the predictions of Epstein and Plessets model were found to be within about 9 of the observed values for the surface tension and saturation level dependencies when the surrounding medium is a simple liquid [3]. Decompression sickness (DCS) is a disorder, seen especially in deep-sea divers, caused by the formation of nitrogen bubbles in the blood and tissues following a sudden drop in the surrounding pressure, as for divers when ascending rapidly from a dive, or people who flight for long distances from the earth, and characterized by severe pains in the joints and chest, skin irritation, cramps, and paralysis. Arterial gas embolism occurs when expanding gas stretches and ruptures alveolar capillaries-pulmonary barotraumas-allowing alveolar gas to enter the arterial circulation [4]. Cerebral arterial gas embolization typically involves the migration of gas to small arteries (average diameter, 30 to 60). The emboli cause pathologic changes by two mechanisms: a reduction in perfusion distal to the obstruction and an inflammatory response to the bubble [10], Fig. 1. Diffusion and perfusion processes thought to govern extra

* Corresponding author e-mail: ahmed.abunab@yahoo.com

vascular gas bubble growth and resolution in tissues. Various studies of gas bubble behavior in animals and humans like [9] have been modeled in terms of ordinary differential equations (ODEs). Two models are used to describe the process of growth and dissolution of a gas bubble in tissue [10]. Some of them whereby the bubble is immersed in a well-stirred tissue compartment but is immediately surrounded by a well defined boundary layer through which diffusion-limited gas exchange between bubble and tissue occurs, are denominated (three region models) [7,8,9,15]; the bubble, boundary layer and tissue region are the three regions. In contrast, models in which the bubble is immersed in an un stirred tissue compartment, and gas exchange between bubble and tissue is limited by bulk diffusion through the tissue are denominated (two-region models) [9,10]; since they consist of only bubble and tissue regions. More literature

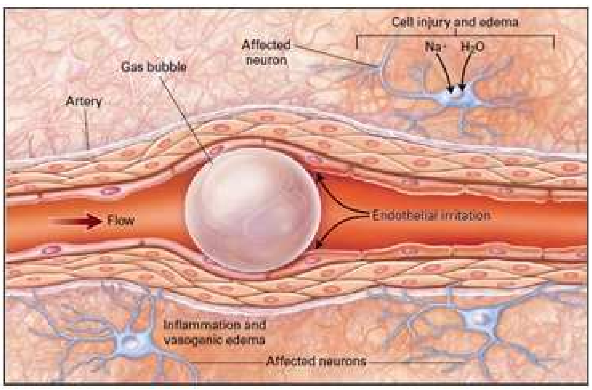

Fig. 1: Obstructing End-Arterial Flow in a Cerebral Vessel with a Diameter of 30 to 60 , Causing Distal Ischemia. The obstruction causes the metabolic processes of neurons to fail. Sodium and water enter the vessel, and cytotoxic edema develops. The surface of the bubble generates a foreign-body response through cellular and humoral immune mechanisms. The bubble also mechanically irritates the arterial endothelium. Both processes result in vasogenic edema and greater impairment of perfusion. The neuronal injury extends beyond the area of obstruction [5].

reviews of previous models that describing the growth of gas bubbles in tissues and blood can be found in [10,16]. Using the three region model, for a stationary gas bubble, several authors like Srinivasan et al. [9,10], Gerhardt [7] and others have solved the problem in the case of quasi-static pressure, while Mohammadein and Mohamed [12] solved the problem in the case of unsteady concentration around the growing bubble. In this study, we have solved the problem in more general case, at which the effect convection is taken into account. The main result is: the growth of a gas bubble enhances the consuming of the over saturated gases in tissue and leads to shorter time to reaching maximum radius rather than the models of stationary gas bubble. Moreover, a formula for concentration distribution around a growing gas bubble is also derived. The results are implemented to explain the effects of given physical parameters on the growth of a gas bubble in a divers tissue that exposes to a sudden decompression in the ambient pressure.

2 Mathematical Model

The bubble is assumed to grow in a well stirred tissue, i.e. three-region model (Gas bubble, thin boundary layer and well-stirred finite tissue) is used. Solvent vapor pressure is neglected, for simplicity and a single diffusible gas is considered. The physical problem is described as illustrated in Fig. 2. A single gas bubble is considered to

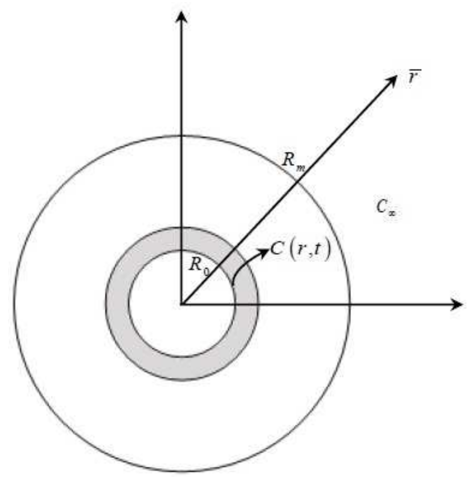

Fig. 2: The problem sketch.

grow inside a tissue between two finite radius boundaries R_o and R , the growth is affected by some parameters such as the pressure difference ΔP between the bubble pressure $P_g(R(t),t)$ and the ambient pressure $P_{amb}(t)$, surface tension of the tissue-bubble interface, concentration difference between the two phases and other physical parameters. The convective term, that taking into account in the diffusion equation, noticeably affect on the growth process, spatially in reducing the growth time. Taking into account the following assumptions:

1. Gases are considered to be ideal.
2. The bubble is assumed to have a spherical geometry.
3. Pressure inside the bubble is assumed to be uniform.
4. Gas density distribution inside the bubble is assumed to be uniform except for a thin boundary layer near the bubble wall.
5. The viscosity of the tissue content is omitted.

