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Mathematical analyses of 2010 FIFA world cup

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Ranking sports teams based on their scorelines is the focus of interest for many sports fan. In this paper, we give a total ranking for 32 teams from the data (only 63 matches) in 2010 FIFA world cup based on the method introduced by Keener. We also show that we can decide whether a given ranking is based on a criterion which puts importance on *win-lose*, or on *scores* by use of a continuous weighted function. Furthermore, we give a subjective measure of *closeness* of matches in each group stage by using the notion of Shannon entropy. We also give some mention on the total ranking based on the Bradley-Terry model.

Keywords: Perron-Frobenius theorem, matrix analysis, estimated ranking, Bradley-Terry model and Shannon entropy

1 Introduction

2010 FIFA World Cup in South Africa came to an end with the first champion, Spain. Due to the tournament system, we exactly find the ranking from the first rank to fourth one. However, we can not find the fifth ranking and under. Motivated by overcoming this situation, we shall give the total ranking for 32 teams using the ranking method introduced by Keener [1]. Although we have data (victory or defeat, and scores for each team) from only 63 matches in 8 Group stages, Round of 16, Quarter-finals, Semi-finals and Final, our trial gives the reasonable estimated ranking (performance) for 32 teams, using the ranking method based on the Perron-Frobenius theorem for an irreducible matrix.

The obtained data from the results in 2010 FIFA World Cup is incomplete, because each team has only 3 matches if they leave at group stage, (otherwise, at least 4 matches) and the matches for teams increase until they lose a match. As for the ranking problem of incomplete tournaments have been studied [1,2] and it has been applied to various problem [3–6]. Ranking problem is also studied in the literature of statistics [7], and the Bradley-Terry model [8] is one of the well-known model for paired comparison data such as sport team matches. The Bradley-Terry model parametrizes the strength of each team and estimate them using the observed match results. Understandable from its nature, it does not work very well for the limited data. Therefore in Section 2 of the present paper, we use the ranking method introduced by Keener [1]. We also apply the continuous weighted functions [3] for the detailed discussion for the rankings in Section 3. Moreover, we give some aspects how close matches were done in group stage in Section 4 by means of Shannon entropy. As concluding remarks, we give some mentions on the results by use of the Bradley-Terry model.

2 The Keener's ranking method

We start from the Perron-Frobenius theorem which plays a fundamental role to obtain our estimated ranking for 32 teams in 2010 FIFA World Cup.

Perron-Frobenius theorem ([1, 9, 10]): *If all elements of the matrix A are nonnegative, then there exists an eigenvector whose elements are nonnegative, associated with a positive eigenvalue. Moreover, if the matrix A is irreducible, then the eigenvector is a unique eigenvector whose elements are positive, and the positive simple eigenvalue associated with its eigenvector is equal to a spectral radius of A.*

This theorem assures that if all elements of a given matrix is nonnegative and it is irreducible (we call it a ranking matrix), then there exists a *unique* eigenvector whose elements are positive. We call it a ranking vector. See the original work by Keener in [1] which studies the ranking problem based on this fact. See also [11] on the definition of the irreducible matrix, for example.

It is suitable and simple to use a power method [10] in order to obtain the ranking vector. Firstly, we must construct a ranking matrix. To do so, we give a setting of the elements of the ranking matrix $R = (r_{ij})$ in 2010 FIFA World Cup. Let S_{ij} be a score that team i obtained from team j. Then we define r_{ij} as

$$r_{ij} = \frac{S_{ij} + 1}{S_{ij} + S_{ji} + 2} \tag{2.1}$$

and we put $r_{ii} = 0$ for $i = 1, \cdots, 32$ since the same team never play a match. We note that if we chose the simple setting $r_{ij} = \frac{S_{ij}}{S_{ij}+S_{ji}}$, it fails in the cases of scoreless draw because we have $S_{ij} = S_{ji} = 0$ in such cases. To take into account of the score at penalty kick shootout (PK), we modify the above r_{ij} as

$$r_{ij} = \frac{S_{ij} + 1 + pP_{ij}}{S_{ij} + S_{ji} + 2 + p(P_{ij} + P_{ji})}, \tag{2.2}$$

