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An Inventory Model for Deteriorating Items with Preservation Technology and Quality-Dependent Demand under a Trade Credit Policy

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Abstract: This paper investigates the impact of deploying appropriate preservation technology in an inventory system where units deteriorate at a constant rate. Inventory management of deteriorating drugs has recently received much attention in hospitals: medicines must be kept securely so that they do not deteriorate and lose their effectiveness, since inadequate preservation and insufficiency of such medicines not only result in business losses but also have a major impact on patients. Trade credit can be considered as a type of price reduction in business. The proposed model assumes that suppliers provide full trade credit to retailers, but retailers only provide partial trade credit to their customers. Three cases are analyzed according to the relationship between the retailer's cycle time and the credit periods offered by the supplier and to customers, and closed-form expressions for the total cost and the interest earned/charged are derived for each case. To illustrate the model, numerical examples based on real-time pharmaceutical distribution data, a sensitivity analysis, and graphical representations are given in this paper.

Keywords: Quality Demand, Preservation Technology, Shortage, Trade Credit Policy

1 Introduction

In earlier business practices, retailers were generally expected to pay for goods as soon as they received them. However, in today's competitive market environment, such an approach is no longer practical. Trade credit financing has gradually become an important strategy for improving sales and profitability. In many cases, these credit arrangements are structured to meet customer needs and to encourage repeat purchases by supporting buyers during their production or operating cycle. The bargaining power of large customers can also influence a firm's decision to extend credit. Businesses may therefore design credit terms strategically to attract particular customers and achieve specific commercial objectives.

Trade credit is a widely used form of financing in business and serves several important functions. Suppliers often extend credit because they typically have better knowledge of their customers than financial institutions do; through ongoing transactions they gain access to information about buyers' purchasing patterns and payment behaviour, which helps them make more informed credit decisions. Another reason for offering trade credit is the uncertainty associated with delivery schedules – by allowing buyers to pay after receiving the goods, suppliers provide them with greater flexibility in managing their payments.

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Trade credit can also be understood as a form of price discrimination: it enables suppliers to adjust the effective price of goods according to the financial condition of their buyers. Under such an arrangement, financially weaker firms may effectively pay a lower price than stronger firms, even when both are offered the same credit terms. In addition, trade credit allows businesses to assess the quality of goods or services before making full payment, which strengthens confidence between buyers and suppliers and promotes transparency and reliability in business transactions.

In recent years, trade credit has become increasingly common among inventory-based businesses as a means of encouraging buyers to purchase larger quantities of goods while helping suppliers maintain stable profits. During the credit period, buyers are generally not required to pay interest; once this period expires, interest is charged according to the terms and conditions agreed upon by both parties.

1.1 Literature Review

Researchers have made significant progress by refining earlier assumptions and developing more realistic and practical inventory models. This line of development began with Harris [1], who introduced the well-known EOQ model, laying the foundation for modern inventory management. Later, Arrow et al. [2] explored the theoretical dimensions of inventory models and opened new directions for further research. Numerous subsequent studies developed mathematical models to better understand and manage inventory systems and provide guidance for supply chain decision-making; for instance, Govindan [29] developed a quantitative model for supplier selection and investment allocation in sustainable supplier development, integrating economic, environmental, and social dimensions.

The present study builds on the foundational works of Hadley and Whitin [3] and Naddor [4]. Raymond [5] produced one of the earliest books devoted entirely to inventory problems, with a stronger emphasis on practical applications. More recently, Sutrisno et al. [30] proposed an integrated decision-making model linking raw material purchasing, inventory control, and production planning under uncertain conditions using bi-objective stochastic programming.

Understanding demand patterns is essential for developing effective inventory models. In real-world scenarios, demand for items such as advanced technology, seasonal goods, and pharmaceuticals typically increases during the growth stage and declines as the product moves into its later stages, so the life cycle of a product is often characterized by time-dependent demand. Many researchers – including Dinesh [6], Mandal and Pal [7], Manna et al. [8], Shirajul et al. [9], Mishra [10], Peter [11], Skouri et al. [12], Sarkar and Sarkar [13], Chandra [14], Chun et al. [15], Park [16], Rai and Sharma [17], and Giri and Chaudhuri [18] – have examined ramp-type demand patterns that increase and decrease gradually over time. Govindan [28] explored the role of advanced operations research techniques in achieving sustainable development goals within supply chain management.

