Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.12785/amis/092L16

Solving a Long-Distance Routing Problem using Ant Colony Optimization

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Received: 15 Jul. 2014, Revised: 16 Oct. 2014, Accepted: 17 Oct. 2014
Published online: 1 Apr. 2015

Abstract: This paper presents a mathematical model and an algorithm based on ant colony optimization to solve a long distance routing problems. The size of freight is relatively small, which uses *Last In First Out "LIFO"* policy and with several time constraints. The objective consists of reducing costs by optimizing the loading of goods in vehicles grouping orders and minimizing number of routes. The performance of the algorithm has been proved using experimental data based on historical data from a large Spanish transport company.

Keywords: Long haul transportation, less than truckload, vehicle routing problem, ant colony optimization

1 Introduction

In operational terms, the distribution of goods has increased in complexity due to increases in the number of operations, the distances involved, and the value of goods and also the reduction in tonnage. The profit margin has been reduced and this has brought about an increase in the volume of freight [1].

This article is focused on Long Haul Transportation (LHT) and its aim is to optimize transport operations. This problem presents different transport systems: "*customized transportation*", where a vehicle is dedicated to each customer, and "*consolidated transportation*", where the demands of several customers are served by the same vehicle by means of the so-called less-than-truckload carrier [2]. Consolidated transportation can use break-bulk terminals, which act as intermediate transshipment points where the freight is unloaded, sorted, consolidated and reloaded, or it can use peddling/collecting routes, where carriers make multiple stops for collecting or delivering goods for grouping orders [3].

Peddling/collecting routes with some special variants of the classic Vehicle Routing Problem (VRP) are considered in [4]. The VRP objective is to determine the optimal set of routes originating and terminating at a single central depot to serve a set of customers by a fleet of vehicles, and where all customers are served only once [5]. The most extensively studied variants of the VRP are capacitated VRP, where each vehicle has a maximum limit of capacity, and VRP with time windows [6], where each customer must be served inside a time window. In [7], waiting time which is restricted and penalized in the objective function is allowed. Another variant is multidepot VRP, and [8] deals with both single and multidepot.

2 Mathematical Formulation

Let $O^* = O \cup \{F1\}$ be the set of orders where $O = \{1 \ldots m\}$ is the set of customer orders to be transported and *F1* is a fictitious order (initial and the final order of every route). Let $G = (N^*, A)$ be a complete graph, with $N = N^* \cup \{n_{o1}, n_{n1}\}$ the set of nodes where $N = \{1 \ldots n\}$ is the set of customers nodes to be served and n_{o1}, n_{n1} are the initial and final fictitious nodes that belong to the order *F1*. Every node $i \in N$ that belong to an order must be distributed by the same vehicle. The set $A = \{(i, j) : i, j \in N^*\}$ contains the directed arcs.

Decission variables:

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$$X_{ok}^{v} = \begin{cases} 1, & \text{if } \ o \in O \text{ in } v \in V \text{ just before } k \in O \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

$$Z_{o}^{v} = \begin{cases} 1, & \text{if } \ o \in O \text{ in } v \in V \\ 0, & \text{otherwise} \end{cases} \quad (2)$$

$$U_{ij}^{v} = \begin{cases} 1, & \text{if } \ arc(i,j) \in A, \text{ is travelled by } v \in V \\ 0, & \text{otherwise} \end{cases} \quad (3)$$

$$a^{v} = \begin{cases} 1, & \text{if } \ v \in V \text{ is activated} \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

