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Thermal Stratification within Mixed Convection Flow of non-Newtonian Fluid over an Inclined Stretching Cylinder

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Abstract: An article is made to study thermal stratification phenomena by way of mixed convection boundary layer flow of Eyring-Powell fluid brought by an inclined stretching cylinder. The temperature adjacent to the surface of cylinder is supposed to be higher than the ambient fluid. Flow directing differential equations are reduced into system of non-linear ordinary differential equations by utilizing appropriate transformations. The numerical solution is obtained with the application of shooting technique conjunction with fifth order Runge-Kutta algorithm. The inspection regarding achieved results shows that the fluid flow is influenced appreciably against thermal stratification parameter. The results are validated by developing comparison with existing published literature and some particular cases are also established. Furthermore, numeric values for two unlike geometries namely, plate and cylinder against skin friction coefficient and Nusselt number are presented with the aid graphs which is very essential regarding industrial application point of view.

Keywords: Inclined Stretching Cylinder; Thermal Stratification; Eyring-Powell Fluid; Mixed Convection; Heat generation/absorption, Shooting Method.

1 Introduction

The investigation of boundary layer flow induced by stretching surfaces of non-Newtonian fluids has been recognized widely by means of engineering and industrial applications. Newtonian fluids are not primarily suitable as compared to non-Newtonian fluids. The flow diversity of non-Newtonian fluids in nature brings uncertainty regarding rheological features and almost impossible to clip complete physical description by a single constitutive expression between shear rate against stress. Due to this fact, a variety of models for non-Newtonian fluids (revealing distinct rheological impacts) are presented in the literature [1,2]. Among those in 1944, Eyring and Powell proposed a distinct fluid model known as Eyring-Powell fluid model [3]. Eyring-Powell model has certain advantages over non-Newtonian model in this sense that it is derived from molecular theory of gases rather than the experimental relation and turn into viscous (Newtonian) mode at low and high shear rates. Even though it is more complex but advantages of this fluid model overcomes its labouring mathematics. For example it can be used to articulate the flows of modern industrial materials such as ethylene glycol and powdered graphite. Heat diffusion through Eyring-Powell fluid plays a vital role in different geophysical, natural and industrial problems namely, moisture and temperature distribution over agricultural pitches, environmental pollution, underground energy transport etc. Although every non-Newtonian fluid model is important with respect to industrial and engineering point of view, so that the researchers identified different effects namely, magnetic field effect, unsteadiness of flow field, thermal radiation effect, heat generation phenomena, porous medium, melting heat transfer effect over a plane and cylindrical stretching geometry like Nadeem et al. [4,5] explored the MHD effects over a exponentially shrinking and linearly stretching sheet by considering Casson fluid model. Ahmed et al. [6] studied the convective heat transfer over a stretching sheet by considering Jeffery fluid model. Shahzad and Ali [7] identified MHD flow over a vertical stretching sheet along convective end by means of power law model. Khan et al. [8] investigated MHD Falkner-Skan flow along mixed convection phenomena

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under convective boundary conditions. Ali et al. [9] studied the axisymmetric flow with partial slip effect in a numerical frame of reference. Recently, MHD free convection dissipative fluid flow over an inclined porous plate was taken by Malik and Khalil [10]. As far as the Eyring-Powell model is concern, during past time most of the researchers owned the importance of Eyring-Powell model and so investigated diverse effects by considering flow of Eyring-Powell fluid brought by different geometries. To mention just a few, Rosca and Pop [11] investigated the heat transfer effect of Eyring-Powell fluid past a shrinking surface in parallel free stream. Panigrahi et al. [12] identified MHD impact under mixed convection flow of Eyring-Powell fluid over a non-linear stretching surface. Recently, Hayat et al. [13] addressed series and numerical solution of Eyring-Powell fluid flow by considering Newtonian heating and heat generation/absorption effect. In addition, Malik et al. [14] pointed the Eyring-Powell fluid over a stretching cylinder with variable viscosity effect. Hayat et al. [15] presented Eyring-Powell MHD nanofluid flow brought by stretching cylinder under thermal radiation impact. More recently, Khan et al. [16] considered the unsteady Eyring-Powell nanofluid past a oscillatory stretching surface by way of generation/absorption effect. Rehman et al. [17] explored Eyring-Powell fluid flow over a vertical cylinder under the region of stagnation point by incorporating heat transfer.

