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Dynamic Programming and Multi Objective Linear Programming approaches

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This paper describes two different methods to deal with the fuzzy shortest path problems. The first one is to study fuzzy shortest path in a network by Bellman dynamic programming approach and the second one is to study same problems by multi objective linear programming (MOLP) technique. It is considered that the edge weights of the network as uncertain. To analyze this idea of uncertainty four examples have been taken with two different network where edge weights have been presented by triangular fuzzy numbers and trapezoidal fuzzy numbers respectively. Both the problems have been solved by the above two methods. In the first method sign distance ranking procedure has been applied to get real value of the fuzzy edge weights where as in the second method MOLP, only 0-1 variables have been considered to get integer solution without using the Branch and Bound technique. It is observed that the length of the shortest paths in fuzzy sense as obtained by both the methods are same/almost same and the shortest path corresponds to the actual path in the network. It is obvious that fuzzy shortest path is an extension of the crisp problem.

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Key words and phrases: Shortest-path, triangular fuzzy number, trapezoidal fuzzy number, multi objective linear programming.

1. INTRODUCTION

The shortest path problem is one of the most important problems in network optimization. The problem of shortest path in a network has attracted many previous researchers since it is important to many transportation problems, routing, communications, economical, and other applications. Graphs/Networks emerge naturally as mathematical model of the observed real world system. The weights of edges can express geographical distances, transportation cost or time between two vertices connected by the edge. While geographical distances can be stated deterministically, cost or time can fluctuate with traffic conditions, pay load and so on. Therefore uncertainty arises to express edge weights of a network through cost or time. A typical way of expressing these uncertainties in the edge weights is to utilize fuzzy numbers. Also in a network fuzziness can be introduced in a variety of ways – through edge weights, edge capacities, vertex restriction, arc lengths [6,7,13, 16, 19].

The concept of fuzzy decision making problems – with maximizing decision was first proposed by Bellman and Zadeh [1]. Zimmerman [2] presented a fuzzy approach to multi objective linear programming problems. He also studied the duality relations in fuzzy linear programming. Fuzzy linear programming problem with fuzzy coefficients was formulated by Negoita is called robust programming. Dubois and Prade [4] investigated linear fuzzy constraints. Liu and Kao's [19] developed an algorithm for finding non-dominated shortest path. In this approach they transform all the fuzzy arcs into crisp arcs by applying Yoger's ranking method and then solve the fuzzy shortest path problem with crisp arcs by using 0-1 variables.

In this paper, we discussed two different methods for the same fuzzy shortest path problems. Both the methods have been executed with the same numerical examples. Two different networks have been considered whose edge weights are fuzzy numbers- triangular fuzzy numbers and trapezoidal fuzzy numbers. The first method is Bellman dynamic programming approach. When solving by this method the fuzzy shortest path problem is first converted into an equivalent crisp problem and then recursion procedure is applied. The second method we use here is the multi objective linear programming method as suggested by [2].

Enough literatures about the fuzzy shortest path can be found, e.g., [4,12,13,16,18,21]. Dubios and Prade [4] first in 1980 treated the fuzzy shortest path problems. It is obvious that fuzzy shortest path can be determined but in reality it may not correspond to an actual path of the network. To analyze this problem Klein [10] proposed a dynamic programming recursion-based fuzzy algorithm that specified each arc length within an integer value from one to a fixed number. In [16], Okada and soper introduced an order relation between fuzzy numbers based on fuzzy min concept and that a fuzzy non dominated path or Pareto optimal path from any node to a specified node of a network.

To deal with imprecision parameters in mathematical programming problems, fuzzy set theory has been applied to real word decision making problems. Fuzzy linear programming models and fuzzy multi objective programming problems are designated for such a purpose in fuzzy set theory, a corresponding membership function is usually employed to quantify the fuzzy objectives and constraints, using the linear membership function, Zimmermann proposed the min operator models to the multi objective linear programming.