Objective function:

$$Min(CP1 + CP2 + CP3 + CP4 + CP5) \quad (5)$$

$$CP1 = \sum_{v\in V}\sum_{o\in O} X_{F_{1O}}^{v} \cdot C1_{b_{(o,1)},d_{(o,total_{o}^{2})}} \quad (6)$$

$$CP2 = C2 \cdot (\begin{matrix} \sum_{v\in V}\sum_{o\in O} Z_{o}^{v} \cdot (\sum_{i\in N} st_{oi}) - \\ \sum_{v\in V}\sum_{o\in O} X_{F_{1O}}^{v} \cdot (st_{ob_{(0,1)}} + st_{od_{(0,total_{0}^{2})}}) \end{matrix}) \quad (7)$$

$$CP3 = C3 \cdot (\begin{matrix} \sum_{v\in V}\sum_{i,j\in N} U_{ij}^{v} \cdot dist_{ij} - \\ \sum_{v\in V}\sum_{o\in O} X_{F_{1O}}^{v} \cdot (dist_{b_{(0,1)},d_{(o,total_{0}^{2})}}) \end{matrix}) \quad (8)$$

$$CP4 = C4 \cdot \sum_{o\in O}\sum_{i\in N} wt_{oi} \quad (9)$$

$$CP5 = \sum_{v\in V} penaltyCost^{v} \quad (10)$$

$$penaltyCost^{v} =$$

$$\left(\begin{matrix} 50, \ if \ 0.5 \le \left(\frac{\sum_{0\in O} Z_{o}^{v} \sum_{i\in N} mp_{oi}}{Q^{v}}\right) \le 0.8 \\ 100, \ if \ 0.25 \le \left(\frac{\sum_{0\in O} Z_{o}^{v} \sum_{i\in N} mp_{oi}}{Q^{v}}\right) \le 0.5 \\ 200, \ if \ 0 \le \left(\frac{\sum_{0\in O} Z_{o}^{v} \sum_{i\in N} mp_{oi}}{Q^{v}}\right) \le 0.25 \\ 0, \ otherwise \end{matrix} \right)$$

Let $dist_{ij}, t_{ij}, c_{ij}$ be the distance, the time and the cost to travel from node $i \in N$ to node $j \in N$ through arc (i,j), where $c_{ij} = C3 * dist_{ij}$, and $C3$ is the cost per kilometer; st_{oi} refers to the service time, wt_{oi} is the waiting time and b_{oind}, and d_{oind} return the identifier node of the picking or the delivery operation of the order $o \in O$ that is placed by the customer in the order $ind \in \{1 \ldots total_{o}^{\alpha}\}$, where $total_{o}^{\alpha}$ is the number of nodes of kind $\alpha \in \{1 = picking, 2 = delivery\}$ of the order $o \in O$. Finally, mp_{oi} is the amount of loaded freight and md_{oi} is the amount of freight to delivery in the node $i \in N, o \in O$.

CP1 is the interregional cost between the zones of the first pickup node and the last delivery node of a route, where the cost between zones is known $C1_{ij}$. *CP2* is the cost due to the stopovers between the first and the last node of a route. C2 is known as the cost of stopping for loading/unloading. *CP3* is the cost of the difference between the calculated distance of a route and the actual distance travelled taking in new orders. *CP4* is the cost of the total waiting time, and C4 is the cost per unit or time. *CP5* is the penalty cost by percentage of filling.

The objective function is subject to constraints from (11) to (39). In (11), $y_{oi}^{\alpha} \in 0,1$ establishes if node $i \in N$ of order $o \in O$ is type $\alpha \in \{1 = picking, 2 = delivery\}$, (12) the relationship between y_{oi}^{α} and $h_{oi} \in \{0,1\}$ that indicates if node $i \in N$ belongs to the order $o \in O$. (13) refers to the total $\alpha \in \{1 = picking, 2 = delivery\}$ nodes of an order $o \in O$. The fictitious nodes $n_{o1} \in N^{*}$ and $n_{n1} \in N^{*}$ belong to $F1 \in O^{*}$ (14). Let $mp_{oi} \geq 0$ and $md_{oi} \geq 0$ be the amount of freight to load/unload in the node $i \in N$ of the order $o \in O$ and M a sufficiently large number satisfying constraints (15). The total load is the same as the total unload for every order $o \in O$ (16).

