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Effects of Wall Properties, Heat Absorption and Chemical Reaction on Peristaltic Motion of Rivlin-Ericksen Fluid through Porous Medium in Vertical Channel

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Abstract: In this paper, we consider the peristaltic motion of Rivlin-Ericksen fluid with heat and mass transfer through porous medium inside a symmetric vertical channel. Heat absorption, chemical reaction and external uniform magnetic field are taken in consideration. This phenomenon is modulated mathematically by a system of partial differential equations which govern the motion of the fluid. The momentum, energy and concentration equations of the fluid motion are illustrated with the appropriate boundary conditions and transformed to non-dimensional form. The perturbation method was used to get the analytical solution under the assumption of long wavelength and low Reynolds number. The velocity, temperature and concentration distributions are obtained as functions of the physical parameters of the problem. The effects of wall properties and other various parameters on these distributions are illustrated graphically.

Keywords: Peristaltic motion, Rivlin-Ericksen fluid, Wall properties, Heat absorption, Chemical reaction, porous medium.

1 Introduction

Peristaltic transport is a form of material transport induced by a progressive wave of area contraction or expansion along the length of a distensible tube, mixing and transporting the fluid in the direction of the wave propagation. This phenomenon is known as peristalsis. In general, during peristaltic the fluid pumped from lower pressure to higher pressure. In addition, peristaltic pumping occurs in many practical applications involving biomedical systems. It has now been accepted that most of the physiological fluids behave in general like suspensions of deformable or rigid particles in a Newtonian fluid. Blood, for example, is a suspension of red cells, white cells, and platelets in plasma. The effects of variable viscosity on the peristaltic flow of non-Newtonian fluid through a porous medium in an inclined channel with slip boundary conditions were done by Ambreen [1]. The effect of heat transfer on peristaltic transport of a Newtonian fluid through a porous medium in an asymmetric vertical channel was studied by Vasudev [2]. Hayat et. al. [3] discussed the heat transfer analysis for peristaltic mechanism in variable viscosity fluid. The peristaltic pumping of a non – Newtonian fluid analyzed by Medhavi [4]. Hayat and Javed [5] considered the exact solutions to peristaltic transport of Power-law fluid in asymmetric channel with compliant walls. Peristaltic flow of Williamson fluid in an asymmetric channel through porous medium analyzed by Kavitha et. al. [6]. Non-linear peristaltic pumping of Johnson-Segalman fluid in an asymmetric channel under effect of magnetic field was investigated by Reddy and Raju [7].

In recent years the flow in porous medium has practical interest due to wide applications which includes heat transfer enhancement, chemical reactors, building insulation, enhanced oil recovery and filtration processes. Most of the previous studies of the flow through porous media are based on the assumption that the fluid is Newtonian. The understanding of non-Newtonian flows though porous media represents interesting challenges in geophysical systems, chemical reactor design, certain separation processes, polymer engineering and in petroleum production. Flow of a visco-elastic fluid in tubes of varying cross-sectionwith suction/injection was

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studied by Parasad and Chandra [8]. The magnetohydrodynamic flow of second order fluid through a porous medium on an inclined porous plane has been studied by Eldabe and Elmohandis [9] .The velocity of second-order Rivlin–Ericksen fluid between two parallel porous plates rotating around two different axes but with the same angular velocity was studied by Unverdi [10]. Effects of chemical reaction on mixed convective Rivlin-Ericksen MHD flow with variable temperature, concentration and suction was obtained by Sravanthi [11]. Heat transfer for flow of a third-grade fluid between two porous plates was considered by Hayat et. al. [12]. Chemical reaction and thermo-diffusion effects on hydromagnetic free convective Walter's memory flow with constant suction and heat sink was discussed by Pavan et. al. [13]. Abd el-Malek and Hassan [14] studied the symmetry analysis for steady boundary layer stagnation-point flow of Rivlin-Ericksen fluid of second grade subject to suction. MHD heat and mass transfer flow of a viscoelastic fluid past an impulsively started infinite vertical plate with chemical reaction was investigated by Reddy et. al. [15]. Saravana et. al. [16] discussed the mass transfer effects on MHD viscous flow past an impulsively started infinite vertical plate with constant Mass flux. The solutions of non-linear equations arising in Rivlin-Ericksen fluids were obtained by Mohyuddin [17]. The effects of Hall current and heat transfer on flow due to a pull of eccentric rotating disks were studied by Asghar [18].

