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Ranked Set Sampling versus Simple Random Sampling in the estimation of the Ratio Estimators

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Abstract: This paper introduces the basic concepts of Ranked Set Sampling (RSS) and compares RSS with Simple Random Sampling (SRS). In this paper, we have presented some improved estimators of population mean using auxiliary variables in RSS. We have derived the expressions for the bias and the mean square errors of the suggested estimators up to the first order of approximation and shown that the proposed estimators under optimum conditions are more efficient than other estimators considered in this paper. In an attempt to verify the efficiencies of proposed estimators, theoretical results are supported by empirical study and simulation study, for which we have considered two populations.

Keywords: Study variable, auxiliary variable, bias, mean square error, and ranked set sampling

1 Introduction

When the population mean of the auxiliary variable is known and positively correlated with the study variable, the ratio method of estimation is a well-established methodology for estimating the population mean of the study variable. Many authors including Sisodia and Dwivedi (1981)[1] , Pandey and Dubey (1988)[2], Bahl and Tuteja (1991)[3], Upadhyaya and Singh (1999)[4], Singh (2003)[5], Khosnevisan et al. (2007)[6], and Singh et al. (2019)[7] have proposed modified ratio estimators for estimating the population mean of a study variable by use of the known value of some population parameters of auxiliary variable.
Ranked Set Sampling (RSS) is an improved sampling method over Simple Random Sampling (SRS). It offers a wide range of methods for utilising auxiliary data to produce estimators that are more effective. McIntyre (1952)[8] was the first to propose RSS and suggested that RSS has better efficiency than SRS estimators of the population mean. Takahashi and Wakimoto (1968)[9] gave a mathematical formulation to RSS to estimate the ratio. Samawi and Muttlak (1996)[10] suggested RSS to define the ratio estimators for the population mean. Shiva (2006)[11] compared RSS with SRS for the estimation of the mean and the ratio. He concluded that RSS gives a better estimate for both the mean and the ratio.

Consider a finite population U= $\{U_1, U_2, \ldots . U_N\}$ of size N. Let Y be a study variable with a value of y_i on the i^{th}unit. Let X be an auxiliary variable with a value of x_i on the i^{th} unit. The following notations are used in this article:
N: Population Size, n: Sample size, λ: $\left(\frac{1}{n}-\frac{1}{N}\right)$, Y: Study Variable, X: Auxiliary Variable
$\overline{X}$,$\overline{Y}$: Population means, $\bar{x}$,$\bar{y}$: Sample means, S_x^2,S_y^2 : Population Variances
s_x^2, s_y^2: Sample Variances, S_{xy} : Population Covariance, C_x, C_y : coefficients of variation
$\beta_1(x)$, $\beta_1(y)$: Population Coefficient of Skewness
$\beta_2(x)$, $\beta_2(y)$: Population Coefficient of Kurtosis, ρ_{xy}:Population Correlation Coefficient, M_d: Median of Auxiliary Variable
Q_r: Inter Quartile Range, $Q_r = Q_3 - Q_1$ where Q_3=lower quartile and Q_1= upper quartile of an auxiliary variable

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2 Ratio type Estimators for Population Mean in SRS

The Classical ratio estimator for the population mean of the study variable Y proposed by Cochran (1940)[12] is, given by

$$t_r = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right) \tag{1}$$

MSE of the estimator t_r up to the first-order approximation is given by

$$MSE(t_r) = \bar{Y}^2\lambda(C_x^2 + C_y^2 - 2\rho_{yx}C_xC_y) \tag{2}$$

Bahl and Tuteja (1991)[3] proposed the exponential ratio type estimator, given by

$$t_{bt} = \bar{y}\exp\left(\frac{\bar{X}-\bar{x}}{\bar{X}+\bar{x}}\right) \tag{3}$$

MSE of the estimator t_{bt} up to the first-order approximation is given by

$$MSE(t_{bt}) = \bar{Y}^2\frac{\lambda}{4}(C_x^2 + 4C_y^2 - 4\rho_{yx}C_xC_y) \tag{4}$$

There are other variations of the ratio estimators of the population mean, but we'll concentrate on the ones shown in table 1 below.

Table 1: Some Commonly Used Estimators

$t_1 = \bar{y}\left(\frac{\bar{X}+C_x}{\bar{x}+C_x}\right)$	$t_8 = \bar{y}\left(\frac{C_x\bar{X}+\rho_{xy}}{C_x\bar{x}+\rho_{xy}}\right)$
$t_2 = \bar{y}\left(\frac{\bar{X}+\rho}{\bar{x}+\rho}\right)$	$t_9 = \bar{y}\left(\frac{C_x\bar{X}+M_d}{C_x\bar{x}+M_d}\right)$
$t_3 = \bar{y}\left(\frac{\bar{X}+\beta_2(x)}{\bar{x}+\beta_2(x)}\right)$	$t_{10} = \bar{y}\left(\frac{\beta_1(x)\bar{X}+\rho_{xy}}{\beta_1(x)\bar{x}+\rho_{xy}}\right)$
$t_4 = \bar{y}\left(\frac{\beta_1(x)\bar{X}+\beta_2(x)}{\beta_1(x)\bar{x}+\beta_2(x)}\right)$	$t_{11} = \bar{y}\left(\frac{\rho_{xy}\bar{X}+\sigma_x}{\rho_{xy}\bar{x}+\sigma_x}\right)$
$t_5 = \bar{y}\left(\frac{\bar{X}+\sigma_x}{\bar{x}+\sigma_x}\right)$	$t_{12} = \bar{y}\left(\frac{\rho_{xy}\bar{X}+M_d}{\rho_{xy}\bar{x}+M_d}\right)$
$t_6 = \bar{y}\left(\frac{\bar{X}+M_d}{\bar{x}+M_d}\right)$	$t_{13} = \bar{y}\left(\frac{\sigma_x\bar{X}+M_d}{\sigma_x\bar{x}+M_d}\right)$
$t_7 = \bar{y}\left(\frac{\beta_2(x)\bar{X}+C_x}{\beta_2(x)\bar{x}+C_x}\right)$	

