Applied Mathematics & Information Sciences
An International Journal

Research and Development of Granular Neural Networks

Li Hui[1], *Ding Shifei*[1,2]

[1]School of Computer Science and Technology, China University of Mining and Technology, Xuzhou Jiangsu 221116 P. R. China
[2]Key Laboratory of Intelligent Information Processing, Institute of Computing Technology, Chinese Academy of Sciences, Beijing 100190 P. R. China

Received: 15 Aug. 2012, Revised: 28 Nov. 2012, Accepted: 23 Jan. 2013
Published online: 1 May 2013

Abstract: Granular neural networks(GNNs) as a new calculation system structure based on Granular Computing(GrC) and artificial neural network can be able to deal with all kinds of granular information of the real world. This article has made the summary on the development and the present situation of GNNs. Firstly, it introduces the basic model of GrC: word calculation model based on fuzzy sets theory, rough sets model, granular computing model based on quotient space theory and so on, summarizes the research progress of fuzzy neural networks(FNNs) and rough neural networks(RNNs), then analyses the ensemble-based methods of GNNs, researches their meeting point of three main GrC methods, and finally points out the research and development direction of GNNs.

Keywords: Artificial Neural Networks, Granular Computing(GrC), Granular Neural Networks(GNNs), Fuzzy Neural Networks(FNNs), Rough Neural Networks(RNNs), Theory of Quotient Space

1. Introduction

Granular Computing (GrC), as a new concept and computing paradigm of information processing, covers all about granularity of the theories, methods, techniques and tools, and is mainly used to deal with uncertain, fuzzy, incomplete and massive information. All the theories and methods of grouping, classification and clustering used for analyzing and solving the problems are belong to the scope of GrC[1,2]. The research of GrC has become one of the recent research focuses of the field of GrC.

The neural network is one kind of network system by simulating human brain information processing mechanism based on the development of modern biology research, which is also one of the soft computing techniques. As artificial neural network has distributed storage of information, massivingly parallel processing, self-adapted and self-learning ability and other advantages, it has inestimable value in the fields of information processing, pattern recognition, intelligent control. But there are many problems in simple neural network, such as easy to fall into local minimum, too long time for training, difficult to obtain those complex and gradient information. With the increasing size of the problem, the growth of the computing complexity is exponentially, which largely restricts the development of the neural network. In order to overcome these shortcomings, many optimized algorithms are introduced into the learning of the neural network weights and structure automatic design, such as the use of evolutionary algorithms to optimize neural network[3,4,5] and particle swarm optimization algorithm based on neural network[6,7]. After verification, these optimized algorithms are feasible and effective, but it will take massive time to process the complex and high-dimensional data.

The GrC theories including fuzzy sets theory, rough sets theory, quotient space theory are an effective tool to deal with uncertain and high-dimensional data. The neural network has good robustness and self-learning ability, but for massive, complex and high-dimensional data, it cannot get good effect: it requires massive time frequently, and has high time complexity for high-dimensional data sets. GrC theories and neural network both have their own advantages in dealing with problems, and both of them have certain supplementary in some degree. The combination between them can solve the complex and high-dimensional problems well. With the further development and needs of GrC, the granular neural networks(GNNs) produced[8]. GNNs as a new calculation system structure based on GrC and artificial neural network can be able to deal with all kinds of

* Corresponding author e-mail: lihui400@163.com, dingsf@cumt.edu.cn

granular information of the real world. The typical GNNs are fuzzy neural networks(FNNs) and rough neural networks(RNNs).

This article has made the summary on the development and the present situation of GNNs. Firstly, it introduces the basic model of GrC: word computing model based on fuzzy sets theory, rough sets model, granular computing model based on quotient space theory and so on, summarizes the research progress of FNNs and RNNs, then analyses the ensemble-based methods of GNNs, researches their meeting point of three main GrC methods, and finally points out the research and development direction of GNNs.

2. Basic model of GrC

The concept of information granularity was proposed by Zadeh in the paper "fuzzy sets and information granularity" [9] in 1979, which causes the widespread interest of researchers. Zadeh thinks there is the concept of information granularity in many areas, which just has different manifestations in different areas. Information granularity is prevalent in our real life, and it is an abstraction of reality. Information granulation is a way of human processing and storage of information. According to his ideas, T. Y. Lin formally proposed the concept of GrC in 1997 [10]. In fact, as early as this, many researchers have proposed a similar concept and obtained the corresponding research results, for example, references [11]. GrC is a very powerful tool to solve problems. Through dividing into the particles, the complex issues can be converted to a number of relative simple problems, which contributes to our analysis and solution of complex problems.

The ideological essence of GrC is to find a simple, easy and low-cost enough satisfaction approximate solution to replace the exact solution, that is to say, it uses the accommodation of inaccurate, incomplete, uncertain, and vast information to reach the aim of easy handling, robustness, low costs of intelligent systems or intelligent control, and better describes the real world. It has become one of the hot research fields of artificial intelligence, soft computing and control science. The basis theory of GrC is the word computing theory based on fuzzy sets theory proposed by Zadeh[12], the rough sets theory proposed by Pawlak[13], quotient space theory proposed by our scholars, Zhang Ling and Zhang Bo[14].

2.1. Theoretical model of computing with word based on fuzzy sets theory

In 1996, Zadeh proposed the theory of computing with word[12] which marks the birth of fuzzy granular theory. Theories and methods of computing with word based on the fuzzy sets theory is one of the important research direction of GrC. Theoretical model of computing with word was proposed by Zadeh [15]: standardized forms, $IfXisR, thenYisB$. It describes that if the language granularity X on the domain U is constrained by R, the language granularity Y is constrainted by B. The generalized forms can be expressed as $XisrR$, where isr denotes the constraint type of R to X. r is a discrete variable, which denotes the type of constraint. Different types of granularity can be obtained by using these constraints. The fuzzy constraint propagation rule can be expressed as

$$\frac{(f(x_1,x_2,\ldots,x_n)isA)}{(q(x_1,x_2,\ldots,x_n)isq(f^{-1}(A)))}$$

Where the numerator represents the introduction of constraints by the initial number, and the denominator represents the constraints of the problem $q(x_1,x_2,\ldots,x_n)$; $f^{-1}(A)$ denotes the pre-image of the fuzzy relation A under the mapping $f: U \to V$. The fuzzy granularity by constraints can be defined as $G = \{X|XisrR\}$, and by the different types of constraints, one can get a variety of granularity. From the simple granularity, we can also get the Descartes product of granularity by the combination of the constraints. Zadeh's model has laid the basis for the theory of computing with words. Fuzzy $if - then$ rules can be formally expressed as: $ifXisr_lA, thenYisr_2B$, where r_1 and r_2 represent different type or the same type of constraints.

