Applied Mathematics & Information Sciences
An International Journal

http://dx.doi.org/10.18576/amis/100327

Thermal Effects on Propagation of Transverse Waves in Anisotropic Incompressible Dissipative Pre-Stressed Plate

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Received: 25 Oct. 2015, Revised: 26 Jan. 2016, Accepted: 27 Jan. 2016
Published online: 1 May 2016

Abstract: In this paper an attempt has been made to find a simple form of the thermal effects on transverse waves propagating in an anisotropic incompressible dissipative pre-stressed plate in the context of Biot's theory. The governing equations are derived and the velocities of propagation as well as damping are discussed. The influences of changes in anisotropy-type, initial and thermal stresses are discussed. Analytical analysis reveals that the velocities of the transverse waves depend upon the anisotropy as well as the initial and thermal stresses present in the medium. Numerical computation shows that the variation in the parameters associated with anisotropy of the medium directly affects the propagation of transverse waves in the medium. The increase of initial stress parameters decreases the phase velocity of transverse wave within the range $(25^o, 65^o)$ and increases the velocity with the initial stress parameter increase within the range $(45^o, 90^o)$. Also, the velocity of transverse waves can be obviously tuned by the thermal effects.

Keywords: Transverse waves, initial stress, anisotropic, thermoelastic, dissipative, plate.

1 Introduction

The problems related to propagation of elastic waves have been investigated in a numerous books and papers (eg. Bromwich [1], Sezawa [2], Haskell [3], Ewing et al [4], Brekhovskikh [5], Bullen [6], Bath [7], Achenbach [8], and others). The term "Initial stress" is meant by stresses developed in a medium before it is being used for study. The earth is an initially stressed medium, due to presence of external loading, slow process of creep and gravitational field, considerable amount of stresses (pre-stresses or initial stresses) remain naturally present in the earth layers. These stresses may have significant influence on elastic waves produced by earthquake or explosions and also in the stability of the medium [9]. An Earlier Biot [10] observed that the initial stresses have notable effect on the propagation on elastic waves in a medium. Several investigators [11]-[17] have studied extensively the propagation of elastic waves using Biot's mechanical deformation theory [18].

The propagation of Rayleigh waves in a viscoelastic half-space under initial hydrostatic stress in presence of the temperature field have studied by Addy et al. [19]. Dey et al. [20] have studied the Edge wave propagation in an incompressible anisotropic initially stressed plate of finite thickness. Dissipation of the plate depends upon its internal structure. A huge amount of mathematical work has been performed for the propagation of elastic waves in dissipative medium (e.g. Norris [21], Singh et al. [22, 23], Dey et al. [24], Shekhar et al. [25] Selim [26,27], and others). Problem of plane waves in anisotropicelastic medium is been very important for the possibility of its extensive application in many branches of Science, particularly in Seismology, Acoustics, Geophysics and nanotechnology. The universal presence of anisotropy is almost observed in many types of rocks in the earth. Prikazchikov et al. [28] and Sharma [29] contribute to the understanding of wave propagation characteristics of anisotropic materials under initial stress. Carcione [30] in his book explains the importance of anisotropy for wave propagation studies in real materials. Temperature gradients play a significant role in the modification of cracks and the flow of fluid [31]. To understand the

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dynamical systems that involve interactions between mechanical work and thermal changes, theory of thermoelasticity were used. A large number of problems have been studied on the propagation of plan waves in generalized thermoelastic media (eg. El-Karamany et al.[32,33], Sharma et al. [34,35] and others). Sharma [36] , considered the general anisotropy in thermoelastic medium and he derived a mathematical model to calculate the complex velocities of four waves in the medium.

In this work, an attempt has been made to find the thermal effects on transverse waves propagating in an anisotropic incompressible dissipative pre-stressed plate in a simple form. Biot [10] incremental deformation theory has been used. Numerical examples are computed to analyze the propagation characteristics of the transverse waves in the plate at different temperatures. Variations of anisotropies and change of initial stress parameter are also analyzed.