Okada and Soper also restricted each fuzzy number of arc length to L-R fuzzy number, thus the four objective functions are required in their approach [16]. Apparently this problem can be formulated as a linear multi objective programming problem without using any 0-1 variable because it belongs to the problem type of network linear programming [14]. The weights (lengths or costs or times) of edges can express geographical distances of the corresponding vertices or transportation costs expended (or times spent) to move between their and vertices. Whole geographical distances can be stated deterministically, cost or times can fluctuate with traffic conditions, pay load and so on. In the last two cases (cost or time), deterministic values for representing the edge weights can not be used. A typical way of expressing these uncertainties in the edge weights is to utilize fuzzy numbers based on fuzzy set theory. In the case we must define an order relation between fuzzy numbers, because the fuzzy variant of the problem evaluates “fuzzy min “operations. As many approaches for the comparison of fuzzy numbers do not guarantee that fuzzy number are totally ordered, they lead to a number of non dominated paths (or pareto optimal paths). In this paper our approach is based on Cheng’s fuzzy ranking method.

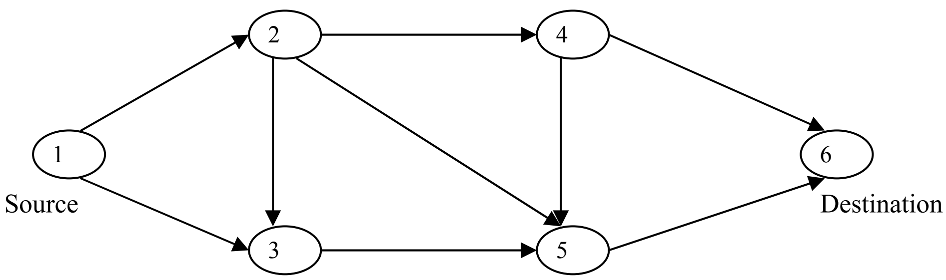

Fig -1

According to bellman’s equation [BL] a Dynamic programming formulation for the shortest path problem can be given as follows, consider a network with an acyclic directed graph $G=(\mathrm{V},\mathrm{E})$ with n vertices number from 1 to n such that 1 is the source node and n is the destination node. Then, by using forward calculation we have,

$$f(1)=0$$
$$f(j)=\min_{l<j}\{f(i)+c_{ij}\} \tag{B4}$$

Where

c_{ij} $\rightarrow$ weight of the directed edge $\langle i, j\rangle$

$f(i)\rightarrow$ length of the shortest-path from the source vertex 1.

2. DEFINATION AND PREREQUISITES

Definition 1: A Fuzzy set is a set whose boundary is not clear, whose elements are characterized by a membership function. Let **X** be a universal set. A fuzzy set Ã define on **X.** A set of order pair of element whose first element $x \in$ **X,** second element $\mu_{\tilde{A}}(x)$ is the membership value of element **x** in the set **Ã .**It is denoted by **Ã** or **A,** and it defined by

$$\tilde{A} = \{(x, \mu_A(x)) \mid x \in X\}$$

Where $\mu_A(x) \rightarrow K$

Signed distance ranking method for fuzzy numbers

In the given network the edge weights are represented as triangular fuzzy numbers. A triangular fuzzy number $\tilde{c}_{ij}$, $\tilde{c}_{ij} = (a,b,c)$ is a fuzzy set. The signed distance of $\tilde{c}$ measured from $\tilde{0}_1$ is defined by

$$d(\tilde{c}\,, \tilde{0}_1) = \frac{1}{4}(a+2b+c)$$

which will map the fuzzy number $\tilde{c}$ on the real line $\Re$.

Prop-1 The sum of two fuzzy numbers $\tilde{A} = (u,v,w)$ and $\tilde{B} = (p,q,r)$ then the binary operation is given by

$$\tilde{A} \oplus \tilde{B} = (u+p, v+q, r+w)$$

and $d(\tilde{A} \oplus \tilde{B}, \tilde{0}_1) = d(\tilde{A}, \tilde{0}_1) + d(\tilde{B}, \tilde{0}_1)$

Prop- 2 The ranking of fuzzy number $\tilde{A} = (u,v,w)$ and $\tilde{B} = (p,q,r)$ is defined by

$$\tilde{A} \propto \tilde{B} \text{ iff } d(\tilde{A}, \tilde{0}_1) < d(\tilde{B}, \tilde{0}_1)$$
$$\tilde{A} = \tilde{B} \text{ iff } d(\tilde{A}, \tilde{0}_1) = d(\tilde{B}, \tilde{0}_1)$$

Fuzzy numbers

Fuzzy number is expressed as fuzzy set defining in the interval of real number $\Re$ .Since the boundary of this interval is ambiguous thus interval is also a fuzzy set.

