

2-Absorbing Intuitionistic Fuzzy Weakly Completely Γ -ideals in Γ -rings

Serkan Onar 

Department of Mathematical Engineering, Yildiz Technical University, Davutpaşa-Istanbul, Turkey

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Abstract: The primary purpose of the work is to present a characterization for a generalisation of intuitionistic fuzzy prime Γ -ideals in Γ -rings by introducing 2-absorbing intuitionistic fuzzy weakly completely Γ -ideals of commutative Γ -rings and to propose their properties. Also, we describe the notion of 2-absorbing intuitionistic K-fuzzy Γ -ideals of Γ -rings. Moreover, we acquire a diagram which transition between definitions of these concepts. Finally, we investigate intuitionistic fuzzy quotient Γ -ring of R induced by the 2-absorbing intuitionistic fuzzy weakly completely Γ -ideal is a 2-absorbing Γ -ring.

Keywords: 2-absorbing, 2-Absorbing Intuitionistic Fuzzy weakly completely Γ -Ideal, 2-Absorbing Intuitionistic K-Fuzzy Γ -Ideal

1 Introduction

In [1] Zadeh introduced the idea of fuzzy subset, and in [2] Rosenfeld investigated how to apply the fuzzy ideology to algebraic systems. Subsequently, numerous researchers were arrested and questioned about this theory, after which Liu [3] investigated the notion of the fuzzy ideal of a ring. Nobusawa [4] described the concept of a Γ -ring, which he defined more broadly as a ring. W.E.Barnes [5] in fact weakened the criteria for describing Γ -rings in Nobusawa's perspective. In [6, 7, 8] the authors cultivated the structure of Γ -rings and made various rationalizations in terms of the units in ring theory. In fuzzy commutative algebra, the prime ideals are the most remarkable structures with much weight. Dutta and Chanda [9] specified fuzzy prime ideals in Γ -rings. Ersoy [10] put fuzzy semiprime ideals in Γ -rings.

Badawi proposed in [11] the concept of a 2-absorbing ideal, which is a prime ideal expansion, and he also explained in [12, 13]. It was elaborated in detail by a number of authors (e.g., [14, 15, 16]). Darani [17] demonstrated the notion of L-fuzzy 2-absorbing ideals and obtained interesting results on these concepts. He then constructed the notion of L-fuzzy 2-absorbing ideals of semiring [18]. Elkettani and Kasem [19] explained the notion of 2-absorbing δ -primary Γ -ideal of Γ -rings and provided food for thought on these concepts. Sönmez [20] described 2-absorbing primary

fuzzy ideals of commutative rings and made connections between 2-absorbing primary fuzzy ideals and 2-absorbing primary ideals.

Atanassov [21] proposed an intuitionistic fuzzy set as an extension of fuzzy sets characterized by the membership's degree and non-membership degree's of an element. Moreover, Kang et al. in [22] focused on the idea of intuitionistic fuzzy subring, and subsequently various authors have attempted to extend the idea of intuitionistic fuzzy subring. The translation of intuitionistic fuzzy subrings was explained by Sharma in [23]. Recently, the application of intuitionistic fuzzy sets to algebraic structures is very popular. In this context, Onar [29] defined the focused concept of intuitionistic fuzzy 2-absorbing semiprimary ideals of commutative rings. In [30] Nakkhasen characterized the notion of intuitionistic fuzzy ideals of ternary near-rings as a generalization of fuzzy ideals. Abbas [31] described the idea of intuitionistic fuzzy ideal topological groups by based on an intuitionistic fuzzy topological groups. In [32] Boudaoud et al. presented the notion of principal intuitionistic fuzzy ideals and filters on lattice and showed any principal intuitionistic fuzzy ideal (resp. filter) coincides with an intuitionistic fuzzy down-set (resp. up-set) generated by an intuitionistic fuzzy singleton. Sharma et al. [33] studied the category of intuitionistic fuzzy modules.

* Corresponding author e-mail: serkan10ar@gmail.com

This paper provides a novel algebraic structure of prime intuitionistic Γ -ideal of commutative Γ -ring by 2-absorbing weakly completely prime ideal theory. This paper is organized as follows: Section 1 contains the literature relevant to the paper and summarizes the relevant studies. Section 2 contains some basic concepts of Γ -ring, 2-absorbing and intuitionistic fuzzy set. Section 3 establishes the notion of 2-absorbing intuitionistic fuzzy weakly complete Γ -ideal of an Γ -ring and explains part of its classification of algebraic properties, explains the description of 2-absorbing intuitionistic K -fuzzy Γ -ideal of a Γ -ring. Moreover, it characterizes image and inverse image of 2-absorbing intuitionistic fuzzy weakly complete Γ -ideals of a Γ -ring and of 2-absorbing intuitionistic K -fuzzy Γ -ideals of a Γ -ring. Moreover, it presents a diagram showing the transition between the relation between these terms and the idea of the 2-absorbing Γ -ideal. Finally, it proposes an intuitionistic fuzzy quotient Γ -ring of R induced by the 2-absorbing intuitionistic fuzzy weakly complete Γ -Ideal is a 2-absorbing Γ -ring. Section 4 summarises the purpose and significance of the study, presents the general portrait of the study, and focuses on the elaboration of future studies.