2 Fundamental equations

We consider an infinite thermally conducting incompressible anisotropic plate of thickness h initially at uniform temperature T_0 under initial stresses S_{11} and S_{33} along x and z directions, respectively (as illustrated in Fig. 1). When the medium is slightly disturbed, the incremental stresses S_{11}, S_{12} and S_{33} are developed and the equations of motion in the incremental state, in the absence of external forces, are [10,37],

$$\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{13}}{\partial z}-P\frac{\partial \omega}{\partial z}-N_t\nabla^2 u=\rho\frac{\partial^2 u}{\partial t^2} \tag{1}$$

$$\frac{\partial S_{31}}{\partial x}+\frac{\partial S_{33}}{\partial z}-P\frac{\partial \omega}{\partial x}-N_t\nabla^2 w=\rho\frac{\partial^2 w}{\partial t^2} \tag{2}$$

where $P=S_{33}-S_{11}$(positive value of P will give compressive stress and negative value of P will give tinsel stress along the said direction), ρ represents the density of the plate, $S_{ij}(i,j=1,2,3)$ are the incremental stress components , ω is rational component about y axis given by

$$\omega=\frac{1}{2}(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}), \tag{3}$$

where u and w are the displacement components in the x and Y directions, respectively and is the thermal stress given by [37],

$$N_t=\frac{\alpha E h}{1-\nu}T, \tag{4}$$

where α is the thermal expansion coefficient, E is the Young's Modulus, ν the Poisson's ratio and T denotes variation in the temperature of the plate in disturbed state.

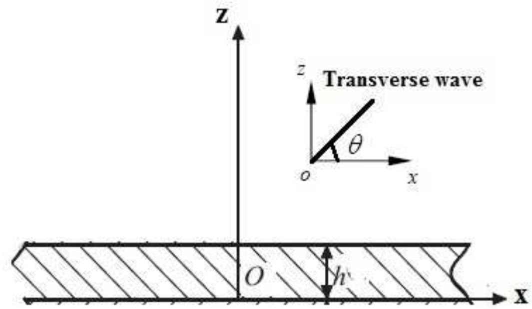

Fig. 1: Transverse waves propagation in pre-stressed dissipative plate.

The stress-strain relations for an incompressible plate may be taken as [10],

$$\begin{aligned} S_{11}&=2Ne_{11}+S,\\ S_{33}&=2Ne_{33}+S,\\ S_{13}&=2Qe_{13}, \end{aligned} \tag{5}$$

where $S=\frac{S_{11}+S_{33}}{2}$, N and Q are the rigidities of the plate.

The incremental strain components $e_{ij}(i,j=1,3)$ are related to the displacement components (u,w) through the relations,

$$\begin{aligned} e_{11}&=\frac{\partial u}{\partial x},\\ e_{33}&=\frac{\partial w}{\partial z},\\ e_{13}&=\frac{1}{2}(\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}). \end{aligned} \tag{6}$$

The incompressibility condition $e_{ii}=0$, is satisfied by

$$u=-\frac{\partial \Omega}{\partial z}, \qquad w=-\frac{\partial \Omega}{\partial x} \tag{7}$$

Using (3), (4), (5), (6) and (7) in equations (1) and (2), we get

$$\frac{\partial s}{\partial x}-\frac{\partial}{\partial z}[(2N-Q-N_t+\frac{P}{2})\frac{\partial^2\Omega}{\partial x^2}+(Q-N_t+\frac{P}{2})\frac{\partial^2\Omega}{\partial z^2}] =-\rho\frac{\partial}{\partial z}\frac{\partial^2\Omega}{\partial t^2} \tag{8}$$

$$\frac{\partial s}{\partial z}-\frac{\partial}{\partial x}[(2N-Q-N_t+\frac{P}{2})\frac{\partial^2\Omega}{\partial z^2}+(Q-N_t+\frac{P}{2})\frac{\partial^2\Omega}{\partial x^2}] =\rho\frac{\partial}{\partial x}\frac{\partial^2\Omega}{\partial t^2} \tag{9}$$

Eliminating s from equations (8) and (9) we obtain

$$(Q-N_t-\frac{P}{2})\frac{\partial^4\Omega}{\partial x^4}+(4N-2Q-2N_t)\frac{\partial^4\Omega}{\partial x^2\partial z^2} +(Q-N_t-\frac{P}{2})\frac{\partial^4\Omega}{\partial z^4}=\rho(\frac{\partial^4\Omega}{\partial x^2\partial t^2}+\frac{\partial^4\Omega}{\partial z^2\partial t^2}) \tag{10}$$

For dissipative plate, the two rigidities coefficients N and Q for anisotropic unstressed state of the medium are replaced by complex constants [38]:

$$N = N_1 + iN_2, \qquad Q = Q_1 + iQ_2 \tag{11}$$

where $i = \sqrt{-1}$, N_i are real and $N_2 \ll N_1$, $Q_2 \ll Q_1$.