The mathematical model describing this problem consists of four main equations: mass, convective diffusion, Fick's and Laplaces equations.

2.1 Mass balance equation

Assuming that, tissue gas tension is in equilibrium with venous blood gas. The rate of gas uptake by the tissue is the amount carried by the blood per unit time less the flux into the gas bubble. Thus, the mass equation has the form [10].

$$\alpha_T V_T \frac{dP_T}{dt} = \alpha_b V_T \dot{Q}(P_a - P_T) - \frac{1}{\Re T}\frac{d}{dt}(P_g V_g). \quad (1)$$

2.2 Diffusion equation

Gas diffusion through the tissue without sources or sinks is described by

$$\frac{\partial C}{\partial t} = D_T\left(\frac{\partial^2 C}{\partial r^2} + \frac{2}{r}\frac{\partial C}{\partial r}\right). \quad (2)$$

2.3 Fick's equation

The rate of change of molar concentration of gas in the bubble equals the molar flux of gas through the bubble surface. Thus [10]

$$\frac{1}{\Re T}\frac{d}{dt}\left(\frac{4}{3}\pi R^3 P_g\right) = 4\pi R^2 D_T\left(\frac{\partial C}{\partial r}\right)_{r=R}. \quad (3)$$

2.4 Pressure balance equation

Taking into account the effect of surface tension at the gas-liquid interface and tissue visco-elastic effects, the Laplaces equation that represents the pressure balance on the gas-liquid interface is [7,17]

$$P_g = P_{amb} + \frac{2\sigma}{R} + \frac{4\pi}{3}MR^3. \quad (4)$$

3 Method of solution

We use the method of combined variables to solve the diffusion equation (2) [8,18]. We assume that

$$C(r,t) = C(s), \quad (5)$$

where

$$s = \frac{\beta r}{f(t)}. \quad (6)$$

At r=R, then s=β and $R = f(t)$ By using the equation (6) into equation (2) , after separating the variables, we have

$$f(t)\dot{f}(t) = \frac{-\beta^2 D_T}{-s}\left(\left(\frac{1}{\frac{dC}{ds}}\right)\frac{d^2C}{ds^2} + \frac{2}{s}\right) = D_T^2\mu. \quad (7)$$

We assumed the separation constant in the form $D_T^2\mu$ where μ is a constant of unit $(s.m^{-2})$. We have two differential equations to be solved

$$f(t)\dot{f}(t) = D_T^2\mu, \quad (8)$$

and

$$\frac{d}{ds}\ln\left(\frac{dC}{ds}\right) = \frac{\mu D_T}{\beta^2}(-s) - \frac{2}{s}. \quad (9)$$

Solving equation(9), and using the initial condition at $t = t_0$, $R = R_0$, we get

$$f^2(t) = R^2 = 2\mu D_T^2 t + k_l, \quad (10)$$

where k_l is a constant. We have refused the negative sign, since the radius must be a positive value. At $t = t_0$, $R = R_0$, thus

$$k_l = R_0^2 - 2\mu D_T^2 t_0, \quad (11)$$

consequently,equation(10)becomes

$$R = \sqrt{2\mu D_T^2(t - t_0) + R_0^2}. \quad (12)$$

Now, integrating equation(9), we obtain

$$\frac{dC}{ds} = \frac{k_2}{s^2}\exp\left(\frac{-\mu D_T}{\beta^2}\left(\frac{s^2}{2}\right)\right), \quad (13)$$

where k_2 is a constant, that can be evaluated as follows: The boundary condition (3), by using of equation (4), is modified to be

$$\frac{1}{3\Re T}\frac{d}{dt}\left(\left(P_{amb} + \frac{2\sigma}{R} + \frac{4\pi}{3}MR_3\right)R^3\right) = R^2 D_T\left(\frac{\partial C}{\partial r}\right)_{r=R}, \quad (14)$$

or

$$\frac{\partial C}{\partial r}|_{r=R} = \frac{R^2\dot{P}_{amb} + 4\sigma\dot{R} + 3P_{amb}R\dot{R} + 8\pi MR_4\dot{R}}{3\Re T D_T R}. \quad (15)$$

Since

$$\begin{aligned}\frac{\partial C}{\partial r}|_{s=\beta} &= \left(\frac{r}{s}\frac{\partial C}{\partial r}\right)_{r=R,s=\beta} \\ &= \frac{R^2\dot{P}_{amb} + 4\sigma\dot{R} + 3P_{amb}R\dot{R} + 8\pi MR_4\dot{R}}{3\beta\Re T D_T R}. \quad (16)\end{aligned}$$