2 Preliminaries

In this part, firstly some important descriptions and the findings are reviewed in order to be comprehensive. Throughout this study, Γ -ring R will be a commutative with $1 \neq 0$ and $L = [0, 1]$ stands for a complete lattice.

Definition 1. [24] Let R and Γ be two abelian additive groups and R is named a Γ -ring if the subsequent mapping exist,

$$R \times \Gamma \times R \rightarrow R \\ (x, \beta, y) \mapsto x\beta y$$

and for all $x, y, z \in R$ and $\beta, \gamma \in \Gamma$ providing the subsequent criterias:

1. $(x + y)\beta z = x\beta z + y\beta z$.
2. $x\beta(y + z) = x\beta y + x\beta z$.
3. $x(\beta + \gamma)y = x\beta y + x\gamma y$.
4. $x\beta(y\gamma z) = (x\beta y)\gamma z$.

A Γ -ring R is named as commutative if $x\beta y = y\beta x$ for any $x, y \in R$ and $\beta \in \Gamma$.

In the following definition, Γ -ideal is given.

Definition 2. [24] A left (resp. right) ideal of a Γ -ring R is a subset A of R which is an additive subgroup of R and $R\Gamma A \subseteq A$ (resp. $A\Gamma R \subseteq A$) where, $R\Gamma A = \{x\beta y \mid x \in R, \beta \in \Gamma, y \in A\}$.

If A is both a left and a right ideal, then A is called a Γ -ideal of R .

Let R and S be two Γ -rings, and ρ be a mapping of R into S . Then ρ is called a Γ -homomorphism [24] if for all $a, b \in R$ and $\beta \in \Gamma$, $\rho(a + b) = \rho(a) + \rho(b)$ and $\rho(a\beta b) = \rho(a)\beta\rho(b)$.

Definition 3. [5] Let R be a Γ -ring. Then a proper ideal P of R is called a prime Γ -ideal if for all pairs of ideals S and T of R , $S\Gamma T \subseteq P$ implies that $S \subseteq P$ or $T \subseteq P$.

The notion of 2-absorbing ideal will be shown in the following definition.

Definition 4. [11] A proper ideal I of commutative ring R is called a 2-absorbing ideal of R if whenever $x, y, z \in R$ and $xyz \in I$, then $xy \in I$ or $xz \in I$ or $yz \in I$.

The concept of 2-absorbing Γ -ideal will be expressed as follows.

Definition 5. [19] A proper Γ -ideal I of a Γ -ring R is called a 2-absorbing Γ -ideal of R ($2\Gamma I(R)$) if whenever $x, y, z \in R$, $\beta, \gamma \in \Gamma$ and $x\beta y\gamma z \in I$, then $x\beta y \in I$ or $x\gamma z \in I$ or $y\gamma z \in I$.

The idea of Atanassov's intuitionistic fuzzy sets will be given in the following definitions.

Definition 6. [21] The intuitionistic fuzzy sets are defined on a non-empty set X as objects having the form,

$$\sigma = \{ \langle x, \theta(x), \xi(x) \rangle \mid x \in X \}, \\ \text{where the functions } \theta : X \rightarrow [0, 1] \text{ and } \xi : X \rightarrow [0, 1] \\ \text{denote the degrees of membership and of non-membership} \\ \text{of each element } x \in X \text{ to set } \sigma, \text{ respectively, } 0 \leq \theta(x) + \xi(x) \leq 1 \text{ for all } x \in X.$$

Definition 7. [21] Let X be a nonempty set and let $\sigma = \langle \theta_\sigma, \xi_\sigma \rangle$ and $\eta = \langle \theta_\eta, \xi_\eta \rangle$ be intuitionistic fuzzy sets in X . Then,

1. $\sigma \subset \eta$ iff $\theta_\sigma \leq \theta_\eta$ and $\xi_\sigma \geq \xi_\eta$.
2. $\sigma = \eta$ iff $\sigma \subset \eta$ and $\eta \subset \sigma$.
3. $\sigma^c = \langle \xi_\sigma, \theta_\sigma \rangle$.
4. $\sigma \cap \eta = (\theta_\sigma \wedge \theta_\eta, \xi_\sigma \vee \xi_\eta)$.
5. $\sigma \cup \eta = (\theta_\sigma \vee \theta_\eta, \xi_\sigma \wedge \xi_\eta)$.

Definition 8. [20] Let σ be an intuitionistic fuzzy set in a set X and $t, s \in L$ such that $t + s \leq 1$. Then, the set $\sigma_{(t,s)} = \{x \in X \mid \theta_\sigma(x) \geq t \text{ and } \xi_\sigma(x) \leq s\}$, is called (t, s) level subset of σ .