where p is a constant less than 1 and P_{ij} represents the scores that team i obtained from team j at PK in the tournaments. In the present paper, we set $p = 0.1$ to take the PK score into account moderately. Base on all results shown in Appendix B, we obtain the ranking matrix R constructed following the above method as

$$R = \begin{pmatrix} A & AB & AC & AD & AE & AF & AG & AH \\ BA & B & BC & BD & BE & BF & BG & BH \\ CA & CB & C & CD & CE & CF & CG & CH \\ DA & DB & DC & D & DE & DF & DG & DH \\ EA & EB & EC & ED & E & EF & EG & EH \\ FA & FB & FC & FD & FE & F & FG & FH \\ GA & GB & GC & GD & GE & GF & G & GH \\ HA & HB & HC & HD & HE & HF & HG & H \end{pmatrix},$$

where the blocks $A, AB, \cdots, H$ of the partitioned matrix R is given in Appendix A. We do not use the actual result of the match for the third place, because it may exchange the 2nd place and the 3rd place in general. We note that in off block-diagonal elements of R such as AB shown in Appendix A, we put a nonnegative constant $h > 0$ into the elements correspond to not played teams. From the following proposition, we see that $h > 0$ is a sufficient condition that the ranking matrix R is irreducible.

Proposition (page 361 in [11]) *Let A be an $n \times n$ matrix whose elements are non-negative. Then A is irreducible if and only if all elements of the matrix $(I + A)^{n-1}$ are positive.*

In this paper, we set $h = 0.1$ which gives a negative effect for the teams that could not launch at Round of 16.

From the ranking matrix R, we calculate the ranking vector in the following way. We assume that $\lambda_1, \cdots, \lambda_{32}$ are the eigenvalues of the ranking matrix R where $|\lambda_1| \geq \cdots \geq |\lambda_{32}|$. Then the power method [10] tells us that for any initial vector $x_0 \neq 0$, by the iteration $x_k = Rx_{k-1}$ for $k = 1, 2, \cdots$, x_k converges to an eigenvector x corresponding to the maximum eigenvalue λ_1. The Perron-Frobenius theorem assures the positivity of λ_1. To avoid the overflow in the calculation on vectors, we often set $y_k = Rx_{k-1}$ and $x_k = \frac{y_k}{||y_k||}$. Thus during the iteration on k, we have only to find the vector x_k such that $||x_k - x_{k-1}|| < \varepsilon$. (It is sufficient to take $\varepsilon = 10^{-8}$ generally.) The obtained vector x is the ranking vector itself. The Perron-Frobenius theorem assures that the ranking vector x is uniquely given and its elements are all positive. For this case, the ranking vector x has 32 elements and the value of the ith element of the ranking vector x corresponds to the point (strength) of the ith team set in the ith row of the ranking matrix R. We show the results of

	Standings by BBC	Ranking vector	$(p = h = 0.1)$			Separated groups	
1	Spain	1.000000	Spain	H	0.84565	Uruguay	A
2	Netherlands	0.934204	Netherlands	E	0.642791	Mexico	A
3	Germany	0.892656	Germany	D	0.571103	South Africa	A
4	Uruguay	0.84565	Uruguay	A	0.566844	France	A
5	Argentina	0.777298	Brazil	G	0.751968	Argentina	B
6	Brazil	0.751733	Argentina	B	0.643342	South Korea	B
7	Ghana	0.737478	Ghana	D	0.543423	Greece	B
8	Paraguay	0.736237	Paraguay	F	0.565094	Nigeria	B
9	Japan	0.687767	Portugal	G	0.656321	United States	C
10	Chile	0.675398	Japan	E	0.646535	England	C
11	Portugal	0.660551	Chile	H	0.592683	Slovenia	C
12	United States	0.655683	United States	C	0.555554	Algeria	C
13	England	0.646202	England	C	0.893764	Germany	D
14	Mexico	0.643384	South Korea	B	0.737478	Ghana	D
15	South Korea	0.642834	Mexico	A	0.579806	Australia	D
16	Slovakia	0.637414	Switzerland	H	0.618878	Serbia	D
17	Ivory Coast	0.631075	Slovakia	F	0.93465	Netherlands	E
18	Slovenia	0.618225	Serbia	D	0.675398	Japan	E
19	Switzerland	0.609342	Ivory Coast	G	0.562358	Denmark	E
20	South Africa	0.604381	New Zealand	F	0.567591	Cameroon	E
21	Australia	0.595709	Italy	F	0.736237	Paraguay	F
22	New Zealand	0.5924	Slovenia	C	0.631142	Slovakia	F
23	Serbia	0.579039	Australia	D	0.604243	New Zealand	F
24	Denmark	0.571035	South Africa	A	0.59557	Italy	F
25	Greece	0.567118	Cameroon	E	1.000000	Spain	H
26	Italy	0.56703	France	A	0.660789	Chile	H
27	Nigeria	0.564888	Nigeria	B	0.637625	Switzerland	H
28	Algeria	0.561879	Denmark	E	0.557468	Honduras	H
29	France	0.557254	Honduras	H	0.777611	Brazil	G
30	Honduras	0.555296	Algeria	C	0.688029	Portugal	G
31	Cameroon	0.54321	Greece	B	0.609593	Ivory Coast	G
32	North Korea	0.503622	North Korea	G	0.503834	North Korea	G