2.2. The model of rough sets

Rough sets theory was proposed by Polish scholar Pawlak in 1982. Rough sets theory has a strong qualitative analysis ability to effectively express uncertain or imprecise knowledge, which is good at accessing knowledge from the data, and can take advantage of the uncertain, incomplete empirical knowledge to reason, so it has been widely applied in the fields of knowledge acquisition, machine learning, rule generation, decision analysis and intelligent control. Here is the model of rough sets. For a given domain U (a limited set of non-empty) and the equivalence relation $R \subseteq U \times U$ on the domain U, the equivalence relation is essentially a mapping from the universe of domain U to the domain U under the power set 2^U, namely, $R: U \to 2^U$, thus (U,R) is an approximation space or knowledge base. In the approximation space, the equivalence relation divides the domain U into union set of the pairwise disjoint equivalence class, namely, $U = \cup_{i\in\tau}[x]_R$ meets: (1) $[x]_R \subseteq U$; (2)$xRy \Leftrightarrow [x] = [y]$, $x\overline{R}y \Leftrightarrow [x] \cap [y] = \phi$. $[x]_R = \{y \in U \mid xRy\}$ represents an equivalence class. The quotient set $U/R = \{[x]_R = \{[x]_R \mid \forall x \in U\}\}$ is a group of knowledge-based of approximation space, which represents the granularity of domain. So all of subsets of domain can be described by upper and lower approximation. $\underline{apr}(X) = \underline{R}(X) = \{x \mid x \in X, [x]_R \subseteq X\}$ is the lower approximate of X, which represents the set of elements belonging to X entirely, and $\overline{apr}(X) = \overline{R}(X) = \{x \mid x \in U, [x]_R \cap X \neq \phi\}$ is the upper approximate, which represents the set of elements belonging to X entirely or probably, it contains the minimum closure of X.

In rough sets theory, whether an object belongs to a set (concept) depends on the degree of our understanding of it, but not the objective nature of elements. Similarly, the nature of equality and contain of the set is no absolute meaning, and also depends on the degree of understanding of the set in our research fields, which is more in line with the process of human cognition. Though it has made great success in the data mining applications of comprehensive information systems, it also exists in ”bottleneck” problem, for example, (1) knowledge of the over-reliance on the domain of relations; (2) limited to set operations, and the lack of algebra system to effectively deal with real world problems; (3) the lack of basic semantic interpretation of granularity; (4) lack of methods to describe the structure of the information granularity.

2.3. Model of GrC based on quotient space

Academician Zhang Bo and Professor Zhang Ling independently proposed the quotient space theory in the study of problem solving, which uses subset to describe the concept. The concept of different granularity reflects the subset of the different granularity. A cluster of concepts constitute a partition of space: Quotient Space (knowledge base), and different concept clusters constitute the quotient space. As for GrC, it is equivalent to study the relationship and conversion between the subsets of a given knowledge base. This model describes the problem with a triple (X,F,T), where X is the domain, F is the set of attributes, T is a topology on X. In this model, granularity of the domain is equivalent to an given equivalence relation R or a partition, so the quotient set corresponding to R is denoted by $[x]$, and the triples $([X],[F],[T])$ can be called the quotient space corresponding to R. The quotient space theory is to study the relationship, synthesis, decomposition, and reasoning between each quotient space. The most important nature is the homomorphism principle, namely, the fidelity principle (or false principle of protection). The so-called “false principle of protection” is that if a proposition of the coarse-granularity space is false, the proposition must have no solution in fine granularity quotient space. Fidelity principle is that if the proposition is true in the two coarse granularity quotient space, under certain conditions, the corresponding problem in the synthetic quotient space is also true. These two principles play an important roles in the quotient space model. If we want to solve a very complicated problem, we often preliminary analysis firstly to get a coarse granularity quotient space, then transform the problem into the corresponding problem of this space to solve. If the problem has no solution in the coarse granularity space, in according to “false principle of protection”, we know immediately that the original problem has no solution. Because of small spatial scale of the coarse granularity computation, we can draw the desired results with a little calculation to yield twice the result with half the effort. Similarly, fidelity principle may also reduce the complexity of problem solving. On this basis, the model of granular world and a series of theories and the corresponding algorithms were established, and applied in the fields of heuristic search, path planning, and achieved good results. Academician Zhang Bo and Professor Zhang Ling introduce fuzzy sets theory to the quotient space again, which achieves the promotion of quotient space model by fuzzy equivalence relations. Fuzzy quotient space theory can better reflect a number of characteristics of human beings to deal with uncertain information, that is to say, that certainty and uncertainty, the concept of clarity and obscurity are relative, and all are related to granularity of the problem. Therefore, how to construct a reasonable stratification delivery order of the granular structure is a key to solve problems and process information efficiently. On the other hand, the quotient space theory also lacks the means and the technical methods to achieve the conversion between granularity and granularity, granularity and granular world, granular world and granular world. Exploring effective techniques and methods to solve this problem will broaden the scope of application of the quotient space, and greatly enriched the theory of GrC.