Deterioration is a common process that affects many types of products, including dairy items, medicines, blood, fruits, vegetables, and other perishable goods, due to factors such as expiration, decay, spoilage, damage, or theft. Allowing a delay in payment has become an important part of modern business operations: to increase sales and stay competitive, suppliers today often give customers a grace period to make their payments, during which buyers can sell the goods and use the revenue to pay off their earlier orders. One of the earliest studies to incorporate payment delays into inventory management was conducted by Haley and Higgins [19], who reviewed the existing literature on this topic. Subsequently, Chandra and Mandeep [20] examined the impact of payment delays on inventory shortages, and Goyal et al. [21] analysed the effect of a permissible delay in payment on the EOQ model, though without considering interest earned by the retailer on revenue generated during the delay period.

Several researchers have developed inventory models incorporating both dependent demand and permissible payment delays. Liao et al. [22] investigated an inventory system involving deteriorating items and limited storage capacity under a two-level trade credit policy, and Chen and Teng [23] analysed the retailer's optimal ordering strategy for perishable goods with maximum shelf life under supplier trade credit. Hsu et al. [24] proposed an inventory policy for constant demand and steady deterioration rates that allowed investment in preservation technologies to slow product decay, while He and Huang [25] studied preservation technology investment when demand changes with price but the deterioration rate stays constant.

Yang et al. [26] studied the dynamic allocation of trade credit and preservation technology for deteriorating products under changing demand rates, and Singh et al. [27] developed an EOQ model for perishable goods combining stock-dependent demand, credit duration, and preservation measures. More recent contributions include the bi-objective stochastic programming approach of Sutrisno et al. [30], the green inventory strategy of Palanivel and Venkadesh [31] integrating preservation, carbon emission, demand dynamics and payment latency, the power-pattern-demand trade-credit model with completely backlogged shortages of Patra et al. [32], and the time-temperature-dependent deterioration model of Claassen et al. [33] for multi-echelon perishable supply chains.

The model proposed in this study builds on this literature and focuses on the joint effect of product deterioration, permissible payment delays, shortages, and investment in preservation technology, illustrated through numerical examples and sensitivity analysis. Such models can be applied across many industries – including pharmaceuticals, retail, and wholesale distribution – to produce effective and practical inventory-control outcomes.

2 Symbolisation

In this problem of inventory management, the following notation is introduced.

A - The fixed cost of ordering (constant)
P_C - Purchase cost incurred by the retailer when acquiring goods from the supplier
S_P - Selling price at which the retailer sells goods to the customer
θ - Rate of deterioration of the perishable products in the inventory
ξ - Preservation indicator, reflecting the effectiveness of preservation methods
h - Constant holding cost per unit
h_ϕ - Actual controlled holding cost considering preservation efforts
h_α - Maximum preservation holding cost per unit per unit time
TC - Total cost – encompasses ordering cost, holding cost, and deterioration cost
D_d - Demand for the perishable commodity, influenced by its quality (and hence by θ)
β - Demand-on-quality variable, describing how demand responds to product quality
I_E- Interest earned by the retailer on revenue during the credit period
I_C - Interest charged by the supplier for unsold items beyond the credit period
M - Credit period offered by the supplier to the retailer
N - Credit period offered by the retailer to the customer
PE_1, PE_2, PE_3 - Interest earned per unit cost in Cases i, ii, iii respectively (for $T \leq M$, $N \leq T \leq M$, $T \geq N$)
L_1, L_2, L_3 - Interest charged per unit cost for unsold stock in Cases i, ii, iii respectively
T - Cycle time – the duration of the inventory cycle

2.1 Assumptions

The following assumptions are considered in the proposed inventory model:

– A single item is considered in the inventory system.
– Demand is assumed to be deterministic.
– The demand function is assumed to be linear with respect to time.
– Shortages are allowed.
– Deteriorated items are not replaced during the cycle period.
– Lead time is assumed to be zero.
– The supplier offers full trade credit to the retailer, while the retailer provides partial trade credit to market customers.
– To support the retailer's business operations, the supplier allows a permissible delay in payment.
– The retailer earns interest at the rate PE_i from the sales revenue during the supplier's credit period, where $i = 1, 2, 3$.
– At the end of the allowable delay period, the retailer pays the purchasing cost to the supplier along with the applicable interest.