$$\sum_{i\in N} y_{oi}^{\alpha} \geq 1, \ \ \forall o \in O, \forall i \in N, \alpha = \{1,2\} \quad (11)$$

$$\sum_{\alpha=1,2} y_{oi}^{\alpha} \leq h_{oi}, \ \ \forall o \in O, \forall i \in N \quad (12)$$

$$total_{o}^{\alpha} = \sum_{i\in N} y_{oi}^{\alpha}, \ \ \forall o \in O, \alpha = \{1,2\} \quad (13)$$

$$y_{F1,n_{o1}}^{1} = 1, \ \ y_{F1,n_{n1}}^{2} = 1 \quad (14)$$

$$mp_{oi} \leq M \cdot y_{oi}^{1}, \ \ md_{oi} \leq M \cdot y_{oi}^{2}, \ \ \forall i \in N, \forall o \in O \quad (15)$$

$$\sum_{i\in N} mp_{oi} - \sum_{i\in N} md_{oi} = 0, \ \ \forall o \in O \quad (16)$$

From (17) to (24) are the constraints for all the activated routes. The order F1 (17) is included in all the activated routes. There is an order $o \in O$ after $F1 \in O^{*}$ (18) in all the activated routes. An order is allocated to only one vehicle except $F1 \in O^{*}$ (19). Before and after an order $o \in O$, there is one and only one order $k \in O^{*}$ (20). Constraint (21) is the flow conservation equation that ensures the continuity of each vehicle route. If an order $o \in O$ is allocated to the vehicle v, then vehicle v is activated (22). $n_{o1} \in N^{*}$ is the first node (23) and $n_{n1} \in N^{*}$ is the last node (24) for all the activated routes.

$$Z_{F1}^{v} \cdot a^{v} = 1, \ \ \forall v \in V \quad (17)$$

$$a^v \cdot \sum_{o\in O} X^v_{F1,o} = 0, \ \ \forall v \in V \tag{18}$$

$$\sum_{v\in V} Z^v_o = 1, \ \ \forall o \in O \tag{19}$$

$$\sum_{k\in O^*,k\neq 0} X^v_{ko} = Z^v_o, \ \sum_{k\in O^*,k\neq 0} X^v_{ok} = Z^v_o, \ \ \forall o \in O, \forall v \in V \tag{20}$$

$$\sum_{k\in O^*,k\neq 0} X^v_{ko} \ - \sum_{k\in O^*,k\neq 0} X^v_{ok} = 0, \ \ \forall o \in O, \forall v \in V \tag{21}$$

$$Z^v_o \cdot a^v \leq 0, \ \ \forall o \in O, \forall v \in V \tag{22}$$

$$\sum_{i\in N} U^v_{in_{o1}} = 0, \ \sum_{i\in N} U^v_{n_{o1},i} \cdot a^v = 0, \ \ \forall v \in V \tag{23}$$

$$\sum_{i\in N} U^v_{n_{n1}i} = 0, \ \sum_{i\in N} U^v_{i,n_{n1}} \cdot a^v = 0, \ \ \forall v \in V \tag{24}$$

From (25) to (31) define the arcs between nodes and ensure the LIFO policy. In (25), the fictitious node $n_{o1} \in N^*$ is linked with the first pickup node b_{o1} of the first order $o \in O$ in route v; (26) and (29), the pickup and delivery nodes of order $o \in O$ in route v follow the sequences introduced by the customer; (27), the last pickup node of order $o \in O$ with the first pickup node of order $k \in O$, if order $k \in O$ is just after order $o \in O$; (28), the last pickup node of the last order $o \in O$, with the first delivery node of order $o \in O$; (30), the last delivery node of order $o \in O$, with the first delivery node of order $k \in O$, if order $k \in O$ is just after order $o \in O$; (31), the fictitious node $n_{n1} \in N^*$ with the last delivery node of the first order $o \in O$ in route v. From (25) to (31) it is considered $\forall v \in V, \forall o,k \in O$ when it is required.