Using equation (16) into equation (13)

$$k_2 = \beta\left(\frac{R^2\dot{P}_{amb} + 4\sigma\dot{R} + 3P_{amb}R\dot{R} + 8\pi MR_4\dot{R}}{3\Re T D_T}\right)\exp\left(\frac{\mu D_T}{2}\right), \quad (17)$$

where

$$\dot{R} = \frac{\mu D_T^2}{R}.$$

From equation (13) into equation (15), we have

$$\begin{aligned}\frac{\partial C}{\partial r} &= \frac{k_2}{rs}\exp\left(\frac{-\mu D_T s^2}{2\beta^2}\right) \\ &= \frac{k_3 R}{r^2}\exp\left(-\mu D_T\left(\left(\frac{r^2}{2R^2} - \frac{1}{2}\right)\right)\right), \quad (18)\end{aligned}$$

where

$$s = \frac{\beta r}{R}$$

and

$$k_3 = \left(\frac{R^2\dot{P}_{amb} + 4\sigma\dot{R} + 3P_{amb}R\dot{R} + 8\pi MR_4\dot{R}}{3\Re TD_T}\right). \quad (19)$$

Integrating the previous equation through the interval from any instant t to t_m at which the bubble reaches its maximum radius R_m,at this instant $C(R_m,t_m) = C_\infty$, that is

$$C(r,t) - C_\infty = -k_3R\int_r^{R_m}\frac{1}{x^2}\exp\left(-\mu D_T\left(\frac{x^2}{2R^2} - \frac{1}{2}\right)\right)dx. \quad (20)$$

At the bubble wall $r = R(t)$ the previous equation becomes

$$C(R(t),t) - C_\infty = -k_3R\int_{R(t)}^{R_m}\frac{1}{x^2}\exp\left(-\mu D_T\left(\frac{x^2}{2R^2} - \frac{1}{2}\right)\right)dx. \quad (21)$$

Putting $y = \frac{x}{R}$, thus the previous equation can be written in the form

$$C(R(t),t) - C_\infty = -k_3\int_1^{\frac{R_m}{R(t)}}\frac{1}{y^2}\exp\left(-\mu D_T\left(\frac{y^2}{2} - \frac{1}{2}\right)\right)dy. \quad (22)$$

Putting $z = 1 - \frac{1}{y}$, then the previous integral can be written as

$$\begin{aligned} C\ (R(t),t) - C_\infty &= \\ -k_3\int_0^{\frac{1-R_m}{R(t)}}&\exp\left(-\mu D_T\left(\frac{1}{2(1-z)^2} - \frac{1}{2}\right)\right)dz \\ = -k_3\int_0^{\frac{1-R_m}{R(t)}}&\exp\left(\frac{-\mu D_T}{2}\left(\frac{1-(1-z)^2}{(1-z)^2}\right)\right)dz. \quad (23) \end{aligned}$$

Since $D_T \leq 1$ and $0 \leq z \leq 1 - \frac{R_0}{R_m}$,we can approximate the integrand, and the previous integral takes the form

$$\begin{aligned} C(R(t),t) - C_\infty = -\ k_3\int_0^{\frac{R_m}{R(t)}}(1 - \mu D_T z \\ -\frac{3}{2}\mu D_T z^2 - \frac{4}{2}\mu D_T z^3 \ldots\Big)dz, \quad (24) \end{aligned}$$

therefore,

$$\begin{aligned} C(R(t),t) - C_\infty &= -k_3\left(z - \frac{\mu D_T}{2}\left(\frac{1}{1-z} - (1+z)\right)\right)_0^{1-\frac{R}{R_m}} \\ &= -k_3\left(\frac{2z(1-z) - \mu D_T z^2}{2(1-z)}\right)_0^{1-\frac{R}{R_m}}, \end{aligned}$$

then,

$$C(R(t),t) - C_\infty = -k_3\left(\frac{2\frac{R}{R_m}(1 - \frac{R}{R_m}) - \mu D_T(1 - \frac{R}{R_m})^2}{2\frac{R}{R_m}}\right). \quad (25)$$

Equation (26)gives an implicit relation between the bubble radius $R(t)$ and the time t. At $t = t_0 \Rightarrow R(t) = R_0$,equation (25) becomes

$$\Delta C_0 = C_\infty - C(R_0,t_0) = k_3\left(\frac{2\frac{R_0}{R_m}(1 - \frac{R_0}{R_m}) - \mu D_T(1 - \frac{R_0}{R_m})^2}{2\frac{R_0}{R_m}}\right), \quad (26)$$

or

$$\Delta C_0 = k_0\left(\frac{2\phi_0^{\frac{1}{3}}(1 - \phi_0^{\frac{1}{3}}) - \mu D_T(1 - \phi_0^{\frac{1}{3}})^2}{2\phi_0}\right), \quad (27)$$

where,

$$k_0 = \left(\frac{R_0^2\dot{P}_{amb0} + 4\sigma\dot{R}_0 + 3P_{amb0}R_0\dot{R}_0 + 8\pi MR_0^4\dot{R}_0}{3\Re TD_T}\right). \quad (28)$$

Equation(29),we get the following expression for μ.

$$\mu = \frac{2\phi_0^{\frac{1}{3}}((1 - \phi_0^{\frac{1}{3}}) - \frac{\Delta C_0}{k_0})}{D_T(1 - \phi_0^{\frac{1}{3}})^2}, \quad (29)$$

Substituting forμ into equation (12) we get the relation of the bubble radius as a function of time. That is

$$R(t) = \sqrt{4D_T\frac{2\phi_0^{\frac{1}{3}}((1 - \phi_0^{\frac{1}{3}}) - \frac{\Delta C_0}{k_0})}{(1 - \phi_0^{\frac{1}{3}})^2}(t - t_0) + R_0^2}. \quad (30)$$

