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Optimum Exponential Ratio Type Estimators for Estimating the Population Mean

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Abstract: This paper proposes modified exponential ratio type estimators for the estimation of finite population mean in case of simple random sampling without replacement (SRSWOR) scheme. The expressions of Bias and mean square error (MSE) for the proposed estimators have been evaluated theoretically. The optimum values of the non zero constants Q and P for the proposed estimators have also been determined. The estimators proposed in this study were found to be more efficient than the mean per unit estimator, ratio type estimator of Cochran [1], exponential ratio type estimators of Bahl & Tuteja [4] and Singh *et al.* [7]. The performance of proposed estimators has also been evaluated empirically.

Keywords: Auxiliary Information, Exponential estimators, Bias, Mean Squared Error, Optimal Condition.

1 Introduction

The information obtained about a variable under study by defining a second variable which is correlated with the study variable is called auxiliary information. The auxiliary information can be the correlation coefficient, mean, median, coefficient of variation, skewness, kurtosis, etc. It is well known that the ratio, product and regression methods of estimation make use of auxiliary information at the stage of estimation for improving the precision of the estimates of population mean. Although, most of the existing ratio and product estimators are biased, biasness problem can be dealt by jackknifing, bootstrapping, increasing the sample size etc. Cochran [1] envisaged the ratio method of estimation, if there is positive correlation between the study and the auxiliary variable and $\frac{C_x}{2C_y} < \rho \leq +1$. However, if the correlation between the variables is negative and $-1 \leq \rho < -\frac{C_x}{2C_y}$, the product method of estimation proposed by Robson [2] and revisited by Murthy [3] can be employed efficiently. Bahl and Tuteja [4] was the pioneer to make use of an exponential type transcendental function in the estimators of population mean. It was observed by using such type of function, a significant reduction in the MSE. Generally the exponential type estimators are found to be estimating the population mean more precisely as compared to conventional/modified ratio and product type estimators and can be used even if there is not a strong/high degree of correlation between the auxiliary and study variable. The authors Kadilar [5] and Zaman [6] proposed modified exponential estimators of population mean. In this paper an attempt has been made to propose optimum exponential ratio type estimators for the estimation of population mean using auxiliary information.

Suppose a random sample of size n is drawn from a population containing N units by SRSWOR scheme. Let Y_i & X_i are i^{th} (i =1, 2, 3, ...) unit of population and y_i & x_i are i^{th} (i =1, 2, 3, ...) unit of sample associated with the study variable which is of main interest and auxiliary variable respectively.

Study Variable

$\bar{Y} = \frac{1}{N}\sum_{i=1}^{N} Y_i$, is the population mean.
$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$, is the sample mean.
$C_y = \frac{S_y}{\bar{Y}}$, is the coefficient of variation.
$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})^2$, is the population mean square.

Auxiliary Variable

: $\bar{X} = \frac{1}{N}\sum_{i=1}^{N} X_i$, is the population mean.
: $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, is the sample mean.
: $C_x = \frac{S_x}{\bar{X}}$, is the coefficient of variation.
: $S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \bar{X})^2$, is the population mean square.

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$s_y^2 = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})^2$, is the sample mean square. : $s_x^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$, is the sample mean square.

Further,
$\gamma = \frac{1-f}{n}$, where f = $\frac{n}{N}$ is the sampling fraction.
$\rho = \frac{S_{xy}}{S_x S_y}$, is the correlation coefficient between X and Y.
$\hat{\beta} = \frac{S_{xy}}{S_x^2}$, is the sample regression coefficient.
$\theta = \gamma \bar{Y}^2$.
$k = \frac{a\bar{X}}{2(a\bar{X}+b)}$.
$S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \bar{Y})(X_i - \bar{X})$.
$s_{xy} = \frac{1}{n-1}\sum_{i=1}^{n}(y_i - \bar{y})(x_i - \bar{x})$.