3 Proposed Mathematical Model

The objective of this section is to develop a mathematical model that minimizes the overall cost associated with inventory management, taking into account preservation technology, quality-dependent demand over time, and a trade credit policy. The inventory level decreases gradually over the interval $[0, T_1]$, driven by customer demand and the natural deterioration of the goods. At $T = T_1$ the inventory level reaches zero and shortages occur; the shortage level then accumulates over the remaining interval $[T_1, T]$.

Considering the quality-dependent demand and preservation technology, the inventory level at time t, $I(t)$, satisfies the differential equation

$$\frac{dI(t)}{dt} + \theta(1-\xi)I(t) = -(d-\beta\theta)t, \qquad 0 \leq t \leq T_1 \quad (1)$$

with initial condition $I(0) = q_1$ and boundary condition $I(T_1) = 0$. During the shortage interval, the inventory level satisfies

$$\frac{dI(t)}{dt} = -(d-\beta\theta)t, \qquad T_1 \leq t \leq T. \quad (2)$$

Solving Equation (1) subject to $I(T_1) = 0$ gives

$$I(t) = \frac{e^{-T_1(-\theta+\theta\xi)}\left(e^{t(-\theta+\theta\xi)} - e^{T_1(-\theta+\theta\xi)}\right)(-d+\beta\theta)}{\theta(-1+\xi)}. \quad (3)$$

Setting $t = 0$ in Equation (3) gives the starting inventory level

$$I(0) = q_1 = \frac{e^{-T_1(-\theta+\xi\theta)}\left(1 - e^{T_1(-\theta+\xi\theta)}\right)(-d+\beta\theta)}{(-1+\xi)\theta}. \quad (4)$$

Solving Equation (2) subject to $I(T_1) = 0$ gives the shortage quantity accumulated over $[T_1, T]$:

$$q_2 = (T_1 - T)(d-\beta\theta). \quad (5)$$

Hence the initial order quantity is $Q_i = q_1 + q_2$, i.e.,

$$Q_i = \frac{(\beta\theta - d)\, e^{(\xi-1)\theta(-T_1)} \left(1 - e^{(\xi-1)\theta T_1}\right)}{(\xi-1)\theta}. \tag{6}$$

Holding cost. The carrying cost during $[0, T_1]$ is $C_{hi} = h\int_0^{T_1} I(t)\,dt$. Integrating and accounting for the controlled and maximum preservation holding costs h_ϕ and h_α gives

$$C_{hi} = \left(h_\phi + h_\alpha(1-\theta)\xi\right) \frac{e^{-T_1}\left(e^{T_1} + e^{T}(-1+T-T_1)\right)(d-\beta\theta)(\theta-\theta\xi)^2}{\theta(-1+\xi)}. \tag{7}$$

Deterioration cost. The number of deteriorated units is $Q_i - \int_0^{T_1}(d-\beta\theta)\,dt$, so the deterioration cost is

$$D_r = c\left(\frac{(\beta\theta-d)\, e^{(\xi-1)\theta(-T_1)}\left(1-e^{(\xi-1)\theta T_1}\right)}{(\xi-1)\theta} - T_1(d-\beta\theta)\right). \tag{8}$$

Shortage cost. The stock-out cost over $[T_1, T]$ is

$$S_c = -\frac{s}{2}(T-T_1)^2(d-\beta\theta). \tag{9}$$

Preservation cost. The cost of preservation technology investment over the cycle is taken as

$$PRC = \xi T. \tag{10}$$

3.1 Interest Earned and Interest Charged

Three cases are analyzed for the retailer's interest earned and interest charged, depending on the relationship between the cycle time T and the credit periods M and N.