Definition 9. [26] Let R be Γ -ring. An intuitionistic fuzzy set,

$$\sigma = \{ \langle x, \theta(x), \xi(x) \rangle \mid x \in R \}$$

, in R is named as an intuitionistic fuzzy Γ -ideal of R ($IF\Gamma I(R)$) for all $x, y \in R$ and $\beta \in \Gamma$ if the following requirements are satisfied:

1. $\theta(x-y) \geq \theta(x) \wedge \theta(y)$.
2. $\xi(x-y) \leq \xi(x) \vee \xi(y)$.
3. $\theta(x\beta y) \geq \theta(x) \vee \theta(y)$.
4. $\xi(x\beta y) \leq \xi(x) \wedge \xi(y)$.

Definition 10. [27] Let $\sigma \in IF\Gamma I(R)$ be such that non-constant is named an intuitionistic fuzzy prime Γ -ideal of R if for $\eta, \sigma \in IF\Gamma I(R)$, $\eta\Gamma\sigma \subseteq \sigma$ implies that either $\eta \subseteq \sigma$ or $\sigma \subseteq \eta$.

An intuitionistic fuzzy ideal σ of R is named an intuitionistic fuzzy weakly completely prime ideal [28] if σ is non-constant function and for $\forall x, y \in R$, $\theta(x, y) = \max\{\theta(x), \theta(y)\}$ and $\xi(x, y) = \min\{\xi(x), \xi(y)\}$.

Let σ be a non-constant intuitionistic fuzzy ideal of R . σ is named be an intuitionistic fuzzy K -prime ideal [28] if for any $x, y \in R$, $\sigma(xy) = \sigma(0)$ implies either $\sigma(x) = \sigma(0)$ or $\sigma(y) = \sigma(0)$.

3 Main Results

In this part, the idea of prime intuitionistic fuzzy weakly completely Γ -ideal (PIFWCFI) and 2-absorbing intuitionistic fuzzy weakly completely Γ -ideals of Γ -ring (2AIFWCFI) are given.

Definition 11. Let $\sigma = (\theta, \xi) \in IF\Gamma I(R)$ be named as a PIFWCFI(R) if σ is non-constant function and for all $x, y \in R$ and $\beta \in \Gamma$, $\theta(x\beta y) = \max\{\theta(x), \theta(y)\}$ and $\xi(x\beta y) = \min\{\xi(x), \xi(y)\}$.

The following definition describes 2AIFWCFI.

Definition 12. Let $\sigma = (\theta, \xi) \in IF\Gamma I(R)$ and σ be called a 2AIFWCFI(R) if for all $x, y, z \in R$ and $\beta, \gamma \in \Gamma$,

$$\begin{aligned}\sigma(x\beta y\gamma z) &= \sigma(x\beta y), \text{ or} \\ \sigma(x\beta y\gamma z) &= \sigma(x\gamma z), \text{ or} \\ \sigma(x\beta y\gamma z) &= \sigma(y\gamma z).\end{aligned}$$

i.e.

$$\begin{aligned}\theta(x\beta y\gamma z) &= \theta(x\beta y), \text{ or} \\ \theta(x\beta y\gamma z) &= \theta(x\gamma z), \text{ or} \\ \theta(x\beta y\gamma z) &= \theta(y\gamma z),\end{aligned}$$

and

$$\begin{aligned}\xi(x\beta y\gamma z) &= \xi(x\beta y), \text{ or} \\ \xi(x\beta y\gamma z) &= \xi(x\gamma z), \text{ or} \\ \xi(x\beta y\gamma z) &= \xi(y\gamma z).\end{aligned}$$

As will be shown in the following proposition, necessary and sufficient conditions for 2AIFWCFI.

Proposition 1. Let $\sigma \in IF\Gamma I(R)$ such that non-constant and then for all $x, y, z \in R$ and $\beta, \gamma \in \Gamma$, $\sigma \in 2AIFWCFI(R) \Leftrightarrow$

$$\begin{aligned}\theta(x\beta y\gamma z) &= \max\{\theta(x\beta y), \theta(x\gamma z), \theta(y\gamma z)\}, \\ \xi(x\beta y\gamma z) &= \min\{\xi(x\beta y), \xi(x\gamma z), \xi(y\gamma z)\}.\end{aligned}$$

The following theorem is derived from Definition 1 and Proposition 1.

Theorem 1. Let $\sigma \in PIFWCFI(R)$, then $\sigma \in 2AIFWCFI(R)$.