Singh and Kumar (2011)[13] have given the justification for using different population parameters in the construction of improved estimators.
To obtain the bias and the MSE of the estimators, we define

$$\bar{y} = \bar{Y}(1+\varepsilon_0)$$

$$\bar{x} = \bar{X}(1+\varepsilon_1)$$

Such that
E (ε_0)=E (ε_1)=0
$E(\varepsilon_0^2) = \lambda C_y^2$, $E(\varepsilon_1^2) = \lambda C_x^2$, $E(\varepsilon_0\varepsilon_1) = \lambda C_{yx}$

The Bias and the MSE of the estimators t_1 to t_{13} up to the first-order approximation are given by

$$Bias(t_i) = \lambda\bar{Y}\left(\theta_i^2C_x^2 - \theta_i\rho_{xy}C_xC_y\right),\ i = 1,2...13 \tag{5}$$

$$MSE(t_i) = \lambda\bar{Y}^2\left(C_y^2 + \theta_iC_x^2(\theta_i - 2K)\right) i = 1,2\ldots13 \tag{6}$$

where

$$C_y = \frac{S_y}{\overline{Y}}, C_x = \frac{S_x}{\overline{X}}, \rho_{yx} = \frac{S_{yx}}{S_y S_x},\ k = \rho_{yx}\frac{C_y}{C_x}$$

$$S_y^2 = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^2, \qquad S_x^2 = \frac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2,\ S_{xy} = \frac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})(X_i - \overline{X})$$

where $\theta_i^{\prime s}$ are given in table 2

Table 2: Value of $\boldsymbol{\theta_i}$

$\theta_1 = \frac{\overline{X}}{\overline{X}+C_x}$	$\theta_8 = \frac{C_x\overline{X}}{C_x\overline{X}+\rho_{xy}}$
$\theta_2 = \frac{\overline{X}}{\overline{X}+\rho_{xy}}$	$\theta_9 = \frac{C_x\overline{X}}{C_x\overline{X}+M_d}$
$\theta_3 = \frac{\overline{X}}{\overline{X}+\beta_2(x)}$	$\theta_{10} = \frac{\beta_1(x)\overline{X}}{\beta_1(x)\overline{X}+\rho_{xy}}$
$\theta_4 = \frac{\beta_1(x)\overline{X}}{\beta_1(x)\overline{X}+\beta_2(x)}$	$\theta_{11} = \frac{\rho_{xy}\overline{X}}{\rho_{xy}\overline{X}+\sigma_x}$
$\theta_5 = \frac{\overline{X}}{\overline{X}+\sigma_x}$	$\theta_{12} = \frac{\rho_{xy}\overline{X}}{\rho_{xy}\overline{X}+M_d}$
$\theta_6 = \frac{\overline{X}}{\overline{X}+M_d}$	$\theta_{13} = \frac{\sigma_x\overline{X}}{\sigma_x\overline{X}+M_d}$
$\theta_7 = \frac{\beta_2(x)\overline{X}}{\beta_2(x)\overline{X}+C_x}$	

3 Ratio Estimators in RSS

In RSS, we rank randomly selected units from the population merely by observation or prior experience after which only a few of these sampled units are measured. RSS is more cost-friendly than SRS because fewer samples need to be collected and measured.

RSS takes the following steps.

1. Select sampling units from the target population.
2. Randomly partitioned sampling units into disjoint subsets each having a pre-defined size (usually taken to be ≤ 4, as it is convenient and minimizes ranking error)
3. Rank each sub-set.
4. Measure one suitable selected unit from each ranked sub-set.

Coming to the mathematical formulation of RSS, if in the RSS scheme, we want to select a sample of size k. We select k random sets each of size k from the target population. Each set is then ranked by observation/inspection/prior information or a convenient/cheap method.

Original observation and after Ranking

$$\begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1k} \\ x_{21} & x_{22} & \cdots & x_{2k} \\ \vdots & \vdots & \vdots & \vdots \\ x_{k1} & x_{k2} & \cdots & x_{kk} \end{bmatrix} \rightarrow \begin{bmatrix} x_{1(1)} & x_{1(2)} & \cdots & x_{1(k)} \\ x_{2(1)} & x_{2(2)} & \cdots & x_{2(k)} \\ \vdots & \vdots & \vdots & \vdots \\ x_{k(1)} & x_{k(2)} & \cdots & x_{k(k)} \end{bmatrix}$$

Here x_{ij} represents the j^{th} observation in the i^{th} set and $x_{i(j)}$ is the j^{th} ordered statistic in the i^{th} set. After Ranking, select the diagonal units.By choosing the smallest ranked unit from the first row, the second smallest ranked unit from the second row, and so on, up until the largest unit from the k^{th} row is chosen, we have now arrived at $x_{1(1)}$, $x_{2(2)}$, ..., $x_{k(k)}$. This will be the Ranked Set Sample (RSS). We can repeat the whole steps r times to obtain an RSS of size $n = rk$.