Table 2.1: FIFA 2010 World Cup standings by BBC and our estimated rankings

ranking analysis in Table 2.1. The left side column in Table 2.1 presents 2010 FIFA World Cup standings by BBC [12], the center column in that does our estimated rankings with the values (which is elements of eigenvector corresponding to the maximum eigenvalue) and the right column in that does our estimated rankings for each group. The BBC standings seems to be decided in the following manner. The top four rankings mirrored the final positions in the tournament. Teams under the 5th rank is set according to FIFA's rankings which takes into account group stage results, progress in the competition and the quality of opposition. We may regard the values in center column as the performance for each team in 2010 FIFA World Cup. For example, we may judge that the performance of Netherlands was approximately 93% for a champion Spain in 2010 FIFA World Cup.

Firstly we observe on our estimated rankings with comparison of FIFA standings by BBC.

(i) Both rankings from 1st place to 4th place are same. It is quite reasonable because the actual rankings from 1st place to 4th place were determined in matches.

(ii) The 5th place and 6th place are exchanged. We consider that Argentina lost a game to Germany by 0-4.

(iii) We pay attentions the 9th place and 10th place. Our estimated rankings tell us that Portugal is higher rank than Japan. It also can be understood that Portugal lost a game to Spain (1st place) by 0-1, while Japan lost a game to Netherlands (2nd place) by 0-1. (In addition, Portugal beat North Korea 7-0, in the group stage. Also Japan lost to Paraguay by 0-0 (PK 3-5) in round of 16. This means that the performance of Japan was pretty close to Paraguay which is one of the best 8 teams.) Our ranking method evaluates the team which lost a close game to the higher ranked team.

(iv) Although Switzerland left in the group stage, it ranked at the 16th place in our our estimated rankings. This is due to the reason why Switzerland beat Spain (1st place) 1-0, in the group stage.

(v) We can estimate how our ranking is consistent to that of given by BBC using rank correlation. The Kendall's τ coefficient is a statistics used to measure the association between two measured quantities [13], and it is in the range $-1 \leq \tau \leq 1$ where $\tau = 1$ means complete agreement of order of the two quantities. The Kendall's τ between our ranking and that of BBC's is 0.8548387, and it can be seen as an collateral evidence of the appropriateness of our ranking method. We shall mention the result of ranking by the Bradley-Terry model based on the Kendall's τ in the concluding remarks.

Secondly, we observe the results from the right column in Table 2.1. We can find that the 3rd place and 4th place were exchanged in three groups, Group B, D and E. Our ranking method tends to give high rank for the team that the total sum of row in the ranking matrix is high. By this tendency, the reversal in Group B and D can be explained. However, as for Group E, there is not such a tendency. The actual rank in this group E is clear. See Appendix B. Why did this reversal in Group E happen? We may answer in the following way. Although Denmark (3rd place in Group E) obtained $1/4$ point from Netherlands (1st place in Group E), Cameroon obtained $2/5$ point from Netherlands (1st place in Group E

). We may claim that our ranking method construct a system that a high evaluation is given for the team which obtained a point from high ranked team. In the next section, for Group E, we show the variational ranking using a continuous weighted function.