Substituting from equation (11) in equation (10) we get

$$\Lambda_1 \frac{\partial^4 \Omega}{\partial x^4} + \Lambda_2 \frac{\partial^4 \Omega}{\partial x^2 \partial z^2} + \Lambda_3 \frac{\partial^4 \Omega}{\partial z^4} = \rho\left(\frac{\partial^4 \Omega}{\partial x^2 \partial t^2} + \frac{\partial^4 \Omega}{\partial z^2 \partial t^2}\right) \tag{12}$$

where

$$\begin{aligned} \Lambda_1 &= (Q_1 - N_t - \frac{P}{2}) + iQ_2, \\ \Lambda_2 &= (4N_1 - 2Q_1 - 2N_t) + i(4N_2 - 2Q_2), \\ \Lambda_3 &= (Q_1 - N_t + \frac{P}{2}) + iQ_2. \end{aligned} \tag{13}$$

3 Solution of the problem

For propagation of transverse waves in any arbitrary direction we take the solution of equations (12) as

$$\Omega(x,z,t) = \Omega_0 e^{ik(\cos\theta x + \sin\theta z - Ct)} \tag{14}$$

where θ is the angle made by the direction of propagation with the x-axis.

Substituting from equation (14) in equations (12), one gets

$$C_T^2 = \frac{1}{\rho}[\Lambda_1(\cos\theta)^4 + \Lambda_2(\sin\theta\cos\theta)^2 + \Lambda_3(\sin\theta)^4] \tag{15}$$

Equation (15) gives the square of velocities of propagation as well as damping. Real parts correspond to phase velocities and the respective imaginary parts correspond to damping velocities of the transverse waves, respectively. From equation (15), we can say that the phase velocity of transverse waves depends on initial stresses, damping, anisotropies and thermal stress N_t.

4 Particular cases

Case (I): For non-dissipative medium, thus the phase velocity of transverse waves propagating along x-axis is given by

$$C_T = \sqrt{\frac{1}{\rho}(Q_1 - N_t - P/2)} \tag{16}$$

Similarly, for transverse waves propagating along z-axis is given by

$$C_T = \sqrt{\frac{1}{\rho}(Q_1 - N_t + P/2)} \tag{17}$$

Case (II): When the effect of initial and thermal stresses is absent (i.e.$P = 0$ and $N_t = 0$), and the medium is non-dissipative. In this case, equation (15) gives

$$C_T = \beta = \sqrt{\frac{Q_1}{\rho}}. \tag{18}$$

where β is the velocity of transverse wave in homogeneous isotropic medium.

5 Numerical Results

The numerical values of the square of the phase velocities of transverse waves have been computed from (15) in non-dimensional form as

$$\left(\frac{C_T}{\beta}\right)^2 = \left(1 - \frac{N_t}{Q_1} - \frac{P}{2Q_1}\right)(\cos\theta)^4 + \left(\frac{2N_1}{Q_1} - \frac{N_t}{Q_1} - 1\right) \times (\sin\theta\cos\theta)^2 + \left(1 - \frac{N_t}{Q_1} + \frac{P}{2Q_1}\right)(\sin\theta)^4 \tag{19}$$

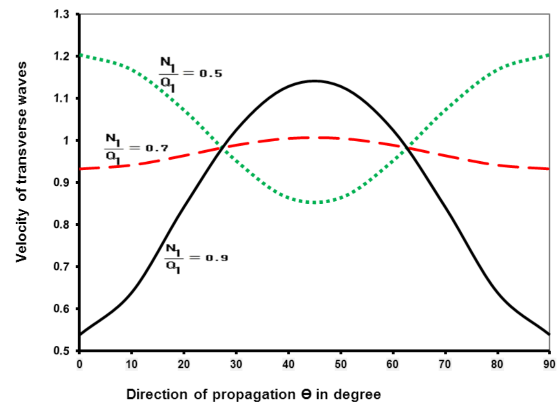

Fig. 2: Variation of phase velocity of transverse waves (C_T) with propagation direction (θ) at different values of anisotropic factor $\frac{N_1}{Q_1} = 0.5, 0.7$ and 0.9 at free of initial and thermal stresses ($\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1} = 0$).

The numerical values of $(\frac{C_T}{\beta})^2$ has been calculated for different values of $\frac{N_1}{Q_1}$, $\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1}$ with respect to the direction of propagation (θ) and the results of computations are presented in Figures 2 through 4.

Figure 2 exhibits the anisotropic variation ($\frac{N_1}{Q_1} = 0.5, 0.7$ and 0.9) of phase velocity of transverse waves (C_T) with propagation direction (θ) at free of initial and thermal stresses ($\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1} = 0$). These variations exhibit the anisotropic character of propagation of transverse waves in the plate considered.

