Quant. Phys. Lett. **10**, No. 3, 49-54 (2021)

Quantum Physics Letters
An International Journal

http://dx.doi.org/10.18576/qpl/100302

The Squeezing Property of Coherently Driven Degenerate Three-Level Atom in a Closed Cavity

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Received: 7 Jun. 2021, Revised: 21 Aug. 2021, Accepted: 23 Aug. 2021
Published online: 1 Dec. 2021

Abstract: The study analyzed the squeezing and statistical properties of the light generated by a degenerate three-level atom driven by coherent light. It also discussed a closed cavity coupled to a vacuum reservoir via a single-port mirror. The analysis is carried out by putting the noise operators associated with the vacuum reservoir in normal order and by applying the large-time approximation. We have found that the photon number statistics is sub-Poissonian for $\eta < 0.15$ and super-Poissonian for $\eta \geq 0.15$. In addition, the quadrature variance and the quadrature squeezing of a cavity mode were determined and the result shows that the cavity mode is in a squeezed state and the squeezing occurs in the minus quadrature. The maximum quadrature squeezing is found to be 43.6%, below the coherent-state level.

Keywords: Degenerate three-level atom, Cavity mode, Quadrature squeezing.

1 Introduction

In recent years, there has been much interest in the interaction of a three-level atom with a cavity mode [1–6]. Quadrature squeezing is a non-classical property of light and a fascinating subject in quantum optics. Several authors have studied the quantum effects of squeezed light extensively [6, 10–14]. The quantum noise in one quadrature is below the coherent state level in a squeezed state, and the product of the uncertainties in the two quadratures meets the uncertainty relation [2]. Squeezed light has potential applications in the detection of weak signals. It also low-noise communications due to the quantum noise reduction accomplished below the coherent state level [9, 12].

A three-level atom makes transitions from the top to the bottom level through the intermediate level generated by two photons. The two photons are strongly correlated, and this correlation leads the light produced by a three-level atom to be squeezed [7, 15]. A subharmonic generator, a three-level laser pumped by electron bombardment, and a three-level laser pumped by coherent light (under some conditions) have all been shown to generate squeezed light, with a maximum quadrature squeezing of 50% [3, 9]. Different authors have investigated the statistical and squeezing properties of light emitted by a three-level atom in which the top and bottom levels are coupled by a coherent light [4, 16]. They believe that such a system can produce squeezed light over a wide range of coherent light amplitudes.In this case, the coupling of the top and bottom levels induced the squeezing.

The study considered a degenerate three-level atom driven by coherent light and a closed cavity coupled to a vacuum reservoir via a single-port mirror. The noise operators associated with the vacuum reservoir have been placed in normal order for the calculations. The probability of the atom being in the top, intermediate, or bottom level has been determined using the cavity mode operators obtained by solving the quantum Langevin equations and using the large-time approximation. The mean and variance of the photon number were also determined using the steady-state solutions of the quantum Langevin equations for the cavity mode operator. Moreover, the researcher calculated the quadrature variance and the quadrature squeezing of a degenerate three-level atom driven by coherent light.

2 Operator Dynamics

We consider here a degenerate three-level atom available in a closed cavity coupled to a vacuum reservoir and

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driven by coherent light. The top, middle and bottom levels of the atom denoted by $|a\rangle$, $|b\rangle$ and $|c\rangle$ respectively. The transitions $|a\rangle \to |b\rangle$ and $|b\rangle \to |c\rangle$ are assumed to be dipole permitted, while the transition $|a\rangle \to |c\rangle$ is assumed to be dipole forbidden [18]. Consider the case where the cavity mode is in resonance with both the $|a\rangle \to |b\rangle$ and $|b\rangle \to |c\rangle$ transitions.

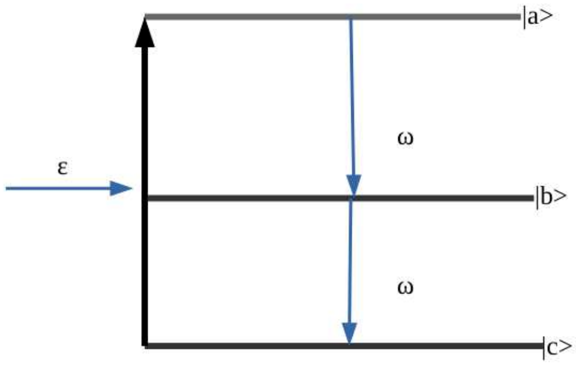

Fig. 1: A three -level atom in a closd cavity.