There are three types of fuzzy number:

1. Interval Fuzzy Number
2. Triangular Fuzzy Number
3. Trapezoidal Fuzzy Number

Triangular Fuzzy Number: It is a fuzzy number represented with three points as follows **A=(** a,b,c **)**

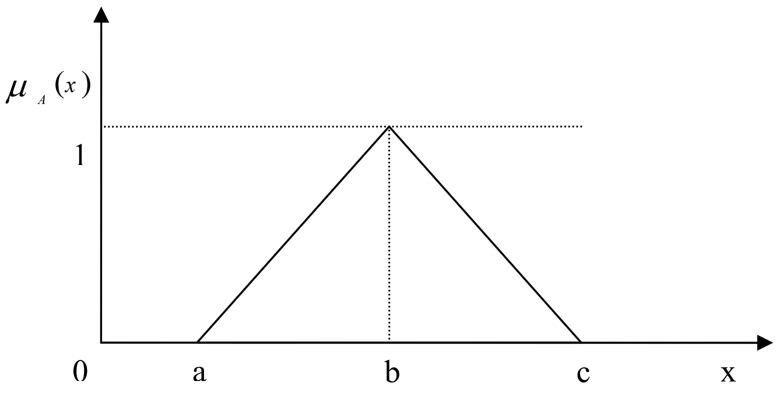

Fig. 1.Traingular fuzzy number

3. Computation of shortest path base on fuzzy numbers

Doubis and Prade [4] first shown how to determine shortest path in a network in fuzzy environment. Fuzziness can be introduced in a network in a variety of ways, e.g., through edge capacities, edge weights, or vertex restrictions. In real life situations, some unexpected events may occur so that the edge weight in the network may changes slightly.

If $\widetilde{c}_{ij} = (c_{ij1}\ ,\ c_{ij},\ c_{ij2})$ then $c^*_{ij} = \frac{1}{4}(2c_{ij} + c_{ij1} + c_{ij2}) > 0$. Therefore, we calculate that $d(\widetilde{c}_{ij}, \widetilde{0}_1) = c^*_{ij}$ is a positive distance measured from $\widetilde{0}_1$ to $\widetilde{c}_{ij}$ and also c^*_{ij} is also a positive number from 0.

Example-1

We consider the following acyclic network $G = (\mathrm{V}, \mathrm{E})$ with topological ordering whose edge weights are given triangular fuzzy numbers.

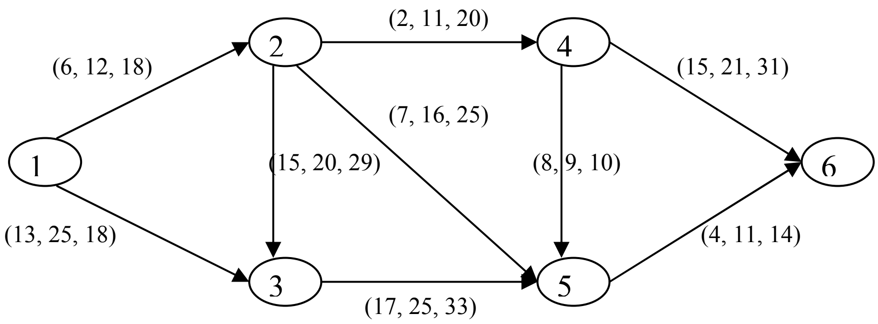

Fig -2

If $\widetilde{c}_{ij} = (c_{ij1}\ ,\ c_{ij},\ c_{ij2})$ then $c^*_{ij} = \frac{1}{4}(2c_{ij} + c_{ij1} + c_{ij2}) > 0$. Therefore, we calculate that $d(\widetilde{c}_{ij}, \widetilde{0}_1) = c^*_{ij}$ is a positive distance measured from $\widetilde{0}_1$ to $\widetilde{c}_{ij}$ and also c^*_{ij} is also a positive number from 0.