3. Fuzzy neural networks(FNNs)

FNNs is the product of combining fuzzy systems and neural network, which brings together the advantages of neural network and fuzzy sets theory, such as learning, association, identification, information processing and so on. The artificial neural network is to simulate the function of thinking of the human brain with a strong self-learning, association, less manual intervention, high accuracy, and makes use of expert knowledge well. But there are many problems, such as difficult to handle and describe fuzzy information, difficult to take advantage of existing experience knowledge, non-explanatory, and its higher sample requirements. Comparing with neural networks, fuzzy systems have many advantages, such as easy to understand the reasoning process, the good use of expert knowledge, the lower sample requirements, and in many fields, having a wide range of applications, such as[16]. But it also has shortcomings of a manual intervention, slow reasoning, and the low accuracy. It is difficult to achieve self-adaptive learning. How to automatically generate and adjust the membership functions and fuzzy rules is a thorny issue. The combination between them can solve the complex problems. Neurons of FNNs can be divided into two categories: The one is the direct fuzzification to the non-fuzzy neuron; The other is fuzzy neuron described by the fuzzy rules.

3.1. Common fuzzy neural networks

This is one way of compromising fuzzy sets and neural network and offen called narrow fuzzy neural network, which add fuzzy ingredients into traditional neural networks. The neuron is obtained by direct fuzzification to non-fuzzy neuron, and the neural network is the weight and input with part or all fuzzification to achieve the mapping between the granularity information and numerical data. The FNNs is composed of common neuron or fuzzy neuron, but they have the same topological structure. There are two kinds of input to handle: handling real input with common neurons and fuzzy input with fuzzy neurons. This neural network not only has the general topology of the neural network, but also can be able to deal with the granular information like values, language, images, sounds, texts and other forms of granularity in the real world.

3.2. Fuzzy neuron based on fuzzy rules

Fuzzy neuron of FNNs based on fuzzy rules, different from the common fuzzy neural networks, is composed of traditional neurons and logical reasoning of GrC theory, which is a kind of granularity neuron with some meanings, and has granular learning algorithms of the cognitive science and computational intelligence. There are many typical fuzzy neurons, such as Glorennec neuron[17], the OWA neuron[18], OR / AND neuron[19], Weak T-norm neuron[20], universal logic neuron[21] and so on.
Table 1 lists several typical fuzzy neurons.

3.2.1. Glorennec neuron

A kind of common neuron was designed by Glorennec, which can realize four basic logic operator of Lukasiewicz logic, such as AND, OR, NOT, I. Figure 1 is the structure of Glorennec neuron. Table 2 shows the Parameters setting.

$$Where, \quad h(u)\begin{cases} 0, & \mathrm{u} < 0 \quad (1) \\ u, & 0 \leqslant \mathrm{u} \leqslant 1 \quad (2) \\ 1, & \mathrm{u} > 1 \quad (3) \end{cases}$$

Namely: $AND(x,y) = max(0, x+y-1); OR(x,y) = min(1, x+y); I(x,y) = min(1, 1-x+y); NOT(x) = 1-x$.

Table 1 Analysis of typical fuzzy neurons

The auther	Fuzzy neuron	Advantages
Glorennec[17]	Glorennec neuron	Can realize Lukasiewicz logic: four basic logic operator: AND, OR, NOT, I.
Yager[18]	OWA neuron	Using a set of OWA fuzzy aggregation operator to construct fuzzy neuron, which can study by BP. The main idea is to input the ordered data, and then gives the weight value addition operation.
Hirota and Pedrycz[19]	OR/AND neuron	Consist of OR and AND neurons, constitute the three-layer topology, can study by BP, and display superior performance and potential in two layer network.
Chen and He[20]	Weak T-norm neuron	Can realize many kinds of fuzzy logic neural network by these neurons.
He and Wang[21]	Universal logic neuron	Study the common rule science of logic in all and not yet proposed, abstract law of logic in all logic from high-level research.

Table 2 Parameters setting

operator and parameter	a	b	θ
AND	1	1	-1
OR	1	1	0
I	-1	1	1
NOT	-1	0	1

3.2.2. The OWA neuron

The OWA neuron is constructed by a set of OWA fuzzy aggregation operator, which can study by BP. The main idea is to input the ordered data, and then gives the weight value addition operation. OWA operator can be defined as: n-dimensional OWA operator is a mapping: $f : R_n \to R$, which has a n-dimensional weight vector

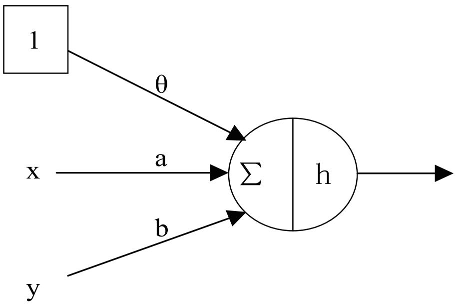

Fig.1 Common logic neuron

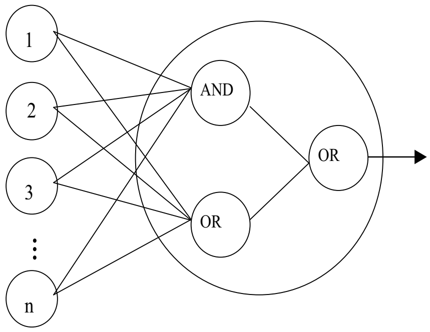

Fig. 3 Structure of OR/AND neuron

$W=[w_1,w_2,\ldots,w_n]$, and meets 1)$w_i \in [0,1]$; 2) $\Sigma w_i = 1$, $f(a_1,a_2,\ldots,a_n) = \Sigma w_i b_j$, where b_j is the maxmum of $(a_1,a_2,\ldots,a_n)$.
Figure 2 is the structure of the OWA neuron.

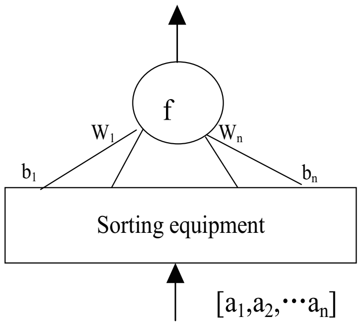

Fig. 2 Structure of OWA neuron

3.2.3. The OR/AND neuron

The OR/AND neuron is the logical connective model based on neural network, which is consist of OR and AND neurons, and constitutes the three-layer topology. OR/AND neuron can study by BP, and display superior performance and potential in two layer network.
Figure 3 is the structure of OR/AND neuron.