Thermal stratification phenomena includes fundamental applications namely, thermal energy storage systems similar to solar ponds, production of sheeting material, annealing and thinning of copper wires, environmental heat rejection such as seas and rivers, etc. So due to huge implementations in fluid mechanics many researchers probed stratification phenomenon. The experimental and analytical investigations have been performed regarding heated surface flows in a stable stratified medium, to mention just a few Jaluria et al. [18], Chen et al. [19] and Ishak et al. [20]. Whereas astrophysics, oceanography, agriculture and many chemical processes also enclosed thermal diffusion mechanism. The analysis of mixed convection in a thermally stratified medium is an important problem because closed containers, ecological heated walls chambers are supported by convectional environments with thermal stratification. Such a stratification of the medium is due to temperature differences, which results density variation in the medium. Stratification along convective heat transfer involves both heat diffusion known as conduction and bulk heat transfer of fluid known as advection. Convection heat transfer may be natural, forced or mixed. Significant applications of mixed convection includes drying of porous solid, chemical transportation in packs bed reactors, solar power collectors etc. In short, stratification along dual convection admits a remarkable role in many industrial and natural phenomena's. So that, it was identified by many scientists as Hayat et al. [21] examined the stagnation point flow of an Oldroyd-B fluid with thermally stratified medium. Mukhopadhyay and Ishak [22] analyzed thermal stratification phenomena with mixed convection along a stretching cylinder. Bachok et al. [23] discussed the characteristics of natural convection, forced convection and surface heat flux over a permeable vertical cylinder. They considered linear variation of heat flux and free stream velocity against distance from leading edge. The surface temperature variation and dual convection that is natural and forced along slender vertical cylinder was taken by Heckel et al. [24]. They assumed arbitrarily surface temperature variations under axial coordinate.

The above assessed literature reflects that most of the investigators identified the comportment of non-Newtonian fluids in a plane geometry by assuming altered effects. The characteristics of Eyring-Powell model over a inclined stretching cylinder in a thermally stratified medium by way of heat generation is not widely investigated as yet. Therefore, the theme of this present work is to fulfil the gap. The temperature is supposed to be variable at the surface of the cylinder and away from it. Numerical computations of the transformed equations are presented by means of shooting technique. The investigation of the results achieved shows that the fluid flow is influenced significantly against thermal stratification parameter. The effect logs for involved physical parameters over velocity and temperature distributions are inspected and plotted graphically. The estimation of local skin friction coefficient and Nusselt number is also presented in this study through figures, which is very essential regarding industrial application point of view. It is trusted that the results found will not only offer convenient information for applications, but also serve as a complement to the preceding studies.

2 Flow Construction

We consider incompressible, two dimensional, steady boundary layer flow of Eyring-Powell fluid taken by an inclined stretching cylinder. Flow analysis is carried out with thermal stratification. Across the surface of cylinder, temperature is assumed higher than the ambient fluid. The rheological equation of state for Eyring-Powell fluid is given by:

$$\Gamma = [\ \mu + \frac{1}{\beta\gamma^1}\sinh^{-1}\left(\frac{1}{c}\gamma^1\right)\mathbf{A}_1, \quad (1)$$

where

$$\gamma^1 = \sqrt{\frac{1}{2}tr(\mathbf{A}_1)^2}, \quad (2)$$

A $\sinh^{-1}$ (\.) function up to second order approximation is measured as:

$$\sinh^{-1}\left(\frac{1}{c}\gamma^{1}\right) \cong \frac{\gamma^{1}}{c} - \frac{\gamma^{1^{3}}}{6c^{3}}, \text{ where } \left|\tfrac{1}{c}\gamma^{1}\right| << 1. \quad (3)$$