$$X^v_{F1,o} \cdot U^v_{n_{o1},b_{o1}} \leq 0 \tag{25}$$

$$Z^v_o \cdot U^v_{b_{(o,ind)},b_{(o,ind+1)}} \leq 0, \ \ ind \in \{1\dots total^1_o\} \tag{26}$$

$$X^v_{o,k} \cdot U^v_{b_{(o,total^1_o)},b_{(k,1)}} \leq 0, \ \ o \neq k \tag{27}$$

$$X^v_{o,F1} \cdot U^v_{b_{(o,total^1_o)},d_{(o,1)}} \leq 0 \tag{28}$$

$$Z^v_o \cdot U^v_{d_{(o,ind)},d_{(o,ind+1)}} \leq 0, \ \ ind \in \{1\dots total^2_o\} \tag{29}$$

$$X^v_{ok} \cdot U^v_{d_{(o,total^2_o)},d_{(k,1)}} \leq 0, \ \ o \neq k \tag{30}$$

$$X^v_{F1,o} \cdot U^v_{d_{(o,total^2_o)},n_{n1}} \leq 0 \tag{31}$$

The service time st_{oi} is composed of a fixed term, st_f, and a variable term, st_{voi}, which is a linear function of demand. Each service of nodes should begin and end within a pre-specified time window $[e_{oi}, l_{oi}]$ for every node $i \in N$ of order $o \in O$. A vehicle is allowed to arrive at the node before the time window starts, wt_{oi}. Let at_{oi}, be the arrival time of node $i \in N$ of order $o \in O$, and E_m and C_m the maximum levels of driving and waiting time. Time constraints are shown from (32) to (36) and it is considered $\forall v \in V$, $\forall i,j \in N$, $\forall o,k \in O$ when it is required.

$$at_{oi} + \left(1 - U^v_{n_{o1},i}\right) \cdot M \geq 0 \tag{32}$$

$$at_{kj} - at_{oi} + \left(1 - U^v_{ij}\right) \cdot M \geq wt_{oi} + st_{oi} + t_{ij} \tag{33}$$

$$at_{F1,n_{n1}} - at_{oi} + \left(1 - U^v_{i,n_{n1}}\right) \cdot M \geq wt_{oi} + st_{oi} + t_{i,n_{n1}} \tag{34}$$

$$at_{oi} + wt_{oi} + (1 - h_{oi}) \cdot M \geq e_{oi} \tag{35}$$

$$at_{oi} + wt_{oi} \leq l_{oi} + (1 - h_{oi}) \cdot M \tag{36}$$

(37) to (39) define the limits for the amount of freight, the total waiting time and the total driving time for all activated routes.

$$\sum_{o\in O} Z^v_o \cdot \left(\sum_{i\in N} md_{oi}\right) \leq Q^v \cdot a^v, \ \ \forall v \in V \tag{37}$$

$$\sum_{i\in N}\sum_{j\in N} U^v_{ij} \cdot t_{ij} \leq C_m \cdot a^v, \ \ \forall v \in V \tag{38}$$

$$\sum_{o\in O} Z^v_o \sum_{i\in N} wt_{oi} \leq E_m \cdot a^v, \ \ \forall v \in V \tag{39}$$

3 Aco Algorithm

Mathematical model is unsolvable for problems of realistic size, so a metaheuristic based on Ant Colony Optimization (ACO) is proposed. ACO is a method inspired by the behavior of real ant colonies. ACO has been successfully applied to several combinatorial optimization problems and has achieved satisfactory performances [9] and [10].

ACO algorithms are construction algorithms where each ant is a solution and its movement depends on two kinds of information. Heuristic information determines the preference of moving and pheromone trail information measures the "learned desirability" of the movement, depending on whether or not the movement in question has been previously carried out.