The objective of the present paper is to investigate the effects of the wall properties, heat absorption and chemical on peristaltic transport of Rivlin-Ericksen fluid through porous medium in a symmetric two-dimension vertical channel with heat and mass transfer in the presence of magnetic field. Under the assumption of long wavelength and low Reynolds number, the equations of momentum, energy and concentration which govern the fluid flow are solved analytically by using perturbation method. The effects of the fluid parameters on the velocity, temperature and concentration distributions have been studied with the help of graphs.

2 Mathematical Analysis

Let us consider the flow of Rivlin-Ericksen fluid through a porous medium in a symmetric two-dimensional vertical channel under the effects of heat absorption and chemical reaction in the presence of magnetic field with flexible walls on which is imposed travelling sinusoidal waves of long wave length. A uniform magnetic field B_o is applied in the transvers direction of the flow. The coordinate system used is given in fig. (1)

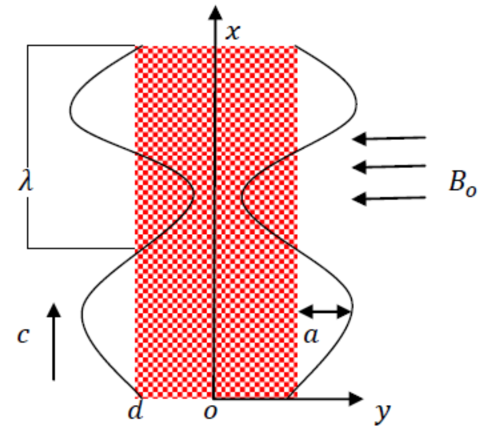

Fig. 1: Geometry of peristaltic transport of Rivlin-Ericksen fluid through porous medium in a symmetric vertical channel.

The travelling waves are represented by:

$$\eta = d + a\sin\left(\frac{2\pi}{\lambda}(x-ct)\right) \tag{1}$$

Where d is the half width of the channel, a is the amplitude of the wave, $\boldsymbol{\lambda}$ is the wave length, t is the time and c is the wave velocity.
The governing equations of motion can be written in the forms:
The continuity equation:

$$\nabla.\boldsymbol{V} = 0 \tag{2}$$

The momentum equation:

$$\rho\left(\frac{\partial \boldsymbol{V}}{\partial t} + (\boldsymbol{V}.\nabla)\boldsymbol{V}\right) = \nabla P + \nabla.\boldsymbol{\tau} - \left(\frac{\mu}{k} + \sigma B_o^2\right)\boldsymbol{V} + \rho g\alpha(T-T_O) + \rho g\alpha^*(\varphi-\varphi_o) \tag{3}$$

The temperature equation:

$$\rho c_p\left(\frac{\partial T}{\partial t} + (\boldsymbol{V}.\nabla)T\right) = k_T\nabla^2 T - Q_o(T-T_O) \tag{4}$$

The concentration equation:

$$\left(\frac{\partial \varphi}{\partial t} + (\boldsymbol{V}.\nabla)\varphi\right) = D_m\nabla^2\varphi + \frac{D_m k_T}{T_m}\nabla^2 T - k_1(\varphi-\varphi_o) \tag{5}$$

Where $\boldsymbol{V}(u,v)$, $\boldsymbol{\tau}$, T and φ are the velocity vector, the stress vector, temperature and concentration, ρ, c_p, P, g, α, α^*, k_T, T_m, $\boldsymbol{q}$ and D_m are the density of the fluid, specific heat, pressure, acceleration due to gravity, coefficient of thermal expansion, coefficient of expansion with concentration, thermal conductivity, mean fluid temperature, the vector rate of heat flux per unit area and the coefficient of mass diffusivity. μ is the viscosity, k is the permeability of porous medium, σ is the Conductivity of the fluid, Q_o is the heat absorption

coefficient and k_1 is the chemical reaction parameter.
The constitute equation for Rivlin-Ericksen fluid is given by (Parasad and Chandra [8]):

$$\boldsymbol{\tau} = \mu \boldsymbol{A}_1 + \mu_1 \boldsymbol{A}_2 + \mu_2 \boldsymbol{A}_1^2 \tag{6}$$