3 The analysis of ranking method by a continuous weighted function

The result obtained in the previous section depends on both *win-lose* and *scores*. In this section, using a parameter as an weight, we show that we can chose the ranking whether we regard *win-lose* as important or we do *scores* as important. We consider Group E as an example. We have the ranking matrix $E = (e_{ij})$ in Appendix A. We define the weighted ranking matrix W by

$$W = (w_{ij}), w_{ij} = \frac{1}{2} + \frac{1}{2}\mathrm{sgn}\left(e_{ij} - \frac{1}{2}\right)|2e_{ij} - 1|^{\frac{1}{w}} \tag{3.1}$$

for a parameter $w > 0$. The parameter $w > 0$ has a definite meaning as follows. We call the weighted function for $0 < w \leq 1$ defined in Eq.(3.1) a reducing difference function and then we attach importance to the *score* for $0 < w \leq 1$ in the group stage of 2010 FIFA World Cup. We also call the weighted function for $w \geq 1$ defined in Eq.(3.1) a expanding difference function and then we attach importance to the *win-lose* for $w \geq 1$ in the group stage of 2010 FIFA World Cup. Here we give some interpretation of the effects of the

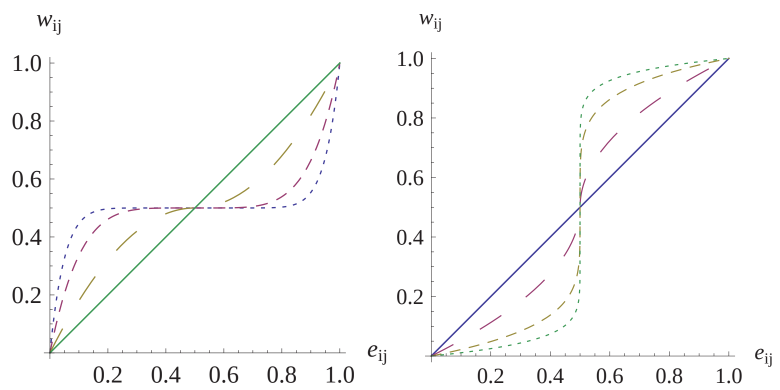

Figure 3.1: Left:The weighted function for $w = 1$ (solid curve), $w = 0.5$ (long dashed curve), $w = 0.2$ (dashed curve) and $w = 0.1$ (dotted curve). Right:The weighted function for $w = 1$ (solid curve), $w = 2$ (long dashed curve), $w = 5$ (dashed curve) and $w = 10$ (dotted curve).

parameter w on the ranking vector of the weighted ranking matrix W. Figure 3.1 (Left) is the function of the weighted elements w_{ij} for the original elements e_{ij} with respect to $w = 1, 0.5, 0.2$ and $w = 0.1$. First of all, a winner i for the team j is given a point such as $e_{ij} > 0.5$ and $e_{ji} = 1 - e_{ij} < 0.5$ from the definition Eq.(2.1) or Eq.(2.2). Of cource, for a drawn match, $e_{ij} = e_{ji} = 0.5$. From this figure, we easily find that the difference between w_{ij} and w_{ji} is getting smaller as the parameter w goes to small. In a similar way, from Figure 3.1, we also find that difference between w_{ij} and w_{ji} is getting larger as the parameter w goes to large.

The following Table 3.2 shows an example of the elements of the weighted ranking matrix W. The values in Table 3.2 represents the elements of the weighted ranking matrix W. From Table 3.2, we find that the difference of the points of two teams are monotone increasing with respect to the parameter w. That is, when $w = 0.1$, the difference is quite small, but when $w = 10$, it is rather large. (When $w = 1$, the points for two teams are original points calculated by the definition Eq.(2.1). That is, 0.75 and 0.25 are the actual elements of the ranking matrix E.) We consider the following two cases for futher

	$w = 0.1$	$w = 0.2$	$w = 0.5$	$w = 1$	$w = 2$	$w = 5$	$w = 10$
Netherlands	0.500488	0.515625	0.625	0.75	0.853553	0.935275	0.966516
Denmark	0.499512	0.484375	0.375	0.25	0.146447	0.064725	0.033484

Table 3.2: An example of our weighted function (1)

understanding our weighted function. For this purpose, we prepare extreme examples such as:

(i) The case of the match (team i vs team j) with scores, 10 vs 9.