Proof. Let $\sigma \in PIFWCFI(R)$. Then for $\forall x, y, z \in R$ and $\beta, \gamma \in \Gamma$,

$$\begin{aligned}\theta(x\beta y\gamma z) &= \theta(x), \text{ or} \\ \theta(x\beta y\gamma z) &= \theta(y), \text{ or} \\ \theta(x\beta y\gamma z) &= \theta(z),\end{aligned}$$

and

$$\begin{aligned}\xi(x\beta y\gamma z) &= \xi(x), \text{ or} \\ \xi(x\beta y\gamma z) &= \xi(y), \text{ or} \\ \xi(x\beta y\gamma z) &= \xi(z).\end{aligned}$$

Presume that

$$\theta(x\beta y\gamma z) = \theta(x) \text{ and } \xi(x\beta y\gamma z) = \xi(x).$$

By

$$\theta(x\beta y\gamma z) \geq \theta(x\beta y) \geq \theta(x),$$

and

$$\xi(x\beta y\gamma z) \leq \xi(x\beta y) \leq \xi(x),$$

it follows that

$$\theta(x\beta y\gamma z) = \theta(x\beta y) \text{ and } \xi(x\beta y\gamma z) = \xi(x\beta y).$$

Likewise, it is demonstrated that if

$$\begin{aligned}\theta(x\beta y\gamma z) &= \theta(y) \text{ or } \theta(x\beta y\gamma z) = \theta(z), \text{ and} \\ \xi(x\beta y\gamma z) &= \xi(y) \text{ or } \xi(x\beta y\gamma z) = \xi(z),\end{aligned}$$

then

$$\begin{aligned}\theta(x\beta y\gamma z) &= \theta(y\gamma z) \text{ or } \theta(x\beta y\gamma z) = \theta(x\gamma z), \text{ and} \\ \xi(x\beta y\gamma z) &= \xi(y\gamma z) \text{ or } \xi(x\beta y\gamma z) = \xi(x\gamma z).\end{aligned}$$

It implies that, $\sigma \in 2AIFWCFI(R)$.

Now, relationship with the level sets of $\sigma \in 2AIFWCFI(R)$ will be given.

Theorem 2. Let $\sigma \in IF\Gamma I(R)$. The subsequent conditions are equivalent:

1. $\sigma \in 2AIFWCFI(R)$.
2. $\sigma_{(t,s)} \in 2A\Gamma I(R)$ for $\forall t, s \in [0, 1]$.

Proof. (1) $\Rightarrow$ (2): Admit that $\sigma \in 2AIFWCFI(R)$ and let $x, y, z \in R$, $\beta, \gamma \in \Gamma$ and $x\beta y\gamma z \in \sigma_{(t,s)}$ for some $t, s \in [0, 1]$. Then,

$$\begin{aligned}\max\{\theta(x\beta y), \theta(x\gamma z), \theta(y\gamma z)\} &= \theta(x\beta y\gamma z) \geq t, \\ \min\{\xi(x\beta y), \xi(x\gamma z), \xi(y\gamma z)\} &= \xi(x\beta y\gamma z) \leq s.\end{aligned}$$

It follows that,

$$\begin{aligned}\theta(x\beta y) &\geq t \text{ or } \theta(x\gamma z) \geq t \text{ or } \theta(y\gamma z) \geq t, \text{ and} \\ \xi(x\beta y) &\leq s \text{ or } \xi(x\gamma z) \leq s \text{ or } \xi(y\gamma z) \leq s,\end{aligned}$$

which infer that

$$x\beta y \in \sigma_{(t,s)} \text{ or } x\gamma z \in \sigma_{(t,s)} \text{ or } y\gamma z \in \sigma_{(t,s)}.$$

Accordingly $\sigma_{(t,s)} \in 2A\Gamma I(R)$.

(2) $\Rightarrow$ (1) : Presume that $\sigma_{(t,s)} \in 2A\Gamma I(R)$ for $\forall t, s \in [0, 1]$. For $x, y, z \in R$ and $\beta, \gamma \in \Gamma$, let $\theta(x\beta y\gamma z) = t$ and $\xi(x\beta y\gamma z) = s$. Then,

$x\beta y\gamma z \in \sigma_{(t,s)}$ and $\sigma_{(t,s)} \in 2A\Gamma I(R)$ and it gives,

$x\beta y \in \sigma_{(t,s)}$ or $x\gamma z \in \sigma_{(t,s)}$ or $y\gamma z \in \sigma_{(t,s)}$. Thus,

$\theta(x\beta y) \geq t$ or $\theta(x\gamma z) \geq t$ or $\theta(y\gamma z) \geq t$

and

$\xi(x\beta y) \leq s$ or $\xi(x\gamma z) \leq s$ or $\xi(y\gamma z) \leq s$

it follows that,

$\max\{\theta(x\beta y), \theta(x\gamma z), \theta(y\gamma z)\} \geq t = \theta(x\beta y\gamma z)$,

and

$\min\{\xi(x\beta y), \xi(x\gamma z), \xi(y\gamma z)\} \leq s = \xi(x\beta y\gamma z)$.

Moreover, by $\sigma \in IF\Gamma I(R)$,

$\theta(x\beta y\gamma z) \geq \max\{\theta(x\beta y), \theta(x\gamma z), \theta(y\gamma z)\}$,

$\xi(x\beta y\gamma z) \leq \min\{\xi(x\beta y), \xi(x\gamma z), \xi(y\gamma z)\}$.