Thus the interaction of coherent light with a three-level atom can be described by the Hamiltonian

$$\hat{H}' = \frac{i\Omega}{2}\left(\hat{\sigma}_{ac}^{\dagger} + \hat{\sigma}_{ac}\right), \tag{1}$$

where $\Omega = 2g\varepsilon$ is the Rabi frequency, with ε being the amplitude of the coherent light and with g being the coupling constant.

Furthermore, the interaction of a degenerate three-level atom with a cavity mode can be described at resonance by the Hamiltonian

$$\hat{H}'' = ig\left(\hat{\sigma}_a^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_a + \hat{\sigma}_b^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_b\right) \tag{2}$$

As a consequence of Eqs. (1) and (2) the interaction of a coherently driven three-level atom with a cavity mode can be described by the total Hamiltonian

$$\hat{H} = \frac{i\Omega}{2}\left(\hat{\sigma}_{ac}^{\dagger} + \hat{\sigma}_{ac}\right) + ig\left(\hat{\sigma}_a^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_a + \hat{\sigma}_b^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_b\right), \tag{3}$$

Where

$$\hat{\sigma}_a = |b\rangle\langle a|, \qquad \hat{\sigma}_b = |c\rangle\langle b| \qquad and \qquad \hat{\sigma}_{ac} = |c\rangle\langle a|, \tag{4}$$

are lowering operator. Moreover, evaluating the time evolution of the atomic operators using the Heisenberg Equations.

$$\frac{d\langle\hat{A}\rangle}{dt} = -i\langle[\hat{A},\hat{H}]\rangle. \tag{5}$$

Employing Eq.(3), along with Eq.(5), one can easily obtain

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = \frac{\Omega}{2}\langle\hat{\sigma}_b^{\dagger}\rangle + g\langle\hat{\eta}_b\hat{a}\rangle - g\langle\hat{\eta}_a\hat{a}\rangle + g\langle\hat{a}^{\dagger}\hat{\sigma}_{ac}\rangle, \tag{6}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = -\frac{\Omega}{2}\langle\hat{\sigma}_a^{\dagger}\rangle + g\langle\hat{\eta}_c\hat{a}\rangle - g\langle\hat{\eta}_b\hat{a}\rangle + g\langle\hat{a}^{\dagger}\hat{\sigma}_{ac}\rangle, \tag{7}$$

$$\frac{d}{dt}\langle\hat{\sigma}_{ac}\rangle = \frac{\Omega}{2}\left(\langle\hat{\eta}_c\rangle - \langle\hat{\eta}_a\rangle\right) + g\langle\hat{a}\hat{\sigma}_d\rangle - g\langle\hat{a}\hat{\sigma}_a\rangle, \tag{8}$$

$$\frac{d}{dt}\langle\hat{\sigma}_{aa}\rangle = \frac{\Omega}{2}\left(\langle\hat{\sigma}_{ac}^{\dagger}\rangle + \langle\hat{\sigma}_{ac}\rangle\right) + g\langle\hat{a}^{\dagger}\hat{\sigma}_a\rangle + \langle\hat{\sigma}_a^{\dagger}\hat{a}\rangle, \tag{9}$$

$$\frac{d}{dt}\langle\hat{\sigma}_{bb}\rangle = g\left(\langle\hat{\sigma}_b^{\dagger}\hat{a}\rangle - \langle\hat{\sigma}_a^{\dagger}\hat{a}\rangle\right) + g\left(\langle\hat{a}^{\dagger}\hat{\sigma}_b\rangle - \langle\hat{a}^{\dagger}\hat{\sigma}_a\rangle\right) \tag{10}$$

$$\frac{d}{dt}\langle\hat{\sigma}_{cc}\rangle = -\frac{\Omega}{2}\left(\langle\hat{\sigma}_{ac}^{\dagger}\rangle + \langle\hat{\sigma}_{ac}\rangle\right) - g\left(\langle\hat{\sigma}_b^{\dagger}\hat{a}\rangle + \langle\hat{a}^{\dagger}\hat{\sigma}_b\rangle\right), \tag{11}$$