(ii) The case of the match (team i vs team j) with scores, 10 vs 1.

In both cases, the team i beat the team j. But if we pay attention to the scores, the team i beat the team j by a narrow margin in case (i) and the team i beat the team j by a large margin in case (ii). The Table 3.3 shows a pair of elements (w_{ij}, w_{ji}) calculated following

	$w = 0.2$	$w = 0.5$	$w = 1$	$w = 2$	$w = 5$	$w = 10$
(w_{ij}, w_{ji}) in (i)	(0.5,0.5)	(0.501,0.499)	(0.52,0.48)	(0.61,0.39)	(0.77,0.23)	(0.87,0.13)
(w_{ij}, w_{ji}) in (ii)	(0.58,0.42)	(0.74,0.26)	(0.85,0.15)	(0.92,0.08)	(0.96,0.04)	(0.98,0.02)

Table 3.3: An example for our weighted function (2)

the definition Eq.(2.1) and Eq.(3.1). ($w_{ij} = e_{ij}$ and $w_{ji} = e_{ji}$ when $w = 1$.) In both cases (i) and (ii), the difference w_{ij} and w_{ji} is getting larger as the parameter w goes to large. This means that the defeated team is assigned a low point. Although the actual difference of the scores is $10-9 = 1$ in the case (i), (then the difference between $w_{ij} = 0.52$ and $w_{ji} = 0.48$ is quite small) the difference between w_{ij} and w_{ji} is rather large for $w = 10$. This shows that the defeated team is given a low evaluation for large parameter w, even if a close match is perfomed. Moreover we compare the case (i) with the case (ii). From Table 3.3, we find that the difference $\Delta \equiv w_{ij} - w_{ji} = 0.04$ for the case (i) and that $\Delta = 0.85 - 0.15 = 0.7$

for the case (ii) in $w = 1$. However, we also find that $\Delta = 0.87 - 0.13 = 0.74$ for the case (i) and $\Delta = 0.98 - 0.02 = 0.96$ for the case (ii) in $w = 10$. While we have a definitive difference between 0.04 and 0.7 (more than 10 times) in $w = 1$, we do not have such a definitive difference between 0.74 and 0.96 in $w = 10$. Therefore if we would like to attach importance to the result of *win-lose*, we take the parameter as $w \geq 1$. The greater w makes the difference for two teams clearer. On the other hand, for the case (i) we have the difference $\Delta \simeq 0$ in $w = 0.2$, while for the case (ii) we have the difference $\Delta = 0.16$ in $w = 0.2$. This means that for the small parameter w, the difference of two teams perfomed a close match vanishes. However we have the difference of two teams having a big lead such as the case (ii), even for a small parameter w. Therefore, if we would like to attach importance to the *score*, we take the parameter as $0 < w \leq 1$. The smaller w makes the difference for two teams clearer. Figure 3.2 shows that the ranking vectors of four teams

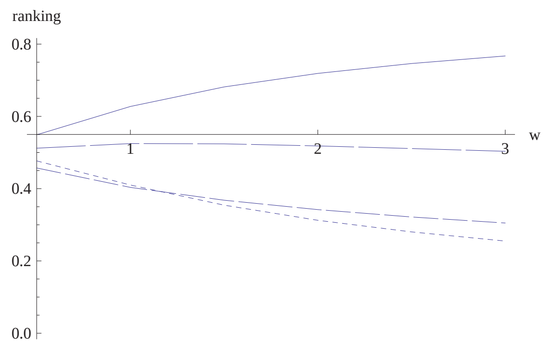

Figure 3.2: Ranking by the proposed weighted function for Netherlands (solid curve), Japan (long dashed curve), Denmark (dashed curve) and Cameroon (dotted curve).