2 Some Ratio type Estimators of population mean

In the absence of auxiliary information, the only estimator satisfying all the properties of a good estimator is the mean per unit estimator given as

$$t_1 = \bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i.$$

The estimator t_1 is unbiased and its MSE is as

$$MSE(t_1) = \theta C_y^2. \tag{1}$$

Cochran [1] used auxiliary information and proposed ratio type estimator as precise estimator of population mean

$$t_2 = \bar{y}\frac{\bar{X}}{\bar{x}}.$$

The estimator is although a biased estimator but found to be more efficient than t_1, if $\rho > \frac{C_x}{2C_y}$. The Bias and MSE of estimator t_2 are as

$$Bias(t_2) = \frac{\theta}{\bar{Y}}(C_x^2 - C_{yx}). \tag{2}$$

$$MSE(t_2) = \theta(C_y^2 + C_x^2 - 2C_{yx}). \tag{3}$$

If the variables Y and X have a low degree of correlation Bahl and Tuteja [4] proposed exponential type ratio estimator as

$$t_3 = \bar{y}\exp\left[\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right].$$

The Bias and MSE of the estimator t_3 are as

$$Bias(t_3) = \frac{\theta}{\bar{Y}}\left(\frac{3}{8}C_x^2 - \frac{1}{2}C_{yx}\right). \tag{4}$$

$$MSE(t_3) = \theta\left(C_y^2 + \frac{1}{4}C_x^2 - C_{yx}\right). \tag{5}$$

Further Singh *et al.* [7] proposed a class of exponential type ratio estimator using the information of auxiliary variable as

$$t_4 = \bar{y}\exp\left[\frac{(a\bar{X}+b) - (a\bar{x}-b)}{(a\bar{X}+b) + (a\bar{x}-b)}\right].$$

Where a($\neq 0$) and b may either be the real numbers or the functions of some known parameters like the coefficient of variation, skewness, correlation etc. For the estimator t_4 Bias and MSE are as

$$Bias(t_4) = \frac{\theta}{\bar{Y}}(k^2 C_x^2 - kC_{yx}). \tag{6}$$

$$MSE(t_4) = \theta(C_y^2 + k^2C_x^2 - 2kC_{yx}). \tag{7}$$

The proposed modified exponential ratio type estimators of population mean $\bar{Y}$ are as

$$t_{pr1} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{Q\bar{x}}\right].$$

$$t_{pr2} = \bar{y}\exp\left[\frac{\bar{X}-\bar{x}}{P\bar{X}}\right].$$

Where, Q and P are non-zero constants.
The Bias and MSE of t_{pr1} & t_{pr2} upto O (n^{-1}) respectively are as

$$Bias(t_{pr1}) = \frac{\theta}{Q\bar{Y}}\left(C_x^2 + \frac{1}{2Q}C_x^2 - C_{yx}\right). \tag{8}$$

$$Bias(t_{pr2}) = \frac{\theta}{P\bar{Y}}\left(\frac{1}{2P}C_x^2 - C_{yx}\right). \tag{9}$$

$$MSE(t_{pr1}) = \theta\left(C_y^2 + \frac{1}{Q^2}C_x^2 - \frac{2}{Q}C_{yx}\right). \tag{10}$$

$$MSE(t_{pr2}) = \theta\left(C_y^2 + \frac{1}{P^2}C_x^2 - \frac{2}{P}C_{yx}\right). \tag{11}$$

For the detailed derivation of the expressions of Bias and MSE of t_{pr1} & t_{pr2} one can see the Appendix-A.

Special Cases
Case1. For Q = P = 1, the MSE of t_{pr1} & t_{pr2} is same as that of ratio type estimator proposed by Cochran [1].
Case2. For Q = P = −1, the MSE of t_{pr1} & t_{pr2} is same as that of product type estimator proposed by Robson [2].
Case3. For Q = P = 2, the MSE of t_{pr1} & t_{pr2} is same as that of the exponential type ratio estimator of Bahl and Tuteja [4].
Case4. For Q = P = −2, the MSE of t_{pr1} & t_{pr2} is same as that of the exponential type product estimator of Bahl and Tuteja [4].