Where μ_1 and μ_2 are the normal stress modules. A_1 and A_2 are the kinematical Rivlin-Ericksen tensors defined through:

$$\boldsymbol{A}_1 = \nabla \boldsymbol{V} + (\nabla \boldsymbol{V})^T \tag{7}$$

$$\boldsymbol{A}_2 = \frac{D}{Dt}\boldsymbol{A}_1 + \boldsymbol{A}_1 . \nabla \boldsymbol{V} + (\nabla V)^T . \boldsymbol{A}_1 \tag{8}$$

Where ∇ is the gradient operator, a subscript T denotes transpose and $\frac{D}{Dt}$ is the material derivative which is defined as follows:

$$\frac{D}{Dt}\boldsymbol{A}_1 = \frac{\partial}{\partial t}\boldsymbol{A}_1 + (\mathrm{V}.\nabla)\boldsymbol{A}_1 \tag{9}$$

The equation of motion of the flexible wall is given by:

$$\mathrm{L}(\eta) = \mathrm{P} - \mathrm{P_O} \tag{10}$$

Where L is an operator that is used to represnt the motion of the stretched membrance with damping forces such that:

$$\mathrm{L} = -\mathrm{T}_2 \frac{\partial^2}{\partial \mathrm{x}^2} + \mathrm{M}_2 \frac{\partial^2}{\partial \mathrm{t}^2} + \mathrm{C}_2 \frac{\partial}{\partial \mathrm{t}} \tag{11}$$

Where T_2 is the tension in the membrane, M_2 is the mass per unit area, C_2 is the coefficient of the damping force and P_O is the pressure on the outside of the wall due to tension in the muscles.
If we assume that $P_O = 0$, then equations (3-5) in two-dimensional form can be written as**:**

$$\rho \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - \left(\frac{\mu}{k} + \sigma B_o^2\right)u + \rho g \alpha (T - T_O) + \rho g \alpha^* (\varphi - \varphi_O) \tag{12}$$

$$\rho \left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} - \frac{\mu}{k}v \tag{13}$$

$$\rho c_p \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = k_T \left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - Q_o (T - T_O) \tag{14}$$

$$\left[\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y}\right] = D_m \left[\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right] + \frac{D_m k_T}{T_m}\left[\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - k_1 (\varphi - \varphi_o) \tag{15}$$

With the appropriate boundary conditions

$$\left.\begin{array}{l} u = -c\,, \quad T = T_o \ \ and \ \ \varphi = \varphi_o \quad at \quad y = -\eta \\ u = -c\,, \quad T = T_1 \ \ and \ \ \varphi = \varphi_1 \quad at \quad y = \eta \end{array}\right\} \tag{16}$$

Let us introduce the following dimensionless quantities as:

$$x^* = \frac{x}{\lambda}, y^* = \frac{y}{d}, \eta^* = \frac{\eta}{d}, t^* = \frac{tc}{\lambda}, u^* = \frac{u}{c}, v^* = \frac{v}{c\delta}, \delta = \frac{d}{\lambda}, \\ p^* = \frac{pd^2}{\mu \lambda c}, T^* = \frac{T - T_o}{T_1 - T_o}, \varphi^* = \frac{\varphi - \varphi_o}{\varphi_1 - \varphi_o}, \quad \tau^* = \frac{\tau d}{\mu c} \tag{17}$$

After substituting from (17), Equations (12-15) can be written in dimensionless form after dropping the star mark as:

$$Re\ \delta \left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = -\frac{\partial P}{\partial x} + \delta \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} - n^2 u + G_{rT} T + G_{rm} \varphi \tag{18}$$

$$Re\ \delta^3 \left[\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y}\right] = -\frac{\partial P}{\partial y} + \delta^2 \frac{\partial \tau_{yx}}{\partial x} + \delta \frac{\partial \tau_{yy}}{\partial y} - \delta^2 a_1^2 v \tag{19}$$

$$Re\, P_r\, \delta \left[\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right] = \left[\delta^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - \gamma P_r T \tag{20}$$

$$\delta \left[\frac{\partial \varphi}{\partial t} + u\frac{\partial \varphi}{\partial x} + v\frac{\partial \varphi}{\partial y}\right] = \frac{1}{S_c}\left[\delta^2 \frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2}\right] + S_r \left[\delta^2 \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right] - S\varphi \tag{21}$$