of group E with respect to the parameter $0 < w \leq 3$. The ranking vectors in the parameter $0 < w \leq 1$ of Figure 3.2 represent the ranking which takes into account much more the scores of the matches than the win-loss result. On the other hand, the ranking vectors in $w \geq 1$ represent the ranking which is considered of importance to the win-loss result more than the score in the matches. From Figure 3.2, we find that our ranking fits the actual rank when the parameter w is greater than around 1.2. In other words, we may state that FIFA's pointing method in Group E corresponds to the parameter w is greater than around 1.2 in our ranking method with a weighted function. This continuous weight parametrization scheme enable us to find appropriate ranking matrices which explain rankings made under unknown standards.

4 Analysis of *closeness* of matches by means of Shannon entropy

In this section, we find the group which was in keen competition in the group stage. For this purpose, we calculate the ranking vectors for matrices $A, B, \cdots, H$ given in Appendix A. And then we obtain the probability vectors by normalizing the ranking vectors by l_1 norm. We next compute the Shannon entropy [14] for the probability vectors. The Shannon entropy is defined for a probability distribution (vector) $\{p_1, \cdots, p_n\}$ by

$$S(p_1, \cdots, p_n) \equiv -\sum_{j=1}^{n} p_j \log_2 p_j,$$

which is always nonnegative and has an inequality $S(p_1, \cdots, p_n) \leq \log_2 n$ with equality if and only if $p_1 = \cdots = p_n = \frac{1}{n}$. Since we have four teams in the group stage, we have $S(p_1, \cdots, p_4) \leq 2.0$. The greater is the Shannon entropy, the closer has a race in group stage. In Table 4.4, we also calculated the distance between minimum element and maximum one in all ranking vectors. From these results, we easily find that Group F ran the closest race. After that, Group D and C were in keen competition. Conversely, Group G was not so in keen competition, since North Korea lost all matches and the maximum difference of the scores was 11.

	Shannon entropy	Distance Min-Max
Group A	1.982399582	0.319891
Group B	1.970466490	0.410982
Group C	1.992161311	0.235491
Group D	1.992224302	0.252678
Group E	1.975109755	0.356906
Group F	1.995540363	0.195682
Group G	1.952120346	0.503972
Group H	1.987297868	0.295993

Table 4.4: Shannon entropy and the distance between minimum element and maximum one in all ranking vectors

We can use this Shannon entropy based analysis on tightness of matches in each group stage to correct the effect of results of group stages to the entire ranking. For example, it might be reasonable to attach some additional weight to the result of high Shannon entropy group stage matches compared to that of low Shannon entropy group stage matches.

5 Concluding remarks

As we have seen, we gave a reasonable ranking for incomplete matches by a simple linear algebraic operation. In particular, we gave the ordering for the teams which have no matches such as Portugal-Japan. Portugal played a match against Spain, Japan did a match against Netherlands and Spain did a match against Netherlands. From these results, our ranking vector gave us the ordering for Portugal and Japan. In addition, applying a continuous weighted function, we showed that we could give a kind of subjective ranking such that we pay much attention on *scores* or *win-lose*. Our weighted function may be applied to the other fields in physics and economy and so on. Furthermore, we proposed an subjective estimation method of closeness of matches in each stage using the notion of Shannon entropy.

As stated in Introduction, Bradley-Terry model is not suitable for this case because there is not enough samples. For the sake of completeness, we calculated the estimated rankings by use of Bradley-Terry model, which is a standard model for such ranking problems in the literature of statistics. We considered two cases in the following way.

(i) We give 3 points for the teams which won the matches, 0 point for teams which lost the matches and 1 point to both teams for draw matches, throughout all matches. This pointing system is the same to that of the group stage in 2010 FIFA World Cup.

(ii) As for the group stage, we use the pointing system above. But we give weighted points for the teams which won in the tournaments. The winners of Round of 16 obtain $3 \times 2!$ points. The winners of Quarter-finals obtain $3 \times 3!$ points. The winners of Semi-finals obtain $3 \times 4!$ points. The winner of final obtains $3 \times 5!$ points. Where $n! = n \cdot (n-1) \cdot \cdots \cdot 2 \cdot 1$.