Thus the large sample properties of ratio and product estimators proposed by Cochran [1], Robson [2] and exponential ratio & product estimators of Bahl and Tuteja [4] are the special cases of the proposed exponential ratio type estimators t_{pr1} and t_{pr2}.

3 Optimality Condition and Bias Comparison

The optimal value for the constants Q and P that will minimize the MSE of the estimators t_{pr1} and t_{pr2} was found to be $\frac{C_x}{\rho C_y} = \delta$ (say). The optimal value δ can be estimated quite accurately from some previously carried surveys or from the experience carried by the researcher in due course of time, for instance, one can refer to Reddy ([8], [9]), Srivenkataramana & Tracy [10], Singh & Vishwakarma [11], Singh & Kumar [12] and Singh & Karpe [13]. Substituting the optimal value of Q and P in equations (8) and (9), the values of Bias (t_{pr1}) and Bias (t_{pr2}) are obtained as

$$Bias(t_{pr1}) = \frac{\theta}{\bar{Y}}\left(C_{yx} - \frac{1}{2}\rho^2C_y^2\right). \tag{12}$$

$$Bias(t_{pr2}) = -\frac{\theta}{2\bar{Y}}\rho^2C_y^2. \tag{13}$$

That proposed estimator t_{pr1} is unbiased, if

$$\rho = \frac{2C_x}{C_y}. \tag{14}$$

Substituting the above optimal values of Q and P in equations (10) and (11), the minimum values of MSE (t_{pr1}) and MSE (t_{pr2}) are obtained and are as

$$MSE_{min}(t_{pri}) = \theta C_y^2(1-\rho^2),\ i = 1,2 \tag{15}$$

Therefore the proposed estimators of the study t_{pr1} and t_{pr2} at their optimal condition are equal efficient as that of the usual regression estimator

$$\bar{y}_{lr} = \bar{y} + \hat{\beta}(\bar{X} - \bar{x}).$$

In case a situation arises when the researcher fails to get the values of δ (the optimum value of Q and P) by utilizing all the available resources at his/her disposal, it is advisable to replace δ by its consistent estimate $\hat{\beta} = \left(\frac{\bar{x}}{\bar{y}}\right)$. Hence the resulting ratio estimators based on the estimated value of δ are as

$$\hat{t}_{pr1} = \bar{y}\exp\left[\frac{\bar{y}(\bar{X} - \bar{x})}{\hat{\beta}\bar{x}^2}\right]. \tag{16}$$

$$\hat{t}_{pr2} = \bar{y}\exp\left[\frac{\bar{y}(\bar{X} - \bar{x})}{\hat{\beta}\bar{X}\bar{x}}\right]. \tag{17}$$

Up to O (n^{-1}) of approximation

$$MSE_{min}(\hat{t}_{pri}) = MSE_{min}(t_{pri}) = \theta C_y^2(1-\rho^2),\ i = 1,2 \tag{18}$$

Further from the equations (2), (4), (6) and (12), (13), the following observations are made

$$|Bias(t_{pr1})| < |Bias(t_2)|,\ if \left|\frac{2C_{yx} - \rho^2 C_y^2}{2C_{yx} - 2C_x^2}\right| < 1.$$

$$|Bias(t_{pr1})| < |Bias(t_3)|,\ if \left|\frac{2C_{yx} - \rho^2 C_y^2}{C_{yx} - \frac{3}{4}C_x^2}\right| < 1.$$

$$|Bias(t_{pr1})| < |Bias(t_4)|,\ if \left|\frac{2C_{yx} - \rho^2 C_y^2}{2kC_{yx} - 2k^2 C_x^2}\right| < 1.$$

$$|Bias(t_{pr2})| < |Bias(t_2)|,\ if \left|\frac{\rho^2 C_y^2}{2C_{yx} - 2C_x^2}\right| < 1.$$

$$|Bias(t_{pr2})| < |Bias(t_3)|,\ if \left|\frac{\rho^2 C_y^2}{C_{yx} - \frac{3}{4}C_x^2}\right| < 1.$$

$$|Bias(t_{pr2})| < |Bias(t_4)|,\ if \left|\frac{\rho^2 C_y^2}{2kC_{yx} - 2k^2 C_x^2}\right| < 1.$$

4 Efficiency Comparision

The efficiency comparison of the study is divided into two parts and has been carried out by using the MSE of the estimators t_{pr1} & t_{pr2} and that of the existing ratio estimators t_1, t_2, t_3 and t_4 considered in this study.