$$\overline{X}_{RSS} = \frac{1}{rk}\sum_{l=1}^{r}\sum_{i=1}^{k} x_{i(i)l} \tag{7}$$

$$var\left(\overline{X}_{RSS}\right) = \frac{\sigma^2}{n} - \frac{1}{rm^2}\sum_{i=1}^{k}\left(\mu_{(i)} - \mu\right)^2 \tag{8}$$

where $\mu_{(i)}$ is the mean of the i^th ranked set and is given by

$$\mu_{(i)} = \frac{1}{r}\sum_{l=1}^{r} x_{i(i)l}$$

In this paper, we take the situation where we use ranking on an auxiliary variable. Let $(y_{[i]}, x_{(i)})$ denote an i^{th} judgment ordering in the i^{th} set for the study variable Y corresponding to the i^{th} order statistic of the i^{th} set for the auxiliary variable X.
Kadilar et al. (2009)[14] defined the ratio estimator for the population mean $\overline{Y}$ in RSS as

$$t_{r,RSS} = \frac{\overline{y}_{[n]}}{\overline{x}_{(n)}}\overline{X} \tag{9}$$

To obtain the bias and the MSE of the estimators, we define

$$\overline{y}_{[n]} = \overline{Y}\left(1+\varepsilon_0\right)$$

$$\overline{x}_{(n)} = \overline{X}(1+\varepsilon_1)$$

E (ε_0)=E (ε_1)=0
$E(\varepsilon_0^2) = C_y^2 - D_{y[i]}^2$
$E(\varepsilon_1^2) = C_x^2 - D_{x[i]}^2$
E $(\varepsilon_0\varepsilon_1) = \eta C_{yx} - D_{yx[i]}$

The Mean Squared Error (MSE) of the estimator $t_{r,RSS}$ is given by

$$MSE(t_{r,RSS}) = \overline{Y}^2[\eta(C_y^2 + C_x^2 - 2\rho C_y C_x) - D_{y[i]}^2 - D_{x[i]}^2 + 2D_{yx[i]}] \tag{10}$$

where

$$\overline{y}_{[n]} = \frac{1}{n}\sum_{i=1}^{n} y_{[i]}$$

$$\overline{x}_{(n)} = \frac{1}{n}\sum_{i=1}^{n} x_{(i)}$$

$$\widehat{\beta} = \frac{\hat{R}(\eta\hat{C}_{yx} - \hat{D}_{yx[i]})}{(\eta\hat{C}_x^2 - \hat{D}_{x[i]}^2)}$$

$$\hat{C}_y = \frac{1}{\overline{y}_{[n]}}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(y_{[i]} - \overline{y}_{[n]})^2}$$

$$\hat{C}_x = \frac{1}{\overline{x}_{(n)}}\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_{(i)} - \overline{x}_{(n)})^2}$$

$$\hat{C}_{yx} = \frac{1}{(n-1)\overline{y}_{[n]}\overline{x}_{(n)}}\sum_{i=1}^{n}(y_{[i]} - \overline{y}_{[n]})(x_{(i)} - \overline{x}_{(n)})$$

$$D_{y[i]}^2 = \frac{1}{k^2 r\overline{y}_{[n]}^2}\sum_{i=1}^{k}(\mu_{[iy]} - \overline{y}_{[n]})^2$$

$$D_{x[i]}^2 = \frac{1}{k^2 r\overline{x}_{(n)}^2}\sum_{i=1}^{k}(\mu_{(ix)} - \overline{x}_{(n)})^2$$

$$D_{yx[i]} = \frac{1}{k^2 r \overline{y}_{[n]} \overline{x}_{(n)}} \sum_{i=1}^{k} (\mu_{[iy]} - \overline{y}_{[n]})(\mu_{(ix)} - \overline{x}_{(n)})$$

$$\eta = \frac{1}{kr} \quad , \qquad \hat{R} = \frac{\overline{y}_{[n]}}{\overline{x}_{(n)}}$$

where $\mu_{[iy]}$ and $\mu_{(ix)}$ are the means of the ith ranked set of Y and X respectively.

4 Suggested Estimator based on RSS

Using RSS, we establish a class of estimators for the population mean Y of the variable in the study Y.

$$t_{rss} = \overline{y}_{[n]} \left[\frac{A\overline{X} + B}{A\overline{x}_{(n)} + B} \right] \tag{11}$$

For the values of A and B refer to Singh and Kumar (2011) [13].
Some estimators of this class of estimators are given below in table 3.

Table 3: Some Estimators in RSS

$t_1 = \overline{y}_{[n]} \left(\frac{\overline{X}+C_x}{\overline{x}_{(n)}+C_x} \right)$	$t_8 = \overline{y}_{[n]} \left(\frac{C_x\overline{X}+\rho_{xy}}{C_x\overline{x}_{(n)}+\rho_{xy}} \right)$
$t_2 = \overline{y}_{[n]} \left(\frac{\overline{X}+\rho}{\overline{x}_{(n)}+\rho} \right)$	$t_9 = \overline{y}_{[n]} \left(\frac{C_x\overline{X}+M_d}{C_x\overline{x}_{(n)}+M_d} \right)$
$t_3 = \overline{y}_{[n]} \left(\frac{\overline{X}+\beta_2(x)}{\overline{x}_{(n)}+\beta_2(x)} \right)$	$t_{10} = \overline{y}_{[n]} \left(\frac{\beta_1(x)\overline{X}+\rho_{xy}}{\beta_1(x)\overline{x}_{(n)}+\rho_{xy}} \right)$
$t_4 = \overline{y}_{[n]} \left(\frac{\beta_1(x)\overline{X}+\beta_2(x)}{\beta_1(x)\overline{x}_{(n)}+\beta_2(x)} \right)$	$t_{11} = \overline{y}_{[n]} \left(\frac{\rho_{xy}\overline{X}+\sigma_x}{\rho_{xy}\overline{x}_{(n)}+\sigma_x} \right)$
$t_5 = \overline{y}_{[n]} \left(\frac{\overline{X}+\sigma_x}{\overline{x}_{(n)}+\sigma_x} \right)$	$t_{12} = \overline{y}_{[n]} \left(\frac{\rho_{xy}\overline{X}+M_d}{\rho_{xy}\overline{x}_{(n)}+M_d} \right)$
$t_6 = \overline{y}_{[n]} \left(\frac{\overline{X}+M_d}{\overline{x}_{(n)}+M_d} \right)$	$t_{13} = \overline{y}_{[n]} \left(\frac{\sigma_x\overline{X}+M_d}{\sigma_x\overline{x}_{(n)}+M_d} \right)$
$t_7 = \overline{y}_{[n]} \left(\frac{\beta_2(x)\overline{X}+C_x}{\beta_2(x)\overline{x}_{(n)}+C_x} \right)$	