We can get the following approximated value for the initial growth velocity to be:

$$\dot{R}_0 \approx \frac{2D_T\phi_0^{\frac{1}{3}}}{(1 - \phi_0^{\frac{1}{3}})}. \quad (31)$$

The constant k_0 has two formulae due to the case of the ambient pressure, for variable ambient pressure at decompression; suppose the ambient pressure linearly decreases with time,i.e. $P_{amb} = P_0 - \dot{\alpha}t$, where $\dot{\alpha}$ is the ascent rate [14],has the formula

$$k_d = \frac{-\dot{\alpha}R_0^2 + 4\sigma\dot{R}_0 + 3(P_0 - \dot{\alpha}t_0)R_0\dot{R}_0 + 8\pi MR_0^4\dot{R}_0}{3\Re TD_T}, \quad (32)$$

and for constant ambient pressure (after decompression),$\dot{P}_{amb} = 0$, i.e, $P_{amb} = Const. = P_\infty$ at diving stops or after finishing diving and reaching the sea level it has the formula

$$k_c = \frac{4\sigma\dot{R}_0 + 3P_\infty R_0\dot{R}_0 + 8\pi MR_0^4\dot{R}_0}{3\Re TD_T}. \quad (33)$$

The time for the bubble to reach its maximum radius, can be calculated by applying the final conditions on equation (12), we get

$$R_m = \sqrt{2\mu D_T^2(t_m - t_0) + R_0^2},$$

from which the time of complete growth will be

$$t_m = \frac{R_m^2 - R_0^2}{2\mu D_T^2} + t_0. \quad (34)$$

4 Concentration distribution gas bubble in the bio tissues of divers

Putting $y = \frac{x}{R}$ into equation (20) it becomes

$$C(r,t) = C_\infty - k_3(t) \int_{\frac{r}{R}}^{\frac{R_m}{R}} \frac{1}{y^2} \exp\left(-\mu D_T \left(\frac{y^2}{2} - \frac{1}{2}\right)\right) dy. \quad (35)$$

Putting $z = 1 - \frac{1}{y}$,then the previous integral can be written as

$$C(r,t) - C_\infty = -k_3 \int_{1-\frac{R}{r}}^{1-\frac{R}{R_m}} \exp\left(\frac{-\mu D_T}{2}\left(\frac{1-(1-z)^2}{(1-z)^2}\right)\right) dz. \quad (36)$$

This integral can be approximated as before to give

$$C(r,t) - C_\infty = -k_3 \left(\frac{2z(1-z)^2 - \mu D_T z^2}{2(1-z)}\right)_{1-\frac{R}{r}}^{1-\frac{R}{R_m}}, \quad (37)$$

that is,

$$\begin{aligned} C(r,t) &= C_\infty - k(t)_3 \frac{\left(2\frac{R}{R_m}\left(1-\frac{R}{R_m}\right) - \mu D_T \left(1-\frac{R}{R_m}\right)^2\right)}{2\frac{R}{R_m}} \\ &\quad - \frac{\left(2\frac{R}{r}\left(1-\frac{R}{r}\right) - \mu D_T \left(1-\frac{R}{r}\right)^2\right)}{2\frac{R}{r}} \\ &= C_\infty - k_3(t)\left(\left(\frac{R}{r} - \frac{R}{R_m}\right) + \frac{\mu D_T}{2R}\left(1-\frac{R}{r}\right)^2 r \right. \\ &\quad \left. - \left(1 - \frac{R}{R_m}\right)^2 R_m\right). \end{aligned} \quad (38)$$

Taking into account the different formulae for $k_3(t)$ in the cases of decompression or constant ambient pressure as follows

$$k_d(t) = \frac{-\alpha R^2 + 4\sigma\dot{R} + 3(P_0 - \alpha t)R\dot{R} + 8\pi M R^4 \dot{R}}{3\Re T D_T}. \quad (39)$$

$$k_c(t) = \frac{4\sigma\dot{R} + 3P_\infty R\dot{R} + 8\pi M R^4 \dot{R}}{3\Re T D_T}. \quad (40)$$

5 Derivation the Radius of gas bubble from Rayleigh equation (Momentum equation)

We can obtained the relation between growth of gas bubble radius $R(t)$ in biotissues and time t from substituting by the equation (38), that represented the concentration of gas bubble in biotissues in the momentum equation at $r = R$.

$$R\ddot{R} + \frac{3}{2}\dot{R}^2 = \frac{1}{\rho}\left(A(C_R - C_s) + b_0 \Delta P_0 - \frac{2\sigma}{R}\right), \quad (41)$$

where, A is the constant,C_R is concentration within the bubble,C_s is saturation concentration at interface of bubble,$\Delta P_0 = P_g(C_0) - P_\infty$,$R_{cr} = \frac{R_0}{1+\delta}$ is the critical bubble radius (unstable equilibrium radius),C_0 is the initial concentration,δ, and $b_0$are the a constant with small value.

$$t = 0, C_R = C_0, R(0) = R_0, \dot{R}(0) = \dot{R}_0, and \ddot{R} = 0. \quad (42)$$