3.2.4. The Weak T-norm neuron

The Weak T-norm neuron is a kind of continuous logic operation neuron model, which can achieve a variety of logical operations, including weak T-norm cluster continuously changing from $min(x,y)$ to $max(0,x+y-1)$, weak S norm cluster continuously changing from $max(x,y)$ to $min(1,x+y)$, the average computing continuously changing from $min(x,y)$ to $max(x,y)$.
We can realize many kinds of fuzzy logic neural network by these neurons. Figure 4 is Weak T-norm neuron model. Table 3 shows weights and the corresponding logical operations of a, b and c.

$$Where, \quad f(u)\begin{cases} 0, & u<0 \quad (4)\\ u, & 0\leqslant u\leqslant 1 \quad (5)\\ 1, & u>1 \quad (6)\end{cases}$$

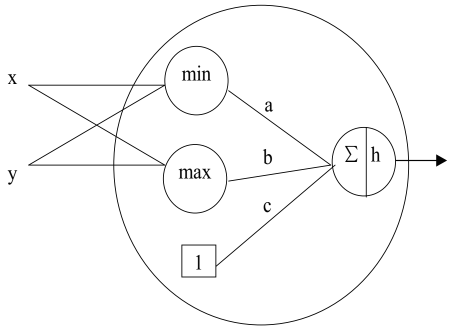

Fig. 4 Model of Weak T-norm neuron

3.3. Fuzzy neural networks(FNNs) based Fuzzy neuron

In recent years, many scholars have proposed a variety of FNNs, such as Fuzzy Multilayer

Table 3 Weights and the corresponding logical operations

operator and parameter	a	b	c
OR(x)	w	1	0
AND(x,y)	1	w	-w
M(x,y)	1-w	w	0
E(x,y)	1	-1	1

Perceptron[22], fuzzy Hopfield neural network[23], the fuzzy artmap[24], fuzzy associative neural network FANN[25], FCM[26] and so on, which is widely used in the fields of pattern recognition, expert systems, identification, intelligent control and intelligent detection. FNNs based on fuzzy neuron is generally divided into five layers: input layer, fuzzification layer, rule layer, clear layer and output layer. Rule layer is the most important part. Rule layer can extract fuzzy rules from the system operation, such as expertise and system operation, and the corresponding fuzzy rules can also be extracted from the control system model and learning algorithm. FNNs based on fuzzy neuron determines the network architecture by fuzzy rules, but this depends on the expert domain knowledge, so the adaptive ability is poor.

With the further development of fuzzy sets and neural network, the GNNs produced. In 2000, Zhang et al.[8] proposed a novel GNNs based on the issue of number in the database and the language data to handle the granular problem in database. It can study the granular contacts and forecast new contacts. It can extract IF-THEN rules, compress database and predict new data. Mubeena Syede [27] proposed parallel GNNs to reduce running time and improve efficiency and successfully applied in detecting fast credit card fraud. Zhang[28] proposed a new interval reasoning method using new granular set based on Yin Yang methodology to deal with different membership function of the same linguistic term, which optimizes the membership function of different interval. Gabrys B. and Bargiela A.[29] proposed the GFMN granular neural network model and this model can be applied to the granular study and classification of pattern recognition. After Verification, the membership function of GFMN can not be able to effectively deal with different levels or size of multi-granularity problem. In order to optimize and improve the membership function of GFMN, Nandedkar A.V. and P.K. Biswas [30]proposed the GrRFMN granular neural network model, which can more efficiently process granular data, and can be able to pocess effectively different levels and the size of multi-granularity by training.

4. Rough neural networks(RNNs)

Rough sets and the neural network complement each other[31]. From the point of view of human thinking simulation, the rough sets method simulates human abstract logical thinking, while the neural network simulates human intuitive thinking. We can combine both of them by using the rough sets to compensate for the lack of neural networks in dealing with the the high dimension data, and by using strong anti-interference characteristics of the neural network to compensate for the noise sensitivity of rough sets. The combination of simulation of abstract thinking and intuitive thinking will be better effect. RNNs is based on the rough granular information and traditional neural network. The granular information can be viewed as a collection with the same or similar properties or characteristics. Coarse granularity measure can solve complex problems in the real world. According to the way of combination, RNNs can be divided into two categories: strong coupling RNNs and weak coupling RNNs.

4.1. The weak coupling RNNs

In an information system, some attributes of objects or objects are unnecessary or redundant, so this unnecessary or redundant information should be removed to improve the efficiency of the system. Rough sets can preprocess the information for the primary information system, remove the redundant attribute and sample, reduce the dimension of the sample, simplify the information system to enhance processing efficiency. In references [32,33], rough sets as a style of weak coupling preprocesses the data. Data preprocessing based on rough sets can be divided into three parts: the formation of decision table, the discretization of serially attribute, attribute reduction[34]. The attribute reduction is made up of condition attribute and decision-making attribute[35], which is called double attribute reduction and make the reduction achieving the best results. The method of attribute reduction can be divided into five categories[36]:based on the discernibility matrix attribute reduction, based on positive region attribute reduction, based on information representation attribute reduction, based on granularity attribute reduction and other attribute reduction. Although these methods can be very intuitive to draw all reduction and nuclear of the information system, it will produce a combinatorial explosion when the scale of problem is large. Therefore, many scholars have proposed a series of improved algorithms for knowledge reduction. References[37,38] optimize attributes and attribute values of rough sets based on neighborhood. Shang et al. [39] proposed a novel attribute reduction method, which improves the reduction performance of data of the continuous properties of knowledge. Because of the shortage of only using rough set, in order to get more stable and generalized reduction, we can use the dynamic technology and the genetic algorithm. Some researchers integrated the sample disturbance boosting and bagging method with input disturbance attribute and proposed the attribute bagging algorithm[40,41], which can get good results. We make use of rough sets to reduce the attributes and their values before the original information, remove redundant information, reduce the dimension of the information space, provide a simpler neural network training sets, and then construct and train the neural network. This method not only shortens the time of study and training of the neural network to improve the system response speed, but also makes full use of the advantages of the neural network to improve the overall performance of neural network.