Using distance ranking method we find the real values of the triangular fuzzy numbers of edge weights c_{ij}^{*} as follows

$$c_{12}^{*} = \frac{1}{4}(6 + 2\times 12 + 18) = 12$$

$$c_{13}^{*} = \frac{1}{4}(13 + 2\times 25 + 33) = 24$$

Similarly

$$c_{23}^{*} = 21 \quad c_{35}^{*} = 25$$

$$c_{24}^{*} = 11 \quad c_{45}^{*} = 9$$

$$c_{25}^{*} = 16 \quad c_{46}^{*} = 22 \quad c_{56}^{*} = 10$$

The solution of the Bellman dynamic programming can be derived as follows,

$$f(1) = 0,\quad f(2) = \min_{1<2}\left\{f(1) + c_{12}^{*}\right\} = \left\{0 + c_{12}^{*}\right\} = c_{12}^{*} = 12$$

$$f(3) = \min_{i<3}\left\{f(i) + c_{i3}^{*}\right\} = \min_{i<3}\left\{f(1) + c_{13}^{*}, f(2) + c_{23}^{*}\right\}$$

$$= \min\{0 + 24,\ 12 + 21\} = 24 = c_{13}^{*}$$

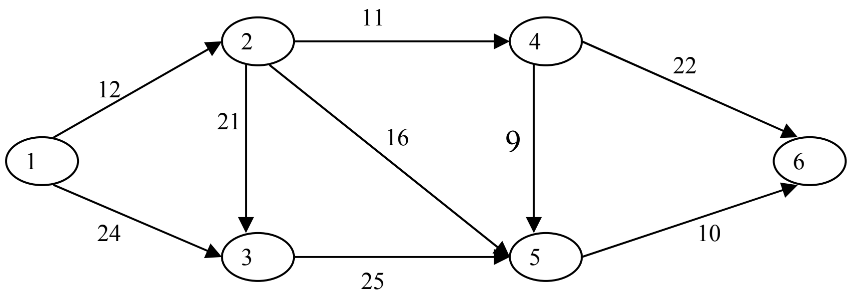

Fig -3

Similarly we get

$$f(4) = \min\left\{f(2) + c_{24}^{*}\right\} = \{12 + 11\} = 23 = c_{12}^{*} + c_{24}^{*}$$

$$f(5) = \min_{i<5}\left\{f(i) + c_{i5}^{*}\right\} = \min\left\{f(2) + c_{25}^{*}, f(3) + c_{35}^{*}, f(4) + c_{45}^{*}\right\}$$

$$= \min\{12 + 16, 24 + 25, 23 + 9\} = 28 = c_{12}^{*} + c_{25}^{*}$$

$$f(6) = \min_{i<6}\left\{f(i) + c_{16}^{*}\right\} = \min\left\{f(4) + c_{46}^{*}, f(5) + c_{56}^{*}\right\}$$

$$= \min\{23 + 22,\ 28 + 10\} = 38 = c_{12}^{*} + c_{25}^{*} + c_{56}^{*}$$

Example-2

We consider the following acyclic network $G = (\mathrm{V}, \mathrm{E})$ with topological ordering whose edge weights are given trapezoidal fuzzy numbers.

