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On Empirical Comparison of Forecast Performance of Autoregressive Moving Average Model and Generalized Autoregressive Conditional Heteroscedasticity Model

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Abstract: The federal Government of Nigeria in a bid to prevent infant mortality introduced Immunization programme whose aim is to prevent infectious diseases among the newly born children. However, the national programme on immunization (NPI) suffers recurrent setbacks due to many factors including ethnicity and religious beliefs. Nigeria is made up of 774 local Governments, 36 states with its federal capital in Abuja. The country is divided into six geo-political zones; north central, North West, North-East, South-East, South-West and South-South. The population is unevenly distributed across the country. The focus of this paper is to provide an understanding about the theoretical and practical application of ARMA and GARCH models to Nigeria immunization data as well as looking at the gains derivable from using either of the models. The paper compares the forecast performance of these two models and used performance measure indices to test the adequacy of the model that perform better. The data used was obtained from University College hospital, Ibadan annual reports on immunization. Augmented-Dickey Fuller test was used as a stationarity test for the series used, at level the series were not stationary but at first difference they were stationary, thereafter, the analysis of the data were performed. The results actually shown that the two models are good for modeling and forecasting the series under investigation, however, GARCH model slightly outperformed ARMA as shown by the analysis.

Keywords: ARMA model, GARCH model, Forecast, Immunization, Performance measure indices.

1 Introduction

Autoregressive Moving Average (ARMA) consists of two parts, an autoregressive (AR) part and a moving average (MA) part. There are five types of traditional time series models that are commonly used in epidemic time series forecasting and in other forecasting areas. They are autoregressive, Moving average (MA), Autoregressive moving average (ARMA), Autoregressive integrated moving average (ARIMA), and Seasonal autoregressive integrated moving average (SARIMA) models. AR model expresses the current value of the time series linearly in terms of its previous values and the current residual; whereas MA model expresses the current value of the time series linearly in terms of its current and previous residual series. ARMA model is a combination of AR and MA models, in which the current value of the time series is expressed linearly in terms of its previous values and in terms of current and previous residual series [1]. The time series defined in AR, MA, and ARMA models are stationary processes, which means that the mean of the series of any of these models and the covariance among its observations do not change with time. For non-stationary time series, transformation of the series to a stationary series has to be performed first. ARIMA model generally fits the non-stationary time series based on the ARMA model, with a differencing process which effectively transforms the non-stationary data into a stationary one. [2] Advanced Engle's ARCH model to Generalized Autoregressive Conditional Heteroskedasticity model (GARCH) to allow for changes in the time dependent volatility, such as decreasing or increasing volatility in the same series. Since then, there have been several derivations of the GARCH model, with letters from alphabet coming before the root name. The volatility clustering implied by ARCH and GARCH models also implies thicker tails than normal [3]. [4] Estimate the volatility of Egyptian and Sudanese markets for the period of January 2006 to November 2010 by employing symmetric and asymmetric GARCH models. They report conditional volatility of returns of explosive and quite persistent nature for both countries. [5] Uses GARCH, GJR-GARCH and EGARCH models to examine the

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volatility of stock indexes from five European emerging markets, namely Turkey, Bulgaria, Czech Republic, Poland and Hungary. They find volatility shocks that are persistent at those markets and conclude that the impact of old news is significant on volatility.