4.2. The strong coupling RNNs

The rough neuron is constituted by traditional neuron and rough neuron. When the input of neural network is not a single but a range, if we use RNNs based rough neuron, we can solve these problems well. Figure 5 is the structure of a rough neuron designed by Lingras[42]. RNNs is a network which neuron is constituted by rough neuron and traditional neuron. Rough neuron contains upper neuron and lower neuon. The connection between neurons denotes the exchange of information. The connection of neuron can be divided into three: full connection, inhibiting connections, activation connections. The output of rough neuron is a pair of values with upper and lower Approximation, but the traditional neuron has only one output value. Experiment [43,44] showed that the neural network based on rough neuron can improve the network performance and reduce errors. To determine the number of hidden layers, hidden layer nodes and the initial weights and network semantics, strong coupling integrated way [45] provides a new idea combinating the advantages of rough sets and neural networks to guide the design of neural network structure and give the interpretation of the semantics of neural network.

4.3. The application of RNNs

The RNNs is made up of rough sets and neural network, so it have the advantages of rough sets and neural network that has learning, association, recognition, self-adapted and processing uncertain information, which is successfully applied in dealing with uncertain and complex problems. Specially, the BP and RBF neural network based on rough sets have a large number of applications. Table 4 only lists the application of some typical RNNs. If you want to know more about attribute eduction, you can see the Thangavel K.'s article[56]. The RNNs also play an important role in our life, Pan [57] have listed these neural network application in intelligent control. Zhang and Wang [58] applied the fuzzy-rough neural network in the vowels recognition. Wang and Mi [59] applied the RNNs in customer management system that predict customer purchasing rule. Stetiawan et al.[60]

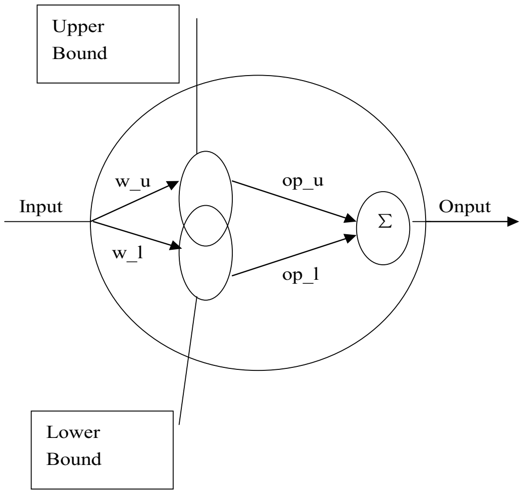

Fig.5 Structure of rough neuron

used RNNs in prediction of missing attribute. Dong et al. [61] applied it in decision analysis and obtained good results. Peters et al.[62] applied the approximate neural network based on granularity in the power system, which is made up of rough approximate neuron and decision-making neuron and get high accuracy in the test of power system fault classification.

From Table 4 we can see researchers often optimize the RNNs with other algorithms. Fuzzy algorithm and genetic algorithm are often used to optimize RNNs, the fuzzy algorithm can handle none-single input[63,64,65], the genetic algorithm speeds up neural network training[34,43,66,67,68].

5. The analyses of GNNs based on the ensemble-based GrC theory

There are three main models of GrC, fuzzy sets, rough sets and quotient space. Analyses and compares among them are benefit for us to understand the relations and differences among them, and benefit for us to find out how to integrate them and how to construct a unified granular neural network model [69,70].

The membership degree of objects in fuzzy sets theory is given directly by the experts. So it's subjective and lack of precision. While the membership degree of objects in rough sets theory depends on Knowledge base, which is objective. Any subset of approximation space is equivalent to a fuzzy sets, and the lower approximation and the upper approximation of rough sets are equal to core and closure of fuzzy sets. So fuzzy sets and rough sets are shown to complement each other, and can be combined together.

Quotient space uses equivalence classes to describe

Table 4 Application of RNNs

Auther	The type of NN	Applications
Ren xiaowen,Zhang Feng [46]	quantum neural network	Make full use of rough sets attribute reduction to classify, which can get good result for fault diagnosis in transformers.
Wang Wenjuan, Xie Bin [47]	BP	The algorithm of evaluation and application research about regional innovation capability is relatively few, the BP based on rough sets reduction has a good prospect.
Chen jaohong, Chen Hueiping, Lin Yimin [48]	Probabilistic network	As using rough sets to classify, probabilistic rough sets neural network can get good results in market forecast.
Xue Feng,Ke Konglin[49]	Elman	Making good using of the reduction of rough sets with none information loss, the NN has been successfully applied to predict the risk of bank loans.
Li Cuiling, Zhang Hao [50]	Heuristic SOM network	Take the advantage of its dealing with continuous attributes and rough sets attribute reduction, it is well applied in pattern recognition.
Dong Ming, Jiang Huiyu [51]	CP neural network	Experiments show that the neuron of rough CP neural network can greatly improve the accuracy of prediction.
Gao Wei, Wen Jingxin [52]	BP	The network successfully applied in data fusion by using rough sets to streamline information and attribute reduction.
Sikant Kumar,Shivam Atri[53]	Hebb net	Using rough sets good at handling large and high-dimensional data, the network has been successfully applied in detecting landmines.
Chiang Junghsien, Ho Shinghua [54]	RBF	The preprocessing based on rough sets feature selection make the network good for classification selection gene expression.
He Haitao, Zhao Na [55]	CMAC network	Make good use of the feature of network and rough set removing redundancy, it was well applied in flatness control.

granularity, rough sets does it either. But quotient space which is the theory describing spatial relationship focus on researching inter-conversion and inter-dependence relations among different granular world. Rough sets focus on granular presentation and the inter-dependence between granularity and conception. So from this point of view, rough sets is a special case of quotient space. We often combine them together.