The dimensionless boundary conditions are:

$$\left.\begin{array}{l} u = -1\,, \quad T = 0 \ \ and \ \ \varphi = 0 \quad at \quad y = -\eta \\ u = -1\,, \quad T = 1 \ \ and \ \ \varphi = 1 \quad at \ \ y = \eta \end{array}\right\} \tag{22}$$

For long wavelength $(i.e,\ \ \delta \ll 1)$ and low Reynolds number $(i.e.,\ \ Re \to 0)$ the system of our equations can be reduced to:

$$\frac{\partial \tau_{xy}}{\partial y} - n^2 u + G_{rT} T + G_{rm} \varphi = \frac{\partial p}{\partial x} \tag{23}$$

$$\frac{\partial P}{\partial y} = 0 \tag{24}$$

$$\frac{\partial^2 T}{\partial y^2} - \gamma P_r T = 0 \tag{25}$$

$$\frac{1}{S_c}\frac{\partial^2 \varphi}{\partial y^2} + S_r \frac{\partial^2 T}{\partial y^2} - S\varphi = 0 \tag{26}$$

From equation (24), it is clear that P is independent of y. Therefore equation (23) can be written as:

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial y} + \gamma_1 \frac{\partial^2 u}{\partial y^2}\right] - n^2 u + G_{rT} T + G_{rm} \varphi = \frac{dP}{dx} \tag{27}$$

By using equations (10) and (11) with the help of the wall equation (1) we can write:

$$\frac{dP}{dx} = -\varepsilon \left[(2\pi)^3 \cos 2\pi (x - t)(E_1 + E_2) - (2\pi)^2\, E_3 \sin 2\pi (x - t)\right] \tag{28}$$

Where $E_1 = -\frac{T_2 d^3}{\mu c \lambda^3}$ is the membrane tension parameter of the wall, $E_2 = \frac{M_2 c d^3}{\mu \lambda^3}$ is the mass characterizing parameter of the wall, $E_3 = \frac{C_2 d^3}{\mu \lambda^2}$ is the damping parameter of the wall, $\varepsilon = \frac{a}{d}$ is the amplitude ratio, $G_{rT} = \frac{\rho g \alpha d^2 (T_1 - T_o)}{\mu c}$ is the local temperature Grashof number, $Re = \frac{\rho c d}{\mu}$ is the Reynolds number , $G_{rm} = \frac{\rho g \alpha^* d^2 (\varphi_1 - \varphi_o)}{\mu c}$ is the local mass Grashof number,$P_r = \frac{\mu c_p}{k}$ is the Prandtl number, $S_c = \frac{cd}{D_m}$ is the Schmidt number, $S_r = \frac{D_m k_T (T_1 - T_o)}{T_m d c (\varphi_1 - \varphi_o)}$ is the Soret number,$\gamma = \frac{Q_o d^2}{\mu c_p}$ is the coefficient of heat absorption, $S = \frac{k_2 d}{c}$ is the coefficient of chemical reaction, $n^2 = a_1^2 + H^2, a_1^2 = \frac{d^2}{k}$ is the porosity parameter, $H^2 = \frac{\sigma B_o^2 d^2}{\mu}$ is the magnetic parameter and $\gamma_1 = \frac{v_o \mu_1}{d \mu}$ is the viscoelastic parameter.

3 Method of Solution

Solving equations (25) and (26) using the boundary conditions (22), we get:

$$T(y) = c_1 \mathrm{e}^{-y\sqrt{\gamma}\sqrt{P_r}} + c_2 \mathrm{e}^{y\sqrt{\gamma}\sqrt{P_r}} \tag{29}$$

$$\varphi(y) = c_3 \mathrm{e}^{-\sqrt{S}y\sqrt{S_c}} c_3 + c_4 3^{\sqrt{S}y\sqrt{S_c}} c_4 - \frac{\mathrm{e}^{-y\sqrt{\gamma}\sqrt{P_r}} \gamma (c_1 + \mathrm{e}^{2y\sqrt{\gamma}\sqrt{P_r}} c_2) P_r S_c S_r}{\gamma P_r - S S_c} \tag{30}$$

