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Performance Analysis of Two-hop Routing in Node-markovian Graph

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Abstract: In order to describe the strong dependence between the existence and absence of nodes in different times, we propose the Node-markovian graph. At every time step, nodes change their state (existence or not) according to a two-state Markov process. If nodes do not discard the messages stored in them when they quit from the graph, we think that they have memory, or they do not have memory, and we study the performance of two-hop routing in both situations. We give the upper and lower bound of the performance in this paper. Surprisingly, simulation and numerical results show that difference of the performance between both cases is small when nodes in the graph exist with bigger probability, but they have bigger difference when the probability is smaller, so we think that it is not necessary for nodes to have memory in some cases.

Keywords: Network Modeling, Temporal Graph, Node-markovian Graph, Two-hop Routing

1 Introduction

In the last decade, graphs and complex networks have been used to study many complex human and communication networks. Typically, graph structures are used to represent interactions between entities such as individuals or organizations, and the central problem in this area is the definition of mathematical models able to capture properties observed in real networks [1]. For example, they use the random graph to indicate the uncertain phenomenon of these relations. But traditional graphs often neglect the time dimension of these interactions which is very common such as in computer networks, social networks, and many other more [2]. A fundamental work on graphs that considered time dimension is in paper [3]. Though it's simple, the formalism introduced therein has been used as framework for many works. Authors in another paper [4] presented theoretical concept of temporal graphs. Tang studied the small world phenomenon in temporal graphs [5], and paper [6] explored the robustness of temporal graphs. Authors in paper [7] use a unified time-varying graph formalism to integrate many dynamic networks and redefine many metrics. Recently, the strong dependence between the existence and absence of a link has been observed. For simplicity, they assume that the state of a link only depends on the state in above step, so the Edge-markovian graph is proposed [8]. Though there are many works oriented to the temporal graph field, most of them focus only on the evolving characteristics of the links. We think that nodes also evolve with time, for example, if we regard peoples in a school as a network, the number of nodes in the system will be different in different times, and nodes will exist in the working day and be absence on the weekend.

In fact, some works have considered the phenomenon that new nodes may born and existing nodes may die such as paper [9,10], but they all focus on that the incoming nodes will be connected to which nodes existing in the graph, and this selection will induce what impact on the graph metrics, such as degree and so on. So they failed to capture the evolving rule of nodes themselves. On the other hand, they assume that a node quit from the graph is dead and can not enter into the graph again, so the information stored in them can not be used by other nodes in future, but in our graph nodes can join in the graph again. Further, the state of the nodes may have strong dependence in different time steps. For example, in wireless sensor networks, nodes may switch off or on to save energy, but because the temporal-correlation of data, if a sensor is on in this time step, it may be off in next step

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with bigger probability, because data may only have little difference and it is not necessary to detect again.

In real environment, the dependence may be very complex and it is hard to model. Even if we can model the phenomenon, we think that the analysis on them will be very complex. For simplicity, we also assume that the state of a node only depends on the state in above step and propose the Node-markovian graph. Because nodes may discard messages when they quit from the graph, we divide the graph into two cases, that is, no-memory (discard) and with-memory (not discard). Based on the model, we study the performance of two-hop routing, because this method is very important in information diffusion. Very recently, there are many papers in this field, such as [11,12]. However, most of them are oriented to the exponential model.

Because many nodes may be absence in the network, the connectivity of the graph cannot be sure all the time. So the end to end path between two nodes may not exist when they want to communicate with each other which is similar to that in Delay Tolerant Networks (DTN) [13]. DTN is a very hot topic recently and it is proposed to support many emerging networking applications, where end-to-end connectivity cannot be assumed, examples include deep-space exploration [14], military networks, etc. Any two nodes can communicate with each other only when they come into the transmission range of each other in DTN. In order to overcome the network partitions, nodes of DTN communicate through a ”store-carry-forward” mode. Due to the node mobility, different links come up and down. If the sequence of connectivity graphs over a time interval is overlapped, then an end-to-end path might exist, so the message should be forwarded over the existing link, stored and carried at the next hop until the next link comes up, and so on and so forth.