Case1. Efficiency comparison of $t_{pr1}(t_{pr2})$ when the value of Q (P) coincides with its optimal value

From the equations (1), (3), (5), (7) and (18), the following conditions have been obtained as

$$MSE_{min}(t_{pri}) < MSE(t_1), if\,\theta\rho^2 > 0. \tag{19}$$

$$MSE_{min}(t_{pri}) < MSE(t_2), if\,(\rho C_y - C_x)^2 > 0. \tag{20}$$

$$MSE_{min}(t_{pri}) < MSE(t_3), if\,(2\rho C_y - C_x)^2 > 0. \tag{21}$$

$$MSE_{min}(t_{pri}) < MSE(t_4), if\,(\rho C_y - kC_x)^2 > 0. \tag{22}$$

Therefore under the conditions (19) to (22), the proposed estimators t_{pr1} & t_{pr2} are more efficient than the considered estimators t_1, t_2, t_3 and t_4.

Case2. Efficiency comparison of $t_{pr1}(t_{pr2})$ when the value of Q (P) does not coincide with its optimal value

By solving the equations (1), (3), (5), (7) and (10), (11), the conditions which have been obtained are as

$$MSE_{min}(t_{pr1})\,(MSE_{min}(t_{pr2})) < MSE(t_1), if$$

$$Q(P) > \left\{ \frac{C_x^2}{2C_{yx}} \right\}. \tag{23}$$

$$MSE_{min}(t_{pr1})\,(MSE_{min}(t_{pr2})) < MSE(t_2), if$$

$$min\left\{1, \frac{C_x^2}{2C_{yx} - C_x^2}\right\} < Q(P) < max\left\{1, \frac{C_x^2}{2C_{yx} - C_x^2}\right\}, \frac{C_{yx}}{C_x^2} > \frac{1}{2}. \tag{24}$$

$$Or\ Q(P) > 1,\ 0 \leq \frac{C_{yx}}{C_x^2} \leq \frac{1}{2}.$$

$$MSE_{min}(t_{pr1})\,(MSE_{min}(t_{pr2})) < MSE(t_3), if$$

$$min\left\{2, \frac{2C_x^2}{4C_{yx} - C_x^2}\right\} < Q(P) < max\left\{2, \frac{2C_x^2}{4C_{yx} - C_x^2}\right\}, \frac{C_{yx}}{C_x^2} > \frac{1}{4}. \tag{25}$$

$$Or\ Q(P) > 2,\ 0 \leq \frac{C_{yx}}{C_x^2} \leq \frac{1}{4}.$$

$$MSE_{min}(t_{pr1})\,(MSE_{min}(t_{pr2})) < MSE(t_4), if$$

$$min\left\{\frac{1}{k}, \frac{C_x^2}{2C_{yx} - kC_x^2}\right\} < Q(P) < max\left\{\frac{1}{k}, \frac{C_x^2}{2C_{yx} - kC_x^2}\right\}, \frac{C_{yx}}{C_x^2} > \frac{k}{2}. \tag{26}$$

$$Or\ Q(P) > \frac{1}{k},\ 0 \leq \frac{C_{yx}}{C_x^2} \leq \frac{k}{2}.$$

Thus under the conditions (23) to (26), the proposed t_{pr1} & t_{pr2} are more efficient than the ratio estimators t_1, t_2, t_3 and t_4 considered.

