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Effect of Thermal Stress and Magnetic Field on Propagation of Transverse Wave in an Anisotropic Incompressible Dissipative Initially Stressed Plate

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Abstract: In the present paper, we extend our previous work of the thermal effects on propagation of transverse waves in anisotropic incompressible dissipative pre-stressed plate, to investigate the effect of both thermal stress and magnetic field on transverse waves propagation in the medium. Biot incremental deformation theory has been used. The governing equations of transverse wave propagation are derived considering the magnetic forces applied on the plate through Maxwell equation. The influences of changes in anisotropy-type, thermal stress, initial stresses and magnetic field are investigated. The velocities of propagation as well as damping are discussed. Analytical analysis and Numerical computations reveal that the velocities of the transverse waves depend upon the anisotropy, thermal stress, initial stresses as well as magnetic field.

Keywords: Transverse waves, anisotropic, initial stress, thermal stress, magnetic field, dissipative, plate

1 Introduction

In recent years, many studies were carried out to solve the problems related to electrically conducting elastic media permeated by uniform magnetic fields. The seismic wave propagation has been used for various studies related to magneto-elasticity on the Earth's mantle and cores. By the knowledge related to the propagation, the transverse waves, are the source of information used to image the Earth's interior. The initial stress in the medium may be developed due to some reasons such as slow process of creep, gravity, external forces, difference in temperature, etc. An earlier Biot [1] observed that the initial stresses have notable effect on the propagation on elastic waves in a medium. Several investigators [2,3,4,5,6,7,8] have studied extensively the effects of initial stresses present in the medium using Biot's mechanical deformation theory [9]. The propagation of Rayleigh waves in a viscoelastic half-space under initial hydrostatic stress in presence of the temperature field have studied by Addy et al. [10]. Dey et al. [11] have studied the Edge wave propagation in an incompressible anisotropic initially stressed plate of finite thickness. Dissipation of the plate depends upon its internal structure. A huge amount of mathematical work has been performed for the propagation of elastic waves in dissipative medium (e.g. Norris [12], Singh et al. [13, 14], Dey et al. [15], Shekhar et al. [16] Selim [17,18], and others). Problem of plane waves in anisotropic elastic medium is been very important for the possibility of its extensive application in many branches of science, particularly in seismology, Acoustics and geophysics. The universal presence of anisotropy is almost observed in many types of rocks in the earth. Prikazchikov et al. [19] and Sharma [20] contribute to the understanding of wave propagation characteristics of anisotropic materials under initial stress. Carcione [21] in his book explains the importance of anisotropy for wave propagation studies in real materials. Temperature gradients play a significant role in the modification of cracks and the flow of fluid [22]. To understand the dynamical systems that involve interactions between mechanical work and thermal changes, theory of thermoelasticity were used. A large number of problems have been studied on the propagation of plane waves in generalized thermoelastic media (eg.

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El-Karamany et al. [23,24], Sharma et al. [25,26] and others). Sharma [27], considered the general anisotropy in thermoelastic medium and he derived a mathematical model to calculate the complex velocities of four waves in the medium. Problem related to the effect of magnetic field on transverse wave propagation in an anisotropic incompressible dissipative medium is very important for the possible application in various branches of science and technology such as earthquake science, acoustic, geophysics and optics etc. Zhu et al. [28] have been discussed wave propagation in non-homogeneous magneto- electro-elastic hollow cylinders. Jiangong, et al. [29] have been studied wave propagation in non-homogeneous magneto- electro-elastic plates. Khojasteh et al. [30] have been studied diffraction- biased shear wave fields generated with longitudinal magnetic resonance elastography drivers. Rayleigh waves in a magnetoelastic initially stressed conducting medium with the gravity field have been investigated by El-Naggar et al. [31]. In recent years, the electromagnetic characteristic of dissipative medium has also attracted considerable interest for theoretical and practical importance in fundamental science and application. The effect of rotation, magnetic field and initial stresses on propagation of plane waves in transversely isotropic dissipative half-space has been studied by Shekhar et al. [32]. The S-wave propagation in non-homogeneous initial stressed elastic medium under the effect of magnetic field has been studied by Kakar et al. [33]. So, studying the effect of magnetic field on the characteristic of transverse wave propagation in dissipative medium is essential and may give a useful help in many applications.