Fuzzy quotient space theory[71] extends quotient space to the fuzzy granular world. There are three ways to realize it: introduce fuzzy sets into universe; introduce fuzzy topology into structure; introduce fuzzy equivalence relation.

Fuzzy sets theory, rough sets theory and quotient space theory which have their own advantages and disadvantages constitute the three main subjects of GrC. Finding their meeting point will help us construct a unified platform to deal with complex and ambiguous information more easily. Li Daoguo [72] and others summed up the three models' advantages and proposed a new model: using rough sets to construct a collection of researching objects (domain size), using fuzzy sets to make sure the semantic interpretation of granularity, using quotient space theory to construct a hierarchical granular structure.

Gong Wei[63] and Zhang Dongbo et al.[43] proposed the model of fuzzy rough neural networks(FRNNs) based fuzzy sets and rough sets, and divided FRNNs into six layers: input layer, fuzzy layer, rule former layer, synthesis layer of reasoning , clear layer, fuzzy incentive output layer. The accurate input attribute values are fuzzification by the corresponding membership function in the fuzzy layer; Rule former layer is one of the most important layers in the FRNN, and implements function of fuzzy "and", and then forms the former of the rules; Synthesis layer of reasoning implements function of fuzzy "or", forms the latter of the rules and then outputs the rules; Clear layer calculates the fuzzy output value by clear rules; Fuzzy incentive output layer realizes the incentive value of the results of precise parameters after clearing by natural function.

6. Summary and Outlook

GNNs is the network based on the traditional artificial neural network and GrC theory. By the fusion of rough sets, fuzzy sets, quotient space granular computing theory, GNNs can handle the value, image, sound, text and many other forms of the granular information. The model of GrC is composed of the particle, granular layer and the granular structure. The research of problem solving between different granular layers and relation of granular layers(namely granular structure) is a subject worthy of further study. Therefore, the GNNs based on GrC theory is also facing a great challenge.

(1) Research and exploration of the granular rules. How to select the different rules to granulate the information and transfer the information into corresponding forms suitable for neural network to learn and construct the GNNs is urgent to resolve.

(2) Research and exploration of muti-granular neural network. How to explore a new neural network model based analysis of multi-granularity and multi-level theory

to deal with uncertain information and also to reduce the computing time and space complexity.

(3) Research and exploration of muti-granular network model and its extreme learning method in the cloud environment. From a theoretical point of view, GrC advocates to aware and explore the knowledge by the different scales. In this sense, GrC includes all methods that can provide flexibility and adaptability in the rules or scale of the extraction and representation of knowledge or information, which are called multi-granular computing. Multi-granular or variable granular computing is the feature of cloud computing. In the cloud environment, how to study and explore the GNNs model and its extreme learning algorithms is also an important research topic.

Acknowledgement

This work is supported by the Fundamental Research Funds for the Central Universities (No. 2012LWB41).

References

[1] Yao Yiyi, Granular computing: basic issues and possible solutions//Wang P P. Proceedings of the 5th Joint Conference on information Sciences, Atlantic City: Association for Intelligent Machinery Press, 2000: 186-189.
[2] Pedrycz W, Granular computing: An Emerging Paradigm, IEEE Trans. On Systems Man and Cybernetics, 2001, 32(2): 212-224.
[3] Eysa S, Saeed G., Optimum design of structures by an improved genetic algorithm using neural networks, Adv Eng Softw, 2005,36(11): 757-767.
[4] Yao Wangshu, Wan Qiong, Chen Zhaoqian, Wang Jingen, The researching overview of evolutionary neural network, Comput Sci, 2004, 31(3): 125-129.
[5] Tang Chunming, Gao Xieping, The researching development of evolutionary neural networks, J Syst Eng Electron , 2001, 23(10): 92-97.
[6] Gao Haibing, Gao Liang, Zhou Chi, Yu Daoyuan, Particle swarm optimization based algorithm for neural network learning, Chin J Electron, 2004, 32(9): 1572-1574.
[7] Chen Guochu, Yu Jinshou, Particle swarm optimization neural network and its application in soft-sening modeling, Lect Notes Comput Sci , 2005, 3611: 610-617.
[8] Zhang Yanqing, Fraser M. D., Gagliano R. A., Kandel A., Granular neural networks for numerical-linguistic data fusion and knowledge discovery, IEEE Trans Neural Netw, 2000, 11(3): 658-667.
[9] Zadeh L. A., Fuzzy sets and information granularity //GUPTAM, RAGADE R,YAGER R. Advances in Fuzzy Set Theory and Applications. Amsterdam: North-Holland Publishing Co, 1979:3-18.
[10] Lin T. Y., Granular Computing: From Rough Sets and Neighborhood Systems to Information Granulation and Computing in Words//European Congress on Intelligent Techniques and Soft Computing, September 8-12, 1997, [s. .l]: [s. n.]: 1602-1606.
[11] Zhang Bo, Zhang Ling, Theory and application of problem solving, Tsinghua University Press, Beijing, 1990.
[12] Zadeh L. A., Fuzzy logic=Computing with words, IEEE Transactions on Fuzzy Systems, 1996, 4(2): 103-111.
[13] Pawlak Z., Rough sets, Computer and Information Science, 1982, 11(5): 341-356.
[14] Zhang Ling, Zhang Bo, Theory of fuzzy quotient space(methods of fuzzy granular computing), Journal of Software, 2003, 14(4): 770-776.
[15] Zadeh L. A., Towards a theory of fuzzy information granulation and its centrality in human reasoning and fuzzy logic, Fuzzy Sets and Systems, 1997, 90(2): 111-127.
[16] Soo Young Moon and Hee Suk Seo, Fuzzy based filtering nodes assigning method for sensor networks, Appl. Math. Inf. Sci. Vol. 6 No. 1S (2012) pp. 285S-291S.
[17] Glorennec P. Y., Neuro-fuzzy logic, In: Proceeding IEEE-FUZZ, New Orleans, 1996: 512-518.
[18] Yager R., OWA neurons: a new of fuzzy neurons, In: Proceeding IEEE-FUZZ. San Diego, 1992: 2326-2340.
[19] Hirota K., Pedrycz W., OR/AND neuron in modeling fuzzy connectives. IEEE Trans Fuzzy Syst, 1994, 2(2): 15l-161.
[20] Chen Dan, He Huacan, Wang Hui, A new neuron model based on weak T-norm cluster, Chin J Comput, 2001, 24(10): 1115-1120.
[21] He Huacan, Wang Hua, Principle of universal logics, Science Press, Beijing, 2001.
[22] Shi Huimin, Zhang Fan, Sun Jixiang, Knowledge-based fuzzy multilayer perceptron, Comput Eng Appl, 2003, 39(1):99-102.
[23] Chen Weijing, Zhang Hongwei, Weng Huairong, Clustering of fuzzy Hopfield neural network and its application in DRP, Comput Eng Appl, 2006, 42(6):215-218.
[24] Zhong Jinhong, Yang Shanlin, Huang Ling, Li Yi, Application of fuzzy artmap for tone recognition of Chinese trisyllabic word, Comput Eng Des, 2004, 25(1): 52-55.
[25] He Fengdao, Handwritten numeral recognition of a fuzzy associative memory neural network, Appl Res Comput, 1994, 11(3): 8-10.
[26] Chen Youling, Hu Chunhua, Peng Jinwen, Dynamic evaluation method study enterprise supply chain performance based on FCM, Appl Res Comput, 2011, 28(1): 185-188.
[27] Syeda M., Zhang Yanqing, Pan Yi, Parallel granular neural networks for fast credit card fraud detection, In: Proceedings of the 2002 IEEE international conference, 2002, 1: 572-577.
[28] Zhang Yanqing, Jin B., Tang Yuchun, Granular neural networks with evolutionary interval learning, IEEE Trans Fuzzy Syst, 2008, 16(2): 309-319.
[29] Gabrys B. and Bargiela A., General fuzzy min-max neural network for clustering and classification, IEEE Trans. Neural Netw, 2000, 11: 769-783.
[30] Nandedkar, A.V. and Biswas P.K., A Granular Reflex Fuzzy Min-Max Neural Network for Classification, IEEE Transactions on Neural Networks, 2009, 20(7):1117-1134.
[31] Ding Shifei, Chen Jinrong, Xu Xinzheng, Li Jianying, Rough neural networks: a review, Journal of Computational International Systems, 2011, 7(7): 2338-2346.
[32] Jelonek J., Krawiec K., Slowinski R., Rough set reduction of attributes and their domains for neural networks, Computational Intelligence, 1995, 11(2): 339-347.
[33] Xu Xinzheng, Ding Shifei, Shi Zhongzhi, Zhu Hong, Optimizing radial basis function neural network based on