The basic routing method in DTN is epidemic routing in which each node receiving the message carries it as it moves, and then forwarding it to all new nodes it encounters which does not have the message yet. Obviously, this would consume large energy, in order to use resource efficiently some economic methods such as two-hop is proposed. Authors in paper [11] study the optimal control problem of two-hop in DTN and [15] study the performance with heterogeneous nodes, a more recent paper [12] consider the impact of node's selfish on two-hop routing. But to the best of our knowledge, none of the papers explore the problem under different message size and they all assume that message can be transmitted instantly. On the other side, DNT was molded as exponentially model before, but this model failed to capture the strong dependence between the existence (or the absence) of a link. To overcome this problem, the edge-Markovian graph is proposed [16].

Most of the works above focused on the evolving rule of links, and they ignored the evolving rule of nodes.

To our best knowledge, we are the first to attempt to model the strong dependence in nodes' evolving process and study the performance of two-hop routing method based on this model.

2 Network Model

Consider a network with a source S, N relay nodes and a destination D, so the total number of nodes is $N+2$. All nodes exist in the graph at the same time can communicate with each other. We adopt a discrete time model. Considering time slot duration Δ, the t-th slot corresponds to interval $[t\Delta,(t+1)\Delta]$. Let $S_i(t)$ denote the state of node i in t-th slot. If $S_i(t)=0$, we say that node i is absence in t-th slot, and if $S_i(t)=1$, node i exists in the graph. Every node changes its state at the beginning of a slot according to a two-state Markov process and keeps invariance in the same slot, that is, if $S_i(t)=0$, $S_i(t+1)=1$ with probability α and equals to 0 with probability $1-\alpha$. Similarly, if $S_i(t)=1$, $S_i(t+1)=0$ with probability β and keeps invariance with probability $1-\beta$. The transition matrix is shown as follows,

$$M=\begin{pmatrix}1-\alpha & \alpha\\ \beta & 1-\beta\end{pmatrix} \tag{1}$$

This is a Bernoulli process and there exists the stationary distribution. Let π_0 and π_1 indicate the probability that a node is in state 0 and 1 when the system went into stable state, separately, we have $\pi_0=\beta/(\alpha+\beta)$ and $\pi_1=\alpha/(\alpha+\beta)$.

If $\alpha+\beta>1$, we have $\alpha>1-\beta$ and $\beta>1-\alpha$, so the network topology changes frequently, and if $\alpha+\beta<1$, the network changes less frequently. If $\alpha+\beta=1$, we can say that at every slot, nodes are in state 1 with probability α, so it equals to the stationary distribution and can be seen as a special case of the case $\alpha+\beta\neq 1$. In this paper, we assume that S is in state 1 all the time, and other nodes are in state 0 at time 0, so in time interval $[0,\Delta]$, they are in state 1 with probability α. For simplicity, we only consider the case $\alpha+\beta<1$. The source node S created message m at time 0 with maximal lifetime $T\Delta$, so the maximal number of slots equals to T. For abuse of language, we will use T to indicate the maximal lifetime of m. Further, we assume that a node that receives a copy during a time slot can forward it starting from the following time slot. If a node received m, it can be thought as infected. In this paper, we explore the two-hop routing method, that is, if S meets another relay node without message m, it forwards m to this relay node, but relay nodes infected can only forward m to D. We assume that S cannot transmit m to D directly.

3 Evolving Process of Two-hop Routing

According to that whether nodes discard messages stored in them when they quit, we study the evolving process in two situations. For simplicity, we further assumed that the forwarding process starts at the beginning of a slot, and the receiving process turns up when the slot is finished. So the number of infected nodes keeps invariance during one slot. Let $X(t)$ indicate the number of infected relay nodes at the beginning of the t-th slot (we have $X(0)=0$), $F(t)$ indicate the probability that D received m at the beginning of the t-th slot, so our object is to get $F(T)$. Let $F(t,t+1)$ denote the conditional probability that D got message in time interval $[t\Delta,(t+1)\Delta]$ known that it didn't infected before, suppose $H(t)=1-F(t)$, we have,

$$H(t+1)=H(t)(1-F(t,t+1)) \tag{2}$$

$$F(t)=1-\Pi_{i=0}^{t-1}(1-F(i,i+1)) \tag{3}$$

In fact, $F(T)$ is a stochastic variable, and what we calculate is its expectation, by abuse of language we didn't distinguish them in the rest part of this paper.