The bias and the MSE for the estimators t_1 to t_{13} up to first-order approximation are given by

$$Bias(t_i) = \overline{Y} \left(\theta_i^2(\eta C_x^2 - D_{x[i]}^2) - \theta_i(\eta C_{xy} - D_{xy[i]}) \right), i = 1,2...13 \tag{12}$$

$$MSE(t_i) = \overline{Y}^2 \left((\eta C_y^2 - D_{y[i]}^2) + \theta_i^2(\eta C_x^2 - D_{x[i]}^2) - \theta_i(\eta C_{xy} - D_{xy[i]}) \right), i = 1,2\ldots13 \tag{13}$$

where $\theta_i^{'s}$ are given in table 4

Table 4: Value of $\theta_i^{'s}$

$\theta_1 = \frac{\overline{X}}{\overline{X}+C_x}$	$\theta_8 = \frac{C_x\overline{X}}{C_x\overline{X}+\rho_{xy}}$
$\theta_2 = \frac{\overline{X}}{\overline{X}+\rho_{xy}}$	$\theta_9 = \frac{C_x\overline{X}}{C_x\overline{X}+M_d}$
$\theta_3 = \frac{\overline{X}}{\overline{X}+\beta_2(x)}$	$\theta_{10} = \frac{\beta_1(x)\overline{X}}{\beta_1(x)\overline{X}+\rho_{xy}}$
$\theta_4 = \frac{\beta_1(x)\overline{X}}{\beta_1(x)\overline{X}+\beta_2(x)}$	$\theta_{11} = \frac{\rho_{xy}\overline{X}}{\rho_{xy}\overline{X}+\sigma_x}$
$\theta_5 = \frac{\overline{X}}{\overline{X}+\sigma_x}$	$\theta_{12} = \frac{\rho_{xy}\overline{X}}{\rho_{xy}\overline{X}+M_d}$
$\theta_6 = \frac{\overline{X}}{\overline{X}+M_d}$	$\theta_{13} = \frac{\sigma_x\overline{X}}{\sigma_x\overline{X}+M_d}$
$\theta_7 = \frac{\beta_2(x)\overline{X}}{\beta_2(x)\overline{X}+C_x}$	

5 Empirical Study

In this section, we compare the performance of the proposed estimators with the other estimators considered in this paper. For comparison, we have taken a population from the book Singh (2003) (page no. 1111(Appendix)) [15].We took ranked set samples with size k=3 and cycles r=4, 5, 6, from the stated population.

Table 5: The Mean Square Errors of the Estimators

	K=3, r=4 , n=rk=12		K=3, r=5 , n=rk=15		K=3, r=6 , n=rk=18		K=3, r=10 , n=rk=30	
	SRS	RSS	SRS	RSS	SRS	RSS	SRS	RSS
t_1	9996.532	7289.907	10908.4	7150.472	13333.97	6324.866	6596.9996	4638.094
t_2	10001.57	7293.819	10917.34	7156.519	13345.57	6325.023	6603.4843	4641.472
t_3	9957.64	7259.57	10839.05	7103.681	13243.76	6323.721	6546.6663	4611.86
t_4	9990.171	7284.964	10897.1	7142.837	13319.3	6324.67	6588.8049	4633.825
t_5	12739.1	6618.599	9053.927	7945.792	7218.921	7190.161	4595.831	3275.291
t_6	9690.421	5994.786	7827.774	5840.974	7934.969	6798.66	4114.73	3224.119
t_7	10007.86	7298.697	10928.48	7164.062	13360.03	6325.222	6611.5668	4645.681
t_8	10003.36	7295.21	10920.52	7158.669	13349.7	6325.079	6605.7883	4642.672
t_9	9361.467	6005.363	7882.372	5683.263	8357.964	6666.869	4216.6336	3307.629
t_{10}	10007.36	7298.308	10927.6	7163.461	13358.88	6325.206	6610.9231	4645.346
t_{11}	13892.69	6947.933	9741.749	8825.355	7362.075	7184.858	4962.1048	3401.291
t_{12}	10189.33	6042.783	7898.639	6137.228	7577.698	6969.272	4086.6456	3170.666
t_{13}	10006.11	7297.334	10925.37	7161.954	13355.99	6325.166	6609.3088	4644.505

From table 5 it can be seen that RSS gives a better estimate when compared to SRS estimate as the Mean Squared Error (MSE) of RSS is always smaller than that of SRS.