The velocity plots show that the velocity of transverse wave decreases with the increase of anisotropy parameters $\frac{N_1}{Q_1}$. This decrease is a largest at $\theta = 0^O$ and $\theta = 90^O$, but the velocity increases with the anisotropy increase at $\theta \in (25^o, 65^o)$. Figure 3 shows the effect of initial stresses on the velocity of propagative of transverse wave at different direction θ with x-axis at different values of $\frac{P}{2Q_1}$ when $\frac{N_1}{Q_1} = 0.5$ and $\frac{N_t}{Q_1} = 0$.

The velocity plots show that the velocity of transverse wave decreases with the increase of initial stress parameters $\frac{P}{2Q_1}$ at $\theta \in (25^o, 65^o)$, but the velocity increases with the initial stress parameter increase at

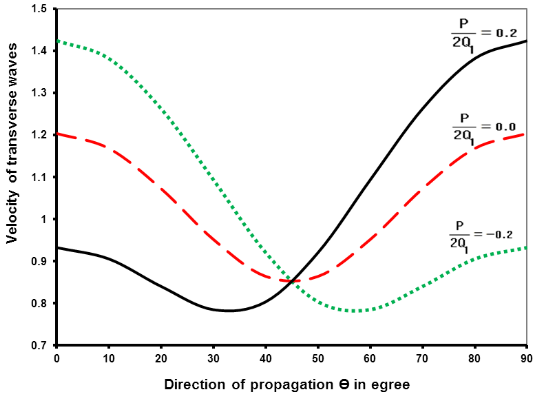

Fig. 3: Variation of phase velocity of transverse waves (C_T) with propagation direction (θ) at different values of initial stress parameter $\frac{P}{2Q_1} = -0.2, 0.0$ and 0.2 when the anisotropic factor $\frac{N_1}{Q_1} = 0.5$ and thermal stresses parameter $\frac{N_t}{Q_1} = 0.0$.

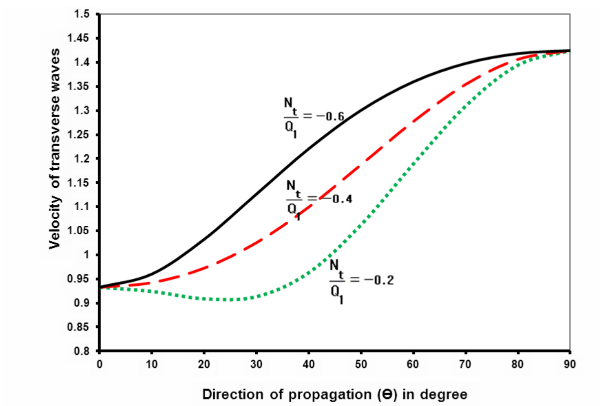

Fig. 4: Variation of phase velocity of transverse waves (C_T) with propagation direction (θ) at different values thermal stress parameter $\frac{P}{2Q_1} = -0.2, -0.4$ and -0.6 when the anisotropic factor $\frac{N_1}{Q_1} = 0.5$ and initial stress parameter $\frac{P}{2Q_1} = 0.0$.

$\theta \in (45^o, 90^o)$. Figure 4 gives the variation in velocities of transverse wave at different direction with x-axis at different values of $\frac{N_t}{Q_1}$ when $\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1} = 0.5$. The velocity plots show that the velocity of transverse wave decreases with the increase of the thermal stress parameters $\frac{N_1}{Q_1}$ at $\theta \in (5^o, 85^o)$.

6 Conclusion

Equation (15) gives the square of velocities of propagation as well as damping. Real parts correspond to phase velocities and the respective imaginary parts correspond to damping velocities of the transverse waves, respectively. From the numerical computation, it can be concluded that the velocities depend on the initial and thermal stresses present in the medium as well as the variations of anisotropy. The variation in parameters associated with anisotropy of the medium directly affects the velocity of the transverse waves. The increase of initial stress parameters decreases the phase velocity of transverse wave with within the range $(25^o, 65^o)$ and increases the velocity with the initial stress parameter increase within the range $(45^o, 9^o)$. Also, the velocity of transverse waves can be obviously tuned by the thermal effects. The velocity of transverse wave decreases with the increase of the thermal stress parameters within the range $(5^o, 85^o)$.

Acknowledgment

The author wish to acknowledge support provided by the Deanship of Scientific Research of Prince Sattam Bin Abdulaziz university.

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