The algorithm has three phases with a different ACO. The choice of the main road depending on the intermediate nodes, traffic, etc. and is selected using the ACO-I. ACO-II selects the first order of a new route and ACO-III adds new orders while possible. The outline of the proposed algorithm is presented in Figure 1.

```
Generate an initial feasible solution of problem
Initialize ACO parameters
While (the stop condition is not verified) do
    While (the number of ants m<M) do
        While (the number of orders o < O) do
            ACO-I: Order-to-road allocation ();
        End do;
        While (the number of orders o < O) do
            ACO-II: First order-in-route insertion ();
            While (not end of route) do
                ACO-III: Rest of the orders-in-route insertion ();
            End do;
        End do;
        Update pheromone trail of selected arcs (local updating);
    End do;
    Update pheromone trail (global updating);
End do;
```

Fig. 1: Proposed algorithm

Each ant builds a solution to the problem according to a pseudo-random-proportional rule. At each step the ant makes a move in order to complete the solution choosing between elements from a set of expansion states, following a probability function that takes into account the following: the attractiveness η_{ou} of the move $o \longrightarrow u$ according to the heuristic information about the problem (ACO-I: cost to carry out order o by road u; ACO-II: available capacity of the vehicle after loading order o and the distance required to dispatch order o; ACO-III: available capacity of the vehicle after loading orders o and u and the distance required to dispatch orders o and u, and the level τ_{ou} of pheromone of the move $o \longrightarrow u$ that indicates how good the move was in the past. As a solution is found, this value is updated.

The pseudo-random-proportional rule combines random selection with the best option. Let q be a random number in $[0,1]$ and q_o, a pre-specified parameter, such that $0 \leq q_o \leq 1$. In ACO I, the order o uses road r following the rule: If $(q \leq q_0)$ then the probability follows the rule in expression (40) else the expression (41) .

$$P_{or} = \{ \begin{array}{l} 1, \; if \;\; r = \underset{u \in Feasible\_roads(o)}{argmax} \;\; ((\tau\_road_{ou})^{\alpha_{road}} \cdot (\eta\_road_{ou})^{\beta_{road}}) \\ 0, \; otherwise \end{array} \} \quad (40)$$

$$P_{or} = \{ \begin{array}{l} \frac{((\tau\_road_{or})^{\alpha_{road}} \cdot (\eta\_road_{or})^{\beta_{raod}})}{\sum_{u \in Feasible\_roads(o)} ((\tau\_road_{ou})^{\alpha_{road}} \cdot (\eta\_road_{ou})^{\beta_{road}})}, \\ if \;\; r \in Feasible\_roads(o) \\ 0, otherwise \end{array} \} \quad (41)$$

In ACO II, order o assigned to road r will be the first on the route.In this case, if $(q \leq q_0)$ then the probability follows the rule in expressions (42)and (43).

$$P_o^r = \{ \begin{array}{l} 1, \; if \;\; o = \underset{o \in rest\_orders(r)}{argmax} \;\; ((\tau\_first_o^r)^{\alpha_{first}} \cdot (\eta\_first_o^r)^{\beta_{first}}) \\ 0, \; otherwise \end{array} \} \quad (42)$$

$$P_o^r = \{ \begin{array}{l} \frac{((\tau\_first_o^r)^{\alpha_{first}} \cdot (\eta\_first_o^r)^{\beta_{first}})}{\sum_{o \in rest\_orders(r)} ((\tau\_first_u^r)^{\alpha_{first}} \cdot (\eta\_first_u^r)^{\beta_{first}})}, \\ if \;\; o \in rest\_orders(r) \\ 0, otherwise \end{array} \} \quad (43)$$

Finally ACO III selects the order k that goes right after order o and the probability rule is shown in expressions (44)and (45).

