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Locking A Three-Mirror Optical Cavity using Negative Imaginary Systems Approach

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Abstract: In this paper, we present an example of control system design based on negative imaginary (NI) system theory. The system under consideration is an optical cavity system. A dynamical model of the cavity system is obtained through system identification of applied to experimental input output frequency response data obtained using a digital signal analyzer (DSA). The identified model satisfies NI property. An integral resonant controller is designed based on the NI system theory.

Keywords: Negative imaginary systems, optical cavity, integral resonant control.

1 Introduction

In many modern quantum physics experiments, the use of an optical cavity has become an important tool for enhancement in detection sensitivity [1,2,3], nonlinear interactions, and quantum dynamics [4], quantum information theory, quantum teleportation [5], coding and quantum cryptography [6]. An important application of optical cavities is in laser physics itself. However, there are many applications of external optical cavities (independent from lasers) that take advantage of the common physical properties associated with resonator physics. For example, cavity locking arises in the frequency stabilization of semiconductor lasers [7], cavity-enhanced spectroscopic techniques [1,2,3], cavity quantum electrodynamics [8], microcavities [9,10], as well as in general atomic, molecular, and optical physics [4]. Also, optical cavities are used in spectroscopy in what is called Cavity ring-down spectroscopy (CRDS). CRDS is a highly sensitive spectroscopy technique that allows measurement of absolute optical extinction by material that scatter and absorb light. CRDS technique has been widely used to study gaseous which absorb light at specific wavelengths, and in turn to determine mole fractions down to the parts per trillion level. The technique is also known as cavity ring-down laser absorption spectroscopy (CRLAS).

The structure of optical cavities involves an arrangement of mirrors that forms a standing wave in the cavity resonator. The light source is usually a continuous or discrete laser source. To form the standing wave inside the cavity, the resonant frequency of the cavity must match the input laser frequency. In this case, the cavity is said to be in *lock* with the input laser frequency. Indeed, the characteristic of the optical cavity allow physicists to study the interaction between matter and the applied field [11]. Also, a cavity allows one to impose a well-defined mode structure on the electromagnetic field [12] and to study manifestly quantum mechanical behavior associated with the modified vacuum structure and/or the large field associated with a single photon confined to a small volume[4].

Feedback control has been playing an important role in controlling the cavity locking. For example, in [13,14,15,16], feedback control is used to stabilize an optical cavity in order to build maximum energy inside these cavities.

The meaning of stability here, is to keep the cavity in lock with the frequency of the input laser.

The difference between the resonant frequency of the cavity and the input laser frequency is characterized in terms of a *detuning parameter* Δ. The control goal is to

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keep this detuning parameter $\Delta = 0$. Since the cavity resonant frequency is directly dependent on the distance between the cavity mirrors, it is possible to force the detuning parameter Δ to zero by varying the distance between these mirrors. The use of piezo-electic transducer (PZT) material can play an important role in moving the cavity mirrors to achieve the required resonant frequency. The control methodology here is to measure the detuning parameter Δ, then using this measured Δ for feedback through a controller to generate voltage applied to a PZT actuator attached to one of the cavity mirrors; see Fig. 1.

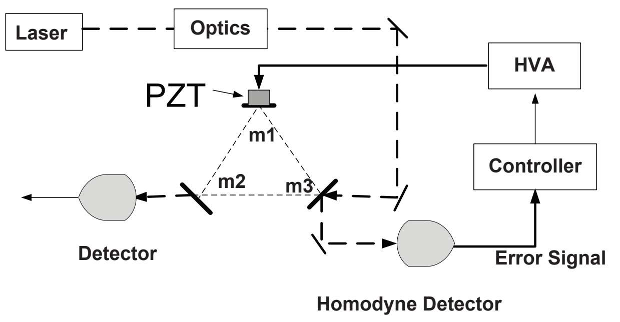

Fig. 1: Locking scheme for a three-mirror ring cavity: The optical signals are represented by dash-dot lines and the electrical signals are represented as solid lines.