Thus,

$\theta(x\beta y\gamma z) = \max\{\theta(x\beta y), \theta(x\gamma z), \theta(y\gamma z)\}$, and

$\xi(x\beta y\gamma z) = \min\{\xi(x\beta y), \xi(x\gamma z), \xi(y\gamma z)\}$,

it is reached $\sigma \in 2AIFWC\Gamma I(R)$.

Then, it will be shown that image of $2AIFWC\Gamma I$ under homomorphism is also $2AIFWC\Gamma I$.

Theorem 3. Let $\varphi : R \rightarrow S$ be an onto Γ -ring homomorphism. If $\sigma \in 2AIFWC\Gamma I(R)$ which is constant on $\text{Ker}\varphi$, then $\varphi(\sigma) \in 2AIFWC\Gamma I(S)$.

Proof. Presume that $\varphi(\lambda)(x\beta y\gamma z) \neq \varphi(\sigma)(x\beta y)$ and $\varphi(\xi)(x\beta y\gamma z) \neq \varphi(\sigma)(x\beta y)$ for any $x, y, z \in S$ and $\beta, \gamma \in \Gamma$. Since φ is an onto Γ -ring homomorphism then $\varphi(a) = x$, $\varphi(b) = y$, $\varphi(c) = z$ for some $a, b, c \in R$.

Thus

$$\begin{aligned}\varphi(\theta)(x\beta y\gamma z) &= \varphi(\theta)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\theta)(\varphi(a\beta b\gamma c)), \\ &\neq \varphi(\theta)(x\beta y), \\ &= \varphi(\theta)(\varphi(a)\beta\varphi(b)), \\ &= \varphi(\theta)(\varphi(a\beta b)), \text{ and}\end{aligned}$$

$$\begin{aligned}\varphi(\xi)(x\beta y\gamma z) &= \varphi(\xi)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\xi)(\varphi(a\beta b\gamma c)), \\ &\neq \varphi(\xi)(x\beta y), \\ &= \varphi(\xi)(\varphi(a)\beta\varphi(b)), \\ &= \varphi(\xi)(\varphi(a\beta b)).\end{aligned}$$

From σ is constant on $\text{Ker}\varphi$,

$$\begin{aligned}\varphi(\theta)(\varphi(a\beta b\gamma c)) &= \theta(a\beta b\gamma c), \text{ and} \\ \varphi(\theta)(\varphi(a\beta b)) &= \theta(a\beta b), \\ \varphi(\xi)(\varphi(a\beta b\gamma c)) &= \xi(a\beta b\gamma c), \text{ and} \\ \varphi(\xi)(\varphi(a\beta b)) &= \xi(a\beta b).\end{aligned}$$

It follows that

$$\begin{aligned}\varphi(\theta)(\varphi(a\beta b\gamma c)) &= \theta(a\beta b\gamma c), \\ &\neq \theta(a\beta b) = \varphi(\theta)(\varphi(a\beta b)), \\ \varphi(\xi)(\varphi(a\beta b\gamma c)) &= \xi(a\beta b\gamma c), \\ &\neq \xi(a\beta b) = \varphi(\xi)(\varphi(a\beta b)).\end{aligned}$$

By $\sigma \in 2AIFWC\Gamma I(R)$, then

$$\begin{aligned}\theta(a\beta b\gamma c) &= \varphi(\theta)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\theta)(x\beta y\gamma z), \\ &= \theta(a\gamma c) = \varphi(\theta)(\varphi(a\gamma c)), \\ &= \varphi(\theta)(\varphi(a)\gamma\varphi(c)) = \varphi(\theta)(x\gamma z),\end{aligned}$$

and

$$\begin{aligned}\xi(a\beta b\gamma c) &= \varphi(\xi)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\xi)(x\beta y\gamma z), \\ &= \xi(a\gamma c) = \varphi(\xi)(\varphi(a\gamma c)), \\ &= \varphi(\xi)(\varphi(a)\gamma\varphi(c)) = \varphi(\xi)(x\gamma z).\end{aligned}$$

So, it is obtained $\varphi(\theta)(x\beta y\gamma z) = \varphi(\theta)(x\gamma z)$ and $\varphi(\xi)(x\beta y\gamma z) = \varphi(\xi)(x\gamma z)$ or

$$\begin{aligned}\theta(a\beta b\gamma c) &= \varphi(\theta)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\theta)(x\beta y\gamma z), \\ &= \theta(b\gamma c) = \varphi(\theta)(\varphi(b\gamma c)), \\ &= \varphi(\theta)(\varphi(b)\gamma\varphi(c)) = \varphi(\theta)(y\gamma z),\end{aligned}$$

and

$$\begin{aligned}\xi(a\beta b\gamma c) &= \varphi(\xi)(\varphi(a)\beta\varphi(b)\gamma\varphi(c)), \\ &= \varphi(\xi)(x\beta y\gamma z), \\ &= \xi(b\gamma c) = \varphi(\xi)(\varphi(b\gamma c)), \\ &= \varphi(\xi)(\varphi(b)\gamma\varphi(c)) = \varphi(\xi)(y\gamma z).\end{aligned}$$

It is arrived $\varphi(\theta)(x\beta y\gamma z) = \varphi(\theta)(y\gamma z)$ and $\varphi(\xi)(x\beta y\gamma z) = \varphi(\xi)(y\gamma z)$. Consequently, $\varphi(\sigma) \in 2AIFWC\Gamma I(S)$.