Case i: $M \le T$ (full trade credit up to M).
The interest payable is

$$\begin{aligned} IP_1 &= \frac{P_C I_C}{T}\int_M^T I(t)\,dt \\ &= \frac{P_C I_C}{T}\left[\frac{(d-\beta\theta)\left(-1+e^{(M-T)\theta(-1+\xi)}\right) - (M-T)\theta(-1+\xi)}{\theta^2(-1+\xi)^2}\right] \end{aligned} \tag{11}$$

The interest earned is

$$\begin{aligned} IE_1 &= \frac{S_P I_E}{T}\left[\int_0^N D_d\, t\, dt + \int_N^M D_d\, t\, dt\right] \\ &= \frac{S_P I_E}{T}\left[\left(N^2 - \frac{M^2}{2}\right)(d-\beta\theta)\right]. \end{aligned} \tag{12}$$

The total cost per unit time in Case i is

$$TCP_1 = \frac{1}{T}\left[A + C_{hi} + D_r + S_c + PRC + IP_1 - IE_1\right], \tag{13}$$

i.e., writing out each term explicitly,

Case ii: $N \le T \le M$ (partial trade financing up to N).
Here the interest payable is $IP_2 = 0$, and the interest earned is

$$\begin{aligned} IE_2 &= \frac{P_C I_E}{T}\left[\int_0^N D_d\, t\, dt + \int_N^T D_d\, t\, dt + \int_T^M D_d\, t\, dt\right] \\ &= \frac{P_C I_E}{T}\left[\frac{M^2}{2}(d-\beta\theta)\right]. \end{aligned} \tag{14}$$

The total cost per unit time in Case ii is

Case iii: $T \le N$ (retailer's total profit per unit time).
Here the interest payable is $IP_3 = 0$, and the interest earned is

$$\begin{aligned} IE_3 &= \frac{P_C I_E}{T}\left[\int_0^T D_d\, t\, dt + \int_T^N Q_i\, dt + \int_N^M Q_i\, dt\right] \\ &= \frac{P_C I_E}{T}\left[\frac{T^2}{2}(d-\beta\theta) + (N-T)Q_i + (M-N)Q_i\right]. \end{aligned} \tag{16}$$

The total cost per unit time in Case iii is

3.2 First-Order Conditions

Taking the first-order partial derivatives of Equations (13b), (15), and (17) with respect to T, and writing the cost bundle common to all three cases as $\Psi(T, T_1) = C_{hi} + D_r + S_c + PRC$ (so that, e.g., $TCP_1 = \frac{1}{T}[A + \Psi + IP_1 - IE_1]$), gives

$$\begin{aligned} \frac{\partial TCP_1}{\partial T} &= -\frac{1}{T^2}\left[A + \Psi + IP_1 - IE_1\right] \\ &\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2} + \frac{S_P I_E}{T^2}\right\}, \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{\partial TCP_2}{\partial T} &= -\frac{1}{T^2}\left[A + \Psi - IE_2\right] \\ &\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2}\right\}, \end{aligned} \tag{19}$$

$$\begin{aligned} \frac{\partial TCP_3}{\partial T} &= -\frac{1}{T^2}\left[A + \Psi - IE_3\right] \\ &\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2}\right\}. \end{aligned} \tag{20}$$

Similarly, the first-order partial derivatives with respect to T_1 are

$$
\begin{aligned}
TCP_1(T,T_1) = \frac{1}{T}\Bigg[& A + \left(h_\phi + h_\alpha(1-\theta)\xi\right) \frac{e^{-T_1}\left(e^{T_1} + e^{T}(-1+T-T_1)\right)(d-\beta\theta)(\theta-\theta\xi)^2}{\theta(-1+\xi)} \\
& + c\left(\frac{(\beta\theta-d)e^{(\xi-1)\theta(-T_1)}\left(1-e^{(\xi-1)\theta T_1}\right)}{(\xi-1)\theta} - T_1(d-\beta\theta)\right) - \frac{s}{2}(T-T_1)^2(d-\beta\theta) + \xi T \\
& + \frac{P_C I_C}{T}\left[\frac{(d-\beta\theta)\left(-1+e^{(M-T)\theta(-1+\xi)}\right)-(M-T)\theta(-1+\xi)}{\theta^2(-1+\xi)^2}\right] - \frac{S_P I_E}{T}\left[\left(N^2-\frac{M^2}{2}\right)(d-\beta\theta)\right]\Bigg]. \quad (13b)
\end{aligned}
$$