$$P_{ok}^r = \{ \begin{array}{l} 1, \; if \;\; k = \underset{k \in rest\_orders(r)}{argmax} \;\; ((\tau\_rest_{ok}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ok}^r)^{\beta_{rest}}) \\ 0, \; otherwise \end{array} \} \quad (44)$$

$$P_{ok}^r = \{ \begin{array}{l} \frac{((\tau\_rest_{ou}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ou}^r)^{\beta_{rest}})}{\sum_{u \in rest\_orders(r)} ((\tau\_rest_{ou}^r)^{\alpha_{rest}} \cdot (\eta\_rest_{ou}^r)^{\beta_{rest}})}, \\ if \;\; u \in rest\_orders(r) \\ 0, otherwise \end{array} \} \quad (45)$$

The parameters α and β establish the relative influence of η versus τ. The initial value of pheromone $\tau_0 = 1/(n \cdot OBJ)$ is equal for the three ACOs, where n is the total number of orders and OBJ the objective of the initial solution. After each ant finds a solution, the pheromone levels of visited arcs are modified (local updating) in order to diversify the solutions obtained by the ants with the formula in (46).

$$\tau = (1 - \varphi) \cdot \tau + \varphi \cdot \tau_0, \;\; 0 \leq \varphi \leq 1 \quad (46)$$

Once all the ants of the iteration have been considered, the pheromone levels are updated (global updating), where $best\_OBJ$ is the best solution obtained, as it is shown in (47).

$$\tau = (1 - \rho) \cdot \tau + \rho \cdot \left(\frac{1}{best\_OBJ} \right), \;\; 0 \leq \rho \leq 1 \quad (47)$$

4 Computational Experiments

4.1 Experimental Data

The instances were based on the daily work of a large transport company in Spain. A random value was assigned to all the parameters considering:

- The fleet is heterogeneous in technical properties. Two types of goods were considered, normal (N) with the 70% and refrigerated (F) the rest. The vehicles have a maximum capacity of 24.000 Kg, 13,5 linear meters, 33 non-stackable pallets and 92,95 m^3.
- There are three types of orders according to the size of goods: 45% small orders (0-10 pallets), 35% medium orders (11-23 pallets) and 20% large orders (24-33 pallets). The time to serve to each customer is the sum of a fixed time (1 hour) and an extra time (0.03 hours * number of pallets).
- Customers are allocated to different zones in Spain. The percentages of pickups and deliveries by zone are shown in Figure 2.
- All pickup operations are made in the morning considering three time ranges: 50% are made from 6:00 to 9:00, 30% from 9:00 to 11:00 and 20% from 11:00 to 13:00.

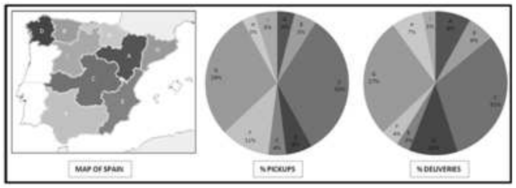

Fig. 2: Percentage of pickups and deliveries by zones

- The distance between the pickup and the delivery point of each order is needed to establish the time range of deliveries. If it is less than 400 kilometers, the range is randomly selected between 17:00 and 21:00 of the same day. If not, it is randomly selected between 6:00 and 13.00 of the next day. The size of each time window is calculated adding a quantity to the lower limit of each time window: one hour is added in 40% of cases, one hour and 30 minutes in 30% of cases and 2 hours in the other 30%.

Euclidean distance from the GPS position of the nodes is used to calculate distances between nodes since the number of available nodes in our database is greater than 400,000. Therefore, it is unfeasible to store a real distance matrix and it is unviable economically to access a geographical information system (GIS). An average speed of 70 km/h is established in accordance with historical data from a large Spanish transport company. In [3], which deal with long distances and routing design for less-than-truckload, the average speed varies between 80 and 75 km/h and with real distances between nodes.

Other restrictions are 9 hours as maximum driving time and a maximum daily waiting time of 3 hours plus 15 minutes per route. The number of roads considered for the Iberian Peninsula is 18; the number of daily orders varies from 100 to 1500. It should be noted that at least double the number of customers will be visited. The influence to allocate the road before or after finding the solution is considered $(0: after, 1: before)$.