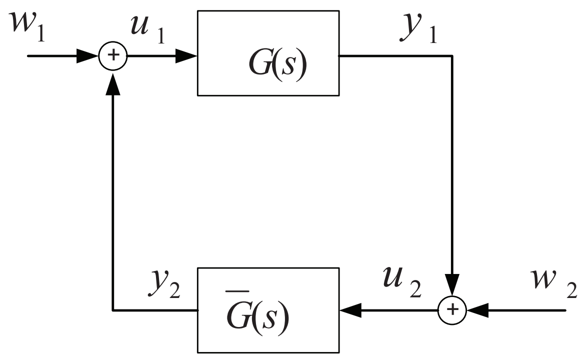

Fig. 2: A negative-imaginary feedback control system. If the plant transfer function matrix $G(s)$ is NI and the controller transfer function matrix $\bar{G}(s)$ is SNI, then the positive-feedback interconnection is internally stable if and only if the DC gain condition, $\lambda_{max}(G(0)\bar{G}(0)) < 1$, is satisfied.

The representation of a cavity system in Fig. 1 involves a collocated force actuator (the PZT actuator) combined with a position sensor (the cavity itself). This implies that the cavity system under consideration can be considered using the recently developed theory called negative imaginary systems theory [17,18,19,20].

Lanzon and Petersen introduced a notion of negative imaginary (NI) systems in [17,18] for the robust control of flexible structures with force actuators combined with position or acceleration sensors. In the single-input single-output (SISO) case, NI systems are defined by considering the properties of the imaginary part of the system frequency response $G(j\omega)$ and requiring the condition $j(G(j\omega) - G(j\omega)^*) \geq 0$ for all $\omega \in (0,\infty)$. The NI property arises in many practical systems. For example, such systems arise when considering the transfer function from a force actuator to a corresponding colocated position sensor (for instance, a piezoelectric sensor) in a lightly damped structure [21,18,22,23,24]. Another area where the underlying system dynamics are NI, are nano-positioning systems; see e.g., [22,25,26,27, 28,29,30,31,32,33]. Also, the positive-position feedback control scheme in [21,34], can be considered using the NI framework. Furthermore, other control methodologies in the literatures such as integral resonant control (IRC) [35] and resonant feedback control [36,37], fit into the NI framework and their stability robustness properties can be explained by NI systems theory.

The stability robustness of interconnected NI systems has been studied in [17,18]. In these papers, it is shown that a necessary and sufficient condition for the internal stability of a positive-feedback control system (see Fig. 2) consisting of an NI plant with transfer function matrix $G(s)$ and a strictly negative imaginary (SNI) controller with transfer function matrix $\bar{G}(s)$ is given by the DC gain condition

$$\lambda_{max}(G(0)\bar{G}(0)) < 1, \tag{1}$$

where the notation $\lambda_{max}(\cdot)$ denotes the maximum eigenvalue of a matrix with only real eigenvalues. This stability result has been used in a number of practical applications. For example in [24], this stability result is applied to the problem of decentralized control of large vehicle platoons. In [22,38], the NI stability result is applied to nanopositioning in an atomic force microscope. A positive position feedback control scheme based on the NI stability result provided in [17,18] is used to design a novel compensation method for a coupled fuselage-rotor mode of a rotary wing unmanned aerial vehicle in [39]. In [33], an IRC scheme based on the stability results provided in [17,18] is used to design an active vibration control system for the mitigation of human induced vibrations in light-weight civil engineering structures, such as floors and footbridges via proof-mass actuators. An identification algorithm which enforces the NI constraint is proposed in [40] for estimating model parameters, following which an Integral resonant controller is designed for damping vibrations in flexible structures. In addition, it is shown in [41] that the class of linear systems having NI transfer function matrices is closely related to the class of linear Hamiltonian input-output systems. Also, an extension of the NI systems theory to infinite-dimensional systems is presented in [42].

The NI framework presented in [17,18] considers systems with poles in the open left half of the complex plane. This theory has been extended in [19] to include NI systems with poles in the closed left half of the complex plane, except at the origin. Also, further extensions to NI systems theory include the study of NI controller synthesis [43,44], connections between NI systems analysis and μ-analysis [45], and conditions for robust stability analysis of mixed NI and bounded-real classes of uncertainties [46]. Furthermore, the concept of lossless NI transfer functions is introduced in [47], and an algebraic approach to the realization of a lossless NI behavior is presented in [48]. The NI systems theory can be extended to nonlinear systems using the concept of counter-clockwise input-output dynamics as presented in [49,50,51]. In [51], a sufficient conditions under which a semilinear Duhem model is counter-clockwise is given, where the counter-clockwise input-output system is restricted to periodic input signals

In this paper, we apply the stability results presented in [17,19] to design a controller for the cavity system shown in Fig. 1. Experimental frequency response data for the cavity system is recorded using a digital signal analyzer and a state space model is obtained using the *System Identification Toolbox* from Matlab®. Then, an integral resonant controller is designed to damp the PZT resonance.