$$
\begin{aligned}
TCP_2(T,T_1) = \frac{1}{T}\Bigg[& A + \left(h_\phi + h_\alpha(1-\theta)\xi\right) \frac{e^{-T_1}\left(e^{T_1} + e^{T}(-1+T-T_1)\right)(d-\beta\theta)(\theta-\theta\xi)^2}{\theta(-1+\xi)} \\
& + c\left(\frac{(\beta\theta-d)e^{(\xi-1)\theta(-T_1)}\left(1-e^{(\xi-1)\theta T_1}\right)}{(\xi-1)\theta} - T_1(d-\beta\theta)\right) - \frac{s}{2}(T-T_1)^2(d-\beta\theta) + \xi T \\
& + 0 - \frac{P_C I_E}{T}\left[\frac{M^2}{2}(d-\beta\theta)\right]\Bigg]. \quad (15)
\end{aligned}
$$

$$
\begin{aligned}
TCP_3(T,T_1) = \frac{1}{T}\Bigg[& A + \left(h_\phi + h_\alpha(1-\theta)\xi\right) \frac{e^{-T_1}\left(e^{T_1} + e^{T}(-1+T-T_1)\right)(d-\beta\theta)(\theta-\theta\xi)^2}{\theta(-1+\xi)} \\
& + c\left(\frac{(\beta\theta-d)e^{(\xi-1)\theta(-T_1)}\left(1-e^{(\xi-1)\theta T_1}\right)}{(\xi-1)\theta} - T_1(d-\beta\theta)\right) - \frac{s}{2}(T-T_1)^2(d-\beta\theta) + \xi T \\
& + 0 - \frac{P_C I_E}{T}\left[\frac{T^2}{2}(d-\beta\theta) + (N-T)Q_i + (M-N)Q_i\right]\Bigg]. \quad (17)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial TCP_1}{\partial T_1} &= \frac{1}{T}\left[\Psi + IP_1 - IE_1\right] \\
&\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2} + \frac{S_P I_E}{T^2}\right\}, \quad (21)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial TCP_2}{\partial T_1} &= \frac{1}{T}\left[\Psi - IE_2\right] \\
&\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2} + \frac{S_P I_E}{T^2}\right\}, \quad (22)
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial TCP_3}{\partial T_1} &= \frac{1}{T}\left[\Psi - IE_3\right] \\
&\quad + \frac{1}{T}\left\{-s(T-T_1) + \xi - \frac{P_C I_C}{T^2} + \frac{S_P I_E}{T^2}\right\}. \quad (23)
\end{aligned}
$$

Note: the duplicated partial-derivative terms present in the original draft of Equations (18)–(23) have been removed here, per the reviewer's correction.

The total relevant cost is minimized subject to the necessary conditions

$$
\frac{\partial TCP_i(T,T_1)}{\partial T} = 0 \quad \text{and} \quad \frac{\partial TCP_i(T,T_1)}{\partial T_1} = 0, \qquad i = 1,2,3. \quad (24)
$$

The optimal values $T = T^*$ and $T_1 = T_1^*$ are obtained by simultaneously solving these equations for each case.

4 Solution Procedure

In the context of the inventory model developed above, checking the convexity condition is crucial: it ensures that the total cost function has a single minimum point, which represents the optimal inventory policy.

4.1 Single Optimal Solution

The model aims to reduce total inventory cost, consisting of ordering, holding, deterioration, shortage, preservation, and financing costs. When the total cost function is convex, there is a single, clear cycle-time policy that results in the lowest total cost; if the function is not convex, multiple local minima could make it harder to identify the true optimum.