5 Numerical Illustration

In order to validate the theoretical results of the study, the data of three populations P1, P2 and P3 has been taken. The population P1 has been taken from Murthy [14], where the study variable (Y) is the fixed capital and the auxiliary variable (X) is the output of 80 factories. The population P2 has been taken from Shabir et. al [15] where the variable (Y) to be studied is the apple production level ('000' tons) and the auxiliary variable (X) is the number of apple trees in 104 villages in the year 1999. The population P3 is also taken from Murthy [14] where the auxiliary variable (X) is the number of workers and the variable of main interest (Y) is the output of 80 factories in a region. The parameter values and constants for the populations are given in below Table-1. The performance of proposed ratio estimators t_{pr1} & t_{pr2} has been compared with the existing estimators.

Table 1: Summary Statistics of the Populations P1, P2 and P3.

Population	N	n	$\bar{Y}$	$\bar{X}$	ρ	C_y	C_x	C_{yx}
P1	80	20	11.264	51.826	0.941	0.750	0.354	0.249
P2	104	20	6.254	13931.680	0.860	1.860	1.650	2.639
P3	80	20	51.826	2.851	0.915	0.354	0.948	0.3071

Table-1 shows that from the populations P1, P2 and P3, a sample of 20 units has been taken. There is a high degree of positive correlation between the variables X and Y in the populations considered, the highest correlation is seen in the population P1 followed by P3 and P2. The coefficient of variation of the study variable is highest for the population P2 followed by P1and P3, while as for the auxiliary variable it is highest for P2 followed by P3 and P1.

Table 2: Values of Q (P) for t_{pr1} (t_{pr2}) to be more efficient than the estimators t_1, t_2, t_3 and t_4.

Estimator	Range of Q (P) for Population		
	P1	P2	P3
t_1	Q (P) > 0.252	Q (P) > 0.516	Q (P) > 1.463
t_2	Q (P) $\in (0.336, 1.000)$	Q (P) $\in (1.000, 1.065)$	Q (P) > 1.000
t_3	Q (P) $\in (0.288, 2.000)$	Q (P) $\in (0.695, 2.000)$	Q (P) $\in (2.000, 5.452)$
t_4	Q (P) $\in (0.287, 2.014)$	Q (P) $\in (0.695, 2.000)$	Q (P) $\in (2.126, 3.157)$
Optimum value (δ)	0.502	1.032	2.927

It can be observed from Table-2 that the common range of Q (P) for t_{pr1} (t_{pr2}) to be more efficient than t_1, t_2, t_3 & t_4 is (0.252, 2.014), (0.516, 2.000) and (1.000, 5.452) for the populations P1, P2 and P3 respectivelly. Further the optimum value for each population data set is also obtained and given in the table.

Table 3: MSE and Bias of the Estimators $t_1, t_2, t_3, t_4, t_{pr1}$ and t_{pr2}.

Estimator	Population					
	P1		P2		P3	
	MSE	$\|Bias\|$	MSE	$\|Bias\|$	MSE	$\|Bias\|$
t_1	2.676	0.000	5.465	0.000	12.622	0.000
t_2	0.895	0.053	1.427	0.021	41.285	1.149
t_3	1.637	0.033	2.371	0.075	4.323	0.357
t_4	1.643	0.039	2.371	0.161	2.111	0.016
t_{pr1}	0.306	0.000	1.423	0.343	2.055	0.495
t_{pr2}	0.306	0.105	1.423	0.323	2.055	0.102

It can be seen from Table-3 that estimators t_{pr1} & t_{pr2} have the minimum MSE among all other estimators. Taking the modulus of the bias of all the estimators, it can be seen that the estimator t_{pr1} has minimum bias for the population P1, while as the estimator t_{pr2} has minimum bias for population P3. It is interesting to note that the estimator t_{pr1} is unbiased for the population P1, the reason being that the condition (14) holds. The results of the above table have also been shown below graphically for quick comparisons and better understanding. Fig.-1 and Fig.-2 respectively shows MSE and Bias comparison of the proposed and the existing estimators considered in the study. It is to be noted that modulus value of the Bias has been taken as it is independent of the sign.