To have a solution of equation (27) subject to the boundary conditions (22) we assume the following perturbation method for small non-Newtonian parameter γ_1 $(i.e \;\; \gamma_1 \ll 1)$ as:

$$u = u_o + \gamma_1 u_1 + O\left(\gamma_1{}^2\right) \tag{31}$$

Substituting from equations (29-31) in equation (27) and collecting the coefficient of various power of γ_1 on both sides, we obtain the flowing equations:
Zero order of $\boldsymbol{\gamma_1}$

$$\frac{\partial^2 u_o}{\partial y^2} - n^2 u_o + G_{rT} T + G_{rm} \varphi - P_1 = 0 \tag{32}$$

Where $P_1 = \frac{dP}{dx}$.
With the corresponding boundary conditions:

$$u_o = -1 \qquad at \quad y = \pm \eta \tag{33}$$

First order of $\boldsymbol{\gamma_1}$

$$\frac{\partial^2 u_1}{\partial y^2} - n^2 u_1 + \frac{\partial^3 u_o}{\partial y^3} = 0 \tag{34}$$

With the corresponding boundary conditions:

$$u_1 = 0 \qquad at \quad y = \pm \eta \tag{35}$$

Solving equations (32) and (34) with the boundary conditions (33)(35), we get velocity as:

$$\begin{aligned}
&u_o(y) = \mathrm{B}_1 \mathrm{e}^{-y\mathrm{B}_2} (-n^2 S S_c (\mathrm{e}^{y\mathrm{B}_3} n^2 c_3 G_{\mathrm{rm}} + \\
&\mathrm{e}^{y\mathrm{B}_4} (\mathrm{e}^{y\mathrm{B}_5} n^2 c_4 G_{\mathrm{rm}} + (\mathrm{e}^{ny} c_1 G_{\mathrm{rt}} + \mathrm{e}^{y\sqrt{\gamma}\sqrt{P_r}} (n^2 (\mathrm{e}^{2ny} c_5 + c_6) + \\
&\mathrm{e}^{ny} (\mathrm{e}^{y\sqrt{\gamma}\sqrt{P_r}} c_2 G_{\mathrm{rt}} - p_1))) \mathrm{D}_1)) - \\
&\mathrm{e}^{y\mathrm{B}_6} \gamma^2 P_r^2 (\mathrm{e}^{ny} n^2 c_3 G_{\mathrm{rm}} + \mathrm{e}^{\sqrt{S}y\sqrt{S_c}} (\mathrm{e}^{y\mathrm{B}_4} n^2 c_4 G_{\mathrm{rm}} + \\
&(n^2 (\mathrm{e}^{2ny} c_5 + c_6) - \mathrm{e}^{ny} p_1) \mathrm{D}_1)) + \\
&\gamma P_r (\mathrm{e}^{y\mathrm{B}_3} n^2 c_3 G_{\mathrm{rm}} \mathrm{D}_2 + \mathrm{e}^{y\mathrm{B}_7} n^2 c_4 G_{\mathrm{rm}} \mathrm{D}_2 + \\
&\mathrm{e}^{y\mathrm{B}_4} \mathrm{D}_1 (\mathrm{e}^{y\sqrt{\gamma}\sqrt{P_r}} (n^2 (\mathrm{e}^{2ny} c_5 + c_6) - \\
&\mathrm{e}^{ny} p_1) \mathrm{D}_2 + \mathrm{e}^{ny} n^2 c_1 \mathrm{D}_3 + \mathrm{e}^{y\mathrm{B}_8} n^2 c_2 \mathrm{D}_4)))
\end{aligned}$$

$$\begin{aligned}
&u_1(y) = \mathrm{B}_9 \mathrm{e}^{-y\mathrm{B}_{10}} (-4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{12} + 4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{13} + \\
&8\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{14} - 4\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{16} + 8\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{17} - 4\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{18} - \\
&8\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{19} - 4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{20} + 4\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{21} - 8\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{22} + \\
&4\mathrm{e}^{y\mathrm{B}_{15}} \mathrm{B}_{23} + 4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{24} + 4\mathrm{e}^{y\mathrm{B}_{25}} \mathrm{B}_{26} + 4\mathrm{e}^{y\mathrm{B}_{27}} \mathrm{B}_{28} + \\
&4\mathrm{e}^{y\mathrm{B}_{29}} \mathrm{B}_{30} + \mathrm{e}^{y\mathrm{B}_{27}} n(-1 + 2ny) \mathrm{B}_{31} + \mathrm{e}^{y\mathrm{B}_{29}} n(1 + 2ny) \mathrm{B}_{32} + \\
&4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{33} - 8\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{34} + 4\mathrm{e}^{y\mathrm{B}_{11}} \mathrm{B}_{35} - 4\mathrm{e}^{y\mathrm{B}_{36}} \mathrm{B}_{37})
\end{aligned}$$

The mathematical formulas of the constants $c_1 - c_8$,$\mathrm{B}_1 - \mathrm{B}_{37}$ *and* $\mathrm{D}_1 - \mathrm{D}_4$ are not included here. However, they are available upon request from the author.