Table 6: The Mean Square Errors, Relative Precision (RP) and Relative Saving (RS) of the Estimators for n=12

Estimators	**MSE(SRS)**	**MSE(RSS)**	**RP**	**RS**
t_1	9996.532	7289.907	1.371284	0.270756
t_2	10001.57	7293.819	1.371239	0.270733
t_3	9957.64	7259.57	1.371657	0.270955
t_4	9990.171	7284.964	1.371341	0.270787
t_5	12739.1	6618.599	1.924743	0.48045
t_6	9690.421	5994.786	1.616475	0.38137
t_7	10007.86	7298.697	1.371185	0.270704
t_8	10003.36	7295.2133	1.371224	0.270724
t_9	9361.467	6005.363	1.558851	0.358502
t_{10}	10007.36	7298.308	1.371189	0.270706
t_{11}	13892.69	6947.933	1.999543	0.499886
t_{12}	10189.33	6042.783	1.686198	0.40695
t_{13}	10006.11	7297.334	1.3712	0.270712

Table 7: The Mean Square Errors, Relative Precision (RP) and Relative Saving (RS) of the Estimators for n=15

Estimators	**MSE(SRS)**	**MSE(RSS)**	**RP**	**RS**
t_1	10908.4	7150.472	1.52555	0.344499
t_2	10917.34	7156.519	1.52551	0.344481
t_3	10839.05	7103.681	1.525836	0.344622
t_4	10897.1	7142.837	1.525599	0.34452
t_5	9053.927	7945.792	1.139462	0.122393
t_6	7827.774	5840.974	1.340149	0.253814
t_7	10928.48	7164.062	1.525459	0.34446
t_8	10920.52	7158.669	1.525495	0.344475
t_9	7882.372	5683.263	1.386945	0.278991
t_{10}	10927.6	7163.461	1.525463	0.344461
t_{11}	9741.749	8825.355	1.103837	0.094069
t_{12}	7898.639	6137.228	1.287004	0.223002
t_{13}	10925.37	7161.954	1.525473	0.344466

Table 8: The Mean Square Errors, Relative Precision (RP) and Relative Saving (RS) of the Estimators for n=18

Estimators	**MSE(SRS)**	**MSE(RSS)**	**RP**	**RS**
t_1	13333.97	6324.866	2.108183	0.525658
t_2	13345.57	6325.023	2.109965	0.526058
t_3	13243.76	6323.721	2.094299	0.522513
t_4	13319.3	6324.67	2.105928	0.52515
t_5	7218.921	7190.161	1.003999	0.000139
t_6	7934.969	6798.66	1.167137	0.143203
t_7	13360.03	6325.222	2.112184	0.526556
t_8	13349.7	6325.079	2.110598	0.526201
t_9	8357.964	6666.869	1.253657	0.202333
t_{10}	13358.88	6325.206	2.112007	0.526517
t_{11}	7362.075	7184.858	1.024665	0.023386
t_{12}	7577.698	6969.272	1.087301	0.080292
t_{13}	13355.99	6325.166	2.111564	0.526417

Table 9: The Mean Square Errors, Relative Precision (RP) and Relative Saving (RS) of the Estimators for n=30

Estimators	MSE(SRS)	MSE(RSS)	RP	RS
t_1	6596.9996	4638.094	1.422351	0.296939
t_2	6603.4843	4641.472	1.422713	0.297118
t_3	6546.6663	4611.86	1.419528	0.295541
t_4	6588.8049	4633.825	1.421893	0.296712
t_5	4595.831	3275.291	1.403182	0.287334
t_6	4114.73	3224.119	1.276234	0.216445
t_7	6611.5668	4645.681	1.423164	0.29734
t_8	6605.7883	4642.672	1.422842	0.297181
t_9	4216.6336	3307.629	1.274821	0.215576
t_{10}	6610.9231	4645.346	1.423128	0.297323
t_{11}	4962.1048	3401.291	1.458889	0.314547
t_{12}	4086.6456	3170.666	1.288892	0.22414
t_{13}	6609.3088	4644.505	1.423038	0.297278

where

$$RP = \frac{MSE\ (SRS)}{MSE\ (RSS)}$$

$$RS = \frac{MSE\ \ (SRS) - MSE(RSS)}{MSE\ (SRS)}$$

$$RP = \frac{1}{1 - RS}$$

The performance of estimators in SRS and RSS is compared using the two aforementioned quantities.
If RS = 0.2969 and RP = 1.42
Relative Saving (RS): it indicates by using RSS, the saving in the sample size with set size three is **30%** as compared to SRS.
Relative Precision (RP): it shows that the accuracy of the population mean estimator using an RSS of size 100 is equivalent to the accuracy of the estimator of population mean using a SRS of size 142.

6 SIMULATION STUDY

We perform some simulation studies in order to verify the proposed estimator's relative efficiency (RE) with the some existing estimators in SRS.
This is done via the following steps
1. Generate two independent random variables X$\sim$ N (μ, σ^2) and $X_1 \sim N(\mu_1, \sigma_1^2)$ using the box Muller method (Jhonson, 1987).
2. Set Y= $\rho X + \sqrt{1-\rho^2}X_1$ where $\rho = 0.6, 0.7, 0.75, 0.8, 0.9 \leq 1$.
3: Return the pair (Y, X).
4: Consider the population-I with the following parameters μ =5, σ =3, and $\mu_1 = 5, \sigma_1$=3 in step-1 and then repeat the steps 1 to 3 a total of 1000 times. For both Y and X, this population will have the same variance.
5: In a similar manner, create the population-II with the parameters μ =3, σ =2, μ_1=5 and σ_1=3 in step-1 and then repeat the steps 1 to 3 a total of 1000 times.For the variables Y and X, this population will have distinct variances.
6. From N=1000, three sets of size three are chosen at random. The xi is used to rank these sets. Choose the smallest value from the first set, the median value from the second group, and the biggest value from the third set.
7. Repeating step 6 r times results in RSS of size n=kr (k=3) r=4, 5,6,10.
8. Step 6 and Step 7 are repeated 10000 times. The mean and the Variance of these values are computed.