The results for these cases are given in Appendix C. The ranking obtained using unweighted points shows relatively high consistency to the standings of BBC in the sense of Kendall's rank correlation ($\tau = 0.8064516$). However, the order of top 4 teams is different from the real World Cup result. On the other hand, the ranking obtained using weighted points shows good consistency for the top 4 teams to the standings of BBC, but the Kendall's τ is relatively small ($\tau = 0.7741935$). This shows that overall, it is not consistent to BBC's standings and, because BBC's standings seems to reflect FIFA's opinion well, it is preferable to use Keener's method for the incomplete matches with insufficient samples such as FIFA World Cup match result.

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A Appendix

The blocks of the ranking matrix R are given by

$$A=\begin{pmatrix}0&2/3&4/5&1/2\\1/3&0&1/2&3/4\\1/5&1/2&0&3/5\\1/2&1/4&2/5&0\end{pmatrix},B=\begin{pmatrix}0&5/7&3/4&2/3\\2/7&0&3/4&1/2\\1/4&1/4&0&3/5\\1/3&1/2&2/5&0\end{pmatrix},C=\begin{pmatrix}0&1/2&1/2&2/3\\1/2&0&2/3&1/2\\1/2&1/3&0&2/3\\1/3&1/2&1/3&0\end{pmatrix},$$

$$D=\begin{pmatrix}0&2/3&5/6&1/3\\1/3&0&1/2&2/3\\1/6&1/2&0&3/5\\2/3&1/3&2/5&0\end{pmatrix},E=\begin{pmatrix}0&2/3&3/4&3/5\\1/3&0&2/3&2/3\\1/4&1/3&0&3/5\\2/5&1/3&2/5&0\end{pmatrix},F=\begin{pmatrix}0&3/4&1/2&1/2\\1/4&0&1/2&4/7\\1/2&1/2&0&1/2\\1/2&3/7&1/2&0\end{pmatrix},$$

$$G=\begin{pmatrix}0&1/2&2/3&3/5\\1/2&0&1/2&8/9\\1/3&1/2&0&4/5\\2/5&1/9&1/5&0\end{pmatrix},H=\begin{pmatrix}0&3/5&1/3&3/4\\2/5&0&2/3&2/3\\2/3&1/3&0&1/2\\1/4&1/3&1/2&0\end{pmatrix},AB=\begin{pmatrix}h&3/5&h&h\\1/3&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$AD=\begin{pmatrix}h&7/13&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},AE=\begin{pmatrix}3/7&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},BA=\begin{pmatrix}h&2/3&h&h\\2/5&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$BD=\begin{pmatrix}1/6&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},CD=\begin{pmatrix}h&2/5&h&h\\2/7&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},DA=\begin{pmatrix}h&h&h&h\\6/13&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$DB=\begin{pmatrix}5/6&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},DC=\begin{pmatrix}h&5/7&h&h\\3/5&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},DH=\begin{pmatrix}1/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$EA=\begin{pmatrix}4/7&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},EF=\begin{pmatrix}h&3/5&h&h\\13/28&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},EH=\begin{pmatrix}1/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$EG=\begin{pmatrix}3/5&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},FE=\begin{pmatrix}h&15/28&h&h\\2/5&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},FH=\begin{pmatrix}1/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$HD=\begin{pmatrix}2/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},HE=\begin{pmatrix}2/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},HF=\begin{pmatrix}2/3&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},$$

$$HG=\begin{pmatrix}h&2/3&h&h\\1/5&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},GE=\begin{pmatrix}2/5&h&h&h\\h&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix},GH=\begin{pmatrix}h&4/5&h&h\\1/3&h&h&h\\h&h&h&h\\h&h&h&h\end{pmatrix}$$

and the other elements of the ranking matrix R are set to be $h>0$.