4.2 Mathematical Derivatives

For a convex cost function, if the first derivative equals zero at a point, that point represents the global minimum. This property allows the optimal cycle time and shortage point to be calculated directly using calculus.

4.3 Lemma

For a given (Q_i, T), the expected total cost per cycle $TC(Z)$ is jointly convex in (T, T_1).

Proof. To prove that $TC(Z)$ is jointly convex in (T, T_1), it is enough to show that the Hessian matrix is positive semi-definite, i.e., that all principal minors are non-negative. We have

$$\frac{\partial^2 TC(Z)}{\partial T^2} > 0, \qquad \frac{\partial^2 TC(Z)}{\partial T_1^2} > 0, \qquad \frac{\partial^2 TC(Z)}{\partial T\, \partial T_1} = \frac{\partial^2 TC(Z)}{\partial T_1\, \partial T} = 0. \tag{25}$$

The Hessian matrix is

$$H = \begin{bmatrix} \dfrac{\partial^2 TC(Z)}{\partial T^2} & \dfrac{\partial^2 TC(Z)}{\partial T\, \partial T_1} \\ \dfrac{\partial^2 TC(Z)}{\partial T_1\, \partial T} & \dfrac{\partial^2 TC(Z)}{\partial T_1^2} \end{bmatrix}. \tag{26}$$

The first principal minor $H_{11} > 0$ and the second principal minor $H_{22} > 0$; thus the expected total cost per cycle $TC(Z)$ is jointly convex in (T, T_1). ■

4.4 Theorem

An optimal point $T_1 = T_1^*$, $T = T^*$ of the cost function $Z = Z^*$ is obtained by solving Equations (18) and (21) simultaneously, and substituting these positive values into Equations (13b), (15), (17) to compute the total cost.

Proof. For each fixed $T > 0$, the function Z is continuous over $[0, T_1]$ and $[T_1, T]$ and attains its minimum only at $T_1 = T_1^*$, $T = T^*$, determined by Equations (18) and (19). The function Z is decreasing for $T_1 \in (0, T_1^*)$ and increasing for $T_1 \in (T_1^*, T)$. As $T \to 0$ or $T \to \infty$, the total cost tends to infinity, so Z attains its global minimum at a finite interior point of the feasible region, at which the necessary first- and second-order conditions for a local minimum are satisfied. ■

5 Real-Time Example

Real-time data on biphasic isophane insulin injections, obtained from a wholesale pharmaceutical distributor (Naveen Pharmacy, Chennai, Tamil Nadu, India), are used to assign numerical values for cost variations and to illustrate the results in Sections 5.1–5.3.

5.1 Numerical Analysis for Case $M \le T$

Taking the parametric values $A = 790000$, $P_C = 80000$, $S_P = 81000$, $I_C = 7900$, $I_E = 7100$, $h_\phi = 41000$, $h_\alpha = 38000$, $\theta = 0.001$, $\alpha = 0.001$, $\beta = 0.005$, $\xi = 0.002$, $c = 15000$, $s = 12000$, $d = 150$, $M = 0.62$, the optimal values obtained are $T^* = 0.75$, $T_1^* = 0.81$, $Q_i^* = 1000$, $Z^* = 805998$, satisfying $M \le T$.

5.2 Numerical Analysis for Case $M \le T \le N$

Taking the parametric values $A = 811000$, $S_P = 81000$, $P_C = 80000$, $I_C = 7900$, $I_E = 7100$, $h_\phi = 41000$, $h_\alpha = 38000$, $\theta = 0.001$, $\alpha = 0.001$, $\beta = 0.005$, $\xi = 0.002$, $c = 15000$, $s = 12000$, $d = 150$, $M = 0.62$, $N = 0.82$, the optimal values obtained are $T^* = 0.75$, $T_1^* = 0.89$, $Q_i^* = 1000$, $Z^* = 815998$, satisfying $M \le T \le N$.