In the present paper, we extend our previous study of the thermal effects on propagation of transverse waves in anisotropic incompressible dissipative pre-stressed plate [34], to investigate the effect of both thermal stress and magnetic field on transverse waves propagation in the considered medium. Biot incremental deformation theory [35] has been used. Transverse wave propagation in the considered medium is derived considering the magnetic forces applied on the plate through Maxwell equation. Numerical examples are computed to analyze the propagation characteristics of the transverse waves in the medium at different temperatures. Variations of anisotropies, change of initial stress parameter and magnetic field are analyzed.

2 Fundamental equations

We consider an infinite thermally and perfectly electric conducting incompressible anisotropic dissipative plate of thickness$'h'$ initially at uniform temperature $'T_0'$ under initial stresses S_{11} and S_{33} along x and z directions, respectively (as illustrated in Fig. 1). When the medium is slightly disturbed, the incremental stresses S_{11}, S_{12} and S_{33} are developed. The Maxwell's equations of electromagnetic field for perfectly electric conducting elastic medium in vacuum are [16],

$$\begin{aligned}
\nabla\cdot\overrightarrow{E} &= \frac{\rho}{\varepsilon_0}\\
\nabla\cdot\overrightarrow{B} &= 0\\
\nabla\wedge\overrightarrow{E} &= -\frac{\partial\overrightarrow{B}}{\partial t}\\
\nabla\wedge\overrightarrow{B} &= \mu_0\overrightarrow{j}+\frac{1}{c^2}\frac{\partial\overrightarrow{E}}{\partial t}\\
\nabla\wedge\overrightarrow{H} &= \overrightarrow{j}+\frac{\partial\overrightarrow{D}}{\partial t}\\
\overrightarrow{j} &= \sigma\overrightarrow{E}\,(Ohm'slaw),
\end{aligned}\tag{1}$$

Where $\nabla=\frac{\partial}{\partial x}\overrightarrow{i}+\frac{\partial}{\partial y}\overrightarrow{j}+\frac{\partial}{\partial z}\overrightarrow{k}$. The $\overrightarrow{D}=\varepsilon_0\overrightarrow{E}$, $\overrightarrow{B}=\mu_0\overrightarrow{H}$ and $\varepsilon_0\mu_0c^2=0.999478\approx 1$. The $\overrightarrow{E}$ is electric field, D is electric displacement, H is magnetic field, B is magnetic flux density, ρ= charge density, j= current density, σ isconductivity of the medium, $\mu_0=4\pi 10^{-7}$ (magnetic permeability), $c=2.99792458 10^8\ m/s$ (speed of light), and $\varepsilon_0=8.854 10^{-12}$ (permittivity). Based on Refs. [1, 9], the equations of motion in the $x-z$ plane for the present problem in the incremental state, can be expressed as

$$\begin{aligned}
&\frac{\partial S_{11}}{\partial x}+\frac{\partial S_{13}}{\partial z}+\mu_0H_0^2\Big(\frac{\partial^2u}{\partial x^2}-\frac{\partial^2w}{\partial x\partial z}\Big)-P\frac{\partial\omega}{\partial z}\\
&\qquad -N_t\nabla^2u=\rho\frac{\partial^2u}{\partial t^2}\\
&\frac{\partial S_{31}}{\partial x}+\frac{\partial S_{33}}{\partial z}+\mu_0H_0^2\Big(\frac{\partial^2w}{\partial x^2}-\frac{\partial^2u}{\partial z\partial z}\Big)-P\frac{\partial\omega}{\partial z}\\
&\qquad -N_t\nabla^2w=\rho\frac{\partial^2w}{\partial t^2}
\end{aligned}\tag{2}$$