4.2 Results and Discussion

The experiments were performed on a PC Intel Core i5 at 4.8GHz with 4GB of RAM under Windows 7. It was necessary to estimate the value of the set of parameters that influences the three ACOs implemented in the algorithm. For this purpose, the literature was reviewed and a set of experiments related to the real problem was carried out. The parameter values for the three ACOs are:$\alpha = 1$, $\beta = 2$ and $\varphi = \rho = 0.1$.

To measure the efficiency of the solution were used measured in terms of the upper and lower value of four performance metrics:

- f_1: Total cost optimized with respect to the cost of the initial solution.
- f_2: Percentage of vehicles involved with respect to total orders.
- f_3: Percentage of vehicles with more than one order with respect to the total vehicles used.
- f_4: Percentage of grouped orders with respect to total orders.

The results of the experiments are listed in Table 1. Regarding the value of the objective function f_1, the results suggest that the allocation of the road a posteriori improves the solution considerably because all orders can be combined mutually, decreasing the number of vehicles required (about 5% in all cases).

The number of vehicles required is not proportional to the number of orders (f_2). The size and the geographical distribution of the orders must be considered.

An important result to note is that solutions with higher percentages of vehicles with two or more orders (f_3) do not imply a lower cost. In the experiment with 500 orders, when the two types of road allocation are compared, the percentage of vehicles with more than one order is greater a priori (11.7 vs. 10.6%). However, the percentage of grouped orders with respect to total orders, f_4, is greater in a posteriori (32.5%) than in a priori allocation (29.6%).Thus, better solutions are obtained when many orders are grouped in fewer vehicles. The last part of the study involved the influence of the time windows strategy because is the most restrictive data for grouping orders. In this case, it is possible to extend the window decreasing the lower limit by two hours. The cases were: $0 = no\ decrease$; $2P = only\ pickups$; $2D = only\ deliveries$; and $2PD = both$. All the experiments were calculated with a posteriori road allocation since the best results are achieved with this option. The results are listed in Table 2.

The TW extension to serve customers has a significant influence on the improvement of solutions. With respect to the total cost, the case $2PD$ (both operations) achieves the best results, decreasing the initial cost up to 41-45%. Note that the total cost is less if the time window is extended in pickups rather than in deliveries because the routes can begin earlier.

5 Conclusions

This paper solves a long-distance routing problem based on an ACO meta-heuristic. Furthermore, several instances have been solved from the historical data of a company with a number of orders between 100 and 1500, in which the influence of road selection has been studied as well as the extending of time windows in terms of the objective function (cost), the vehicles involved with respect to total orders, vehicles with more than one order with respect to the total vehicles used and grouped orders with respect to total orders.

Future research could address the possibility of using the proposal outlined in this paper to solve more general forms of the vehicle routing problem, which would contain more real-world objectives and constraints. The powerful capacity of the algorithm to find excellent solutions to a difficult combinatorial optimization problem should make it a useful model for solving many other problems in transportation and logistics.

Table 1: Results of algorithm experiments with different road allocations

Number of Orders	Roads	% Total cost / Cost initial solution			% Vehicles / Orders			% Vehicle with more than one order			% Grouped orders / Total orders		
		f_1^{max}	f_1^{min}	f_1^{ave}	f_2^{max}	f_2^{min}	f_2^{ave}	f_3^{max}	f_3^{min}	f_3^{ave}	f_4^{max}	f_4^{min}	f_4^{ave}
100	0	68.3	67.4	67.9	68.0	67.0	67.1	23.5	16.4	20.5	49.0	43.0	46.7
	1	76.5	72.6	75.1	74.0	71.0	72.5	24.3	19.7	21.9	46.0	42.0	43.4
200	0	77.8	77.7	77.7	74.5	74.5	74.5	20.1	18.7	19.1	40.5	39.5	39.8
	1	83.0	80.0	81.9	81.5	78.5	79.6	15.9	12.1	15.4	34.5	29.0	32.3
500	0	80.1	79.7	80.0	75.8	75.2	75.5	11.3	10.3	10.6	33.2	32.0	32.5
	1	84.7	84.1	84.5	80.0	79.4	79.6	12.7	10.5	11.7	30.6	28.6	29.6
1000	0	70.1	69.5	69.9	66.6	66.0	66.2	21.0	20.4	20.7	47.7	47.1	47.4
	1	76.9	75.7	76.3	73.1	71.9	72.5	20.3	18.3	19.4	42.9	40.8	41.5
1500	0	66.8	66.3	66.6	62.3	61.7	62.0	29.1	28.9	29.0	56.3	55.5	55.9
	1	72.9	71.8	72.4	68.6	67.2	67.8	27.4	25.4	26.6	51.1	48.5	50.2