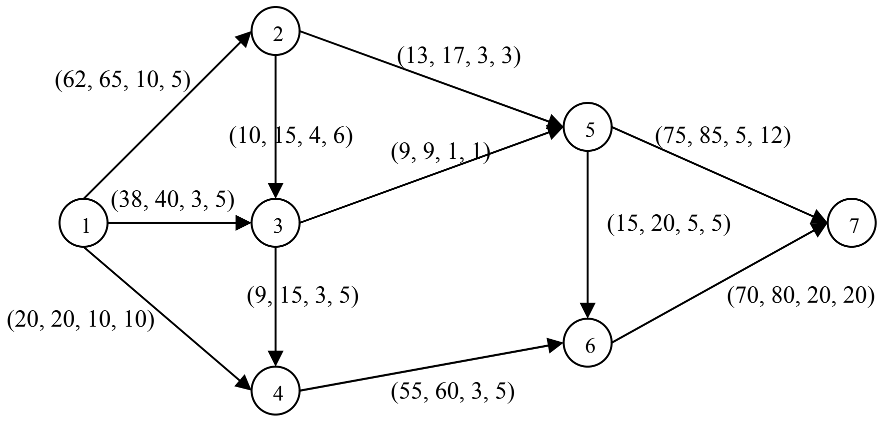

Fig -4

The second network $G=(\mathrm{V},\mathrm{E})$ is consists of 7 nodes and 11 edges with topological ordering of nodes. The edge weights have been consider as L-R trapezoidal fuzzy numbers $\tilde{c}_{ij}$ and are represented as $\left(\underline{c}_{ij}\ , \overline{\mathrm{c}}_{\mathrm{ij}}\ , \alpha_{\mathrm{ij}}, \beta_{\mathrm{ij}}\right)_{LR}$

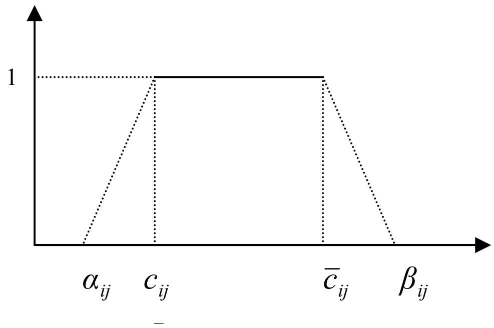

Fig- 5

If the edge weights are triangular fuzzy numbers, like (l, m, n) then we can transform the numbers from triangular type to L-R trapezoidal type such as

$(l,\ m,\ n)\ =\ (m,\ n,\ m\text{-}l,\ n\text{-}m)_{LR}$

by using Yager's [Bel 13 'A procedure for ordering] center of gravity or centroid defuzzification method the L-R trapezoidal numbers can be transformed into real numbers or utility values as follows

$$\left(\underline{c}_{ij}\ , \overline{c}_{ij},\ \alpha_{ij},\ \beta_{ij}\right) \cong \frac{1}{4}\left[\underline{c}_{ij}\ +\ \overline{c}_{ij}\ +\ \left(\underline{c}_{ij}-\alpha_{ij}\right)\ +\ \left(\overline{c}_{ij}\ +\ \beta_{ij}\right)\right]$$

Therefore

$$(62,\ 65,\ 10,\ 5) \cong \frac{1}{4}\left[62\ +65\ +(62\ \text{-}10\ )+(65\ +5)\right]\ 62.25\ = c_{12}^{*}$$

$$(38, 40, 3, 5) \cong \frac{1}{4}\left[38+40+(38\text{-}3)+(40+5)\right] = 39.5 = c_{13}^{*}$$

Similarly the other numbers are

$(20, 20, 10, 10) \cong 20 = c_{14}^{*}$ $\quad$ $(10, 15, 4, 6) \cong 13 = c_{23}^{*}$

$(9, 15, 3, 5) \cong 12.5 = c_{34}^{*}$ $\quad$ $(55, 60, 3, 5) \cong 58 = c_{46}^{*}$

$(13,17,3,3) \cong 15 = c_{25}^*$ $\quad$ $(9,9,1,1) \cong 9 = c_{35}^*$

$(15,20,5,5) \cong 17.5 = c_{56}^*$ $\quad$ $(75,85,5,12) \cong 81.75 = c_{57}^*$