where $P=S_{33}-S_{11}$(positive value of P will give compressive stress and negative value of P will give tinsel stress along the said direction), ρ represents the density of the plate, $S_{ij}(i,j=1,2,3)$ are the incremental stress components, H_0 is the intensity of the uniform magnetic field (along y-axis), and ω is rational component along y axis given by

$$\omega=\frac{1}{2}\Big(\frac{\partial w}{\partial x}-\frac{\partial u}{\partial z}\Big),\tag{3}$$

where u and w are the displacement components in the x and z directions, respectively and N_t is the thermal stress given by [36],

$$N_t=\frac{\alpha Eh}{1-v}T,\tag{4}$$

where α is the thermal expansion coefficient, E is the Young's Modulus, v the Poisson's ratio and T denotes variation in the temperature of the plate in disturbed state. The stress-strain relations for an incompressible plate may be taken as [1],

$$\begin{aligned}
S_{11}&=2Ne_{11}+S,\\
S_{33}&=2Ne_{33}+S,\\
S_{13}&=2Qe_{13},
\end{aligned}\tag{5}$$

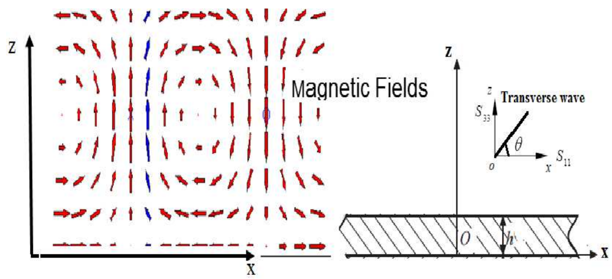

Fig. 1: Transverse waves propagation in initially stressed dissipative plate.

where $s = \frac{S_{11}+S_{33}}{2}$, N and Q are the rigidities of the plate.

The incremental strain components $e_{ij}(i, j = 1,3)$ are related to the displacement components (u, w) through the relations,

$$e_{11} = \frac{\partial u}{\partial x},\quad e_{33} = \frac{\partial w}{\partial z},\quad e_{13} = \frac{1}{2}\left(\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}\right). \tag{6}$$

The incompressibility condition $e_{ii} = 0$, is satisfied by

$$u = -\frac{\partial \zeta}{\partial z}, \qquad w = \frac{\partial \zeta}{\partial x} \tag{7}$$

Substituting (3)-(7) in Eq. (2) , we get

$$\frac{\partial s}{\partial x} - \frac{\partial}{\partial z}[(2N + 2\mu_0 H_0^2 - Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial x^2} + (Q - N_t + \frac{P}{2})\frac{\partial^2 \zeta}{\partial z^2}] = -\rho\frac{\partial}{\partial z}\frac{\partial^2 \zeta}{\partial t^2} \tag{8}$$

$$\frac{\partial s}{\partial z} - \frac{\partial}{\partial x}[(2N + 2\mu_0 H_0^2 - Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial z^2} + (Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial x^2}] = \rho\frac{\partial}{\partial x}\frac{\partial^2 \zeta}{\partial t^2} \tag{9}$$

Differentiating (8) with respect to z and (9) with respect to x, we obtain

$$\frac{\partial^2 s}{\partial z\partial x} - \frac{\partial^2}{\partial z^2}\ [\ (2N + 2\mu_0 H_0^2 - Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial x^2} + (Q - N_t + \frac{P}{2})\frac{\partial^2 \zeta}{\partial z^2}] = -\rho\frac{\partial^2}{\partial z^2}\frac{\partial^2 \zeta}{\partial t^2} \tag{10}$$

$$\frac{\partial^2 s}{\partial x\partial z} - \frac{\partial^2}{\partial x^2}\ [\ (2N + 2\mu_0 H_0^2 - Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial z^2} + (Q - N_t - \frac{P}{2})\frac{\partial^2 \zeta}{\partial x^2}] = \rho\frac{\partial^2}{\partial x^2}\frac{\partial^2 \zeta}{\partial t^2} \tag{11}$$