Table 10: Comparison of MSE of the estimators of Population 1

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
0.8	12	t_1	0.006915	0.006408
		t_2	0.011512	0.008481
		t_3	0.078791	0.044021
		t_4	0.245205e	0.133914
		t_5	0.023205	0.014397
		t_6	0.067527	0.037940
		t_7	0.006795	0.006413
		t_8	0.085540	0.047714
		t_9	0.197643	0.108285
		t_{10}	0.237785	0.130378
		t_{11}	0.029346	0.017596
		t_{12}	0.081513	0.045474
		t_{13}	0.039596	0.023041
	15	t_1	0.005895	0.005717
		t_2	0.009068	0.007215
		t_3	0.059857	0.034176
		t_4	0.187230	0.104265
		t_5	0.017676	0.011609
		t_6	0.051237	0.029508
		t_7	0.005860	0.005732
		t_8	0.065064	0.037167
		t_9	0.150890	0.084219
		t_{10}	0.181466	0.101134
		t_{11}	0.022275	0.014014
		t_{12}	0.061921	0.035291
		t_{13}	0.030026	0.018183
	18	t_1	0.005008	0.005002
		t_2	0.007603	0.006067
		t_3	0.054514	0.028063
		t_4	0.174260	0.085694
		t_5	0.015360	0.009566
		t_6	0.046432	0.024207
		t_7	0.005048	0.005072
		t_8	0.059324	0.030456
		t_9	0.139902	0.069229
		t_{10}	0.169440	0.083008
		t_{11}	0.019581	0.011515
		t_{12}	0.056407	0.028967
		t_{13}	0.026710	0.014898
	30	t_1	0.003650	0.003312
		t_2	0.004829	0.003773
		t_3	0.029653	0.016763
		t_4	0.095139	0.051995
		t_5	0.008780	0.005740
		t_6	0.025302	0.014450
		t_7	0.003706	0.003375
		t_8	0.032386	0.018241
		t_9	0.076326	0.041874
		t_{10}	0.092644	0.050628
		t_{11}	0.109791	0.006874
		t_{12}	0.030682	0.017323
		t_{13}	0.014788	0.008888
0.75	12	t_1	0.037091	0.036390
		t_2	0.038900	0.036489
		t_3	0.097191	0.065852
		t_4	0.251570	0.148685
		t_5	0.048319	0.040645
		t_6	0.087044	0.060452

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
		t_7	0.037383	0.036715
		t_8	0.099447	0.067101
		t_9	0.206999	0.124751
		t_{10}	0.244235	0.145012
		t_{11}	0.055222	0.044021
		t_{12}	0.103558	0.069194
		t_{13}	0.062244	0.047609
	15	t_1	0.031635	0.028896
		t_2	0.031768	0.028881
		t_3	0.072982	0.050804
		t_4	0.189702	0.114992
		t_5	0.037636	0.031862
		t_6	0.065419	0.046716
		t_7	0.032094	0.029154
		t_8	0.074692	0.051907
		t_9	0.155854	0.096341
		t_{10}	0.183943	0.111919
		t_{11}	0.042382	0.034349
		t_{12}	0.077682	0.053356
		t_{13}	0.047380	0.037094
	18	t_1	0.025335	0.024702
		t_2	0.024991	0.024264
		t_3	0.062890	0.041468
		t_4	0.173210	0.094059
		t_5	0.030083	0.026358
		t_6	0.055775	0.038129
		t_7	0.025869	0.024998
		t_8	0.064437	0.042272
		t_9	0.141035	0.078735
		t_{10}	0.168175	0.091355
		t_{11}	0.034402	0.028272
		t_{12}	0.067267	0.043527
		t_{13}	0.038989	0.030375
	30	t_1	0.015903	0.013915
		t_2	0.015613	0.013483
		t_3	0.035602	0.023520
		t_4	0.095678	0.055685
		t_5	0.018182	0.014572
		t_6	0.031781	0.021534
		t_7	0.016197	0.014130
		t_8	0.036545	0.024050
		t_9	0.078115	0.046263
		t_{10}	0.093172	0.054274
		t_{11}	0.020427	0.015667
		t_{12}	0.037954	0.024786
		t_{13}	0.022870	0.016930
0.7	12	t_1	0.052087	0.051382
		t_2	0.052304	0.050677
		t_3	0.105441	0.077393
		t_4	0.253550	0.157381
		t_5	0.060315	0.054033
		t_6	0.095884	0.072284
		t_7	0.052631	0.051847
		t_8	0.103759	0.076576
		t_9	0.210488	0.134128
		t_{10}	0.245983	0.153499
		t_{11}	0.068303	0.057940
		t_{12}	0.115456	0.082718
		t_{13}	0.072831	0.060306

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
	15	t_1	0.044537	0.040943
		t_2	0.043207	0.040350
		t_3	0.079316	0.060218
		t_4	0.190630	0.122059
		t_5	0.047511	0.042721
		t_6	0.072300	0.056347
		t_7	0.045257	0.041302
		t_8	0.078111	0.059730
		t_9	0.158034	0.103959
		t_{10}	0.184674	0.118847
		t_{11}	0.052762	0.045586
		t_{12}	0.086652	0.064251
		t_{13}	0.055900	0.047413
	18	t_1	0.035511	0.034008
		t_2	0.033871	0.032894
		t_3	0.067225	0.047486
		t_4	0.172857	0.097312
		t_5	0.037536	0.034171
		t_6	0.060583	0.044432
		t_7	0.036280	0.034430
		t_8	0.066038	0.047005
		t_9e	0.141764	0.082627
		t_{10}	0.167547	0.094511
		t_{11}	0.042346	0.036210
		t_{12}	0.074106	0.050684
		t_{13}	0.045247	0.037508
	30	t_1	0.022044	0.019836
		t_2	0.021076	0.019015
		t_3	0.038535	0.027519
		t_4	0.095895	0.058109
		t_5	0.022866	0.019611
		t_6	0.034986	0.025691
		t_7	0.022462	0.020127
		t_8	0.037998	0.027273
		t_9	0.078961	0.049044
		t_{10}	0.093291	0.056650
		t_{11}	0.025343	0.020777
		t_{12}	0.042224	0.029474
		t_{13}	0.026887	0.021572
0.6	12	t_1	0.141893	0.138594
		t_2	0.134668	0.133143
		t_3	0.151760	0.136890
		t_4	0.261354	0.194154
		t_5	0.130482	0.128441
		t_6	0.145964	0.134095
		t_7	0.144099	0.140162
		t_8	0.145793	0.134109
		t_9	0.227447	0.176178
		t_{10}	0.254652	0.190678
		t_{11}	0.134120	0.128983
		t_{12}	0.164344	0.143131
		t_{13}	0.134008	0.129025
	15	t_1	0.122378	0.107124
		t_2	0.114144	0.102675
		t_3	0.116610	0.103332
		t_4	0.195163	0.145835
		t_5	0.106582	0.098370
		t_6	0.112968	0.101410