Now, it will be demonstrated that preimage of $2AIFWC\Gamma I$ under homomorphism is also $2AIFWC\Gamma I$.

Theorem 4. Let $\varphi : R \rightarrow S$ be a Γ -ring homomorphism. If $\sigma_1 = \langle \theta_1, \xi_1 \rangle \in 2AIFWC\Gamma I(S)$, then $\varphi^{-1}(\sigma_1) \in 2AIFWC\Gamma I(R)$.

Proof. Presume that $\varphi^{-1}(\theta_1)(x\beta y\gamma z) \neq \varphi^{-1}(\theta_1)(x\beta y)$ and $\varphi^{-1}(\xi_1)(x\beta y\gamma z) \neq \varphi^{-1}(\xi_1)(x\beta y)$ for any $x, y, z \in R$ and $\beta, \gamma \in \Gamma$. Then,

$$\begin{aligned}\varphi^{-1}(\theta_1)(x\beta y\gamma z) &= \theta_1(\varphi(x\beta y\gamma z)), \\ &= \theta_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)), \\ &\neq \varphi^{-1}(\theta_1)(x\beta y), \\ &= \theta_1(\varphi(x\beta y)) = \theta_1(\varphi(x)\beta\varphi(y)),\end{aligned}$$

and

$$\begin{aligned}\varphi^{-1}(\xi_1)(x\beta y\gamma z) &= \xi_1(\varphi(x\beta y\gamma z)), \\ &= \xi_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)), \\ &\neq \varphi^{-1}(\xi_1)(x\beta y), \\ &= \xi_1(\varphi(x\beta y)) = \xi_1(\varphi(x)\beta\varphi(y)).\end{aligned}$$

By $\sigma_1 \in 2AIFWCFI(S)$ it is achieved that

$$\begin{aligned}\theta_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)) &= \varphi^{-1}(\theta_1)(x\beta y\gamma z), \\ &= \theta_1(\varphi(x)\gamma\varphi(z)) = \theta_1(\varphi(x\gamma z)), \\ &= \varphi^{-1}(\theta_1)(x\gamma z), \text{ and}\end{aligned}$$

$$\begin{aligned}\xi_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)) &= \varphi^{-1}(\xi_1)(x\beta y\gamma z), \\ &= \xi_1(\varphi(x)\gamma\varphi(z)) = \xi_1(\varphi(x\gamma z)), \\ &= \varphi^{-1}(\xi_1)(x\gamma z),\end{aligned}$$

or

$$\begin{aligned}\theta_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)) &= \varphi^{-1}(\theta_1)(x\beta y\gamma z), \\ &= \theta_1(\varphi(y)\gamma\varphi(z)) = \theta_1(\varphi(y\gamma z)), \\ &= \varphi^{-1}(\theta_1)(y\gamma z), \text{ and}\end{aligned}$$

$$\begin{aligned}\xi_1(\varphi(x)\beta\varphi(y)\gamma\varphi(z)) &= \varphi^{-1}(\xi_1)(x\beta y\gamma z), \\ &= \xi_1(\varphi(y)\gamma\varphi(z)) = \xi_1(\varphi(y\gamma z)), \\ &= \varphi^{-1}(\xi_1)(y\gamma z).\end{aligned}$$

Therefore $\varphi^{-1}(\sigma_1) \in 2AIFWCFI(R)$.

Based on Theorem 3 and Theorem 4, the following correspondence theorem is verified.

Corollary 1. Let φ be a Γ -ring homomorphism from R onto S . φ induces a 1-1 inclusion preserving correspondence between $2AIFWCFI(S)$ to this extent that whether $\sigma \in 2AIFWCFI(R)$ which constant on $\text{Ker}\varphi$, then correspondingly $\varphi(\sigma) \in 2AIFWCFI(S)$, and if $\sigma_1 \in 2AIFWCFI(S)$, then correspondingly $\varphi^{-1}(\sigma_1) \in 2AIFWCFI(R)$.

Now, the notion of prime intuitionistic K -fuzzy Γ -ideal ($PIKF\Gamma I(R)$) and 2-absorbing intuitionistic K -fuzzy Γ -ideal of Γ -ring R ($2AIKF\Gamma I(R)$) will be given.