[5] Furthermore investigated whether the Nigerian stock market followed a random walk using GARCH model. The results showed that it followed a random walk and persistent volatility clustering suggested weak-form efficiency in the market. [6] Applied GARCH, EGARCH and TGARCH models to illustrate volatility of four Borsa Istanbul sub-indexes for the period of 2011-2014. They find no significant asymmetric impact of shocks on the volatility of banking shares, while all other sub-indices exhibit asymmetry. [7] Used random level shift model (incorporated into GARCH) to simulate and forecast volatility of four US stock market indices including Nasdaq. Their findings show that level shift model successfully captures long-memory and conditional heteroscedasticity, and it outperforms GARCH (1,1) model in forecasting. On their part, [8] addressed limitations of [9] sequential estimation method for modeling an intraday volatility process. With 10-min returns of the Nasdaq composite stock index from15 August 2005 to 12 September 2008, they search for better ARCH parameters, and propose an approach that considers the interaction effect between the periodicity and the heteroscedasticity. [10] Study volatility of thirty most actively traded Nasdaq stocks with after-hours information added to GARCH model. They find pre-open coefficients in the model to be positive and significant for 23 of the 30 stocks, however the post-close variance to have less power in predicting the future conditional volatility. [11] Examined volatility persistence and long-memory property of Nasdaq-100 index with daily data from January 2, 2001 to February 20, 2004. [12] Aimed in finding the response of Pakistani and Indian stock markets to global financial crisis which started from last half of 2007 got severity in 2008. EGARCH model been applied for econometric analysis which illuminated that inertia of volatility clustering prevailed in the stock markets of both countries. The study also revealed that negative shocks have more pronounced impact on the volatility than positive shocks. [13] Examined the behaviour of tests of fit for the hypothesis of normality of innovations in GARCH models. The procedures were natural extensions of well-known tests for normality, which included classical goodness-of-fit tests based on the empirical distribution function.

The vision of EPI in Nigeria is to improve the health of Nigerian children by eradicating all the six killer diseases, which are polio, measles, diphtheria, whooping cough, tuberculosis, and yellow fever. Between 1985 and 1990, as outlined in the national health plan for that period, the objectives of EPI were to strengthen immunization, accelerate disease control and introduce new vaccines, relevant technologies and tools. In1995 in line with the above, Nigeria became a signatory to the World Health Assembly, adopted the World Health Assembly Resolution (WHAR) and United Nations General Assembly Special Session (UNGASS) goals for all countries to achieve by 2005 (i) polio eradication, (ii) measles mortality reduction and (iii) maternal and neonatal tetanus elimination (MNTE). Nigeria also adopted the millennium development goals (MDGs) calling for a two-third reduction in child mortality, as compared to 1990, the year 2005. In addition to the above, the country ratified the United Nations General Assembly Special Session (UNGASS) goals urging Nigeria to achieve by 2010 (i) ensure full immunization of children under one year of age at 90% coverage nationally with at least 80% coverage in every district or equivalent administrative unit, and (ii) vitamin A deficiency elimination. Immunization against childhood diseases such as diphtheria, pertussis, tetanus, polio and measles is one of the most important means of preventing childhood morbidity and mortality. Achieving and maintaining high levels of immunization coverage must therefore be a priority for all health systems. In order to monitor progress in achieving this objective, immunization coverage data can serve as an indicator of a health system’s capacity to deliver essential services to the most vulnerable segment of a population [13]. It is therefore the objective of this paper to determine the forecast ability of Autoregressive moving average (ARMA) and Generalized Autoregressive conditional heteroscedasticity (GARCH) models. At the empirical illustration level we want to determine which of these two models will give better forecast accuracy using some competitive criteria.

The remaining part of the paper is organized as follow section 2 covers theory and methods on ARMA and GARCH models. Section 3 looks at model evaluation indices like RMSE, MAPE, MAD, MAE, and Theil’s U inequality (Bias, variance and covariance). Section 4 is results and discussion like descriptive statistics, Augmented Dickey Fuller tests (level and first difference), Estimation of ARCH and GARCH models, forecast performance of ARCH and GARCH models, forecast evaluation of ARCH and GARCH models, their variances and the optimal variance of the ARCH and GARCH models. Section 5 and 6 are Conclusion and References.