4 Results and Discussion

This study consider the effects of heat absorption and chemical reaction with wall properties on peristaltic transport of Rivlin-Ericksen fluid through porous medium inside a symmetric vertical channel with heat and mass transfer in the presence of magnetic field. The effects of the flow parameters entering the problem on the velocity, temperature and concentration distribution of the flow field are shown graphically.

4.1 Velocity Distribution. The velocity of the flow is found to change more or less with the variation of the flow parameters. The effect of the flow parameters on the velocity distribution is analyzed with the help of figures (2)-(8).

Fig. (2) illustrates the velocity distribution against y for several values of the viscoelastic parameter γ_1. It's found that the increase of the vescoelastic parameter decrease the velocity of the flow field. This is due to the fact that as the fluid viscosity increases, the fluid in both regions of the channel becomes thicker and hence the flow velocity is reduced.

Fig. (3) depicts the effect of the magnetic parameter H on the velocity distribution. Applications of magnetic field to an electrically conducting flow give rise to a resistive type of force called Lorenz force. This force has the tendency to slow down the motion of the fluid. As expected, as H increase the velocity decrease.

Figs. (4)-(6) display the effects of wall parameters on the velocity distribution. The rigid nature of the wall is represented by the parameter E_1, which depends on the wall tension and E_2 represents the stiffness property of the wall. E_3 represents the dissipative feature of the wall. The choice $E_3 = 0$ implies that the wall moves up and down with no damping force on it, and therefore, indicates the case of elastic walls. The effects of the rigid nature and the stiffness property of the walls on the velocity distribution for the elastic walls ($E_3 = 0$) is shown in figures (4) and (5). It can be seen from these figures that the velocity increase by increasing the tension parameter E_1 and the mass characterizing parameter E_2. The effect of the dissipative walls on the velocity is given in figure (6). This figure show that, as the dissipative nature of the walls E_3 increase, the velocity decreased.

Figs. (7) and (8) illustrate the effects of the thermal Grashof number G_{rt} and the mass Grashof number G_{rm}. The thermal Grashof number signifies the relative effect of the thermal buoyancy force to the viscous hydrodynamic force in the flow field. The positive values of G_{rt}correspond to cooling of the wall by natural convection. Heat is therefore conducting away from the vertical wall into the fluid which increases temperature and thereby enhances the buoyancy force. It observed that the velocity accelerates due to enhancement in thermal buoyancy force. The mass Grashof number G_{rm} defines the ratio of the species buoyancy force to the viscous hydrodynamic force. It is noticed that the velocity increases with a rise in the species buoyancy force.

4.2 Temperature Distribution. The temperature of the flow field is mainly affected the heat absorption coefficient γ. The effect of this parameter on the temperature of the flow field is shown in figure (9). It is noticed that an increase in the values of γ leads to decrease in the temperature.

4.3 Concentration Distribution. The concentration distribution of the flow field is affected by the chemical reaction coefficient S.

Fig. (10) Shows the effect of the coefficient of chemical reaction S. It is clear that the concentration decrease as the coefficient of chemical reaction S increase.

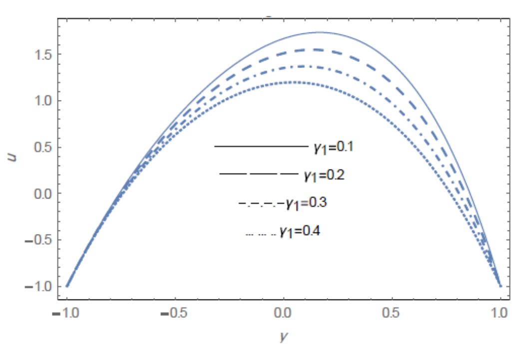

Fig. 2: The velocity distribution u is plotted against y for different values of γ_1 when $a_1 = 0.2, H = 1.5, x = \pi, t = \pi, \varepsilon = 0.5, P_r = 0.71, \gamma = 2, S_c = 0.15, S_r = 0.5, S = 2, E_1 = 0.01, E_2 = 0.01, E_3 = 0, G_{rt} = 6, G_{rm} = 10$.