3.1 No-memory

First, we can have,

$$X(t+1)=\Sigma_{i\in\Omega(t)}e_{t+1}(i)+\Sigma_{i\in\Gamma(t)}\delta_{t+1}(i) \tag{4}$$

The expression of $X(t)$ is shown as formula 4. $\Omega(t)$ indicates the set in which every node has been infected before time $t\Delta$, so the number of elements in this set is $X(t)$. Similarly, $\Gamma(t)$ indicates the set which includes all nodes uninfected and it has $N-X(t)$ elements. Symbol $e_{t+1}(i)$ indicates the event that node i infected before time $t\Delta$ did not discard the message in time interval $[t\Delta,(t+1)\Delta]$, and $\delta_{t+1}(i)$ indicates the event that node i uninfected before received m in time interval $[t\Delta,(t+1)\Delta]$. Because nodes are no-memory, they need to keep in state 1, and we have $p(e_{t+1}(i)=1)=1-\beta$. For abuse of language, in next section, $X(t)$ also denotes the set of infected relay nodes at the beginning of the t-th slot. So nodes in $N-X(t)$ denotes the set of nodes not infected at time $t\Delta$. Because S is in state 1 all the time, we can say that nodes in set $N-X(t)$ was in state 0 at $(t-1)$-slot, so we have $p(\delta_{t+1}(i)=1)=\alpha$. Now, we can get the expectation of $X(t)$ denoted as $E(X(t))$ as follows,

$$\begin{aligned}E(X(t+1)) &= E(\Sigma_{i\in\Omega(t)}e_{t+1}(i))+E(\Sigma_{i\in\Gamma(t)}\delta_{t+1}(i))\\ &= E(\Sigma_{i=1}^{X(t)}(1-\beta))+E(\Sigma_{i=1}^{N-X(t)}\alpha)\\ &= (1-\beta)E(X(t))+\alpha(N-E(X(t)))\\ &= (1-\alpha-\beta)E(X(t))+N\alpha.\end{aligned} \tag{5}$$

Further, we can get that

$$E(X(t+1))-N\pi_1=(1-\alpha-\beta)(E(X(t))-N\pi)$$

Therefore, we can get the expression of $E(X(t))$ now which is shown as follows

$$E(X(t))=N\pi_1-N\pi_1(1-\alpha-\beta)^t \tag{6}$$

Because nodes received a copy during a time slot can forward it only from the next slot, only nodes in set $X(t)$ can forward m to D in t-th slot, so the expectation of $F(t,t+1)$ is,

$$\begin{aligned}&F(t,t+1)=E(p(S_D(t)=1)\mu(t))\\ &=p(S_D(t)=1)(1-E(\Pi_{i=1}^{E(X(t))}p(p(S_i(t)=0))))\end{aligned} \tag{7}$$

This formula means that D will be infected in t-th slot only when D is in state 1 and at least one of the node in set $X(t)$ is also in sate 1 in this slot which is denoted as $\mu(t)$. Because nodes are *no-memory*, nodes in $X(t)$ are in state 1 in $(t-1)$-slot, so it translates to state 0 in next slot with probability β, that is $p(S_i(t)=1)=\beta$. Now our main problem is how to get probability $p(S_D(t)=1)$, and it needs to keep track its value in previous steps, so it's hard to get the accurate value and we give the upper and lower bound in this paper

First, we compute the lower bound and divide it into two cases

Case 1: $X(t)\cap X(t-1)\neq\varnothing$, $\varnothing$ denotes the empty set, so some nodes infected before are in state 1 in $(t-1)$-th slot. If D is also in state 1 in $(t-1)$-th slot, D will be infected. Because we known D is not infected in this slot, we can say that D is in state 0 in $(t-1)$-th slot. So we have $p(S_D(t)=1)=\alpha$.