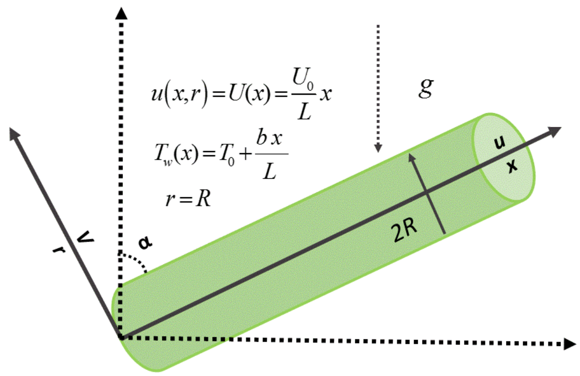

Fig. 1(a) Geometry and coordinate's system of flow model.

The mass conservation, momentum and energy equations via boundary layer approximation reduces to:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0, \quad (4)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial r} = \left(\nu + \frac{1}{\beta\rho c}\right)\frac{\partial^2 u}{\partial r^2} - \frac{1}{2\beta c^3\rho}\left(\frac{\partial u}{\partial r}\right)^2\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\left(\nu + \frac{1}{\beta\rho c}\right)\frac{\partial u}{\partial r} - \frac{1}{6\beta r\rho c^3}\left(\frac{\partial u}{\partial r}\right)^3 + (g\beta_T(T - T_\infty))\cos\alpha, \quad (5)$$

$$u\frac{\partial T}{\partial x} + u\frac{\partial T}{\partial r} = \frac{\alpha'}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{Q_o}{c_p\rho}(T - T_\infty) \quad (6)$$

Subjected to endpoint conditions:

$$u(x,r) = U(x) = \frac{U_0}{L}x, \quad v(x,r) = 0 \text{ at } r = R,$$
$$u(x,r) \to 0 \text{ at } r \to \infty,$$
$$T(x,r) = T_w(x) = T_0 + \frac{b\,x}{L}, \quad \text{at } r = R,$$
$$T(x,r) \to T_\infty(x) = T_0 + \frac{c\,x}{L}, \quad \text{as } r \to \infty. \quad (7)$$

We consider x-axis as a axial line of cylinder and radial direction upright to x-axis is taken as r-axis. For the solution of Eqs. (5)-(6) under end point conditions Eq. (7), we considered following transformations:

$$u = \frac{U_0 x}{L}F'(\eta),\ v = -\frac{R}{r}\sqrt{\frac{U_0\nu}{L}}F(\eta),\ \eta = \frac{r^2 - R^2}{2R}\left(\frac{U_0}{\nu L}\right)^{\frac{1}{2}},$$
$$\psi = \left(\frac{U_0\nu x^2}{L}\right)^{\frac{1}{2}} R\,F(\eta),\ \ T(\eta) = \frac{T - T_\infty}{T_w - T_0}, \quad (8)$$

$$u = \frac{1}{r}\left(\frac{\partial\psi}{\partial r}\right), \quad v = \frac{-1}{r}\left(\frac{\partial\psi}{\partial x}\right), \quad (9)$$

by incorporating Eqs. (8)-(9) in Eqs. (5)-(6), the resulting reduced forms are given by:

$$FF'' - (F')^2 + (1 + 2K\eta)(1 + M)F''' + 2K(1 + M)F'' - \frac{4}{3}\lambda MK(1 + 2K\eta)(F'')^3 - M\lambda(1 + 2K\eta)^2(F'')^2F''' + \lambda_T T(\eta)\cos\alpha = 0, \quad (10)$$

$$2KT' + (1 + 2K\eta)T'' + \Pr(FT' - F'T - F\delta_1 + \delta T) = 0 \quad (11)$$

subjected to the transformed endpoint conditions:

$$F' = 1, \quad F = 0, \quad T = 1 - \delta_1 \quad \text{at} \quad \eta = 0,$$
$$F' \to 0, \quad T \to 0 \quad \text{as} \quad \eta \to \infty, \quad (12)$$

where K, M, λ, λ_T, Pr, δ_1, δ, defined as follows:

$$K = \frac{1}{R}\sqrt{\frac{\nu}{a}},\ M = \frac{1}{\mu\beta c},\ \lambda = \frac{a^3x^2}{2c^2\nu},\ \lambda_T = \frac{Gr}{\mathrm{Re}_x^{\,2}},$$
$$\Pr = \frac{\nu}{\alpha'},\ \delta_1 = \frac{c}{b},\ \text{and } \delta = \frac{LQ_0}{U_0\rho c_p}. \quad (13)$$