where

$$\langle\hat{\eta}_a\rangle = \langle|a\rangle\langle a|\rangle = \rho_{aa}, \langle\hat{\eta}_b\rangle = \langle|b\rangle\langle b|\rangle = \rho_{bb}, \qquad and$$
$$\langle\hat{\eta}_c\rangle = \langle|c\rangle\langle c|\rangle = \rho_{cc}, \tag{12}$$

with ρ_{aa}, ρ_{bb}, and ρ_{cc} being the probability for the atom to be the top, intermediate and bottom levels, respectively. We assume that the cavity modes are coupled to a vacuum reservoir via a single-port mirror. Furthermore, by putting the noise operator associated with the vacuum reservoir in normal order. As a result, the noise operators would have no effect on the cavity mode operator's dynamics. As a consequence, the noise operator can be dropped and the quantum Langevin equation for the operator $\hat{a}$ can be written as

$$\frac{d\hat{a}}{dt} = -i[\hat{a},\hat{H}] - \frac{\kappa}{2}\hat{a} \tag{13}$$

where κ is the cavity damping constant. Using Eq.(3), we can now easily find

$$\frac{d\hat{a}}{dt} = -\frac{\kappa}{2}\hat{a} - g(\hat{\sigma}_a + \hat{\sigma}_b) \tag{14}$$

We have seen that Eqs. (6-11) are nonlinear and coupled differential equations and it is difficult to find the time dependent solutions of these equations. The large-time approximation scheme [17] can be used to solve this problem. When this approximation is added to Eq. (14) the result is

$$\hat{a} = -\frac{2g}{\kappa}(\hat{\sigma}_a + \hat{\sigma}_b) \tag{15}$$

With the aid of (15) (with the time argument suppressed), the aforementioned equations can be rewritten as

$$\frac{d}{dt}\langle\hat{\sigma}_a\rangle = -\gamma_c\langle\hat{\sigma}_a\rangle + \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle, \tag{16}$$

$$\frac{d}{dt}\langle\hat{\sigma}_b\rangle = \gamma_c\langle\hat{\sigma}_a\rangle - \frac{\gamma_c}{2}\langle\hat{\sigma}_b\rangle - \frac{\Omega}{2}\langle\hat{\sigma}_a^\dagger\rangle, \tag{17}$$

$$\frac{d}{dt}\rho_{ac} = \frac{\Omega}{2}(\rho_{cc}-\rho_{aa}) - \frac{\gamma_c}{2}\rho_{ac}, \tag{18}$$

$$\frac{d}{dt}\rho_{aa} = -\gamma_c\rho_{aa} + \frac{\Omega}{2}(\rho_{ca}+\rho_{ac}), \tag{19}$$

$$\frac{d}{dt}\rho_{bb} = \gamma_c(\rho_{aa}-\rho_{bb}), \tag{20}$$

$$\frac{d}{dt}\rho_{cc} = -\gamma_c\rho_{bb} + \frac{\Omega}{2}(\rho_{ca}+\rho_{ac}), \tag{21}$$

where

$$\gamma_c = \frac{4g^2}{\kappa}, \tag{22}$$

which is the stimulated emission decay constant. The steady-state solutions of Eqs. (16-21),
are given by

$$\langle\hat{\sigma}_a\rangle = \frac{\Omega}{2\gamma_c}\langle\hat{\sigma}_b^\dagger\rangle, \tag{23}$$

$$\langle\hat{\sigma}_b\rangle = \frac{\Omega}{\gamma_c}\langle\hat{\sigma}_a^\dagger\rangle, \tag{24}$$

$$\rho_{ac} = \frac{\Omega}{\gamma_c}(\rho_{cc}-\rho_{aa}), \tag{25}$$

$$\rho_{aa} = \frac{\Omega}{\gamma_c}\rho_{ac}, \tag{26}$$

$$\rho_{aa} = \rho_{bb}, \tag{27}$$

since ρ_{ac} is real, we can see that $\rho_{ac} = \rho_{ca}$. With the aid of the identity

$$\rho_{aa}+\rho_{bb}+\rho_{cc} = 1, \tag{28}$$

and using Eq. (25), and Eq. (27), over come to :