Case 2: $X(t)\cap X(t-1)=\varnothing$, in this situation, we can not judge the state of D directly. Suppose $p(S_D(t-1)=1)=p1$, we have $p(S_D(t)=1)=p1(1-\beta)+(1-p1)\alpha=p1(1-\alpha-\beta)+\alpha$, because $\alpha+\beta<1$, $p(S_D(t)=1)$ is increasing with $p(S_D(t-1)=1)$ and we know that $p(S_D(t)=1)\geq\alpha$.

From formula 7 we can see that if we let $p(S_D(t)=1)=\alpha$, we can get the lower bound of $F(t)$. To get the upper bound, first we let $\omega(t)$ denote the event that D is not infected in t-th slot. In fact, symbol $F(t,t+1)$ denotes the conditional probability that D is not infected before. So we have,

$$\begin{aligned}&(p(S_D(t)=0),p(S_D(t)=1))=\\ &(p(S_D(t-1)=0|\omega(t-1)),p(S_D(t-1)=1|\omega(t-1)))M\end{aligned} \tag{8}$$

If A is a vector, we use symbol $A[2]$ to denote the second element of the vector. According to the Bayesian formula, we have,

$$\begin{aligned}&p(S_D(t)=1|\omega(t))\\ &=\frac{p(\omega(t)|S_D(t)=1)p(S_D(t)=1)}{p(\omega(t)|S_D(t)=1)p(S_D(t)=1)+p(\omega(t)|S_D(t)=0)p(S_D(t)=0)}\\ &<p(S_D(t)=1)\\ &=\{(p(S_D(t-1)=0|\omega(t-1)),p(S_D(t-1)=1|\omega(t-1)))M\}[2]\\ &<\{(p(S_D(t-1)=0),p(S_D(t-1)=1))M\}[2]\\ &=\{(p(S_D(t-2)=0|\omega(t-2)),p(S_D(t-2)=1|\omega(t-2)))M^2\}[2]\\ &\dots\\ &<\{(p(S_D(0)=0),p(S_D(0)=1))M\}(2)=\{(1,0)M^{t+1}\}[2]\end{aligned} \tag{9}$$

So we have next formula and we can get the upper bound according to this formula.

$$p(S_D(t)=1) \leq \{(1,0)M^{t+1}\}[2] \tag{10}$$

3.2 With-memory

Similar to above case, we can first get next equation,

$$X(t+1) = X(t) + \Sigma_{i\in\Gamma(t)}\delta_{t+1}(i) \tag{11}$$

The evolving rule of $X(t)$ is shown in formula 11. Symbol $\Gamma(t)$ and $\delta_{t+1}(i)$ have the same meanings as above case, that is, $p(\delta_{t+1}(i)=1)=\alpha$. So we can get the expectation of $E(X(t))$ as follows,

$$E(X(t)) = N - N(1-\alpha)^t \tag{12}$$

Because nodes have memory, at $(t-1)$ slot, we can only know nodes in set $X(t)-X(t-1)$ is in state 1, so we get the expectation of $F(t,t+1)$ as follows,

$$F(t,t+1) = \\ p(S_D(t)=1)(1-\beta^{E(X(t)-X(t-1))}\Pi_{i=1}^{E(X(t-1))}p(S_i(t)=0)) \tag{13}$$

In fact, formula 10 is right for all the nodes, so we have,

$$\begin{aligned} p(S_i(t)=0) &= 1-p(S_i(t)=1) \\ &\geq 1-\{(1,0)M^{t+1}\}[2], i=1,2,\ldots,N \end{aligned} \tag{14}$$

Because $F(t,t+1)$ is decreasing with $p(S_i(t)=0)$, we can get the upper bound by combing 10 and 14.

$$\begin{aligned} &p(S_i(t)=0) \\ &= p(S_i(t-1)=0)(1-\alpha)+(1-p(S_i(t-1)=0))\beta \\ &= p(S_i(t-1)=0)(1-\alpha-\beta)+\beta \geq 1-\alpha. \end{aligned} \tag{15}$$

From analysis above, we know $p(S_D(t)=1) \geq \alpha$, so we can get the lower bound by making $p(S_i(t)=0)=1-\alpha$ and $p(S_D(t)=1)=\alpha$.