In addition, Gr denotes thermal Grashof number which is defined as:

$$Gr=\frac{g\beta_T(T_w-T_0)x^3}{\nu^2}. \tag{14}$$

The skin friction coefficient at the surface of cylinder is considered as:

$$C_f=\frac{\tau_w}{\rho\frac{U^2}{2}},$$

$$\tau_w=\left[\mu\left(\frac{\partial u}{\partial r}\right)+\frac{1}{\beta c}\frac{\partial u}{\partial r}-\frac{1}{6\beta c^3}\left(\frac{\partial u}{\partial r}\right)^3\right]_{r=R}, \tag{15}$$

where τ_w is the shear stress and μ denotes viscosity of fluid. The dimensionless form of skin friction coefficient is prearranged as:

$$C_f\,\mathrm{Re}_x^{1/2}=2(1+M)F''(0)-\frac{2M\lambda}{3}\left(F''(0)\right)^3, \tag{16}$$

with $\mathrm{Re}_x=\frac{U_0x^2}{\nu L}$ as a local Reynolds number.
The local Nusselt number is defined as:

$$Nu_x=\frac{xq_w}{k(T_w-T_\infty)},\ q_w=-k\left(\frac{\partial T}{\partial r}\right)_{r=R}, \tag{17}$$

in dimensionless form it can be written as:

$$Nu_x\,\mathrm{Re}_x^{-1/2}=-T'(0). \tag{18}$$

3 Particular Cases

Case -1

Consider incompressible steady flow of Eyring-Powell fluid induced by inclined stretching cylinder whose mathematical formulation are given by Eq. (10). Incorporating curvature parameter equal to zero, we may re-trace the incompressible steady flow of Eyring-Powell fluid induced by inclined stretching plate. A comparative study of cylindrical and plane geometry is examined and plotted via Figs. 9 -10 regarding skin friction coefficient and heat transfer rate. Mathematically by using K = 0, Eq. (10) reduces to:

$$FF''-(F')^2+(1+M)F'''+M\lambda(F'')^2F'''+\lambda_T T(\eta)\cos\alpha=0,$$

$$T''+\Pr\left(FT'-F'T-F\delta_1+\delta T\right)=0, \tag{19}$$

$$F'=1\ ,\quad F=0\ ,\quad T=1-\delta_1\quad \text{at}\quad \eta=0,$$

$$F'\to 0\ ,\quad T\to 0\quad \text{as}\quad \eta\to\infty. \tag{19a}$$

Case-2

Furthermore, by using inclination $\alpha=0$, in the absence of mixed convection parameter the problem reduces to:

$$FF''-(F')^2+(1+M)F'''+M\lambda(F'')^2F'''$$

$$F'=1\ ,\quad F=0\ ,\quad \text{at}\quad \eta=0,$$

$$F'\to 0\ ,\quad \text{as}\quad \eta\to\infty. \tag{20}$$

Which is already discussed by Javed at al. [25]