Table 2: Results of algorithm experiments with the time windows extension

Number of Orders	ΔTW	% Total cost / Cost initial solution			% Vehicles / Orders			% Vehicle with more than one order			% Grouped orders / Total orders		
		f_1^{max}	f_1^{min}	f_1^{ave}	f_2^{max}	f_2^{min}	f_2^{ave}	f_3^{max}	f_3^{min}	f_3^{ave}	f_4^{max}	f_4^{min}	f_4^{ave}
100	0	68.3	67.4	67.9	68	67	67.1	23.5	16.4	20.5	49.0	43.0	46.7
	2P	65.5	65.4	65.5	64	63	63.3	32.8	26.9	31.2	58.0	53.0	56.5
	2D	67.3	67.3	67.3	67.0	66.0	66.4	25.3	18.2	22.6	51.0	45.0	48.6
	2PD	59.1	58.9	59.0	57.0	57.0	57.0	47.3	45.6	46.8	70.0	69.0	69.7
200	0	77.8	77.7	77.7	74.5	74.5	74.5	20.1	18.7	19.1	40.5	39.5	39.8
	2P	74.8	73.7	74.0	72.0	70.5	71.1	25.0	22.7	23.9	47.5	44.0	45.9
	2D	77.3	76.4	76.6	74.5	73.5	73.9	20.1	19.7	19.9	41.5	40.0	40.8
	2PD	55.1	55.1	55.1	59.5	59.5	59.5	29.3	29.4	29.4	58.0	58.0	58.0
500	0	80.1	79.7	80.0	75.8	75.2	75.5	11.3	10.3	10.6	33.2	32.0	32.5
	2P	76.2	75.7	76.0	70.2	69.6	69.9	19.6	18.6	19.0	44.0	42.8	43.4
	2D	76.5	76.3	76.4	73.4	73.2	73.3	13.3	12.8	13.0	36.6	36.0	36.2
	2PD	58.1	57.9	58.0	55.2	55.0	55.1	38.4	37.4	27.6	66.2	65.4	65.6
1000	0	70.1	69.5	69.9	66.6	66.0	66.2	21.0	20.4	20.7	47.7	47.1	47.4
	2P	66.8	66.3	66.5	62.0	61.5	61.8	28.8	28.1	28.5	56.2	55.5	55.8
	2D	66.0	65.7	65.8	63.8	63.5	63.6	24.1	23.9	24.0	51.9	51.5	51.7
	2PD	56.2	55.9	56.1	53.5	53.1	53.2	43.1	37.8	39.5	68.1	66.6	67.5
1500	0	66.8	66.3	66.6	62.3	61.7	62.0	29.1	28.9	29.0	56.3	55.5	55.9
	2P	64.5	64.1	64.3	59.2	58.9	59.9	35.9	34.0	35.3	62.2	61.0	61.8
	2D	62.5	62.2	62.4	59.0	58.6	58.8	34.4	33.4	33.9	61.6	60.6	61.1
	2PD	55.4	55.1	55.2	52.2	51.6	51.8	46.8	44.5	46.0	72.8	70.8	72.0

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