4 Simulation and Numerical Results

In this section, we will check the accuracy of our model, and we run several simulations using the Opportunistic Network Environment (ONE) simulator [17]. We set $\alpha=0.05$, $\beta=0.57$, $N=50$. Symbol $F(T)$ is also called the delivery ratio in this paper. In the simulation, the number of nodes is 50, and the nodes evolve according to above settings, we run the simulation 20 times. Numerical and simulation results are shown in Figure. 1.

We can see that the simulation results are among the theoretical upper and lower bound, so our analysis is right. When T is smaller than 20 or bigger than 50, the upper and lower bound are tight, but in interval [20, 50] they have bigger difference, so we may get more tight result in future. From the theoretical results, it is hard to say whether the case of *with-memory* is better than the case of *no-memory*. Though simulation result shows that *with-memory*case may have better performance, the difference is very small. From formula 6 and 12 we can see that in the with-memory case the number of infected nodes is very bigger, and it will occupy much more storage space. So we think that nodes do not need to have memory when they quit from the graph in this situation.

Nowwe will check the impact of the parameters α and β, for simplicity we set $\alpha=0.1$, and increase β from 0 to 0.9, because we assume $\alpha+\beta<1$, we do not consider the case of $\beta \geq 0.9$. Result is shown in Figure. 2.

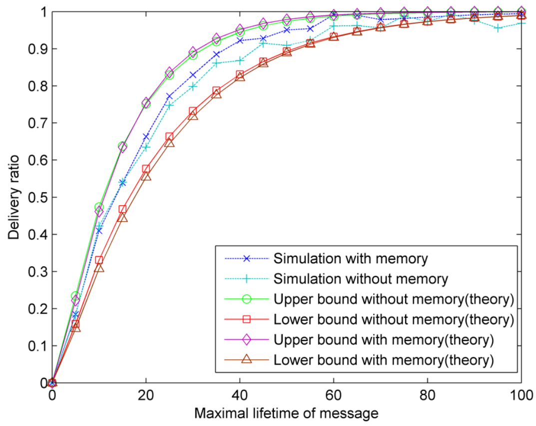

Fig. 1 Delivery ratio with different maximal lifetime T

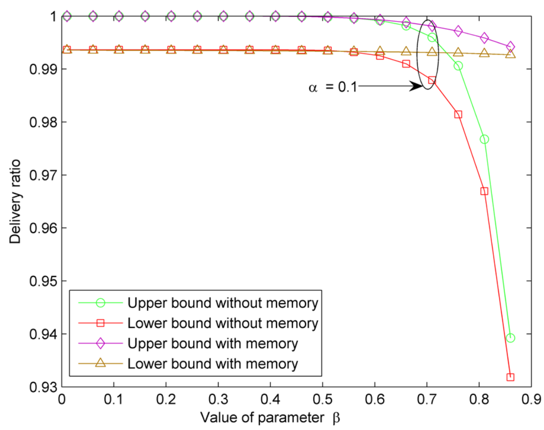

Fig. 2 Delivery ratio with different transitive probability $\beta(\alpha=0.1)$

Figure. 2 shows that the delivery ratio is decreasing with the increasing of β, and this is because when β is bigger, nodes have much bigger probability in state 0 and the graph is very sparse. When $\beta \geq 0.7$, the performance between the two cases has much bigger difference, and the delivery ratio in the case of *with-memory* is higher than the other case, obviously. So we think that when the graph is sparse, the case of *with-memory* may have some superiority.

5 Conclusion

We proposed the *Node*-markovian graph to describe the evolving rule of nodes in this paper. According to that whether nodes discard the messages when they quit from the graph, we divided the graph into two cases: *with-memory* and *no-memory*, and we studied the performance of two-hop routing in both cases.

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