Subtracting (10) from (11), we can eliminate s, and the equation of motion becomes

$$(Q - N_t - \frac{P}{2})\frac{\partial^4 \zeta}{\partial x^4} + (4N + 4\mu_0 H_0^2 - 2Q - 2N_t)\frac{\partial^4 \zeta}{\partial x^2 \partial z^2} + (Q - N_t + \frac{P}{2})\frac{\partial^4 \zeta}{\partial z^4} = \rho(\frac{\partial^4 \zeta}{\partial x^2 \partial t^2} + \frac{\partial^4 \zeta}{\partial z^2 \partial t^2}) \tag{12}$$

For dissipative plate, the two rigidities coefficients N and Q for anisotropic unstressed state of the medium are replaced by complex constants [37]:

$$N = N_1 + iN_2, \qquad Q = Q_1 + iQ_2 \tag{13}$$

where $i = \sqrt{-1}$, N_i are real and $N_2 \ll N_1$, $Q_2 \ll Q_1$.

From Eqs. (12)-(13) we obtain

$$L_1\frac{\partial^4 \zeta}{\partial x^4} + L_2\frac{\partial^4 \zeta}{\partial x^2 \partial z^2} + L_3\frac{\partial^4 \zeta}{\partial z^4} = \rho(\frac{\partial^4 \zeta}{\partial x^2 \partial t^2} + \frac{\partial^4 \zeta}{\partial z^2 \partial t^2}) \tag{14}$$

where

$$L_1 = (Q_1 - N_t - \frac{P}{2}) + iQ_2,\quad L_2 = (4N_1 + 4\mu_0 H_0^2 - 2Q_1 - 2N_t) + i(4N_2 - 2Q_2),\quad L_3 = (Q_1 - N_t + \frac{P}{2}) + iQ_2. \tag{15}$$

3 Solution of the problem

For propagation of transverse waves in any arbitrary direction we take the solution of equations (14) as

$$\zeta(x, z, t) = \zeta_0 e^{ik(x\cos\theta + z\sin\theta - Ct)} \tag{16}$$

where θ is the angle made by the direction of propagation with the x-axis (Fig.1).

Substituting Eq. (16) into Eq. (14) and equating real and imaginary parts separately, one gets

$$C_{TR}^2 = \frac{1}{\rho}[(Q_1 - N_t - \frac{P}{2})(\cos\theta)^4 + (Q_1 - N_t + \frac{P}{2})(\sin\theta)^4 + (4N_1 + 4\mu_0 H_0^2 - 2Q_1 - 2N_t)(\sin\theta\cos\theta)^2] \tag{17}$$

$$C_{TI}^2 = \frac{1}{\rho}[Q_2(\cos^2\theta - \sin^2\theta)^2 + 4N_2\sin^2\theta\cos^2\theta] \tag{18}$$

Eqs (17) and (18) give the phase velocities (real parts) and the damping velocities (imaginary parts) of the transverse waves, respectively. From the above equation (17), we can say that the phase velocity of transverse waves depends on initial stresses, anisotropies, thermal stress, magnetic field and the direction of propagation θ.

4 Numerical Results

The numerical values of the square of the phase velocities of transverse waves have been computed from (17) in non-dimensional form as

$$(\frac{C_T}{\beta})^2 = (1 - \frac{N_t}{Q_1} - \frac{P}{2Q_1})\cos^4\theta + (\frac{2N_1}{Q_1} + \frac{2\mu_0 H_0^2}{Q_1} - \frac{N_t}{Q_1} - 1) \times 2\sin^2\theta\cos^2\theta + (1 - \frac{N_t}{Q_1} + \frac{P}{2Q_1})\sin^4\theta, \tag{19}$$

where $\beta = \sqrt{\frac{Q_1}{\rho}}$ is the velocity of transverse wave in homogenous isotropic medium.