$(70,80,20,20) \cong 75 = c_{67}^*$

with this real values the above network can be rewritten as

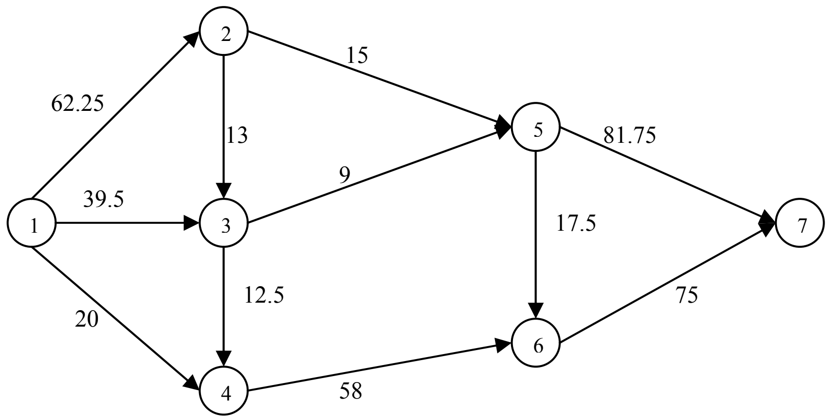

Fig -6

Now we use Bellman method of dynamic programming we have

$$f(1)=0$$

$$f(2)=\min_{1<2}\{f(1)+c_{12}^*\}=\{0+62.25\}=62.25=c_{12}^*$$

$$f(3)=\min_{i<3}\{f(i)+c_{i3}^*\}=\min_{i<3}\{f(1)+c_{13}^*, f(2)+c_{23}^*\}$$

$$=\min\{0+39.5,\ 62.25+13\}=39.5=c_{13}^*$$

$$f(4)=\min_{i<4}\{f(i)+c_{i4}^*\}=\min_{i<4}\{f(1)+c_{14}^*, f(3)+c_{34}^*\}$$

$$=\min\{0+20,\ 39.5+12.5\}=20=c_{14}^*$$

$$f(5)=\min_{i<5}\{f(i)+c_{i5}^*\}=\min_{i<5}\{f(2)+c_{25}^*, f(3)+c_{35}^*\}$$

$$=\min\{62.25+15,\ 39.5+9\}=48.5=c_{13}^*+c_{35}^*$$

$$f(6)=\min_{i<6}\{f(i)+c_{i6}^*\}=\min_{i<6}\{f(4)+c_{46}^*, f(5)+c_{56}^*\}$$

$$=\min\{20+58,\ 48.5+17.5\}=66=c_{13}^*+c_{35}^*+c_{56}^*$$

$$f(7)=\min_{i<7}\{f(i)+c_{i7}^*\}=\min_{i<7}\{f(5)+c_{57}^*, f(6)+c_{67}^*\}$$

$$=\min\{48.5+81.75,\ 66+75\}=130.25=c_{13}^*+c_{35}^*+c_{57}^*$$

4. Computation of shortest path by using multi objective linear programming

In this section we purpose a simple multi objective linear programming to deal with the fuzzy shortest path problem. In this approach 0-1 variables are not required to obtain shortest path in a network [Yu &Wei; solving the F. S. path…M. O. P] .Yu and Wei obtained a compromising non-dominated integer optimal solution of fuzzy shortest path without adding extra constraints.

Weighted Additive Method to solve Multi Objective Linear Programming

Chen et al have used weighted average in 'Fuzzy goal programming with different importance and properties'. Tiwari, Dharmar and Rao have mentioned an additive model in fuzzy goal programming which incorporates each goal's weight U_K into the corresponding objective function

ie $Z = \sum_{K=1}^{m} W_K U_K$

Where W_K denotes the K^{th} fuzzy goal and $\sum_{K=1}^{m} U_K$.

In the additive model weights show the relative importance of the goals. Now for simplicity the importances of these objectives (goals) are assumed as the same. Hence all the objective function s can be reformulated as a single objective function without adding the constraints.

$$\min Z = U_1 * W_1(x) + U_2 * W_2(x) + U_3 * W_3(x) + U_4 * W_4(x)$$

such that

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ji} = \begin{cases} 1, & i = 1 \\ -1, & i = n \\ 0, & \text{otherwise} \end{cases}$$

for $i = 1, 2, 3, \ldots, n$ and $j = 1, 2, 3, \ldots, n$

$$\sum_{K=1}^{n} U_K = 1$$

The description of the network LPs constraints properly is as follows

A linear programming is said to be a network

1. If except for simple upper and lower bound constraints (such as $x \le 3$), each variable appears in at most two constraints.
2. If each variable appear in two constraints, its coefficients in the two arc +1 and -1. If the variable appears in one constraint, its coefficient is either +1 or -1