According to the results in [17,19], the positive-feedback interconnection of an NI system and an SNI system is internally stable if and only if the DC gain is less than one. The identified model for the cavity satisfies the NI property.

Unlike the control techniques applied to experimental quantum optics presented in [13,14,15,16], where the linear quadratic Gaussian (LQG) controller synthesis was discussed, the proposed NI technique guarantees the robustness of the closed-loop with respect to changes in the plant resonant frequencies.

The rest of the paper is organized as follows: Section 2 discusses the structure of the three-mirror ring cavity. Section 3 describes the process of obtaining a state space model for the cavity system, starting with the experimental frequency response data and shows that the system model is NI. The design of an integral resonant controller for the cavity system is discussed in Section 4. The paper is concluded with final remarks in Section 5.

2 Cavity System

The cavity system under consideration is a three-mirror open-air ring cavity as shown in Fig. 1, a PZT is mounted on mirror m1 which is used to vary the length of the cavity hence the resonant frequency of the cavity. The laser source in this experiment is continuous-wave 1550 nm diode laser. Before coupling the light into the cavity, it is modified using optics such as isolators, mode matching optics, half wave plates, and beam splitters. Then the light is propagated to the mirror m3. There are two outputs of this cavity, the first output is the transmitted beam, which is detected at the transmitted port by a photodetector at the output of mirror m2. This output is not used for cavity locking. The second output is the error signal which is detected at mirror m3 using homodyne detection; see e.g., [15,16]. This error signal, detected using a homodyne detection, is fed to the integral resonant controller and then to a high voltage amplifier (HVA) before providing the necessary control signal to the PZT actuator.

The control objective is to drive the error signal to zero so as to achieve cavity locking ($\Delta \approx 0$) while maintaining the transmitted signal at a maximum. This ensures that the cavity operates in a linear region as depicted in the calculated plot for the error signal in Fig. 3.

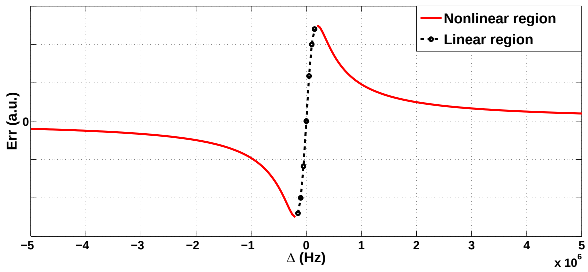

Fig. 3: Calculated variation of the error signal (output of homodyne detector) with the detuning parameter Δ.

3 Cavity Model and System Identification

To obtain a dynamic model for the cavity system, a system identification method is used to determine a state-space model from the experimental input output frequency response data. We record the frequency response of the cavity system using a digital signal analyzer as shown in Fig. 4. The cavity was held in lock using a manually tuned analog PI controller when the frequency response data was collected as shown in Fig. 4.

The identified state space model of the cavity system is given as following;

$$\dot{x}(t) = Ax(t) + Bu(t), \quad (2)$$
$$y(t) = Cx(t) + Du(t). \quad (3)$$

This model can be written as a sum of a second order systems as following;

$$G(s) = \sum_{i=1}^{3} \frac{k_i}{s^2 + 2\zeta\omega_i + \omega_i^2}, \quad (4)$$

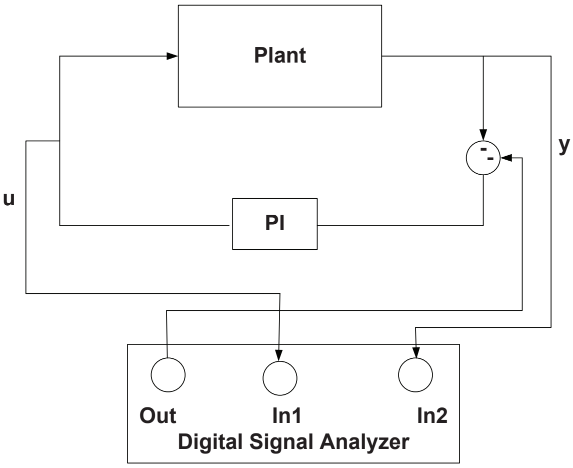

Fig. 4: Block diagram of the the digital signal analyzer setup used to obtain the frequency response data for the plant.