rough sets and affinity propagation clustering algorithm, Zhenjiang Univ-Sci(Comput & Electron), 2012, 13(2): 131-138.

[34] Liao Ying, Zhang Shaoyong, Yi Dawei, Yi Jiawa, Faults diagnosis of diesels based on rough set genetic neural network, Control Eng China, 2009, 16(6): 709-712.

[35] Zhang Xibin, Cheng Li, Yu Jiangmin, Study of fault diagnosis based on new rough set neural networks, Appl Res Comput, 2006, 23(5): 156-158.

[36] Ding Hao, Ding Shifei, Hu Lihua, Research progress of attribute reduction based on rough sets, Comput Eng Sci, 2010, 32(6): 92-94.

[37] Hu Qinghua, Yu Daren, Xie Zongxia, Numberical attribute reduction based on neighborhood granulation and rough approximation, Journal of Software, 2008, 19(3): 640-649.

[38] Hu Qinghua, Zhao Hui, Yu Daren, Efficient symbolic and numberical attribute reduction with neighborhood rough sets, PR & AI, 2008, 21(6): 732-738.

[39] Shang Lin, Wan Qiong, Yao Wangshu, Wang Jingen, Chen Shifu, An approach for reduction of continuous -valued attributes, Journal of Rearch and Developments, 2005, 42(7): 1217-1224.

[40] Bryll R., Gutierrez-Osuna R., Quek F., Attribute bagging: improving accuracy of classifier ensembles by using random feature subset, Pattern Recognit, 2003, 36(3): 1291-1302.

[41] Ling Jinjiang, Chen Zhaoqian, Zhou Zhihua, Feature selection based neural network ensemble method, J Fudan University(Nature Sci), 2004, 43(5): 685-688.

[42] Lingras P., Rough neural networks //Proc of the 6th International Conference on Information Processing and Management of Uncertainty in Knowledge-based Systems, 1996: 1445-1450.

[43] Zhang Dongbo, Wang Yaonan, Filtering image impulse noise based on fuzzy rough neural network, Journal of Computer Application, 2005, 25(10):2336-2338.

[44] Chang Zhiling, Wang Quanxi, A new rough neural network construction algorithm, Comput Ear, 2009, 4:51-53.

[45] Zhang Dongbo, Wang Yaonan, Yi Lingzhi, Rough neural network and its application to intelligent information procession.control and decision, 2005, 20(2): 121-126.

[46] Ren Xiaowen, Zhang Feng, Application of quantum neural network based on rough set in transformer fault diagnosis, In: Second International Conference On Machine Learning and Computing. 2010: 978-980.

[47] Wang Wenjuan, Xie Bin, The evaluation and application research about regional innovation capability based on rough set and BP neural network, In: Second International Conference on Information and Computing Science, 2009: 308-311.

[48] Cheng Jaohong, Chen Hueiping, Lin Yimin, A hybrid forecast marketing timing model based on probabilistic network,rough set and C4.5, Expert systems with Applications, 2010,37(3):1814-1820.

[49] Xue feng, Ke kong-lin, Five-category evaluation of commercial bank's load by the integration of rough set and neural network, Systems engineering theory & practice, 2008, 28(1): 40-45.

[50] Li Cuiling, Zhang Hao, Wang Jian, Zhao Rongyong, A new pattern recognition model based on heuristic SOM network and rough set theory, IEEE, 2006: 45-48.