B Appendix

Group A

	Uruguay	Mexico	South Africa	France
Uruguay		1-0	3-0	0-0
Mexico	0-1		1-1	2-0
South Africa	0-3	1-1		2-1
France	0-0	0-3	1-2	

Group B

	Argentina	South Korea	Greece	Nigeria
Argentina		4-1	2-0	1-0
South Korea	1-4		2-0	2-2
Greece	0-2	0-2		2-1
Nigeria	0-1	2-2	1-2	

Group C

	United States	England	Slovenia	Algeria
United States		1-1	2-2	1-0
England	1-1		1-0	0-0
Slovenia	2-2	0-1		1-0
Algeria	0-1	0-0	0-1	

Group D

	Germany	Ghana	Australia	Serbia
Germany		1-0	4-0	0-1
Ghana	0-1		1-1	1-0
Australia	0-4	1-1		2-1
Serbia	1-0	0-1	1-2	

Group E

	Netherlands	Japan	Denmark	Cameroon
Netherlands		1-0	2-0	2-1
Japan	0-1		3-1	1-0
Denmark	0-2	1-3		2-1
Cameroon	1-2	0-1	1-2	

Group F

	Paraguay	Slovakia	New Zealand	Italy
Paraguay		2-0	0-0	1-1
Slovakia	0-2		1-1	3-2
New Zealand	0-0	1-1		1-1
Italy	1-1	2-3	1-1	

Group G

	Brazil	Portugal	Ivory Coast	North Korea
Brazil		0-0	3-1	2-1
Portugal	0-0		0-0	7-0
Ivory Coast	1-3	0-0		3-0
North Korea	1-2	0-7	0-3	

Group H

	Spain	Chile	Switzerland	Honduras
Spain		2-1	0-1	2-0
Chile	1-2		1-0	1-0
Switzerland	1-0	0-1		0-0
Honduras	0-2	0-1	0-0	

Round of 16

Uruguay	2-1	South Korea
USA	1-2	Ghana
Germany	4-1	England
Argentina	3-1	Mexico
Netherlands	2-1	Slovakia
Brazil	3-0	Chile
Paraguay	0-0	Japan
	PK5-3	
Spain	1-0	Portugal

Quarter-finals

Netherlands	2-1	Brazil
Uruguay	1-1	Ghana
	PK4-2	
Argentina	0-4	Germany
Paraguay	0-1	Spain

Semi-finals

Uruguay	2-3	Netherlands
Germany	0-1	Spain

Final

Netherlands	0-1	Spain

Match for third place

Uruguay	2-3	Germany

Table B.5: The results of all matches in 2010 FIFA World Cup

C Appendix

(i) unweighted	BT Parameter	(ii) weighted ($n!$)	BT Parameter
Netherlands	0.0785696	Spain	0.09004876
Germany	0.06199517	Netherlands	0.08157827
Spain	0.05430948	Germany	0.06769848
Uruguay	0.0489373	Uruguay	0.04443869
Brazil	0.04584088	Brazil	0.04093729
Argentina	0.0432712	Portugal	0.03885925
Japan	0.03750989	Japan	0.03685436
Portugal	0.03731276	Argentina	0.03468679
Paraguay	0.03363823	Ghana	0.03189228
Ghana	0.03216234	Paraguay	0.03137611
United States	0.03196856	Australia	0.0301546
England	0.03160605	England	0.02997656
Australia	0.03049108	United States	0.02996679
Chile	0.03040081	Chile	0.02829329
Ivory Coast	0.02899632	Ivory Coast	0.02805113
South Africa	0.02858492	New Zealand	0.02738536
New Zealand	0.02844685	South Africa	0.02733383
Slovenia	0.02762225	Slovenia	0.02667925
Mexico	0.02618416	Switzerland	0.02390486
Slovakia	0.02519541	Denmark	0.02366336
Switzerland	0.0242085	Mexico	0.02340646
Denmark	0.02415991	Slovakia	0.02322214
South Korea	0.02357287	Italy	0.02136651
Italy	0.02227126	South Korea	0.02055562
Greece	0.0216447	Greece	0.0201977
Serbia	0.01964293	Serbia	0.01922685
Honduras	0.01871237	Honduras	0.01896782
France	0.01776596	Algeria	0.01696716
Algeria	0.01761953	France	0.01693236
Nigeria	0.01720105	Nigeria	0.01607847
Cameroon	0.01523751	Cameroon	0.01487298
North Korea	0.01492014	North Korea	0.01442664

Table C.6: The rankings by Bradley-Terry model

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