2 Theory and Methods

2.1 ARMA Process

ARMA model is expressed in terms of both p and q. The model parameters relate what happens in period t to both the past value and the random errors that occurred in past time periods. A general ARMA model can be written as follow:

$$Y_t = \varphi_1 Y_{t-1} + \phi_2 Y_{t-2} + \cdots + \phi_p Y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} + \theta_2 \varepsilon_{t-2} + \cdots \theta_q \varepsilon_{t-q} \quad (1)$$

Equation above of the time series model will be simplified by a back a backward shift operator B to obtain $\Phi(B)Y_t = \theta(B)\varepsilon_t$

The ARMA model is stable i.e., it has a stationary 'solution' if all roots of $\Phi(B)=0$ are larger than one in absolute value. The representation is unique if all root $\Phi(B)=0$ lie outside the unit circle and $\Phi(B)$ and $\theta(B)$ do not have common roots. Stable ARMA models always have an infinite order. The process is invertible only when the roots of $\theta(B)$ lie outside the unit circle. Furthermore, a process is said to be causal when the root of $\Phi(B)$ lie outside the unit circle. To have $ARMA(p,q)$ model, both ACF and PACF should show the pattern of decaying to zero. The eventually the ACF consists of mixed damped exponentials and sine terms. Similarly, the partial autocorrelation of an $ARMA(p,q)$ process is determined at greater lags by the $MA(q)$ part of the process. Thus, eventually the partial autocorrelation function will also consist of a mixture of damped exponentials and sine waves.

Behaviour of the ACF and PACF for ARMA models.

	$AR(P)s$	$MA(Q)s$	$ARMA(P,Q)s$
ACF	Tails off at lag ks $k=1,2,3,\cdots$	Cuts off after lag Qs	Tails off at lag ks
$PACF$	Cuts off after lag Ps	Tails off at lag ks $k=1,2,3,\cdots$	Tails off at lag ks

An ARMA model in forecasting may be represented in three forms as follows:

$$\alpha(L)y(t) = \mu(L)\varepsilon(t) \quad \text{Difference operator} \quad (2)$$

$$y(t) = \frac{\mu(L)}{\alpha(L)} y(t) = \psi(L)\varepsilon(t) \quad \text{Moving average form} \quad (3)$$

$$\psi(L) = \frac{\alpha(L)}{\mu(L)} y(t) = \pi(L)y(t) = \varepsilon(t) \quad \text{Autoregressive form} \quad (4)$$

Here

$$\psi(L) = \left[1 + \psi L_1 + \psi L^2 + \ldots\right] \text{ and } \pi(L) = \left[1 - \pi L_1 + \pi L^2 + \ldots\right]$$

stand for the series expansions of the respective rational operators.

In developing the theory of forecasting, we may consider infinite information

Set $I_t = (y_t, y_{t-1}, y_{t-2}\ldots)$ Knowing the parameters in $\alpha(L)$ and $\mu(L)$ enables us to recover the sequence $(\varepsilon_t, \varepsilon_{t-1}, \varepsilon_{t-2}\ldots)$ from the sequence $(y_t, y_{t-1}, y_{t-2}\ldots)$ and vice versa; so either of these constitute the information set. This equivalence implies that the forecasts may be expressed in terms (y_t) or in terms (ε_t) or as a combination of the elements of both sets.

2.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

One class of models which have proved useful in forecasting volatility in so many sphere of life is GARCH model and its extensions.

The GARCH (p,q) model is formulated as follows:

let $\left(y_{(t)}\right)$ be the time series of an exchange rate return, then

$$y_t = \sigma_t \varepsilon_t \tag{5}$$

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{p} \alpha_i y_{t-1}^2 + \sum_{j=1}^{q} \beta_j \sigma_{t-j}^2 \tag{6}$$

Where $\alpha_0 > 0, \alpha_i \geq 0$ and innovation sequence $\{\varepsilon\}_{i=-\infty}^{\infty}$ is independent and identically distributed (iid) with $E(\varepsilon_0)$=0 and $E(\varepsilon_0^2)$=1. The main idea is that σ_t^2, the conditional variance of y_t given information available up to time $t-1$ has an autoregressive structure and is positively correlated to its own recent past and to recent values of the squared return y_t^2. This captures the idea of volatility being "persistent", large (small) values of y_t^2 are likely to be followed by large (small) values. The GARCH model formulation captures the fact that volatility is changing in time. The change corresponds to a weighted average among the long term average variance, the volatility in the previous period, and the fitted variance in the previous period as well. The model described in equation (6) is used to parameterize financial time series, and shall be used to compare the forecasting of equation (1) through (6) using the following performance adequacy measures