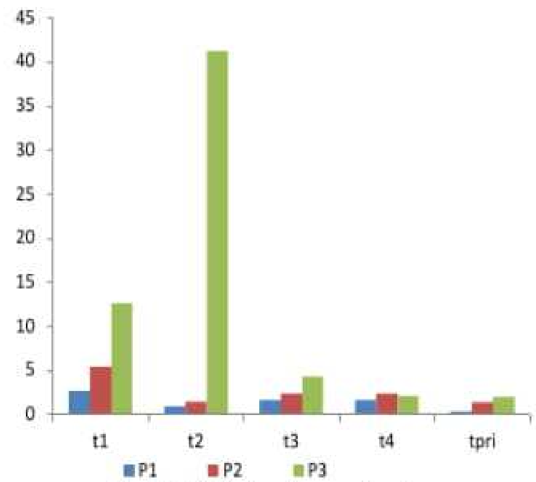

Fig. 1: MSE of various estimators.

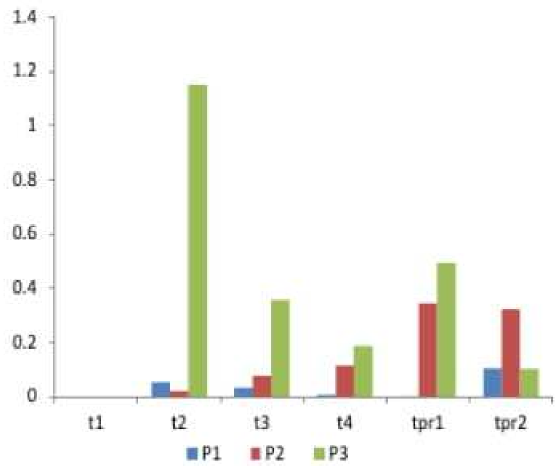

Fig. 2: Bias of various estimators.

Table 4: Percent relative efficiency of various estimators w.r.t mean per unit estimator $\bar{y}$.

Estimator	Population		
	P1	P2	P3
t_1	100.000	100.000	100.000
t_2	298.994	382.971	30.573
t_3	163.469	230.493	291.973
t_4	162.873	230.493	597.916
t_{pri}	874.509	384.048	614.209

Table-4 clearly highlights the efficiency of the proposed estimators. The efficiency is found to be highest for the population P1 followed by P3 and P2. The results of the table have also been shown graphically as

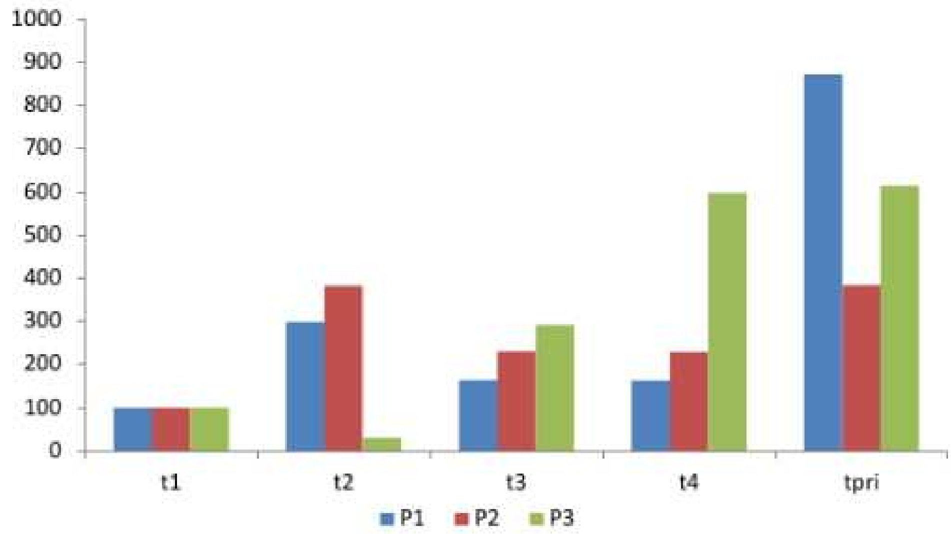

Fig. 3: Percent relative efficiency (PRE) of various estimators.