On the basis of equations (9, 10, and 11), the constant A becomes

$$A = \frac{H\rho_l}{(C_0 - C_s)}, \quad (43)$$

where

$$H = \frac{1}{3}\dot{R}_0^2 + \frac{2\sigma}{\rho_l R_0} - \frac{b_0 \Delta P_0}{\rho_l}. \quad (44)$$

Thus, equation (41) can be rewritten in the form

$$\begin{aligned} R\ddot{R} + \frac{3}{2}\dot{R}^2 &= \frac{1}{\rho}\left(A(C_\infty - k_3(t)(1 - \frac{R}{R_m}) + \frac{\mu D_T k_3(t) R_m}{2R}\right. \\ &\quad \left. \times (1 - \frac{R}{R_m})^2 + b_0 \Delta P_0 - \frac{2\sigma}{R}\right). \end{aligned} \quad (45)$$

$$\begin{aligned} \frac{1}{2\dot{R}R^2}\frac{d}{dt}(R^3\dot{R}^2) &= \frac{1}{\rho}\left(A(C_\infty - k_3(t)(1 - \frac{R}{R_m})\right. \\ &\quad + \frac{\mu D_T k_3(t) R_m}{2R} \\ &\quad \left.(1 - \frac{R}{R_m})^2 + b_0 \Delta P_0 - \frac{2\sigma}{R}\right). \end{aligned} \quad (46)$$

Integrated the above equation w. r. to R,from $R = R$ to $R = R_0$, then

$$\dot{R} = \sqrt{A2 + A3R + \frac{A4}{R} + \frac{A1}{R^3}}, \quad (47)$$

where,

$$\begin{aligned} A1 &= {R_0}^3 {\dot{R}_0}^2 - \frac{1}{\rho_l}\left(A\left(\frac{2}{3}C_\infty {R_0}^3 - 2K_3\left(\frac{{R_0}^3}{3} - \frac{{R_0}^4}{4R_m}\right)\right.\right. \\ &\quad \left. + \bar{\mu} D_T K_3 R_m \left(\frac{{R_0}^2}{2} + \frac{2{R_0}^3}{3R_m} + \frac{{R_0}^4}{4{R_m}^2}\right)\right) \\ &\quad \left. + \frac{2}{3} b_0 \Delta P_0 {R_0}^3 - 2\sigma {R_0}^2\right). \end{aligned} \quad (48)$$

The approximately solution for the relation between the growth of gas bubble radius $R(t)$ and time t is defined as follows

$$\begin{aligned} t - t_0 &= \left(\left(R - \frac{1}{A1}\left(\frac{A2}{4}R^2 + \frac{A3}{2}(1+R) - \frac{A4}{4R^2}\right)\right)\right. \\ &\quad - \left(R_0 - \frac{1}{A1}\left(\frac{A2}{4}{R_0}^2 + \frac{A3}{2}(1+R_0)\right.\right. \\ &\quad \left.\left.\left. - \frac{A4}{4{R_0}^2}\right)\right)\right), \end{aligned} \quad (49)$$

where, $A2$,$A3$,$A4$ are constant defined as follows:

$$A2 = \frac{1}{3\rho_l}\left(2AC_\infty - 2AK_3 - 2A\bar{\mu} D_T K_3 + 2b_0 \Delta P_0\right); \quad (50)$$

$$A3 = \frac{AK_3}{2R_m\,\rho_l}\left(1+\frac{\bar{\mu}D_T}{2}\right); \tag{51}$$

$$A4 = \frac{A\bar{\mu}D_T K_3 R_m}{2\rho_l} - \frac{2\sigma}{\rho_l}; \tag{52}$$

Table 1: The data which is used to get the graphs needed to show the effect of the physical parameters on the growth of the gas bubble.

Parameter (P)	Value	(P)	Value
R_0	$1.0\times10^{-6}m$	T	$310(37^0c)K$
P_0	$200000N/m^2$	$\dot{\alpha}$	$3066.67N/m^2.s$
ΔC_0	$0.7mpl/m^3$	$\Re$	$8.314472N.m/mol.K$
σ	$0.03N/m$	t_0	$0.0s$
D_T	$2.2\times10^{-12}m^2/s$	M	$34473.79Pa.m^{-3}$

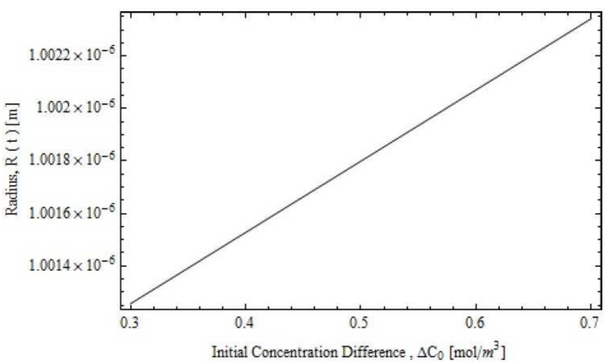

Fig. 3: The relation between the bubble radius $R(t)$ and the initial concentration difference ΔC_0, for the range$0.3 \le \Delta C_0 \le 0.7$.