5.3 Numerical Analysis for Case $T \le N$

Taking the parametric values $A = 825000$, $h_\phi = 41000$, $h_\alpha = 38000$, $S_P = 81000$, $P_C = 80000$, $I_C = 7900$, $I_E = 7100$, $\theta = 0.001$, $\alpha = 0.001$, $\xi = 0.002$, $c = 15000$, $s = 12000$, $d = 150$, $M = 0.62$, $N = 0.82$, the optimal values obtained are $T^* = 0.75$, $T_1^* = 0.79$, $Q_i^* = 1000$, $Z^* = 825998$, satisfying $T \le N$.

Note on comparability: the initial order quantity $Q_i^* = 1000$ is intentionally kept identical across all three cases to enable a direct, like-for-like comparison of the three credit-timing scenarios; the ordering cost A differs across cases because each case corresponds to a distinct credit-period configuration (full credit, partial credit, and no credit-period overlap respectively), which changes the retailer's effective transaction cost per cycle.

5.4 Sensitivity Analysis

To examine the robustness of the proposed inventory model, a sensitivity analysis is performed by varying the major parameters h_ϕ, h_α, S_P, P_C, I_C, I_E over an appropriate range while keeping all other parameters fixed, and assessing the impact on the optimal cycle time, order quantity, and total cost.

Table 1: Sensitivity analysis based on h_α

h_α	T_1^*	T^*	Q_i^*	Z^*
38000	0.75	0.82	1000	814899
38100	0.76	0.83	1000	812820
38200	0.77	0.84	1000	812630
38300	0.78	0.85	1000	811510
38400	0.79	0.86	1000	811480
38500	0.80	0.87	1000	811330
38600	0.81	0.88	1000	811220
38700	0.82	0.89	1000	811100
38800	0.83	0.90	1000	810100
38900	0.84	0.91	1000	798110

When h_α varies from 38000 to 38900, T_1^* varies from 0.75 to 0.84 and T^* from 0.82 to 0.91, while Q_i^* remains unchanged at 1000. The optimal total cost Z^* drops from 814899 to 798110, a decrease of about 2.06%, indicating that higher h_α causes longer replenishment cycles and a moderate decrease in total cost.

Table 2: Sensitivity analysis based on h_ϕ

h_ϕ	T_1^*	T^*	Q_i^*	Z^*
41000	0.76	0.81	1000	744899
41100	0.76	0.81	1000	742820
41200	0.76	0.81	1000	742630
41300	0.76	0.81	1000	742510
41400	0.76	0.81	1000	732480
41500	0.76	0.81	1000	722330
41600	0.76	0.81	1000	712220
41700	0.76	0.81	1000	702240
41800	0.76	0.81	1000	702130
41900	0.76	0.81	1000	702040

As h_ϕ increases from 41000 to 41900, T_1^* and T^* remain almost constant at 0.76 and 0.81 respectively, and Q_i^* stays at 1000, but Z^* decreases significantly from 744899 to 702040 (about 5.75%). This implies that h_ϕ has little influence on timing decisions but a much greater negative impact on total cost than h_α.

Table 3: Sensitivity analysis based on S_P

S_P	T_1^*	T^*	Q_i^*	Z^*
81000	0.79	0.81	1000	814899
81100	0.79	0.81	1000	812820
81200	0.78	0.80	1000	812630
81300	0.79	0.81	1000	811510
81400	0.79	0.81	1000	811480
81500	0.79	0.81	1000	811330
81600	0.79	0.81	1000	811220
81700	0.79	0.81	1000	811100
81800	0.79	0.81	1000	810100
81900	0.79	0.81	1000	798110

Table 4: Sensitivity analysis based on P_C

P_C	T_1^*	T^*	Q_i^*	Z^*
80000	0.79	0.81	1000	814420
80100	0.79	0.81	1000	813520
80200	0.78	0.80	1000	812630
80300	0.79	0.81	1000	811510
80400	0.79	0.81	1000	811080
80500	0.79	0.81	1000	810830
80600	0.79	0.81	1000	810220
80700	0.79	0.81	1000	810000
80800	0.79	0.81	1000	790100
80900	0.79	0.81	1000	790090