$$\rho_{ac} = \frac{\gamma_c\Omega}{\gamma_c^2+3\Omega^2}, \tag{29}$$

$$\rho_{aa} = \frac{\Omega^2}{\gamma_c^2+3\Omega^2}, \tag{30}$$

and

$$\rho_{cc} = \frac{\gamma_c^2+\Omega^2}{\gamma_c^2+3\Omega^2}. \tag{31}$$

For strong coherent light ($\Omega \ggg \gamma_c$) there is an equal probability to found the atom in all levels. That is, $\rho_{aa} = \rho_{bb} = \rho_{cc} = \frac{1}{3}$. For weak coherent light ($\Omega \lll \gamma_c$) there is a high probability to found the atom in the bottom level than the top and middle levels. Which means

$$\rho_{aa} = \rho_{bb} \simeq 0 \qquad and \qquad \rho_{cc} = 1. \tag{32}$$

3 Photon Statistics

The mean and variance of the photon number define the photon statistics of light emitted by a three-level atom. The relationship between the mean and variance of the photon number will be important in classifying the photon statistics of a light mode. Thus the photon statistics of light mode for which $(\Delta n)^2 = \bar{n}$ is referred to as Poissonian and the photon statistics of a light mode for which $(\Delta n)^2 > \bar{n}$ is called Supper-Poissonian. Otherwise the photon statistics is said to be Sub-Poissonian [2].

3.1 *The mean of the photon number*

For the cavity mode, we specified the photon number operator as

$$\hat{n} = \hat{a}^\dagger\hat{a}. \tag{33}$$

At steady-state, the three-level atom's mean photon number is written as

$$\bar{n} = \langle\hat{a}^\dagger\hat{a}\rangle. \tag{34}$$

When Eqs. (15), (27), and (30) are introduced into Eq.(34), the result is

$$\bar{n} = \frac{\gamma_c}{\kappa}2\rho_{bb} = \frac{\gamma_c}{\kappa}\left(\frac{2\Omega^2}{\gamma_c^2+3\Omega^2}\right). \tag{35}$$

We let that, $\eta = \frac{\Omega}{\gamma_c}$, the mean photon number of the cavity light turns to be;

$$\bar{n} = \frac{\gamma_c}{\kappa}\left(\frac{2\eta^2}{1+3\eta^2}\right). \tag{36}$$

3.2 *The variance of the photon number*

The photon number variance for cavity light can be expressed as

$$(\Delta n)^2 = \langle\hat{a}^\dagger\hat{a}\hat{a}^\dagger\hat{a}\rangle - \langle\hat{a}^\dagger\hat{a}\rangle^2. \tag{37}$$

Since $\hat{a}$ is Gaussian variable with zero mean, the variance of the photon number can be
written as

$$(\Delta n)^2 = \langle\hat{a}^\dagger\hat{a}\rangle\langle\hat{a}\hat{a}^\dagger\rangle + \langle\hat{a}^2\rangle\langle\hat{a}^{\dagger 2}\rangle. \tag{38}$$

When you plug Eq. (15) and its adjont into Eq. (38), you get

$$\langle\hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}(\rho_{bb}+\rho_{cc}), \tag{39}$$

and

$$\langle\hat{a}^2\rangle\langle\hat{a}^{\dagger 2}\rangle = \left(\frac{\gamma_c}{2}\right)^2\rho_{ac}^2. \tag{40}$$

Equations (35), (39) and (40) are now used to obtain

$$\begin{aligned}(\Delta n)^2 &= \frac{\gamma_c}{\kappa}2\rho_{bb}\frac{\gamma_c}{\kappa}(\rho_{bb}+\rho_{cc})+\left(\frac{\gamma_c}{\kappa}\right)^2\rho_{ac}^2,\\ &= \left(\frac{\gamma_c}{\kappa}\right)^2\left(\frac{2\Omega^2}{\gamma_c^2+3\Omega^2}\right)\left(\frac{\Omega^2}{\gamma_c^2+3\Omega^2}+\frac{\gamma_c^2+\Omega^2}{\gamma_c^2+3\Omega^2}\right)\\ &+\left(\frac{\gamma_c}{\kappa}\right)^2\left(\frac{\gamma_c\Omega}{\gamma_c^2+3\Omega^2}\right)^2,\\ &= \frac{4\gamma_c^2}{\kappa^2}\left(\frac{\Omega^2}{\gamma_c^2+3\Omega^2}\right)^2+\frac{3\gamma_c^2}{\kappa^2}\left(\frac{\Omega\gamma_c}{\gamma_c^2+3\Omega^2}\right)^2,\\ &= \bar{n}^2\left(1+\frac{3}{4\eta^2}\right). \end{aligned} \tag{41}$$