6 Results and discussion

The diffusion equation (2), for the growing gas bubble in tissue through decompression in ambient pressure is solved by the method of combined variables. The solution of the problem, equation (30), gives explicitly the instantaneous bubble radius as a function of time combined with the physical parameters that affect on the growth process. The dominant parameter is the initial void fraction ϕ_0 . The growth formula can be used for calculating the bubble radius in both cases of ambient pressure through decompression or after decompression (constant ambient pressure), that is by using the equations(32),(33). The time of complete growth can be calculated by the equation (34). Moreover, the

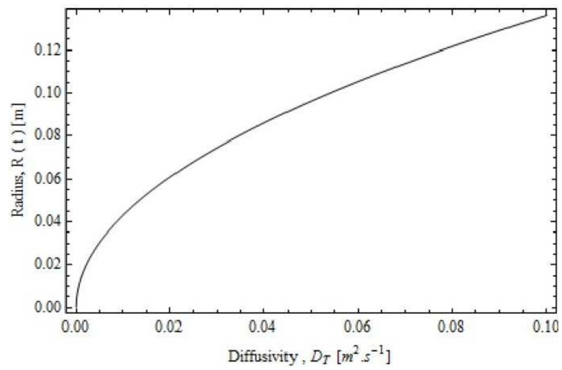

Fig. 4: The relation between the bubble radius $R(t)$ and the diffusivity of gas in tissue D_T, for the range$10^{-12} \le \Delta C_0 \le 10^{-10}m^2.s^{-1}$.

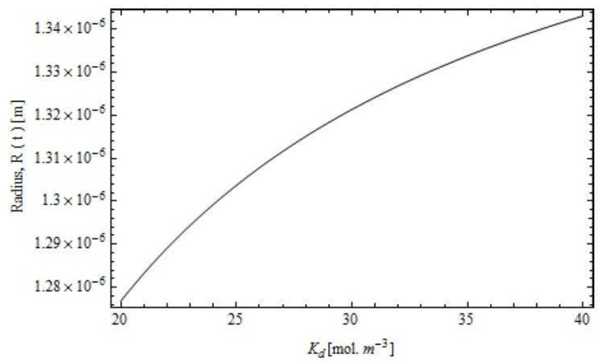

Fig. 5: The relation between the bubble radius $R(t)$ and the constant K_d at decompression, for the range$20 \le K_d \le 40mol.m^{-3}$.

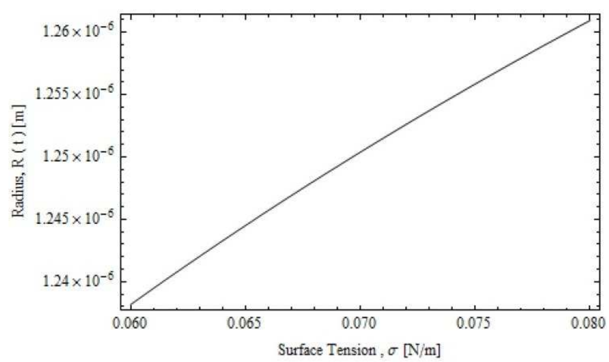

Fig. 6: The relation between the bubble radius $R(t)$ and the surface tension σ, for the range$20 \le \sigma \le 40mol.m^{-3}$.

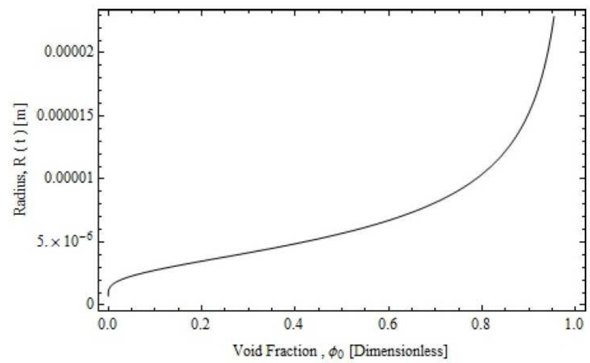

Fig. 7: The relation between the bubble radius $R(t)$ and the initial void fraction ϕ_0, through its range$0 \leq \phi_d \leq 1$.

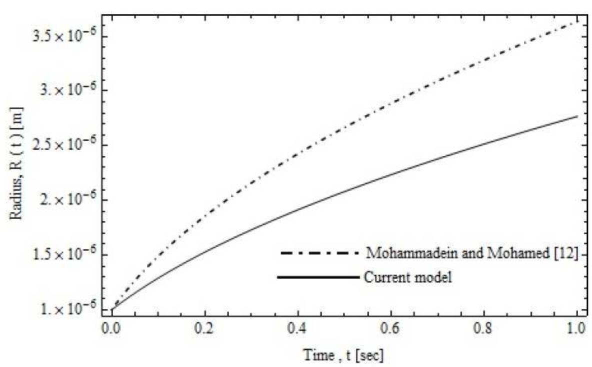

Fig. 8: Comparison between the bubble radius $R(t)$ for the current model and Mohammadein and Mohammed model [12].