4 Numerical Results By Shooting

The system of governing coupled non-linear ordinary differential equations i-e Eq. (10) and (11) subjected to endpoint conditions Eq. (12) solved by employing shooting method with the aid of fifth order Runge-Kutta scheme. Firstly, reduction has been done in a system of five first order simultaneous equations by letting

$$x_2=F',$$
$$x_3=x'_2=F'',$$
$$x_5=T',$$

equivalent form of Eqs. (10) and (11) under new variables is given by:

$$\begin{bmatrix} x_1'=x_2 \\ x_2'=x_3 \\ x_3'=\frac{(x_2)^2-x_1x_2-(2K)(1+M)x_3+\frac{4}{3}\lambda M(1+2K\eta)x_3^3-\lambda_T x_4\cos\alpha}{(1+2K\eta)(1+M)-M\lambda(1+2K\eta)^2x_3^2} \\ x_4'=x_5 \\ x_5'=\frac{\Pr(x_2x_4+\delta_1x_2-x_1x_3-\delta x_4)-2Kx_5}{1+2K\eta}\end{bmatrix} \tag{21}$$

the corresponding endpoint conditions in new variables are given as follows:

$$x_1(0)=0,$$
$$x_2(0)=1,$$
$$x_3(0)=unknown,$$
$$x_4(0)=1-\delta_1,$$
$$x_5(0)=unknown.$$

In order to integrate Eq. (21) as a IVP we required values for $x_3(0)$ i.e. $F''(0)$, and $x_5(0)$ i.e. $T'(0)$. We observed that the initial conditions $x_3(0)$, and $x_5(0)$, are not given but we have additional conditions:

$$x_2(\infty) = 0,$$
$$x_4(\infty) = 0. \quad (23)$$

By electing complimentary values of $F''(0)$, and $T'(0)$ so that, the integration of system of first order differential equations carried out in a such a way that the endpoint conditions in Eq. (23) holds absolutely. Step size $\Delta\eta = 0.025$ under tolerance 10^{-6} is used to obtain the numerical solution with four decimal accuracy as convergence standards.

Table 1: Comparison value of $-T'(0)$ against Pr.

Pr	Hayat et al.,[15]	Ishak and Nazar [26]	Vajravelu et al. [27]	Current results
1	1.00000	1.0000	1.000002	1.0309
1.1	-	-	-	1.1008
1.2	1.120662	-	-	1.1317
1.4	1.2135	-	-	1.3420

4.1 *Velocity distribution*

The implemented parameter values for current computational analysis are specified as mixed convection parameter λ_T = 0.1, thermal stratification parameter δ_1 = 0.1, heat generation/absorption parameter δ = 0.1, curvature parameter K = 0.1, fluid parameters λ = M = 0.1, Prandtl number Pr = 1.5. The results are obtained by maintaining these values otherwise indicated on graphs. Table 1 shows the assessment of heat transfer rate for the altered values of Prandtl number Pr with results obtained in [15], [26-27]. An excellent match has been found among existing literature, which yields conformity of present results. Fig. 1 is used to explain the impact of mixed convection parameter λ_T on velocity profile. It is observed that for larger values of mixed convection parameter λ_T velocity of fluid increases. Physically, it is due to inciting of thermal buoyancy force. From Fig. 2, it is examined that velocity profile decreases for increasing values of thermal stratification parameter δ_1. Essentially, this effect is due to drop of convective potential between surface of cylinder and ambient temperature. Furthermore, Fig. 3 shows that the higher values of curvature parameter K is the cause of increase in velocity profile. Curvature parameter K is inversely proportional to radius of curvature. So when we increase curvature parameter, the radius of cylinder decreases and hence contact surface area of cylinder with fluid reduces which offers less resistance to fluid flow. So increase in curvature parameter K causes increase in velocity profile. From Fig. 4, it is evident that the velocity of fluid increases for increasing values of fluid parameter M because fluid parameter M has inverse relation with the viscosity of fluid, so increase in fluid parameter M turns fluid to be less viscous and hence deformation rate increases which yields increase in velocity.