3 Model Evaluation Indices

Several error indices are commonly used in model evaluation; these include mean absolute error (MAE), root mean square error (RMSE), mean absolute deviation (MAD), mean absolute precision error (MAPE) and **THEIL U**. These indices are valuable because they indicate error in the units (or squared units) of the constituent of interest, which aids in analysis of the results. RMSE, MAE, MAPE, MAD and Theil U values of 0 indicate a perfect fit.

3.1 Root Mean Square Error (RMSE)

The **root-mean-square error (RMSE)** is a good measure of accuracy, used to measure the differences between values predicted by a model and the values actually observed. These individual differences are called errors when the calculations are performed over the data sample that was used for estimation, and are called prediction errors when computed out-of-sample. The RMSE serves to aggregate the magnitudes of the errors in predictions for various times into a single measure of predictive power. Although it is commonly accepted that the lower the RMSE the better the model performance.

RMSE is given as:

$$RMSE = \sqrt{\left\{T^{-1} \sum_{t=1}^{T} \left(Y_t - \hat{Y}_t\right)\right\}} \tag{7}$$

3.*2 Mean Absolute Percentage Error (MAPE)*

This is frequently used to evaluate cross-sectional forecasts So also MAPE has indispensable statistical properties in that it makes use of all observations and has the smallest variability from sample to sample. It is also useful for purposes of reporting because it is expressed in generic percentage terms that will be understandable to a wide range of users and very simple to calculate and easy to understand which attest to its popularity. It is given as:-

$$MAPE = T^{-1} \sum_{t=1}^{T} \left| \frac{Y_t - \hat{Y}_t}{Y_t} \right| X\ 100 \tag{8}$$

3.3 Mean Absolute Error (MAE)

The simplest measure of forecast accuracy is called Mean Absolute Error (MAE). MAE is simply, as the name suggests the mean of the absolute errors. The absolute error is the absolute value of the difference between the forecasted value and the actual value. MAE tells us on the average how big of an error we can expect from the forecast. This is mathematically given as:-

$$MAE = T^{-1}\sum_{t=1}^{T}\left|Y_t - \hat{Y}_t\right| \tag{9}$$

3.4 Theil's U Inequality Coefficient

A more useful measure to evaluate the predictive accuracy of a model is Theil's U inequality coefficient [14], which measures the root mean square error in relative terms, and is defined as

$$U = \frac{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t^s - Y_t^a)^2}}{\sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t^s)^2} + \sqrt{\frac{1}{n}\sum_{t=1}^{n}(Y_t^a)^2}} \tag{10}$$

The denominator imposes an upper bound to the U coefficient, which is bounded above by 1 and bounded below by 0, that is, $0 \le U \le 1$. This is particularly useful since it gives a threshold to evaluate the accuracy of a model and not only compare it to other models. The closer to 0 the coefficient is, the more accurate the model is, while a coefficient equal to 1 indicates that the forecast performance of the model is as bad as it could be. The U coefficient can be decomposed into three proportions that provide useful additional information on the performance of the model.

Bias,

$$U^M = \frac{(\overline{Y}^s - \overline{Y}^a)^2}{\frac{1}{n}\sum_{t=1}^{n}(Y_t^s - Y_t^a)^2} \tag{11}$$

Variance,

$$U^S = \frac{(\sigma_s - \sigma_a)^2}{\frac{1}{n}\sum_{t=1}^{n}(Y_t^s - Y_t^a)^2} \tag{12}$$

Covariance,

$$U^C = \frac{2(1-\rho)\sigma_s\sigma_a}{\frac{1}{n}\sum_{t=1}^{n}(Y_t^s - Y_t^a)^2} \tag{13}$$

The bias proportion measures the systematic error of the forecast; it gathers the share of the simulation error that comes from bias, that is, the difference between the averages of the forecasted series and the actual series. The variance proportion is intended to provide a measure of how well our forecast replicates the volatility of the actual series. The covariance proportion offers a measure of the unsystematic error in the forecast. The ideal distribution of proportions for any non-zero inequality coefficient would be $U^m = U^s = 0$ and $U^c = 1$.