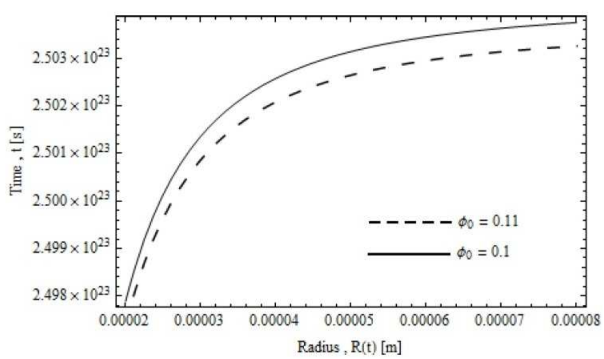

Fig. 9: The growth of gas bubble radius $R(t)$ is plotted as a function of time with two different values of the initial void fraction ϕ_0 , through its range $0 \leq \phi_0 \leq 1$.

concentration distribution around a growing gas bubble in tissue is presented by equation (38). The relation between the growth of gas bubble radius $R(t)$ with the time t is obtained by equation (48). Under decompression growth $(P_{amb}(t)) = P_0 - \dot{\alpha} t$, Figs. 3, 4, 5, 6, and 7explain the effect of changing the values of the parameters ΔC_0,D_T

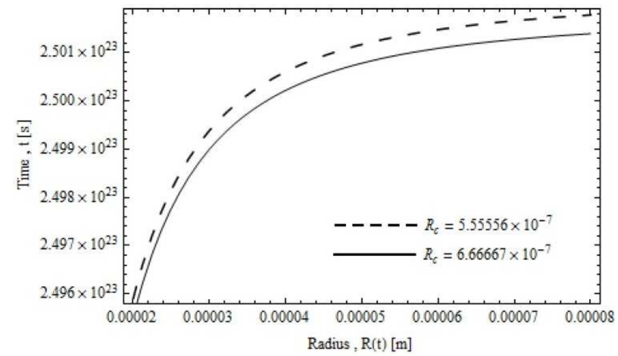

Fig. 10: The growth of gas bubble radius $R(t)$ is plotted as a function of time with two different values of the critical bubble radius R_c.

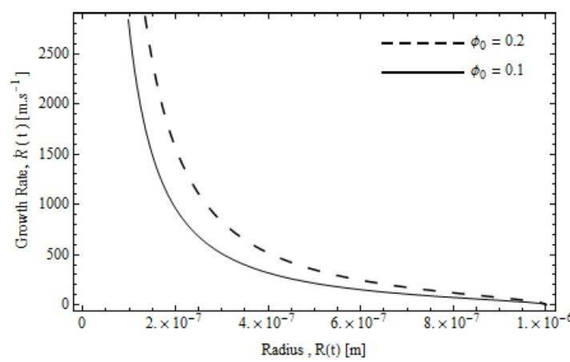

Fig. 11: The growth rate of gas bubble radius $\dot{R}$ is plotted as a function of gas bubble radius $R(t)$ with two different values of the initial void fraction ϕ_0, through its range $0 \leq \phi_d \leq 1$.

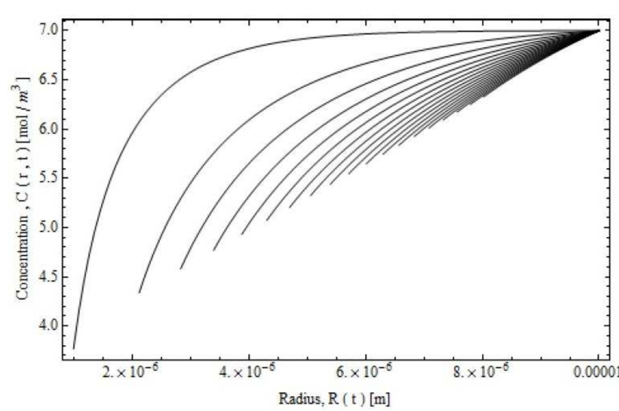

Fig. 12: The concentration distribution around a growing gas bubble in tissue for the case of constant ambient pressure stage at $C_\infty = 0.7 mol./m^3$,$\Delta C_o = 6.4 mol./m^3$,$P_\infty = 101235 N/m^2$and $\phi_0 = 1.0 \times 10^{-3}$.

,k_d, and ϕ_0 respectively on the bubble radius at some instant $(t = 1.0s)$ for some intervals of these parameters. It is noticed that, the growth of the gas bubble is proportional to the parameters ΔC_0,D_T ,k_d, and ϕ_0.The comparison between the present model and Mohammadein and Mohammed model [12] is illustrated in Fig. 8. It is clearly the present model taken the lower values more than Mohammadein and Mohammed model [12], the reason for this is due to me Mohammadein and Mohammed model [12] is studied with the convective bubble growth. Moreover, the growth process is sensitive to slight changes in value of the initial void fraction near the value one. In the Fig. 9, it is clearly the relation between the growth of gas bubble radius $R(t)$ and time t is proportional inversely with the small increases of the initial void fractionϕ_0. The effect of the critical bubble radius R_c on the growth process is studied in Fig. 10, and which taken the same behavior of the initial void fraction ϕ_0 . Fig. 11, show that, the growth rate is proportional with the increasing of the initial void fraction ϕ_0. Fig. 12 explains the concentration distribution around a growing gas bubble in tissue for the case of constant ambient pressure, the sea level pressure, which shows that the concentration gradient decreases while the growth process is taking place until it vanishes at the complete growth of the bubble. This model can be used in several applications; it can be used to predict the time that a growing gas bubble takes to reach a fixed radius R_m by using equation (39). This may be help in predicting the time that the bubble takes to reach some critical radius that can close some blood vessel, if its radius is known, which may cause an embolism. The concentration around the growing bubble at any instant can be calculated by the relations (38,39, and 40). The oversaturated concentration,C_∞ can be calculated by Henrys law formula,$P_\infty = K_H C_\infty$. For nitrogen gas, $K_H = 1.63934 m^3.atm.mol^{-1}$,for example in diving: P_∞equals nearly 0.78 bar for each 10 meters depth plus 0.78 bar for the atmospheric pressure. That is, for 20 meters depth $P_\infty = 2.34 bar$ and $C_\infty = 1.427 mol.m^{-3}$.