Definition 13. Let $\sigma \in IF\Gamma I(R)$ and σ is named as a $PIKF\Gamma I(R)$ if for $x, y \in R$ and $\beta, \gamma \in \Gamma$,

$$\sigma(x\beta y) = \sigma(0) \text{ asserts that } \sigma(x) = \sigma(0) \text{ or } \sigma(y) = \sigma(0).$$

Definition 14. Let $\sigma \in IF\Gamma I(R)$ and σ is named as a $2AIKF\Gamma I(R)$ if for all $x, y, z \in R$ and $\beta, \gamma \in \Gamma$,

$\sigma(x\beta y\gamma z) = \sigma(0)$ implies that

$$\sigma(x\beta y) = \sigma(0), \text{ or}$$

$$\sigma(x\gamma z) = \sigma(0), \text{ or}$$

$$\sigma(y\gamma z) = \sigma(0).$$

In the following theorem, the relationship between $2AIFWCFI$ and $2AIKF\Gamma I$ are characterized.

Theorem 5. Let $\sigma \in 2AIFWCFI(R)$, then $\sigma \in 2AIKF\Gamma I(R)$.

Proof. Presume that $\sigma \in 2AIFWCFI(R)$. If $\sigma(x\beta y\gamma z) = \sigma(0)$ for any $x, y, z \in R$ and $\beta, \gamma \in \Gamma$, then since $\sigma \in 2AIFWCFI(R)$,

$$\sigma(0) = \sigma(x\beta y\gamma z) \leq \sigma(x\beta y) \leq \sigma(0), \text{ or}$$

$$\sigma(0) = \sigma(x\beta y\gamma z) \leq \sigma(x\gamma z) \leq \sigma(0), \text{ or}$$

$$\sigma(0) = \sigma(x\beta y\gamma z) \leq \sigma(y\gamma z) \leq \sigma(0).$$

For this reason, the following outcome is acquired,

$$\sigma(x\beta y) = \sigma(0) \text{ or } \sigma(x\gamma z) = \sigma(0) \text{ or } \sigma(y\gamma z) = \sigma(0).$$

It implies that $\sigma \in 2AIKF\Gamma I(R)$.

The subsequent illustration demonstrates that the inverse of the above theorem is require not to be right.

Example 1. Let $R = \mathbb{Z}$ and $\Gamma = 2\mathbb{Z}$, so R is a Γ -ring. Describe $\sigma \in IF\Gamma I(\mathbb{Z})$ by

$$\theta(x) = \begin{cases} 1, & \text{if } x = 0; \\ 1/3, & \text{if } x \in 27\mathbb{Z} - \{0\}; \\ 1/4, & \text{if } x \in \mathbb{Z} - 27\mathbb{Z}. \end{cases}, \text{ and}$$

$$\xi(x) = \begin{cases} 0, & \text{if } x = 0; \\ 1/4, & \text{if } x \in 27\mathbb{Z} - \{0\}; \\ 1/3, & \text{if } x \in \mathbb{Z} - 27\mathbb{Z}. \end{cases}.$$

Then $\sigma \in 2AIKF\Gamma I(\mathbb{Z})$. However for $\beta, \gamma \in 2\mathbb{Z}$,

$$\xi(3\beta 3\gamma 15) = 1/4 < 1/3 = \min\{\xi(3\beta 3), \xi(3\gamma 15), \xi(3\gamma 15)\}.$$

Thus $\sigma \notin 2AIFWCFI(\mathbb{Z})$.

Theorem 6. Let $\sigma \in PIKF\Gamma I(R)$ and then $\sigma \in 2AIKF\Gamma I(R)$.

Proof. Suppose $\sigma \in PIKF\Gamma I(R)$. Then for $\forall x, y, z \in R$ and $\beta, \gamma \in \Gamma$,

$\sigma(x\beta y\gamma z) = \sigma(0)$ implies

$$\sigma(x) = \sigma(0), \text{ or}$$

$$\sigma(y) = \sigma(0), \text{ or}$$

$$\sigma(z) = \sigma(0).$$

Suppose that $\sigma(x) = \sigma(0)$ then by,

$$\sigma(0) = \sigma(x) \leq \sigma(x\beta y) \leq \sigma(x\beta y\gamma z) = \sigma(0),$$

it is obtained $\sigma(x\beta y) = \sigma(0)$ or in the similar wice it can be affirmed that $\sigma(x\gamma z) = \sigma(0)$ or $\sigma(y\gamma z) = \sigma(0)$. As a result, $\sigma \in 2AIKF\Gamma I(R)$.

Right now, it will be indicated that image of $2AIKF\Gamma I$ under homomorphism is also $2AIKF\Gamma I$.

Theorem 7. Let $\varphi : R \rightarrow S$ be an onto Γ -ring homomorphism. If $\sigma \in 2AIKF\Gamma I(R)$ which is constant on $\text{Ker}\varphi$, then $\varphi(\sigma) \in 2AIKF\Gamma I(S)$.

Proof. The proof is nearly identical to the Theorem 3's proof, thus it is removed.

After that, it will be expressed that preimage of $2AIKFGI$ under homomorphism is also $2AIKFGI$.

Theorem 8. Let $\varphi : R \rightarrow S$ be a Γ -ring homomorphism. If $\sigma_1 \in 2AIKFGI(S)$, then $\varphi^{-1}(\sigma_1) \in 2AIKFGI(R)$.