[51] Dong Ming, Jiang Huiyu, Li Xiangpeng, A rough CP neural network model based on rough set, In: Third international conference on natural computation, 2007, 1: 735-739.

[52] Gao Wei, Wen Jingxin, Jiang Nan, Zhao Hai, A study of data fusion based on combining rough set with BP neural network, In: Ninth International Conference on Hybrid Intelligent Systems, 2009, 3: 103-106.

[53] Kumar S., Atri S., Mandoria H. L., A combined classifier to detect landmines using rough set theory and Hebb net learning & fuzzy filter as neural networks, In: International Conference on Processing Systems, 2009: 423-427.

[54] Chiang Junghsien, Ho Shinghua, A combination of rough-based feature selection and RBF neural network for classification selection gene expression, IEEE Transactions on nanobioscience, 2008, 7(1): 91-99.

[55] He Haitao, Zhao Na, YaoLiu, The research on CMAC network model based on rough set for flatness control, In: Fourth International Conference on Innovative Computing, Information and Control, 2009: 1252-1254.

[56] Thangavel K., Pethalakshmi A., Dimensionality reduction based on rough set theory: a review, Appl Soft Comput, 2009, 9(1): 1-12.

[57] Pan Wei, Yi Jinhui, San Ye, Rough set theory and its application in the intelligent systems, In: Proceedings of the 7th world congress on intelligent control and automation, 2008: 3706-3711.

[58] Zhang Dongbo, Wang Yaonan, Fuzzy-rough neural network and its application to vowel recognition, Control Decis, 2006, 21(2): 221-224.

[59] Wang Wei, Mi Hong, The application of rough neural network in RMF model, In: 2010 2nd international Asia conference on informatics in control, automation and robotics, 2010: 210-213.

[60] Setiawan N. A., Venkatachalam P. A., Hani A. F. M., Missing attribute value prediction based on artificial neural network and rough set theory, In: 2008 international conference on bioMedical engineering and informatics, 2008: 306-310.

[61] Dong Chengxi, Wu Dewei, He Jing, Decision analysis of combat effectiveness based on rough set neural network, In: Fourth international conference on natural computation, 2008: 227-231.

[62] Pedrycz W., Vukovich G., Granular neural networks, Neurocomputing, 2001, 36(1-4): 205-224.

[63] Gong Wei, Application of rough set and fuzzy neural network in information handling, In: International Conference on Networking and Digital Society, 2009, 2: 36-39.

[64] Hong Jing, Zhao Yi, The study of the gas emission prediction model based on fuzzy-rough set nerural network, In: Second International on Machine Learning and Computing, 2010: 313-316.

[65] Zhang Xiaoguang, Sun Zheng, Ruan Dianxu, Xu Guiyun, Liu Xiaoping, Research and application of integration of RS and FNN in defect recognition of welding, Journal of Harbin institute technology, 2009, 41(1): 141-143.

[66] Wu Bin, Guo Xiansheng, Design of neural network controller based on rule encoded by rough sets, Journal of University of Electronic Science and Technology of CHINA, 2006, 35(5): 798-781.

[67] Han Lin, Xue Jing, Zhang Tong, Fault diagnosis of rotating machine based on rough set and neural network, Computer Measurement & control, 2010, 15(1): 64-67.
[68] Dou Yudan, Yuan Yongbo, Zhang Mingyuan, A model for assessment of earthquake damages and losses based on rough genetic BP networks, Construction Management Modernization, 2010, 24(1): 29-32.
[69] Li Daoguo, Miao Duoqian, Zhang Hongyun, The theory, model and method of Granular Computing, Journal of Fudan (Natural Science), 2004, 43(5), 837-841.
[70] Zhu Hong, Ding Shifei, Li Xu, and Zhang Liwen, Research and Development of Granularity Clustering, Communications in Computer and Information Science, 2011, 159(5): 253-258.
[71] Li Daoguo, Miao Duoqian, et al. The Theory, Model and Method of Granular Computing. Journal of Fudan (Natural Science) 43(5), 837-841 (2004).
[72] Li Daoguo, Miao Duoqian, Zhang Dongxing, et al., An overview of granular computing, Computer Science, 2005, 32(9): 1-12.

Li Hui was born in Xuzhou, China, in 1978, received his MS degree from Suzhou University of Science and Technology in 2007. He is a doctoral student of China University of Mining and Technology and his advisor was Professor Ding Shifei. His research interests are in the areas of Artificial Intelligence, Pattern Recognition, Neural networks, Granular Computing et al.

Ding Shifei received his bachelor's degree and master's degree from Qufu Normal University in 1987 and 1998 respectively. He received his Ph.D degree from Shandong University of Science and Technology in 2004. He received postdoctoral degree from Key Laboratory of Intelligent Information Processing, Institute of Computing Technology, and Chinese Academy of Sciences in 2006. And now, he works in China University of Mining and Technology as a professor and Ph.D supervisor. His research interests include intelligent information processing, pattern recognition, machine learning, data mining, and granular computing et al. He has published 3 books, and more than 100 research papers in journals and international conferences.

In addition, Prof. Ding is a senior member of China computer Federation (CCF), and China Association for Artificial Intelligence (CAAI). He is a member of professional committee of Artificial Intelligence and Pattern Recognition, CCF, professional committee of distributed intelligence and knowledge engineering, CAAI, professional committee of machine learning, CAAI, and professional committee of rough set and soft computing, CAAI. He is an associate Editor-in-Chief, Topic Editor-in-Chief (TEIC) for International Journal of Digital Content Technology and its Applications (JDCTA), acts as an editor for Journal of Convergence Information Technology (JCIT), International Journal of Digital Content Technology and its Applications (JDCTA). Meanwhile, he is a reviewer for Journal of Information Science (JIS), Information Sciences (INS), Computational Statistics and Data Analysis (CSTA), IEEE Transactions on Fuzzy Systems (IEEE TFS), Applied Soft Computing (ASOC), Computational Statistics and Data Analysis (CSDA), International Journal of Pattern Recognition and Artificial Intelligence (IJPRAI) et al.