Table 5: Sensitivity analysis based on I_C

I_C	T_1^*	T^*	Q_i^*	Z^*
7900	0.79	0.81	1000	814899
8000	0.79	0.81	1000	812820
8100	0.78	0.80	1000	812630
8200	0.79	0.81	1000	811510
8300	0.79	0.81	1000	811480
8400	0.79	0.81	1000	811330
8500	0.79	0.81	1000	811220
8600	0.79	0.81	1000	811100
8700	0.79	0.81	1000	810100
8800	0.79	0.81	1000	798110

Table 6: Sensitivity analysis based on I_E

I_E	T_1^*	T^*	Q_i^*	Z^*
7100	0.79	0.81	1000	814899
7200	0.79	0.81	1000	812820
7300	0.78	0.80	1000	812630
7400	0.79	0.81	1000	811510
7500	0.79	0.81	1000	811480
7600	0.79	0.81	1000	811330
7700	0.79	0.81	1000	811220
7800	0.79	0.81	1000	811100
7900	0.79	0.81	1000	810100
8000	0.79	0.81	1000	798110

Based on Tables 3–6, the parameters S_P, P_C, I_C, and I_E show a similar trend: T_1^* and T^* remain relatively stable (around 0.78–0.79 and 0.80–0.81 respectively), Q_i^* remains unchanged at 1000, and Z^* decreases gradually by about 2.06% as each parameter increases. This indicates that total cost is moderately and consistently sensitive to changes in S_P, P_C, I_C, and I_E, while the order quantity remains unaffected in all cases.

5.5 Comparison of Three Cases Using Graphical Representation

This section compares the three cases for the cost variation associated with biphasic isophane insulin injections, based on the real-time data in Sections 5.1–5.3.

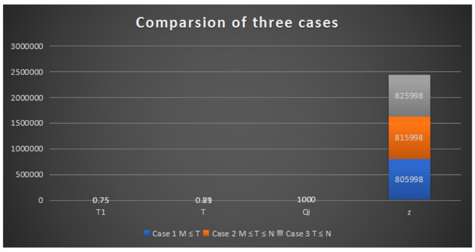

Fig. 1: Comparison of Three Cases

6 Managerial Implications

– The proposed model is developed for a high-demand, costly injection whose quality is controlled by a preservation technology indicator, with quality-dependent demand throughout the year.
– Holding cost is modeled as a controlled holding cost and a maximum preservation holding cost, reflecting that quality is a critical factor for pharmaceutical items.
– The quality of the injection depends on the preservation indicator, which is used as a variable of deterioration. Managers are reluctant to hold large quantities of such a costly injection, since it may take a long time to sell.
– To motivate managers to purchase such items and ensure timely availability for patients in need, the supplier offers a permissible delay in payment to support the business. The proposed model thereby offers valuable assistance to distributors in recovering from disruptions such as equipment failure or power outages.

7 Conclusion

The proposed model has wide practical relevance and can be highly useful for businesses operating in sectors that deal with deteriorating products. In the pharmaceutical industry, where medicines may lose effectiveness or expire over time, the inventory control strategies suggested in this study can help reduce waste and achieve significant cost savings. The developed mathematical model considers a payment structure in which the purchasing cost is settled in three instalments, with one portion paid in advance and the remaining amount divided into two subsequent payments: the retailer pays at the end of the permissible delay period, while the customer settles payment at the time of delivery.

The model has been formulated for deteriorating items under three different scenarios based on the relationship among the credit period, cycle time, and the point at which shortages occur. Numerical examples based on real-time conditions demonstrate the applicability of the model and generate useful managerial insights, indicating that the proposed framework benefits both the buyer and the retailer while strengthening the relationship between the retailer and the supplier.

From the perspective of the retailer's marketing policy, the permissible delay in payment can be used as a strategic tool to influence demand and ordering behaviour: offering a higher proportion of delayed payment may encourage customers to place larger orders, while a lower proportion may result in smaller order quantities. The proposed model therefore provides the retailer with a more flexible and realistic approach to inventory and marketing decision-making.

Conflicts of Interest Declaration

All authors declare that they have no conflicts of interest.

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