4 Results and Discussions

The data for the study comprises of BCG (Bacilli Calmette Guerin) OPV (Oral Polio Vaccine) and DPT (Diphtheria, pertusis, tetanus), were obtained from annual records of University College Hospital, U.C.H, Ibadan reports on immunization. The table below (Table 1) provides summary statistics of all the series used in the paper, it reveals the presence of excess kurtosis and very large size of Jarque-Bera statistics which made us to conclude that the series are not normally distributed. The unit root tests conducted using Augmented Dickey-Fuller test (tables 2 and 3), shows that at level all series are not stationary but stationary at first difference, with this we proceeded to the analysis of the data.

Table 1: Descriptive Statistics

Statistics	DPT3	MV	OPV3
Mean	33677.47	44881.30	34153.28
Median	25118.00	37393.00	25645.00
Maximum	208403.0	166007.0	150570.0
Minimum	28.00000	2774.000	1277.000
Std. Dev.	31444.66	31115.53	28439.66
Skewness	2.794408	1.578633	1.891450
Kurtosis	13.80567	5.981015	7.078726
Jarque-Bera	696.8211	88.77458	145.7057
Probability	0.000000	0.000000	0.000000
Observations	113	113	113

Table 2: Augmented Dickey-Fuller Test (Level)

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob ●
DPT3	-2.964652	-3.4900	0.0415
MV	-9.0413	-3.4900	0.0000
OPV3	-9.5818	-3.4900	0.0000

Table 3: Augmented Dickey-Fuller Test (First Difference)

Series	ADF-Test statistic	Critical value (5%)	Mackinnon prob ●
DPT3	-16.1913	-2.8892	0.0000
MV	-6.7093	-2.8892	0.0000
OPV3	-7,2178	-2.8892	0.0000

4.1 Estimation of ARMA Model

Table 4: ARMA Analysis of DPT3.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	0.992074	0.015529	63.88554	0.0000
MA(1)	-0.814379	0.064260	-12.67320	0.0000

S.E. of regression 29109.98

Table 5: ARMA Analysis of MV.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.004819	0.002610	384.9656	0.0000
MA(1)	-0.978341	0.015745	-62.13584	0.0000

S.E. of regression 30661.77

Table 6: ARMA Analysis of OPV3.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
AR(1)	1.006448	0.003354	300.0400	0.0000
MA(1)	-0.978296	0.019613	-49.88028	0.0000

S.E. of regression 27716.24

4.2 Estimation of Classical GARCH Model

Table 7: GARCH Analysis of DPT3

	Coefficient	Std. Error	z-Statistic	Prob.
DPT3	0.000679	7.33E-05	9.272139	0.0000
	Variance Equation			
C	3.574598	6.042990	0.591528	0.5542
RESID(-1)^2	0.111640	0.177275	0.629758	0.5289
GARCH(-1)	0.931875	0.170281	5.472559	0.0000

S.E. of regression 47.13951

Table 8: GARCH Analysis of MV

	Coefficient	Std. Error	z-Statistic	Prob.
MV	0.000924	6.50E-05	14.21751	0.0000
	Variance Equation			
C	1043.421	371.4334	2.809175	0.0050
RESID(-1)^2	0.524634	0.249033	2.106682	0.0351
GARCH(-1)	-0.112744	0.223894	-0.503561	0.6146

S.E. of regression 40.83912

Table 9: GARCH Analysis of OPV3

GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

	Coefficient	Std. Error	z-Statistic	Prob.
OPV3	0.000452	4.99E-05	9.042898	0.0000
	Variance Equation			
C	3.505743	6.550124	0.535218	0.5925
RESID(-1)^2	0.228558	0.348418	0.655988	0.5118
GARCH(-1)	0.822089	0.348350	2.359950	0.0183