Multi objective linear programming formulation of Example-1

The edge weights of the network in example-1 are triangular fuzzy number. Therefore, the corresponding fuzzy shortest path problem can be formulated with three objectives as follows

$$\min Z = W_1 U_1 + W_2 U_2 + W_3 U_3$$

we assume equal weights $U_1 = U_2 = U_3 = \frac{1}{3}, \ \sum_{i=1}^{3} U_i = 1$

$$\min Z = \frac{1}{3} * \begin{bmatrix} (6x_{12} + 13x_{13} + 15x_{23} + 2x_{24} + 7x_{25} + 17x_{35} + 8x_{45} + 15x_{46} + 4x_{56}) + \\ (12x_{12} + 25x_{13} + 20x_{23} + 11x_{24} + 16x_{25} + 25x_{35} + 9x_{45} + 21x_{46} + 11x_{56}) + \\ (18x_{12} + 33x_{13} + 29x_{23} + 20x_{24} + 25x_{25} + 33x_{35} + 10x_{45} + 31x_{46} + 14x_{56}) \end{bmatrix}$$

Where, $W_1 = 6x_{12} + 13x_{13} + 15x_{23} + 2x_{24} + 7x_{25} + 17x_{35} + 8x_{45} + 15x_{46} + 4x_{56}$

$$W_2 = 12x_{12} + 25x_{13} + 20x_{23} + 11x_{24} + 16x_{25} + 25x_{35} + 9x_{45} + 21x_{46} + 11x_{56}$$

$$\mathrm{W}_3 = 18x_{12} + 33x_{13} + 29x_{23} + 20x_{24} + 25x_{25} + 33x_{35} + 10x_{45} + 31x_{46} + 14x_{56}$$

such that

$$x_{12} + x_{13} = 1 \quad -x_{24} + x_{46} + x_{45} = 0$$

$$-x_{12} + x_{24} + x_{25} + x_{23} = 0 \quad -x_{25} - x_{35} - x_{45} + x_{56} = 0$$

$$-x_{13} - x_{23} + x_{35} = 0 \quad -x_{46} - x_{56} = 0$$

The optimal solution is Z = 37.66

$\mathrm{W}_1 = 17, \mathrm{W}_2 = 39, \mathrm{W}_3 = 57$

and $x_{12} = 1$, $x_{25} = 1$, $x_{56} = 1$, $x_{13} = 0$, $x_{23} = 0$, $x_{24} = 0$, $x_{35} = 0$, $x_{45} = 0$, $x_{46} = 0$

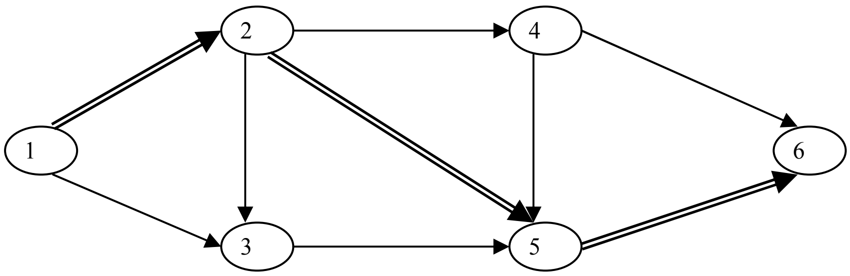

Fig 7

The shortest path is shown in figure-7 $\{1 \to 2 \to 5 \to 6\}$ and its fuzzy shortest path length is (17, 39, 57)

Multi objective linear programming solution of Example-2

The edge weights of the network in example-2 are L-R trapezoidal fuzzy number. So, the corresponding multi objective fuzzy shortest path problem can be formulated with four objectives as follows

$\min \mathrm{Z} = \frac{1}{4} * [\mathrm{W}_1 + \mathrm{W}_2 + \mathrm{W}_3 + \mathrm{W}_4]$, when weight are equal $\mathrm{U}_1 = \mathrm{U}_2 = \mathrm{U}_3 = \mathrm{U}_4 = \frac{1}{4}$