4.2 *Temperature Distribution*

From Fig. 5, it is noted that the temperature profile decreases for increase in thermal stratification parameter δ_1. This is due to decline in temperature difference between surface of cylinder and ambient fluid therefore, temperature profile decreases. Influence of mixed convection parameter λ_T on temperature profile is depicted in Fig. 6. It is noticed that increasing values of mixed convection parameter λ_T corresponds decrease in temperature profile. When mixed convection parameter λ_T increases, thermal buoyancy forces enhanced which results inciting in heat transfer rate and hence temperature profile decreases. Fig. 7 is plotted to examine the effects of heat generation/absorption parameter δ on temperature profile. It is clearly seen that for different values heat generation/absorption parameter δ temperature of fluid increases. During heat generation process energy is produced which brings enhancement in temperature distribution. In addition, for higher values of heat generation/absorption parameter δ, over shoot in temperature profile is observed. Such type of overshoot can be controlled by introducing heat sink which helps to reduce temperature of fluid. Fig. 8 shows that temperature distribution increases due to increase in curvature parameter K. As Kelvin temperature is defined as an average kinetic energy so when we increase curvature of cylinder, velocity of the fluid increases, resultantly kinetic energy increases and due to this temperature increases. Note that temperature of fluid decreases near the surface of cylinder and increases away from the surface. Fig. 9 witnessed that the skin friction decreases (in absolute sense) for larger values of mixed convection parameter λ_T and opposite impact against heat generation\absorption parameter δ. It is also observed that the magnitude in case of cylinder is higher than the plate. Fig. 10 is the evident that heat transfer rate decreases in absolute sense for larger values of inclination α and heat generation/absorption parameter δ. The significant enhancement of local Nusselt number is observed in case of cylinder as compare to plate.

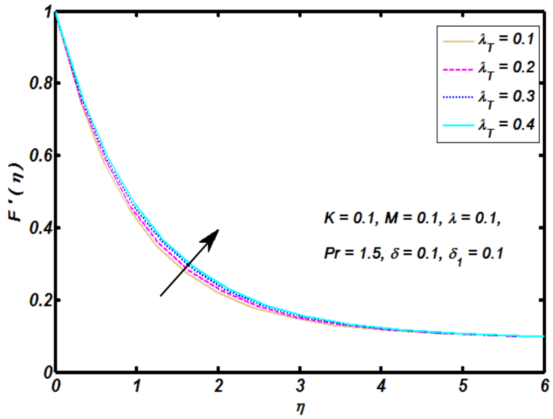

Fig. 1. Influence of λ_T over velocity distribution.

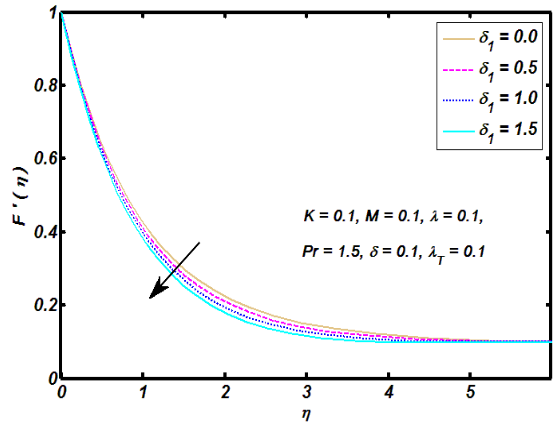

Fig. 2. Influence of δ_1 over velocity distribution.

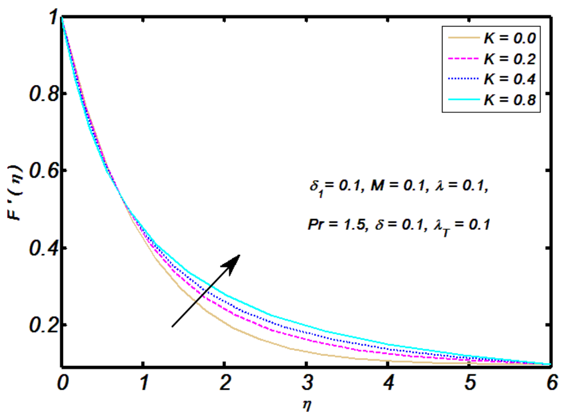

Fig. 3. Influence of K over velocity distribution.