Proof. The proof is removed as it is pretty much identical to the proof of Theorem 4.

Now, the result follows by combining Theorem 7 and Theorem 8.

Corollary 2. Let φ be a Γ -ring homomorphism from R onto S . φ elicits a 1-1 inclusion preserving correspondence between $2AIKFGI(S)$ in this wise that if $\sigma \in 2AIKFGI(R)$ which constant on $\text{Ker}\varphi$, then correspondingly $\varphi(\sigma) \in 2AIKFGI(S)$ and if $\sigma_1 \in 2AIKFGI(S)$, then correspondingly $\varphi^{-1}(\sigma_1) \in 2AIKFGI(R)$.

Remark. The following diagram summarizes results of $2AIFWCGI(R)$.

$$\begin{array}{ccc} PIFWCGI(R) & \implies & 2AIFWCGI(R) \\ \downarrow & & \downarrow \\ PIKFGI(R) & \implies & 2AIKFGI(R). \end{array}$$

Now, I will mention about intuitionistic fuzzy quotient Γ -ring of R ($IFQGR$) induced by $2AIFWCGI(R)$. I recall the notion of $IFQGR$ induced by $IFGI(R)$. Let $\sigma \in IFGI(R)$ and for $\forall x, y \in R$, describe a binary relation $\sim$ on R which is a congruence relation of R by $x \sim y$ if and only if

$$\sigma(x - y) = \sigma(0).$$

Let $\sigma[x] = \{y \in R \mid y \sim x\}$ be the equivalence class containing x and $R/\sigma = \{\sigma[x] \mid x \in R\}$ the set of all equivalence classes of R . Describe two operations by for $x, y \in R, \beta \in \Gamma$,

$$\sigma[x] + \sigma[y] = \sigma[x + y], \text{ and}$$

$$\sigma[x] \beta \sigma[y] = \sigma[x \beta y].$$

Then R/σ is an $IFGR$ with two operations and call it $IFQGR$ induced by $\sigma \in IFGI(R)$.

Now, it is characterized that $2AIGR$ by using $2AIKFGI$.

Theorem 9. Let $\sigma \in IFGI(R)$. Then $\sigma \in 2AIKFGI(R) \Leftrightarrow R/\sigma \in 2AIGR$.

Proof. Presume that $\sigma \in 2AIKFGI(R)$ and let $\sigma[x], \sigma[y], \sigma[z] \in R/\sigma$ be such that $\sigma[x] \beta \sigma[y] \gamma \sigma[z] = \sigma[0]$.

By $\sigma[x] \beta \sigma[y] \gamma \sigma[z] = \sigma[x \beta y \gamma z]$ then it is accomplished,

$$\sigma(x \beta y \gamma z) = \sigma(x \beta y \gamma z - 0) = (1, 0) = \sigma(0).$$

Since $\sigma \in 2AIKFGI(R)$,

$$\sigma(x \beta y) = \sigma(0) = (1, 0), \text{ or}$$

$$\sigma(x \gamma z) = \sigma(0) = (1, 0), \text{ or}$$

$$\sigma(y \gamma z) = \sigma(0) = (1, 0).$$

It follows that

$$\sigma[x \beta y] = \sigma[x] \beta \sigma[y] = \sigma[0], \text{ or}$$

$$\sigma[x \gamma z] = \sigma[x] \gamma \sigma[z] = \sigma[0], \text{ or}$$

$$\sigma[y \gamma z] = \sigma[y] \gamma \sigma[z] = \sigma[0].$$

So, $R/\sigma \in 2AIGR$. Otherwise, presume that $R/\sigma \in 2AIGR$ and let $\sigma(x \beta y \gamma z) = \sigma(0) = (1, 0)$ for $x, y, z \in R$ and $\beta, \gamma \in \Gamma$. Then it is obtained,

$$\sigma[x] \beta \sigma[y] \gamma \sigma[z] = \sigma[x \beta y \gamma z] = \sigma[0].$$

As $R/\sigma \in 2AIGR$, then

$$\sigma[x \beta y] = \sigma[0] \text{ or } \sigma[x \gamma z] = \sigma[0] \text{ or } \sigma[y \gamma z] = \sigma[0],$$

which implies that $\sigma \in 2AIKFGI(R)$.

These consideration lead us to result a new corollary.

Corollary 3. If $\sigma \in 2AIFWCGI(R)$, then $R/\sigma \in 2AIGR$.

4 Conclusion

The notions of $2AIFWCGI(R)$ and $2AIKFGI(R)$ have been characterized. In this paper. The transition between these algebraic structures has been depicted in a diagram. The statement $\sigma \in 2AIFWCGI(R)$, then $IFQGR$ induced by $\sigma \in IFGI(R)$ is an $R/\sigma \in 2AIGR$ has been investigated. It can be given to open problem and discuss 2-absorbing vague Γ -ideal of a Γ -ring and a 2-absorbing vague weakly completely Γ -ideal of a Γ -ring in ongoing research.

Availability of data and materials

All data are included within this paper.

Conflict of interest

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