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
		t_7	0.124721	0.108379
		t_8	0.112897	0.101528
		t_9	0.170107	0.132201
		t_{10}	0.189892	0.143051
		t_{11}	0.106667	0.098202
		t_{12}	0.124931	0.107708
		t_{13}	0.106635	0.098285
	18	t_1	0.096927	0.093065
		t_2	0.089288	0.088685
		t_3	0.093792	0.086761
		t_4	0.171326	0.120847
		t_5	0.082749	0.084101
		t_6	0.090027	0.085363
		t_7	0.099113	0.094270
		t_8	0.090019	0.085244
		t_9	0.146739	0.109714
		t_{10}	0.166077	0.118383
		t_{11}	0.083341	0.083348
		t_{12}	0.102103	0.090125
		t_{13}	0.083360	0.083353
	30	t_1	0.058998	0.049214
		t_2	0.054836	0.046945
		t_3	0.056000	0.047789
		t_4	0.097018	0.070877
		t_5	0.050932	0.044823
		t_6	0.054097	0.046724
		t_7	0.060170	0.049859
		t_8	0.054125	0.046766
		t_9	0.083866	0.063516
		t_{10}	0.094585	0.069578
		t_{11}	0.050912	0.044859
		t_{12}	0.060252	0.050220
		t_{13}	0.050916	0.044905

Table 11: Comparison of MSE of the estimators of Population 2

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
0.9	12	t_1	0.078432	0.066783
		t_2	0.052092	0.051160
		t_3	0.103298	0.076338
		t_4	0.250956	0.156770
		t_5	0.048981	0.048560
		t_6	0.064502	0.055693
		t_7	0.095038	0.075816
		t_8	0.078780	0.063284
		t_9	0.158745	0.106578
		t_{10}	0.246215	0.154323
		t_{11}	0.049405	0.048570
		t_{12}	0.069108	0.058070
		t_{13}	0.053492	0.050306
	15	t_1	0.068809	0.051628
		t_2	0.045161	0.039714
		t_3	0.077574	0.057516
		t_4	0.189514	0.119568
		t_5	0.040841	0.037474
		t_6	0.049614	0.042268
		t_7	0.082350	0.058308
		t_8	0.059693	0.047945
		t_9	0.119243	0.080642
		t_{10}	0.185723	0.117536
		t_{11}	0.040691	0.037400
		t_{12}	0.052780	0.043984
		t_{13}	0.042563	0.038530
	18	t_1	0.059973	0.047421
		t_2	0.036628	0.035482
		t_3	0.066637	0.046861
		t_4	0.173508	0.096770
		t_5	0.032141	0.032734
		t_6	0.040096	0.035211
		t_7	0.073316	0.053785
		t_8	0.049611	0.039331
		t_9	0.106240	0.065247
		t_{10}	0.170043	0.094955
		t_{11}	0.031928	0.032456
		t_{12}	0.043077	0.036457
		t_{13}	0.033576	0.032710
	30	t_1	0.034788	0.025956
		t_2	0.022571	0.019026
		t_3	0.037830	0.026923
		t_4	0.095665	0.058603
		t_5	0.200695	0.017497
		t_6	0.023873	0.019400
		t_7	0.041481	0.029705
		t_8	0.028918	0.022129
		t_9	0.059209	0.038655
		t_{10}	0.093960	0.057649
		t_{11}	0.019902	0.017372
		t_{12}	0.025410	0.020220
		t_{13}	0.020597	0.017715
0.8	12	t_1	0.141807	0.125811
		t_2	0.108235	0.104482
		t_3	0.128986	0.109129
		t_4	0.254751	0.176445
		t_5	0.096418	0.095747
		t_6	0.100710	0.095242
		t_7	0.162071	0.137760