6 Conclusion

In this paper two optimum exponential ratio type estimators of population mean t_{pr1} and t_{pr2} are proposed in simple random sampling using information on single auxiliary variable. The proposed estimators have the minimum MSE among mean per unit estimator, ratio type estimator of Cochran [1], exponential ratio type estimators of Bahl & Tuteja [4] and Singh *et al.* [7]. Further the proposed estimator t_{pr1} is unbiased, if $\rho = \frac{2C_x}{C_y}$.

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Appendix-A

A detailed derivation of Bias and MSE for the estimators proposed in the present study is as follows
Let

$$e_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}} \;\&\; e_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}.$$

So that the expected values of different quantities are obtained as

$$E(e_0) = E(e_1) = 0.$$

$$E(e_0^2) = \frac{\theta}{\bar{Y}^2}C_y^2,\; E(e_1^2) = \frac{\theta}{\bar{Y}^2}C_x^2,\; E(e_0e_1) = \frac{\theta}{\bar{Y}^2}C_{yx}.$$

Now on writing the proposed estimator t_{pr1} in terms of e_0 and e_1, the obtained expression is as

$$t_{pr1} = \bar{Y}(1+e_0)\exp\left[\left(\frac{-e_1}{Q}\right)(1+e_1)^{-1}\right]. \tag{27}$$

After solving the equation (27) and keeping the terms upto 2^{nd} degree only, the expression will get reduced to the equation

$$t_{pr1} = \bar{Y}(1+e_0)\exp\left(\frac{-e_1+e_1^2}{Q}\right)$$

$$\Rightarrow t_{pr1} = \bar{Y}\left(1+e_0-\frac{e_1}{Q}+\frac{e_1^2}{Q}+\frac{e_1^2}{2Q^2}-\frac{e_0e_1}{Q}\right)$$

$$\Rightarrow t_{pr1}-\bar{Y} = \bar{Y}\left(e_0-\frac{e_1}{Q}+\frac{e_1^2}{Q}+\frac{e_1^2}{2Q^2}-\frac{e_0e_1}{Q}\right). \tag{28}$$

Taking expectation for obtaining Bias (t_{pr1}) and squaring & then taking expectation for obtaining MSE (t_{pr1}) from the equation (28), we get

$$Bias(t_{pr1}) = \frac{\theta}{Q\bar{Y}}\left(C_x^2+\frac{1}{2Q}C_x^2-C_{yx}\right)$$

$$MSE(t_{pr1}) = \theta\left(C_y^2+\frac{1}{Q^2}C_x^2-\frac{2}{Q}C_{yx}\right).$$

Now writing the proposed estimator t_{pr2} in terms of e_0 and e_1, the following expression for t_{pr2} will be obtained as

$$t_{pr2} = \bar{Y}(1+e_0)\exp\left(\frac{-e_1}{P}\right). \tag{29}$$

After solving the equation (29) and keeping the terms upto 2^{nd} degree only, the expression will get reduced to the equation

$$t_{pr2} = \bar{Y}\left(1+e_0-\frac{e_1}{P}+\frac{e_1^2}{2P^2}-\frac{e_0e_1}{P}\right)$$

$$\Rightarrow t_{pr2}-\bar{Y} = \bar{Y}\left(e_0-\frac{e_1}{P}+\frac{e_1^2}{2P^2}-\frac{e_0e_1}{P}\right). \tag{30}$$

Taking expectation for obtaining Bias and squaring & then taking expectation for obtaining MSE of (t_{pr2}) from the above equation (30), we get

$$Bias(t_{pr2}) = \frac{\theta}{P\bar{Y}}\left(\frac{1}{2P}C_x^2 - C_{yx}\right).$$

$$MSE(t_{pr2}) = \theta\left(C_y^2 + \frac{1}{P^2}C_x^2 - \frac{2}{P}C_{yx}\right).$$

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