The numerical values of C_T has been calculated for

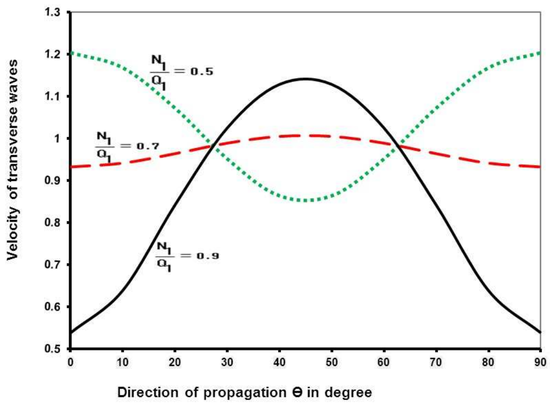

Fig. 2: Variation of phase velocity of transverse waves (C_T) with propagation direction at different values of anisotropic factor $\frac{N_1}{Q_1} = 0.5, 0.7$ and 0.9 at $(\frac{P}{2Q_1} = 0$, $\frac{N_t}{Q_1} = 0$ and $\frac{\mu_0 H_0^2}{Q_1} = 0)$ [34].

different values of $\frac{N_1}{Q_1}$, $\frac{P}{2Q_1}$, $\frac{N_t}{Q_1}$ and $\frac{\mu_0 H_0^2}{Q_1}$ with respect to the direction of propagation θ and the results of computations are presented in Figures 2-5.

Figure 2 exhibits the anisotropic variation $(\frac{N_1}{Q_1} = 0.5, 0.7$ and $0.9)$ of phase velocity of transverse waves (C_T) with propagation direction (θ) at free of initial and thermal stresses and no effect of magnetic field $(\frac{P}{2Q_1} = 0$, $\frac{N_t}{Q_1} = 0$ and $\frac{\mu_0 H_0^2}{Q_1} = 0)$. The velocity plots show that the velocity of transverse wave decreases with the increase of anisotropy parameters $\frac{N_1}{Q_1}$. This decrease is a largest at $\theta = 0^O$ and $\theta = 90^O$, but the velocity increases with the anisotropy increase at $\theta \in (25^o, 65^o)$.

Figure 3 shows the effect of initial stresses on the velocity of propagative of transverse wave at different direction θ with x-axis at different values of $\frac{P}{2Q_1}$ when $\frac{N_1}{Q_1} = 0.5$, $\frac{N_t}{Q_1} = 0$ and $\frac{\mu_0 H_0^2}{Q_1} = 0$. The velocity plots show that the velocity of transverse wave decreases with the increase of initial stress parameters $\frac{P}{2Q_1}$ at $\theta \in (25^o, 65^o)$, but the velocity increases with the initial stress parameter increase at $\theta \in (45^o, 90^o)$.

Figure 4 gives the variation in velocities of transverse wave at different direction with x-axis at different values of $\frac{N_t}{Q_1}$ when $\frac{P}{2Q_1} = 0$, $\frac{N_1}{Q_1} = 0.5$ and $\frac{\mu_0 H_0^2}{Q_1} = 0$. The velocity plots show that the velocity of transverse wave decreases with the increase of the thermal stress parameters $\frac{N_1}{Q_1}$ at $\theta \in (5^o, 85^o)$.

Figure 5 gives the variation in velocities of transverse wave at different direction with x-axis at different values

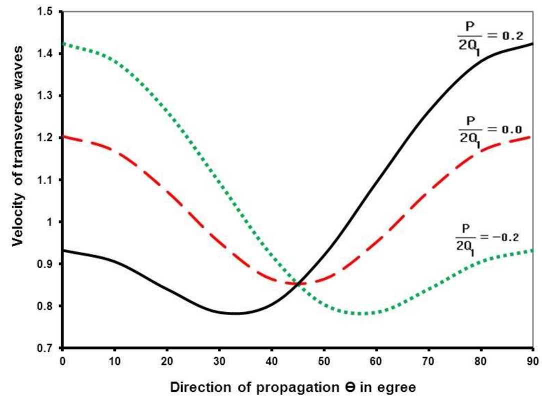

Fig. 3: Variation of phase velocity of transverse waves (C_T) with propagation direction θ at different values of initial stress parameter $\frac{P}{2Q_1} = -0.2, 0.0$ and 0.2 at $\frac{N_1}{Q_1} = 0.5$, $\frac{N_t}{Q_1} = 0.0$ and $\frac{\mu_0 H_0^2}{Q_1} = 0$ [34].