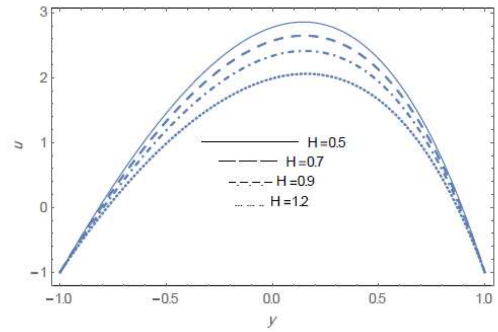

Fig. 3: The velocity distribution u is plotted against y for different values of H when $\gamma_1 = 0.1,\ a_1 = .2,\ x = \pi,\ t = \pi, \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ E_1 = .01,\ E_2 = .01,\ E_3 = 0,\ G_{rt} = 6,\ G_{rm} = 10$.

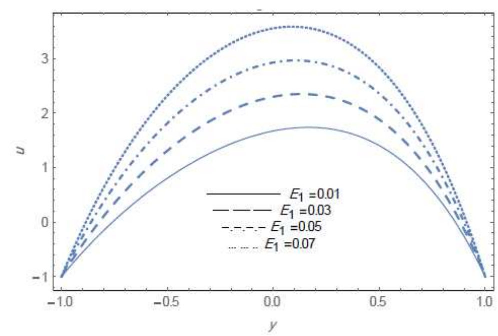

Fig. 4: The velocity distribution u is plotted against y for different values of E_1 when $a_1 = 0.2,\ H = 1.5,\ x = \pi,\ t = \pi,\ \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ \gamma_1 = 0.1,\ E_2 = .01,\ E_3 = 0,\ G_{rt} = 6,\ G_{rm} = 10$.

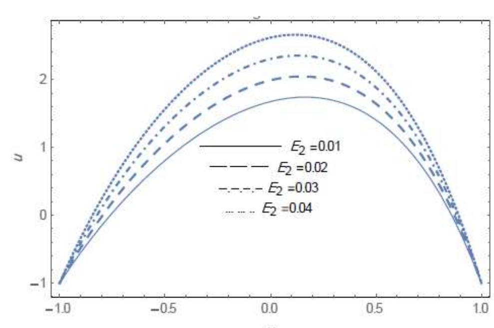

Fig. 5: The velocity distribution u is plotted against y for different values of E_2 when $a_1 = 0.2,\ H = 1.5,\ x = \pi,\ t = \pi,\ \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ \gamma_1 = 0.1,\ E_1 = .01,\ E_3 = 0,\ G_{rt} = 6,\ G_{rm} = 10$.

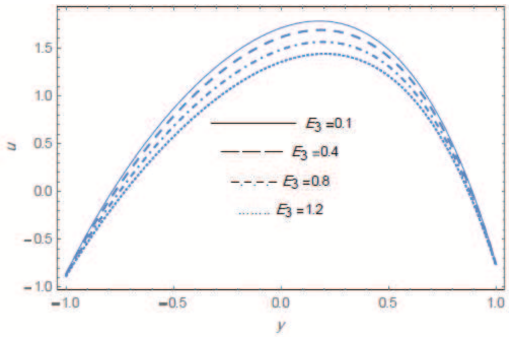

Fig. 6: The velocity distribution u is plotted against y for different values of E_3 when $a_1 = 0.2,\ H = 1.5,\ x = \pi,\ t = \pi,\ \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ \gamma_1 = 0.1,\ E_1 = .01,\ E_2 = 0.01,\ G_{rt} = 6,\ G_{rm} = 10.$