S.E. of regression 52.26624

Table 10: Forecast Performances of ARMA Model

SERIES	RMSE	MAE	MAPE	THEIL-U	U-BIAS	VAR.	COVAR.
DPT3	41475.96	27698.76	469.5084	0.7667	0.3990	* 0.5029	0.0981
MV	35827.60	23737.75	* 58.2654	* 0.4426	0.2874	0.5682	* 0.1443
OPV3	* 32176.23	* 19709.62	68.2440	0.5143	* 0.2696	0.5898	0.1405

Table 11: Forecast Performances of GARCH Model

SERIES	RMSE	MAE	MAPE	THEIL-U	U-BIAS	VAR.	COVAR.
DPT3	51.3244	42.5593	73.1627	* 0.0602	0.6377	0.1447	0.2178
MV	* 40.1032	* 31.4955	111.3275	0.3467	* 0.1367	* 0.0084	* 0.8549
OPV3	46.2901	37.6460	* 68.7438	0.4799	0.5253	0.0567	0.4198

4.3 The Forecast Evaluation Indices of the two Models

In tables 10 and 11, the asterisks numbers are selected for being the best among the various forecast performance listed in the tables for the two models used. From these tables (10 and 11), table 12 was formed, from where we picked our optimum model.

4.4 Forecast Evaluation

Forecast performance of the ARMA and GARCH models for the series under investigation were evaluated using the measures of comparison described in section 3.0 above and are presented below:

Table 12: Forecasts Evaluation for ARMA and GARCH Models

FORECAST INDICES	ARMA	GARCH
Root mean square error	32176.23	40.1032
Mean absolute error	19709.62	31.4955
Mean absolute percent error	58.2654	68.7438
Theil inequality coefficient	0.4426	0.3467
Bias proportion	0.2696	0.1367
Variance proportion	0.5029	0.0002
Covariance proportion	0.0084	0.8549

From the table 12, the value of Theil inequality for the two models are 0.4426 for ARCH and 0.3467 for GARCH showing that the two models have good fit, the values of both bias proportion and variance proportion are somehow close to zero (0.2696 for ARCH) and (0.1367 for GARCH), indicating that level of bias are very negligible, the variance

proportion of ARMA (0.5029) is very high compare to that of GARCH which is just (0.0002). The covariance proportion of ARCH model (0.0088) is very poor compare to the variance of GARCH model (0.8549). Showing that the forecast performance of ARCH model is very poor and that the performance of GARH is very good.

4.5 Variances of the Models

All asterisks minimum variances for these two models were pooled together in table (14), from this we select the optimum for GARCH model. The optimum variance value here is 40.83912, produced by GARCH Model. The minimum variance for ARCH model was 27716.24 as shown in table 13 while the minimum variance for GARCH model was 40.83912 as shown in table 14. The two minimum variance pooled together produced table 15. The minimum of these two variances produced the optimal results, which is 40.83912.

Table 13: Variances of ARMA model for all series.

SERIES	MODEL VARIANCE
DPT3	29109.98
MV	30661.77
OPV3	* 27716.24

Table 14: Variances of GARCH model for the series

SERIES	MODEL VARIANCE
DPT3	47.13951
MV	* 40.83912
OPV3	52.266124

Table 15: Optimal variance for the two models

SERIES	MODEL VARIANCE
ARMA	27716.24
GARCH	* 40.83912

5 Conclusions

This paper focuses on modeling and forecasting immunization in Nigeria using ARMA and GARCH models, we examined the stationarity of the series by both ADF test and Graph, from where we discovered that the series are non-stationary at level and appeared stationary after differencing the data, thereafter we analyzed the data using ARMA (1,1) and GARCH (1,1). We equally subjected the forecast power of the models to adequacy tests like RMSE, MAE, MAD, MAPE and others, from where we discovered that GARCH model is far better than ARCH model as shown by tables (12) and (15).

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