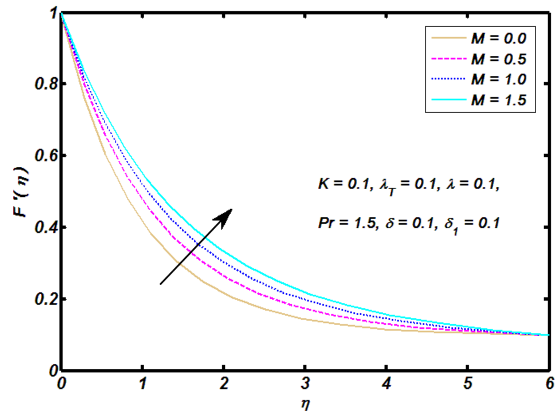

Fig. 4. Influence of M over velocity distribution.

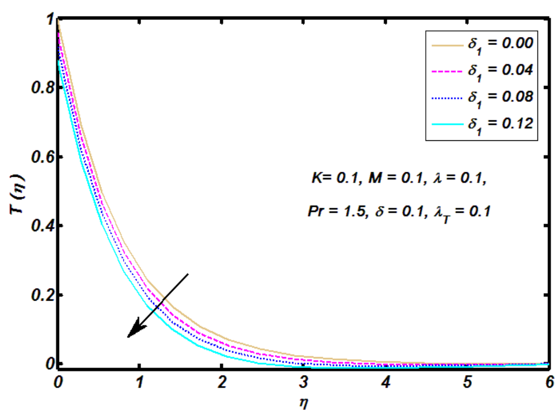

Fig. 5. Influence of δ_1 over temperature distribution.

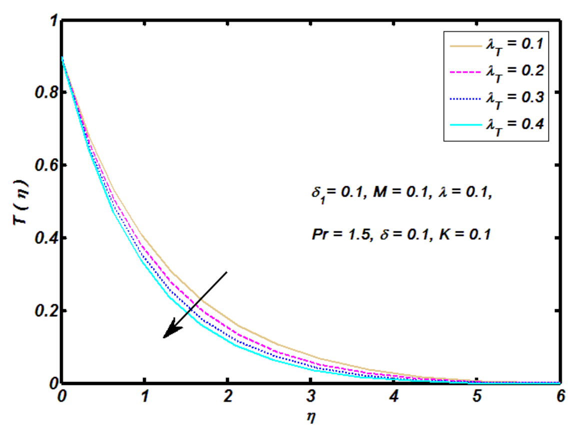

Fig. 6. Influence of λ_T over temperature distribution.

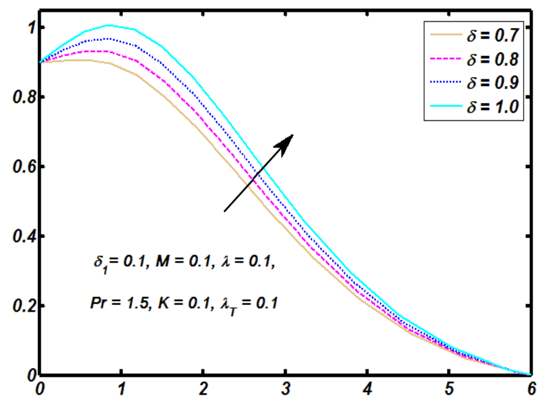

Fig. 7. Influence of δ over temperature distribution.

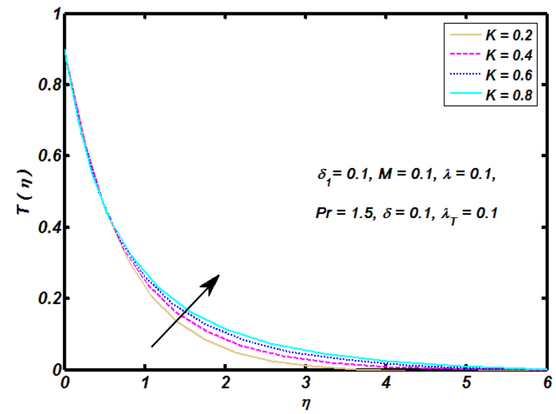

Fig. 8. Influence of K over temperature distribution.