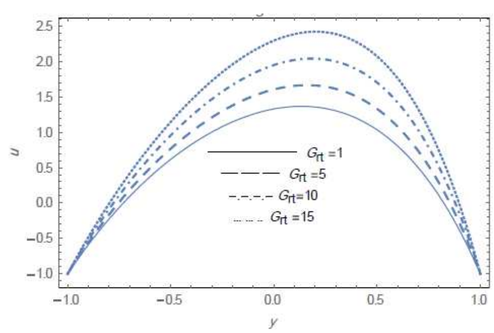

Fig. 7: The velocity distribution u is plotted against y for different values of G_{rt} when $a_1 = 0.2,\ H = 1.5,\ x = \pi,\ t = \pi,\ \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ \gamma_1 = 0.1,\ E_1 = .01,\ E_3 = 0,\ E_2 = .01,\ G_{rm} = 10.$

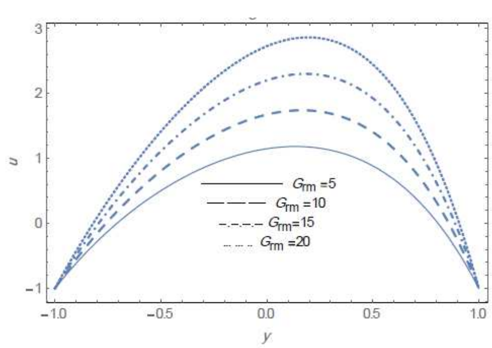

Fig. 8: The velocity distribution u is plotted against y for different values of G_{rm} when $a_1 = 0.2,\ H = 1.5,\ x = \pi,\ t = \pi,\ \varepsilon = .5,\ P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5,\ S = 2,\ \gamma_1 = 0.1,\ E_1 = .01,\ E_3 = 0,\ E_2 = .01,\ G_{rt} = 6.$

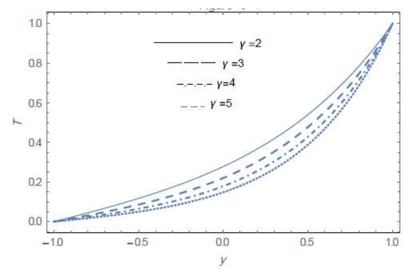

Fig. 9: The temperature distribution T is plotted against y for different values of γ when $P_r = .71,\ x = \pi,\ t = \pi, \varepsilon = .5$.

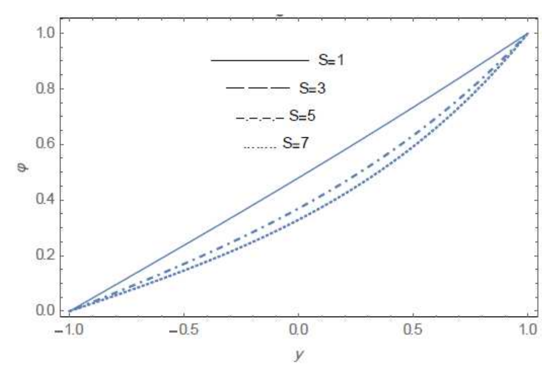

Fig. 10: The concentration distribution φ is plotted against y for different values of S when $P_r = .71,\ \gamma = 2,\ S_c = 0.15,\ S_r = 0.5, x = \pi,\ t = \pi,\ \varepsilon = .5$

5 Conclusion

In the present paper we investigated the effect of heat absorption and chemical reaction on peristaltic transport of Rivlin-Ericksen fluid through a symmetric vertical channel in the presence of magnetic field with heat and mass transfer. The resulting equations which control the motion of a non-Newtonian Rivlin-Ericksen fluid are solved analytically by using the perturbation technique. In the case of long wave length and low Reynolds number, calculations are presented for the velocity, temperature, concentration distributions and their dependence on the material parameters of the fluid. The effect of problem's parameters such as the tension parameter E_1, the mass parameter E_2, the damping parameter E_3, the magnetic parameter H, the non-Newtonian parameter γ_1, the coefficient of heat absorption γ, the coefficient of chemical reaction S, the local temperature Grashof number G_{rT} and the local mass Grashof number G_{rm} on these distributions are discussed by a set of graphs. Peristaltic transport plays an important role in transporting many physiological fluids in the body such as urine transport from the kidney to the bladder through the ureter, food mixing and chimes movement in the intetire, movement of eggs in the fallopian tube, the transport of the spermatozoa in the cervical canal and transport of bile in the bile duct. Also many modern mechanical devices have been designed on the principle of peristaltic pumping for transporting fluids without internal moving parts, for example, the blood pumping in the heart-lung and the peristaltic transport of noxious fluid in nuclear industry.

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