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
		t_8	0.105964	0.097691
		t_9	0.174692	0.133388
		t_{10}	0.250157	0.174013
		t_{11}	0.094786	0.093914
		t_{12}	0.107633	0.098374
		t_{13}	0.095070	0.093448
	15	t_1	0.125549	0.096438
		t_2	0.094108	0.080148
		t_3	0.097458	0.081180
		t_4	0.191307	0.132120
		t_5	0.080966	0.073024
		t_6	0.078979	0.071561
		t_7	0.143184	0.105324
		t_8	0.082038	0.073279
		t_9	0.130827	0.099226
		t_{10}	0.187599	0.130156
		t_{11}	0.078035	0.071347
		t_{12}	0.083029	0.073600
		t_{13}	0.076989	0.070774
	18	t_1	0.105169	0.085723
		t_2	0.075581	0.070355
		t_3	0.081082	0.067986
		t_4	0.172854	0.109623
		t_5	0.063372	0.063361
		t_6	0.062398	0.060787
		t_7	0.122040	0.093935
		t_8	0.065648	0.061815
		t_9	0.113766	0.082501
		t_{10}	0.169171	0.107843
		t_{11}	0.060811	0.061536
		t_{12}	0.066565	0.062175
		t_{13}	0.060125	0.060648
	30	t_1	0.061313	0.045804
		t_2	0.045786	0.037361
		t_3	0.047183	0.038455
		t_4	0.096080	0.065921
		t_5	0.039082	0.033695
		t_6	0.037806	0.050420
		t_7	0.069785	0.033136
		t_8	0.039392	0.034080
		t_9	0.064464	0.048226
		t_{10}	0.094315	0.065009
		t_{11}	0.037522	0.032860
		t_{12}	0.039802	0.034286
		t_{13}	0.036939	0.032621
0.7	12	t_1	0.190415	0.165128
		t_2	0.154338	0.142302
		t_3	0.147692	0.131039
		t_4	0.256787	0.189828
		t_5	0.132207	0.127149
		t_6	0.127575	0.121554
		t_7	0.213615	0.179036
		t_8	0.128131	0.121758
		t_9	0.185897	0.151365
		t_{10}	0.252135	0.187333
		t_{11}	0.126619	0.122485
		t_{12}	0.135431	0.124871
		t_{13}	0.126180	0.122102
	15	t_1	0.168686	0.131594
		t_2	0.134530	0.114435

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
		t_3	0.112200	0.102774
		t_4	0.192396	0.146233
		t_5	0.111218	0.102388
		t_6	0.100962	0.096888
		t_7	0.189514	0.141833
		t_8	0.101061	0.097088
		t_9	0.139279	0.117426
		t_{10}	0.188639	0.144162
		t_{11}	0.103548	0.098314
		t_{12}	0.104526	0.098705
		t_{13}	0.102858	0.098015
	18	t_1	0.139073	0.116093
		t_2	0.107807	0.099855
		t_3	0.092042	0.086159
		t_4	0.172494	0.122121
		t_5	0.086895	0.088191
		t_6	0.079266	0.082040
		t_7	0.158553	0.125661
		t_8	0.079630	0.082025
		t_9	0.119566	0.098016
		t_{10}	0.168535	0.120233
		t_{11}	0.080541	0.083990
		t_{12}	0.083705	0.083057
		t_{13}	0.080164	0.083591
	30	t_1	0.081340	0.059201
		t_2	0.064849	0.050695
		t_3	0.054158	0.046548
		t_4	0.096342	0.071014
		t_5	0.053368	0.044861
		t_6	0.048228	0.042722
		t_7	0.091179	0.064328
		t_8	0.048348	0.042859
		t_9	0.068343	0.054962
		t_{10}	0.094455	0.070096
		t_{11}	0.049526	0.043045
		t_{12}	0.050076	0.044058
		t_{13}	0.049190	0.042968
0.6	12	t_1	0.290450	0.247681
		t_2	0.249113	0.220727
		t_3	0.184523	0.170812
		t_4	0.259603	0.208118
		t_5	0.204827	0.190528
		t_6	0.181429	0.172445
		t_7	0.319947	0.266278
		t_8	0.182989	0.174020
		t_9	0.207322	0.181338
		t_{10}	0.255552	0.206084
		t_{11}	0.186089	0.176484
		t_{12}	0.181649	0.169916
		t_{13}	0.188959	0.178912
	15	t_1	0.256371	0.197140
		t_2	0.216784	0.176464
		t_3	0.141376	0.132999
		t_4	0.194067	0.159151
		t_5	0.172147	0.152256
		t_6	0.144859	0.136328
		t_7	0.283885	0.211143
		t_8	0.147328	0.137942
		t_9	0.155704	0.139580
		t_{10}	0.190849	0.157516
		t_{11}	0.151168	0.140200

r_{yx}	n	estimators	SRS(MSE)	RSS (MSE)
		t_{12}	0.140402	0.132836
		t_{13}	0.154740	0.142342
	18	t_1	0.207050	0.162151
		t_2	0.172411	0.144437
		t_3	0.114175	0.108810
		t_4	0.172008	0.132951
		t_5	0.134171	0.123927
		t_6	0.113196	0.110893
		t_7	0.231698	0.110894
		t_8	0.115030	0.174194
		t_9	0.131358	0.112074
		t_{10}	0.168188	0.115221
		t_{11}	0.117521	0.131139
		t_{12}	0.112159	0.113946
		t_{13}	0.120431	0.115712
	30	t_1	0.121678	0.090070
		t_2	0.103157	0.080398
		t_3	0.068017	0.062516
		t_4	0.096759	0.079083
		t_5	0.081993	0.069292
		t_6	0.069043	0.096706
		t_7	0.134321	0.062747
		t_8	0.070198	0.063335
		t_9	0.076010	0.067319
		t_{10}	0.094904	0.078238
		t_{11}	0.719900	0.064140
		t_{12}	0.067252	0.062067
		t_{13}	0.073697	0.065111

Table 10 and Table 11 also show that our proposed estimators (RSS) perform better than the existing estimators (SRS). The MSE of the estimators decreases when the correlation and sample size increases the population 1 and 2.

7 Conclusion

Using the information from auxiliary variables, we have developed estimators for the population mean in ranked set sampling in this article. Up to the first order of approximation, the expressions for the suggested estimators' bias and MSE have been determined. of approximation. An empirical and simulation study for comparing the efficiency of the proposed estimators with other estimators have been used.
The results have been shown in Tables 1 to 7 respectively. The Tables show that the proposed estimators turn out to be more efficient as compared to the other estimators for both populations. Based on our empirical study and simulation study, we can conclude that our proposed estimators can be preferred over the other estimators taken in this paper in several real situations

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