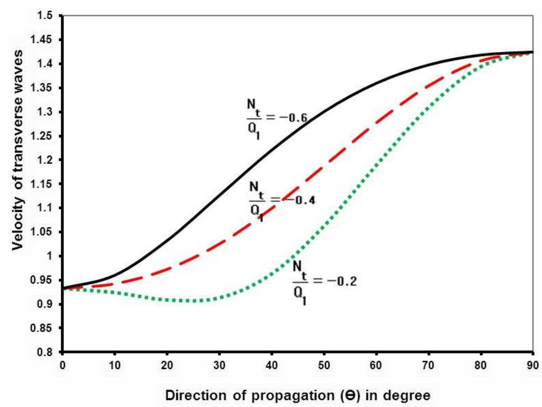

Fig. 4: Variation of phase velocity of transverse waves (C_T) with propagation direction (θ) at different values of thermal stress parameter $\frac{P}{2Q_1} = -0.2, -0.4$ and -0.6 at $\frac{N_1}{Q_1} = 0.5$, $\frac{P}{2Q_1} = 0.0$ and $\frac{\mu_0 H_0^2}{Q_1} = 0$ [34].

of $\frac{\mu_0 H_0^2}{Q_1}$ when $\frac{N_t}{Q_1} = 0$, $\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1} = 0.5$. The velocity plots show that the velocity of transverse wave increases with the increase of the magnetic field $\frac{\mu_0 H_0^2}{Q_1}$ and vice versa at $\theta \in (5^o, 87^o)$, but this effect disappear in the ranges of $\theta \in (0^o, 4^o)$ and $\theta \in (88^o, 90^o)$.

5 Conclusion

Equations (17) and (18) give the phase velocities (real parts) and the damping velocities (imaginary parts) of the transverse waves, respectively. From the numerical computation, it can be concluded that the phase velocity

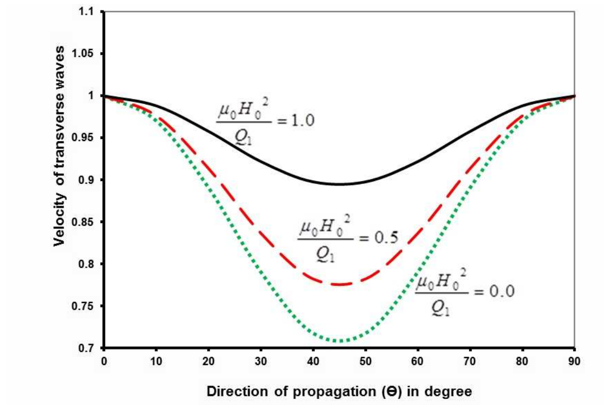

Fig. 5: Variation of phase velocity of transverse waves with propagation direction at different values of magnetic field parameter $\frac{\mu_0 H_0^2}{Q_1} = 0.0, 0.5, 1$ at $\frac{N_t}{Q_1} = 0$, $\frac{P}{2Q_1} = 0$ and $\frac{N_1}{Q_1} = 0.5$.

of transverse waves depends on initial stresses, anisotropies, thermal stress, magnetic field and the direction of propagation θ. The variation in parameters associated with anisotropy of the medium directly affects the velocity of the transverse waves. The increase of initial stress parameters decreases the phase velocity of transverse wave within the range of $(25^o, 65^o)$ and increases the velocity with the initial stress parameter increase within the range of $(45^o, 9^o)$. The velocity of transverse wave decreases with the increase of the thermal stress parameters within the range of $(5^o, 85^o)$ but this effect disappear in the ranges of $(0^o, 5^o)$ and $(85^o, 90^o)$. Also, the velocity of transverse waves can be obviously tuned by the magnetic field effects. The velocity of transverse wave increases with the increase of the magnetic field within the range of $(5^o, 87^o)$, but the effect of the magnetic field disappear in the ranges of $(0^o, 4^o)$ and $(88^o, 90^o)$.

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