$$= \frac{1}{4} * \begin{bmatrix} (62x_{12} + 38x_{13} + 20x_{14} + 10x_{23} + 9x_{24} + 13x_{25} + 9x_{35} + 55x_{46} + 15x_{56} + 75x_{57} + 70x_{67}) + \\ (65x_{12} + 40x_{13} + 20x_{14} + 15x_{23} + 15x_{34} + 17x_{25} + 9x_{35} + 60x_{46} + 20x_{56} + 85x_{57} + 80x_{67}) + \\ (52x_{12} + 35x_{13} + 10x_{14} + 6x_{23} + 6x_{34} + 10x_{25} + 8x_{35} + 52x_{46} + 10x_{56} + 70x_{57} + 50x_{67}) + \\ (70x_{12} + 45x_{13} + 30x_{14} + 21x_{23} + 20x_{34} + 20x_{25} + 10x_{35} + 65x_{46} + 25x_{56} + 97x_{57} + 100x_{67}) \end{bmatrix}$$

such that

$$x_{12} + x_{13} + x_{14} = 1, \quad -x_{12} + x_{23} + x_{25} = 0, \quad -x_{13} - x_{23} + x_{35} + x_{34} = 0$$

$$-x_{14} - x_{34} + x_{46} = 0, \quad -x_{25} - x_{35} + x_{56} + x_{57} = 0$$

$$-x_{46} - x_{56} + x_{67} = 0, \quad -x_{57} - x_{67} = -1$$

the optimal solution is Z = 130.25

$x_{13} = 1$, $x_{35} = 1$, $x_{57} = 1$, $x_{12} = 0$, $x_{23} = 0$, $x_{14} = 0$, $x_{34} = 0$, $x_{25} = 0$, $x_{46} = 0$, $x_{56} = 0$, $x_{67} = 0$, $\mathrm{W}_1 = 122, \mathrm{W}_2 = 113, \mathrm{W}_3 = 134, \mathrm{W}_4 = 152$

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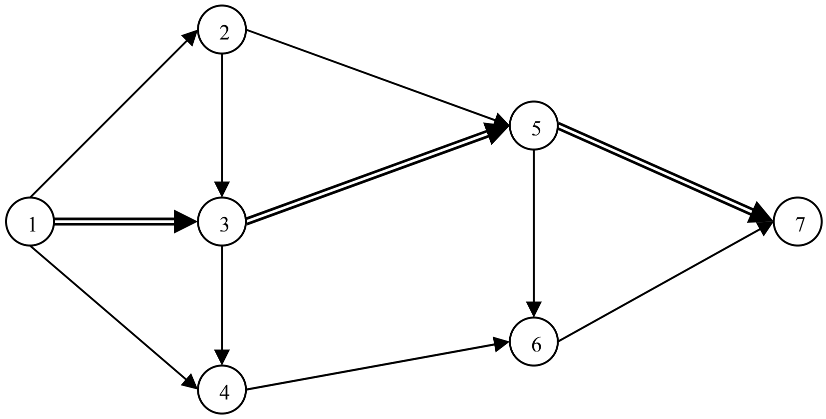

Fig-8 Fuzzy shortest path

The shortest path is shown in fig-8 as $\{1 \to 3 \to 5 \to 7\}$ and the corresponding fuzzy shortest path length is (122, 134, 9, 18). The optimal value of Z is 130.25, that mean the shortest path length is 130.25 which is the same as obtained by Bellman dynamic programming method.

Examples	Bellman Dynamic Programming Method	Multi Objective Linear Programming Method
S. P. length in Ex-1	Length→ 38 Path : 1→2→5→6	Optimal value = 37.66 S. Path $\{1 \to 2 \to 5 \to 6\}$ F.S.P. length is (17, 39, 57)
S. P. length in Ex-2	Length→ 130.25 Path : 1→3→5→7	Optimal value = 130.25 S. Path $\{1 \to 3 \to 5 \to 7\}$ F.S.P. length is (122,134,9, 18)

5. CONCLUSIONS

In this paper some fuzzy problems in network have been presented through two different methods –first one is Bellman Dynamic Programming method and the second one is Multi Objective Linear Programming method. In the second method non-dominated integer optimal solution is obtained without using 0-1 variables. Also this method reduces the complexity of solving shortest path in a network. Two different networks have been considered whose edge weight triangular fuzzy numbers and trapezoidal fuzzy numbers. Both the networks have been solved by each of the above two methods. It is observed that shortest path length in Bellman method is same/nearly same with the optimal value in the Multi Objective Linear Programming and also the corresponding fuzzy numbers also shown.

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