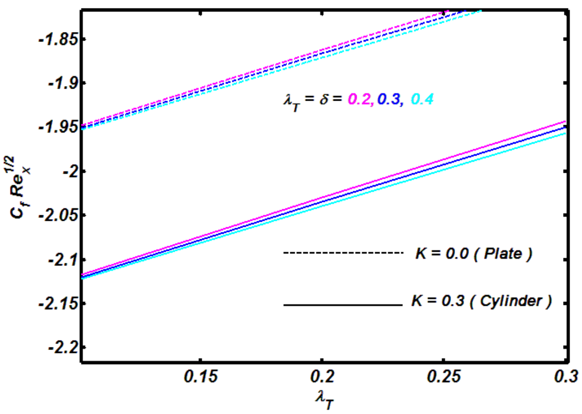

Fig. 9. Influence of λ_T and δ on skin friction coefficient.

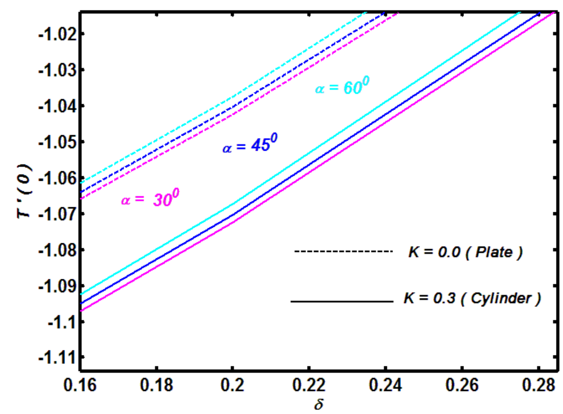

Fig. 10. Effect of α and δ on local Nusselt number.

5 Conclusion

Thermally stratified mixed convection boundary layer flow of Eyring-Powell fluid brought by inclined stretching cylinder with heat generation/absorption was examined. The computations of transformed coupled system of non-linear ordinary differential equations are performed successfully by fifth order Runge-Kutta algorithm with shooting technique. The behaviour of dimensionless velocity and temperature distributions are identified under the influence of various physical parameters. The summarized results of present study are itemized as follows:

–A comparison among previously published results [15], [26,27] are validated for heat transfer rate against different values of Prandtl number Pr.

–The velocity profile shows remarkable increase for increasing values of mixed convection parameter λ_T, fluid parameter M and curvature parameter K whereas it shows decline via thermal stratification parameter δ_1.

–The temperature profile increases for higher values of curvature parameter κ and heat generation/absorption parameter δ while it shows opposite behaviour towards thermal stratification parameter δ_1, mixed convection parameter λ_T.

–The skin friction coefficient is decreasing function of mixed convection parameter λ_T and increasing against heat generation/absorption parameter δ while the local Nusselt number shows decreasing attitude towards mixed convection parameter λ_T and heat generation/absorption parameter δ.

–Magnitude of skin friction and heat transfer rate is significantly large for cylinder as compare to plane geometry (plate).

Nomenclature

u , v	Velocity components	α	Inclination
v	Kinematic viscosity	ρ	Fluid density
g	Gravity	μ	Dynamic viscosity
tr	Trace	K	Curvature parameter
c , β	Fluid parameters	M , λ	Fluid parameters
$T_w(x)$	Prescribed surface temperature	$T_\infty(x)$	Variable ambient temperature
T_0	Reference temperature	U_0	Free stream velocity
L	Reference length	$F(\eta)$	Dimensionless variable
$F'(\eta)$	Velocity of fluid	b, c	Positive constants
α'	Thermal diffusivity	Pr	Prandtl number
ψ	Stream function	η	Similarity variable
δ_1	Thermal stratification parameter	Gr	Thermal Grashof number
β_T	Coefficient of thermal expansion	c_p	Specific heat at constant pressure
A_1	First Rivlin-Ericksen tensor	λ_T	Mixed convection parameter
δ	Heat generation/absorption parameter	Q_0	Heat generation/absorption coefficient

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