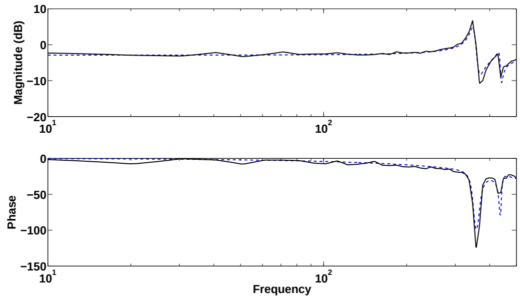

Fig. 5: The solid line is the frequency response data for the plant obtained from the DSA, and the dashed line is the identified model.

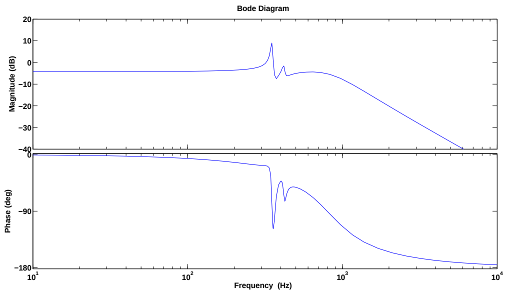

Fig. 6: Bode plot of identified cavity model which shows that the phase lies between 0 and $-\pi$ for all $\omega > 0$.

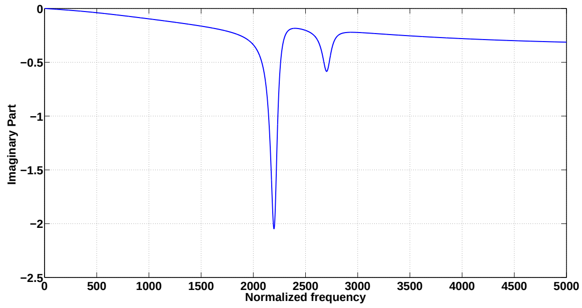

Fig. 7: The imaginary part of the model transfer function, which shows that it is negative over the bandwidth of interest.

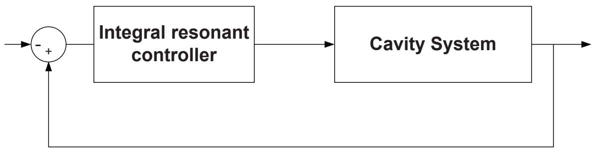

Fig. 8: Integral resonant controller with the cavity model.

where the parameters ζ_i, ω_i and k_i are given in the Table 1.

Table 1: Model parameters

i	1	2	3
ζ_i	25×10^{-3}	40.2×10^{-3}	99.2×10^{-2}
ω_i	22×10^{2}	26.3×10^{2}	51×10^{2}
k_i	3×10^{5}	11×10^{4}	14×10^{6}

The state space model (2)-(3) is satisfies the NI property. This can be verified from the Bode plot of the model in Fig. 6, since the phase lies between 0 and $-\pi$ for all $\omega > 0$, see [18].

Also, the imaginary part of the model transfer function is plotted in Fig.7, which shows that it is negative over the bandwidth of interest.

4 Controller Design

In this paper, an integral resonant controller [52,18] with a feed-through is used, which has a transfer function as follows:

$$C(s) = \frac{\Gamma}{s+\Phi\Gamma} - D. \tag{5}$$

The controller in (5) is SNI for any $\Gamma > 0$ and $\Phi > 0$.

The proposed integral resonant control scheme is shown diagrammatically in Fig. 8. According to the stability results in [17,19,20], the closed loop system for the cavity system and the integral resonant controller in (5) is internally stable providing the DC gain condition $\lambda_{max}(C(0)G(0)) < 1$ is satisfied.

The DC gain condition $\lambda_{max}(C(0)G(0)) < 1$ can be guaranteed with the feed-through gain D. Also, the large

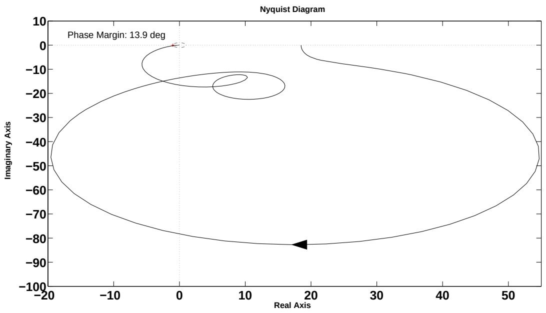

Fig. 9: Nyquist plot of $-C(0)G(s)$ which shows that the phase margin is 14°.