7 Conclusion

The growth of gas bubble in tissue is discussed, based on the three-region model. The growth of the bubble radius is proportional to the initial concentration differenceΔC_0,the diffusivity of the tissue D_T , the concentration constant k_d, and the initial void fraction ϕ_0 . Moreover, the concentration distribution around the growing gas bubble $R(t)$ in tissue is obtained analytically for the two main growth stages by equations. (38, 39, and 40). We have shown that the concentration gradient decreases while the growth process is taking place until it vanishes at the complete growth of the bubble.

References

[1] WEN-JEI YANG, Dynamics of Gas Bubbles in Whole Blood and Plasma, J. Biomechanics**4**,119-125 1971.

[2] J. M. Solano-Altamirano, J. D. Malcolm, S. Goldman. Gas bubble dynamics in soft materials. Soft Matter **11**, 202 (2015).

[3] P. B. Duncan, D. Needham. Microdroplet Dissolution into a Second Phase Solvent Using a Micropipet Technique: Test of the Epstein Plesset Model for an Aniline Water System. Langmuir **22**, 4190 4197 (2006).

[4] R. D. Vann, F. K. Butler, S. J. Mitchell, R. E. Moon. Decompression illness, The Lancet **377**, 153-164 (2011).

[5] C. M. Muth, E. S. Shank, E. S. Gas Embolism. New England Journal of Medicine **342**, 476-482 (2000).

[6] M. A. Chappell, S. J. Payne. A physiological model of the release of gas bubbles from crevices under decompression. Respiratory Physiology and Neurobiology **153**, 166-180 (2006).

[7] M. L. Gernhardt. Development and evaluation of a decompression stress index based on tissue bubble dynamics. Ph. D. Thesis, University of Pennsylvania (1991).

[8] S. A. Mohammadein, K. G. Mohamed. Concentration Distribution around a Growing Gas Bubble in Tissue. Mathematical Biosciences **225**, 11-17 (2010).

[9] R. S. Srinivasan, W. A. Gerth, M. R. Powell. A Mathematical Model of Diffusion Limited Gas Bubble Dynamics in Tissue with Varying Diffusion Region Thickness. Respiration Physiology **123**, 153164 (2000).

[10] R. S. Srinivasan, W. A. Gerth, M. R. Powell. Mathematical Models of Diffusion-Limited Gas Bubble Dynamics in Tissue. J. Appl. Physiol. **86**, 732741 (1999).

[11] R. S. Srinivasan, W. A. Gerth, M. R. Powell. Mathematical Model of Diffusion-Limited Gas Bubble Dynamics in Unstirred Tissue with Finite Volume. Annals of Biomedical Engineering **30**, 232246 (2002).

[12] S. A. Mohammadein, K. G. Mohamed. Growth of gas bubbles in the biotissues with convective acceleration. Int. J. of Biomath. **7** (6) (2014).

[13] P. Tikuisis, K. A. Gault, R. Y. Nishi. Prediction of Decompression Illness Using Bubble Models. Undersea Hyperb.Med. **21**, 129143 (1994).

[14] H. D. Van Liew, M. E. Burkard. Density of Decompression Bubbles and Competition for Gas among Bubbles, tissue and blood. J. Appl. Physiol. **75**, 22932301 (1993).

[15] J. Zueco, J. A. Hern?ndez - Gonz?lez. Network simulation method applied to models of diffusion-limited gas bubble dynamics in tissue. Acta Astronautica, **67**, 344352 (2010).

[16] V. Papadopoulou, R. J. Eckersley, C. Balestra, T. D. Karapantsios, Meng-Xing, Tang. A critical review of physiological bubble formation in hyperbaric decompression. Advances in Colloid and Interface Science **191**, 2230 (2013).

[17] R. S. Srinivasan, W. A. Gerth. M. R. Powell. Mathematical Model of Diffusion-Limited Evolution of Multiple Gas Bubbles in Tissue, Annals of Biomedical Engineering**31**, 471481 (2003).

[18] C. R. Gutvik, A. O. Brubakk. A dynamic two-phase model for vascular bubble formation during decompression of divers. IEEE Transactions on Biomedical Engineering, **56**, 884 889 (2009).

A. K. Abu-Nab received the B.Sc. from Menoufia University, Faculty of Science, Department of Mathematics and Computer Science; and M.Sc.degree of applied mathematics (2015)at Menoufia University; and Ph.D. degree of applied mathematics (2017) at Menoufia University. Currently,he is a lecturer of applied Mathematics at Menoufia University, Faculty of Science, Department of Mathematics and Computer Science. His research interests in the areas of applied mathematics, heat and mass transfer and bubble dynamics. He has published research articles in reputed international journals of mathematical and engineering sciences. He is referee of some mathematical journals.

S. A. Mohammadein received the B.Sc. and M.Sc. degrees from Tanta University, Faculty of Science; and Ph.D. degree from the Polish Academy of Sciences (1994). Currently, he is professor of applied mathematics at Tanta University, Faculty of Science. He has published papers in the field of Bubble Dynamics specially growth of gas or vapour bubbles and relaxation times for the systems containing bubbly flow. He is also a reviewer of some journals as Springers Journal of Heat and Mass Transfer.