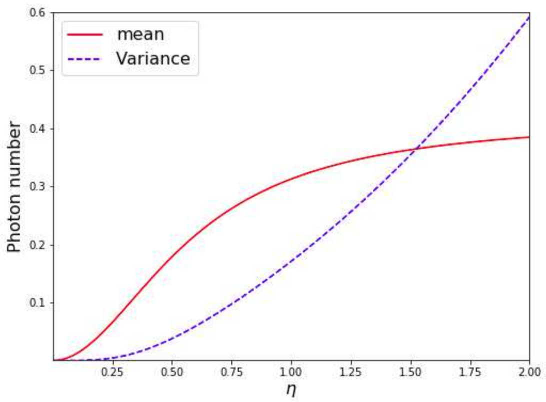

Fig. 2: Plots of the mean of the photon number $\bar{n}$ [Eq. (36) solid curve] and the variance of the photon number $(\Delta n)^2$ [Eq. (41) dashed curve] versus η for $\gamma_c = 0.5$ and $\kappa = 0.8$.

It can be seen from the plots in figure 2,that the photon number statistics are sub-Poissonian for $\eta < 0.15$ and super-Poissonian for $\eta \geq 0.15$. It also showed that, for strong coupling ($\eta \gg 1$) the mean and variance of the photon number was independent of external field radiation(Ω) and for weak coupling ($\eta \ll 1$) the mean and variance of the photon number influenced by external field radiation(Ω) and the photon number variance of the cavity light increases with η.

4 Quadrature squeezing of the cavity mode

The quantum properties of a cavity mode powered by coherent light will be investigated. To this end, first calculate the plus and minus quadrature variances and the quadrature squeezing relative to the quadrature variance of the coherent state was estimated using the results obtained. Then determined the quadrature variance for the cavity mode formed by a degenerate three-level atom deriven by coherent light.
The cavity mode's squeezing properties are represented by two quadrature operators defined by

$$\hat{a_+} = \hat{a^\dagger} + \hat{a}, \tag{42}$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{43}$$

One can easily verify that

$$[\hat{a}_+, \hat{a}_-] = 2i[\hat{a}, \hat{a}^\dagger]. \tag{44}$$

On the other hand, the steady-state solution of Eq. (14) is given by

$$\hat{a} = -\frac{2g}{\kappa}(\hat{\delta}_a + \hat{\delta}_b), \tag{45}$$

and by combining this result with its adjoint, we arrive at

$$[\hat{a}, \hat{a}^\dagger] = \frac{\gamma_c}{\kappa}(\hat{\eta}_c - \hat{\eta}_a). \tag{46}$$

In view of this relation, Eq. (44) becomes

$$[\hat{a}_+, \hat{a}_-] = 2i\frac{\gamma_c}{\kappa}(\hat{\eta}_c - \hat{\eta}_a). \tag{47}$$

Using this result and Eqs. (30) and (31) as a guide, it can be determined that

$$\begin{aligned}\Delta a_+ \Delta a_- &\geqslant \frac{\gamma_c}{\kappa}\left(\frac{\gamma_c^2}{\gamma_c^2+3\Omega^2}\right),\\ &\geqslant \frac{\gamma_c}{\kappa}\left(\frac{1}{1+3\eta^2}\right).\end{aligned} \tag{48}$$

Thus, for the quadrature operators, Eq.(48) represents the uncertainty relation.

4.1 Quadrature variance

The cavity mode's quadrature variance is expressed as

$$(\Delta a\pm)^2 = \langle\hat{a}^\dagger\hat{a}\rangle + \langle\hat{a}\hat{a}^\dagger\rangle \pm \langle\hat{a}^{\dagger 2}\rangle \pm \langle\hat{a}^2\rangle \mp \langle\hat{a}^2\rangle \mp \langle\hat{a}^\dagger\rangle^2 \pm \langle\hat{a}\rangle^2 - 2\langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle.$$