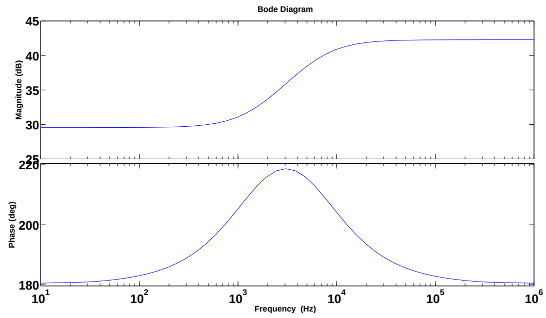

Fig. 10: Bode plot of the integral resonant controller with a transfer function $C(s) = \frac{\Gamma}{s+\Phi\Gamma} - D$, where $\Phi = 0.01, D = 130$ and $\Gamma = 40 \times 10^5$.

D is used to reduce the steady state error. The controller (5) can be considered as a lead compensator since it have one pole and one zero in the left half of the complex plane.

The integral resonant controller design process that we used can be summarized as following:

– Determine the DC gain that required to give the desired steady state error, where $erorr = \frac{1}{1-G(0)C(0)}$ and $C(0) = \frac{1}{\Phi} - D$.
– Using Nyquist plot to determine the phase margin of the loop gain $-C(0)G(s)$.
– Choose the location of the zero and the pole of the controller such that the phase margin is greater that 30° and the maximum phase lead of the controller occurs at the gain crossover frequency.

In our case, we chose the steady state error to be less than 0.06 and to achieve that, a DC gain for the controller is chosen to be $C(0) = -30$ dB. The resulting phase margin of the loop-gain $-C(0)G(s)$ is 14° as shown in the Nyqust plot given in Fig. 9.

Then, the controller parameters were chosen such that the controller has a maximum phase shift 35° as shown Bode plot given in Fig. 10.

Finally, to verify the performance of the closed loop system, the step response and Bode plot of the closed loop are plotted in Fig. 11 and Fig. 12 respectively.

5 Conclusion

In this paper, a dynamical model of a cavity system is obtained through system identification applied to experimental input output frequency response data. The data was recorded using a digital signal analyzer. The system model is shown to be negative imaginary (NI) system. An integral resonant controller is designed based on the NI system theory. Simulation results for the closed-loop have been provided. Future research will be directed towards implementing this controller experimentally.

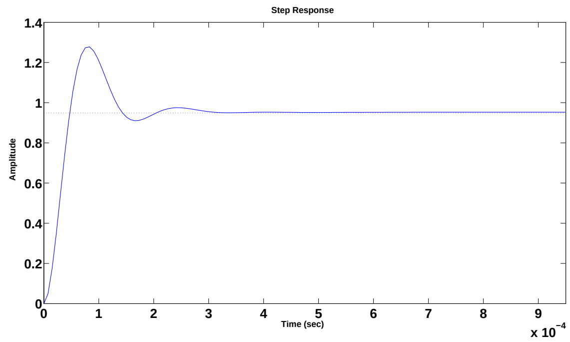

Fig. 11: The step response of the closed-loop system corresponding to an integral resonant controller with a transfer function $C(s) = \frac{\Gamma}{s+\Phi\Gamma} - D$, where $\Phi = 0.01, D = 130$ and $\Gamma = 40 \times 10^5$.

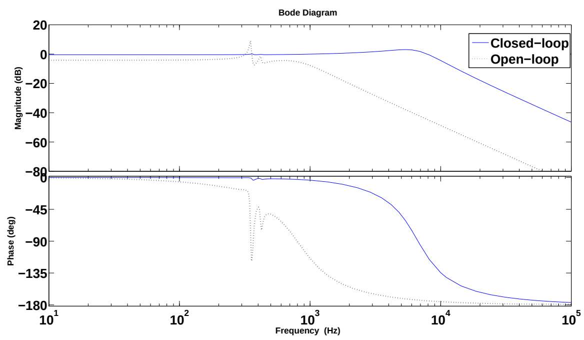

Fig. 12: Open- and closed-loop frequency responses for the cavity system and an integral resonant controller with a transfer function $C(s) = \frac{\Gamma}{s+\Phi\Gamma} - D$, where $\Phi = 0.01, D = 130$ and $\Gamma = 40 \times 10^5$. These parameters are chosen to provide adequate damping of the resonant mode.

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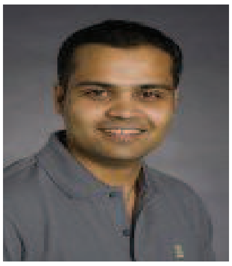

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