Applying Eq.(45) and its adjoint Once more, one gets

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{\gamma_c}{\kappa}(\rho_{aa}+\rho_{bb}) = \frac{2\gamma_c}{\kappa}\rho_{aa} \tag{49}$$

and

$$\langle\hat{a}\hat{a}^\dagger\rangle = \frac{\gamma_c}{\kappa}(\rho_{bb}+\rho_{cc}). \tag{50}$$

On account of Eqs. (30) and (31), We easily get

$$\langle\hat{a}^\dagger\hat{a}\rangle = \frac{\gamma_c}{\kappa}\left(\frac{\Omega^2}{\gamma_c^2+3\Omega^2}\right) \tag{51}$$

and

$$\langle \hat{a}\hat{a}^{\dagger} \rangle = \frac{\gamma_c}{\kappa} \left(\frac{2\Omega^2 + \gamma_c^2}{\gamma_c^2 + 3\Omega^2} \right). \tag{52}$$

Eq. (45) can also be used to write

$$\langle \hat{a}^2 \rangle = \frac{\gamma_c}{\kappa} (\langle \hat{\sigma}_a \hat{\sigma}_a \rangle + \langle \hat{\sigma}_a \hat{\sigma}_b \rangle + \langle \hat{\sigma}_b \hat{\sigma}_a \rangle + \langle \hat{\sigma}_b \hat{\sigma}_b \rangle). \tag{53}$$

Now with the help of the identity

$$\hat{\sigma}_a \hat{\sigma}_a = 0 \qquad and \qquad \hat{\sigma}_b \hat{\sigma}_b = 0, \tag{54}$$

and using Eq. (29), we were able to arrive at

$$\langle \hat{a}^2 \rangle = \frac{\gamma_c}{\kappa} \left(\frac{\gamma_c \Omega}{\gamma_c^2 + 3\Omega^2} \right), \tag{55}$$

and

$$\langle \hat{a}^{\dagger 2} \rangle = \frac{\gamma_c}{\kappa} \left(\frac{\gamma_c \Omega}{\gamma_c^2 + 3\Omega^2} \right) \tag{56}$$

Now in view of Eq. (45) and the fact that the cavity light is initially in a vacuum state,

$$\langle \hat{a} \rangle^2 = \langle \hat{a}^{\dagger} \rangle^2 = \langle \hat{a} \rangle \langle \hat{a}^{\dagger} \rangle = 0. \tag{57}$$

Incorporating Eqs. (52), (53), (56), (57), and (58) into Eq. (49), one obtains

$$\begin{aligned} (\Delta a \pm)^2 &= \frac{\gamma_c}{\kappa} \left(\frac{4\Omega^2 + \gamma_c^2}{\gamma_c^2 + 3\Omega^2} \pm \frac{2\gamma_c \Omega}{\gamma_c^2 + 3\Omega^2} \right), \\ &= \frac{\gamma_c}{\kappa} \left(\frac{4\Omega^2 + \gamma_c(\gamma_c \pm \Omega)}{\gamma_c^2 + 3\Omega^2} \right). \end{aligned} \tag{58}$$

Involve the relation $\eta = \frac{\Omega}{\gamma_c}$ in Eq. (59),the quadrature variance can be written as

$$(\Delta a_+)^2 = \frac{\gamma_c}{\kappa} \left(\frac{4\eta^2 + 2\eta + 1}{1 + 3\eta^2} \right), \tag{59}$$

and

$$(\Delta a_-)^2 = \frac{\gamma_c}{\kappa} \left(\frac{4\eta^2 - 2\eta + 1}{1 + 3\eta^2} \right). \tag{60}$$

From the above, we observed that only $(\Delta a_-)^2$ is less than the quadrature variance of the coherent state.
For $\gamma_c \gg \Omega$,

$$(\Delta a_+)(\Delta a_-) \geqslant \frac{\gamma_c}{\kappa}. \tag{61}$$

For a coherent light, this is the quadrature uncertainty relation. Coherent light defined as a light mode in which the uncertainties in two quadratures are equal and satisfy minimum uncertainty relation [2].

The plot in figure 3, showed that $(\Delta a_-)^2 < \frac{\gamma_c}{\kappa}$ where $\frac{\gamma_c}{\kappa} = 0.625$ and we have seen that coherently driven degenerate three level atom is in squeezed state and squeezing occurs for $\Omega < \gamma_c$, with maximum squeezing being stable against small fluctuation of Ω.

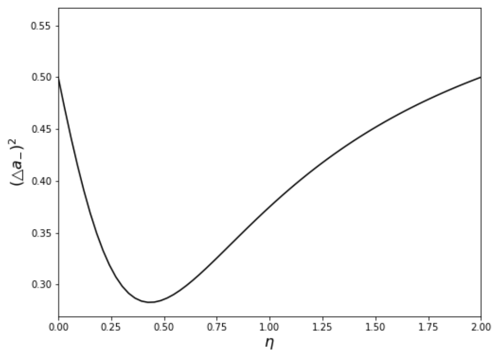

Fig. 3: Plot of the mins quadrature variance $(\Delta a_-)^2$ [Eq. (61)], at steady state versus η for $\kappa = 0.8$ and $\gamma_c = 0.5$.

4.2 *Quadrature squeezing*

We characterize quadrature squeezing in terms of the coherent state's quadrature variance as [11]

$$s = \frac{(\Delta a_{\pm})^2 c - (\Delta a_-)^2}{(\Delta a_{\pm})^2 c} = 1 - \frac{(\Delta a_-)^2}{(\Delta a_-)^2 c}. \tag{62}$$

We recall that the light emitted by a three-level atom well above threshold is coherent.

$$(\Delta a_{\pm})^2 c = \frac{\gamma_c}{\kappa}, \tag{63}$$

where $(\Delta a_{\pm})^2 c$ is the quadrature variance of coherent light. The qudrature squeezing is defined as [15]

$$S = 1 - \frac{(\Delta a_-)^2}{(\Delta a_-)^2_c} \tag{64}$$

With the help of E.qs. (61) and (64), we arrived at

$$S = \frac{\frac{\gamma_c}{\kappa} \left(1 - \left[\frac{4\eta^2 - 2\eta + 1}{1 + 3\eta^2} \right] \right)}{\frac{\gamma_c}{\kappa}} = \frac{2\eta - \eta^2}{1 + 3\eta^2} \tag{65}$$

The maximum quadrature squeezing of the cavity mode is 43.6% below the coherent-state level, as shown in Fig. (4), and this occurs at $\eta = 0.42$. We discovered that the degree of squeezing for a cavity mode increases between 0 and 0.42, but decreases between 0.42 and 2.

5 Conclusion

The study showed the statistical and squeezing properties of light produced by a coherently driven degenerate three-level atom in a closed cavity coupled to a vacuum reservoir through a single-port mirror. Employing the solutions of the quantum Langevin equations, the

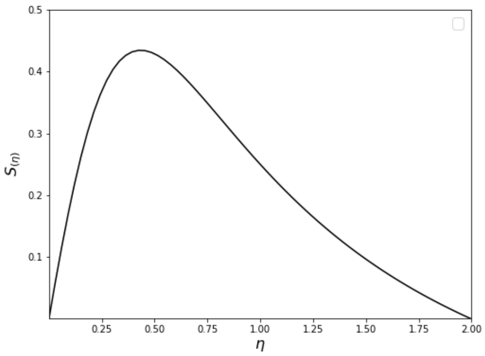

Fig. 4: Plot of the quadrature squeezing s(η) versus η

probability for the atom to be in the top, intermediate, or bottom level were determined by putting the noise operators associated with the vacuum reservoir in normal order. Our result shows that at a steady- state the atom has an equal probability to be in the top or intermediate level.

On the other hand, the mean and variance of the photon number have been determined using the steady-state solutions of the quantum Langevin equations for the cavity mode operator. According to the findings, the photon number statistics were sub-Poissonian for $\eta < 0.15$ and super-Poissonian for $\eta \geq 0.15$. Also, the quadrature variance and the quadrature squeezing of a cavity mode were calculated and our analysis shows that, the cavity mode is in a squeezed state and the squeezing occurs in the mins quadrature. The maximum quadrature squeezing being 43.6% below the coherent-state level. This occurs at $\eta = 0.42$. We have also seen that the degree of squeezing for a cavity mode increases between 0 and 0.42 but, decreases between 0.42 and 2.

Acknowledgments

It is my pleasure to thank Dr. Hulugirgesh Degefu and Mr.Dejene Zewdie for their help,contribution of valuable suggestions and encouragement during my research.

I am also grateful, to Quantum Physics Letters (QPL)- for accepting this paper, and to the anonymous referee- for a careful checking of the details and for helpful comments that improved this paper.

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