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Rapidly Convergent Approximation Method to Chiral Nonlinear Schrodinger's Equation in (1+2)-dimensions

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Abstract: In this paper, rapidly convergent approximation method (RCAM) is applied for seek of exact solutions of chiral nonlinear Schrodinger's equation (NLSE) in (1+2)-dimensions. Application of this method gives series solution which converges to the exact analytic solution of NLSE from which several kinds of exact solutions are constructed. The solutions comprise bright, dark, kink, soliton, singular solution and periodic solutions. The parametric conditions for existence of chiral solutions are also presented.

Keywords: Chiral Nonlinear Schrodinger Equation, Solitons and singular periodic solutions, Rapidly Convergent Approximation Method

1 Introduction

Nowadays, the research of finding solutions of nonlinear evolution equations (NLEEs) which model several physical phenomena are challenging and fascinating task. NLEEs arises in various fields of science and engineering, especially in mathematical physics. Their closed form solutions provide us a complete concept of the physical phenomena, which help to analyze and understand the physical processes that are modeled by NLEEs. So finding their solution have great importance in mentioned fields. There are several solutions like, shock wave, solitons, plane wave, cuspons, peakons, rogons that can be constructed from this equations. All this various types solutions cannot be obtained by one method. Hence developing a new method which able to provide various types of solutions of NLEEs have drowned a lot of interests of researchers. The nonlinear Schrodinger equation (NLSE) is the generic model in nonlinear physics governing wave evolution in optics [1, 2], Bose-Einstein condensates [3–5], plasma physics [6, 7], fluid dynamics and nuclear physics [11]. Recently, it is observed that chiral NLSE plays important role in Hall effect and optical fiber. Many result [12–18] have been reported for this model in past those are on soliton perturbation theory, application of semi inverse variational principle and Lie symmetry analysis.

Many powerful methods have been developed to construct exact solutions of nonlinear evolution equations, such as the Jacobi elliptic function expansion [8], the tanh-function expansion [9], ansatz method [10], and so on. One of the most effective simple flexible methods to develop the solitary wave solutions of nonlinear evolution equations is the Adomian decomposition method [20, 21]. Which is a simple and effective iterative method for the computation of series solutions, was first proposed by Adomian [20]. This method was further developed by many authors among them one is P. K. Das and M. M. Panja [22]. Recently, same authors [23] have proposed a rapidly convergent approximation method (RCAM) as a modification of previous work and applying this scheme they obtained 1-soliton solution of many important well known nonlinear mathematical physics models. And they have shown that this scheme is very useful and efficient tool for constructing soliton solution of single as well as system of nonlinear differential equations. In this paper, RCAM is applied to chiral NlSE in (1+2)-dimensions to look for exact solution.

The objective of this paper is to investigate the chiral NLSE in (1+2)-dimensions by RCAM. Using this method derived several kinds of exact travelling wave solution of NLSE. Furthermore, the condition on parameters for existence of this solutions presented. Here, for the first time to my knowledge, chiral NLSE in (1+2)-dimensions

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is solved by RCAM and this method can construct eight types of solution is shown.

The present paper is organized as follows. After this introduction Section 2 contains a brief review of RCAM. In Section 3 we presented conversion of chiral NLSE to two different ODEs. Section 4 and 5 contains solution obtaining technique of these two ODEs by RCAM and finally using these solutions we obtained eight different solutions of chiral NLSE. Last Section 6 is devoted to research perspective.

2 The Rapidly Convergent Approximation Method

First consider a second order differential equation of the form

$$u''(\xi)-\lambda^2u(\xi)=\mathscr{N}[u](\xi)+g(\xi),\ \xi\in(-\infty,\infty), \qquad (1)$$

where $\mathscr{N}[u]$ is an nonlinear term in u, and $g(\xi)$ is the inhomogeneous or source term, continuous over the infinite domain of independent variable. Now (1) can now be recast into the form

$$\hat{\mathscr{O}}[u](\xi)=\mathscr{N}[u]+g(\xi), \qquad (2)$$

where the linear operator $\hat{\mathscr{O}}[\cdot]$ given by

$$\hat{\mathscr{O}}[\cdot](\xi)=e^{\lambda\ \xi}\tfrac{d}{d\xi}(e^{-2\lambda\ \xi}\tfrac{d}{d\xi}(e^{\lambda\ \xi}[\cdot]). \qquad (3)$$

One may reinterpret the inverse operator $\hat{\mathscr{O}}^{-1}$ as a twofold integral operator given by

$$\hat{\mathscr{O}}^{-1}[\cdot](\xi)=e^{-\lambda\ \xi}\int e^{2\lambda\ \xi}\int e^{-\lambda\ \xi}[\cdot](\xi)\ d\xi\ d\xi. \qquad (4)$$

In this case operation of $\hat{\mathscr{O}}^{-1}$ on $u''(\xi)-\lambda^2u(\xi)$ gives

$$\begin{aligned}&\hat{\mathscr{O}}^{-1}(u''(\xi)-\lambda^2u(\xi))\\&=e^{-\lambda\ \xi}\int e^{2\lambda\ \xi}\int e^{-\lambda\ \xi}(u''(\xi)-\lambda^2u(\xi))\ d\xi\ d\xi\\&=e^{-\lambda\ \xi}\int e^{2\lambda\ \xi}(e^{-\lambda\ \xi}u'(\xi)+\lambda\ e^{-\lambda\ \xi}u(\xi)+c)d\xi\\&=u(\xi)-c_+\ e^{\lambda\ \xi}-c_-\ e^{-\lambda\ \xi},\end{aligned} \qquad (5)$$

where c, c_+ and c_- integration constants. Operating $\hat{\mathscr{O}}^{-1}$ on both sides of (2) and use of (5), leads to

$$u(\xi)=c_+\ e^{+\lambda\ \xi}+c_-\ e^{-\lambda\ \xi}+\hat{\mathscr{O}}^{-1}[\mathscr{N}[u]](\xi)+\hat{\mathscr{O}}^{-1}[g](\xi). \qquad (6)$$

Assuming $\lambda>0$ and using the vanishing boundary condition $u(\infty)=0$ for localized solution of (1) within $[0,\infty)$ we can obtain $c_+=0$, thus

$$u(\xi)=c_-\ e^{-\lambda\ \xi}+\hat{\mathscr{O}}^{-1}[\mathscr{N}[u]](\xi)+\hat{\mathscr{O}}^{-1}[g](\xi),\ \xi\in[0,\infty). \qquad (7)$$

To evaluate terms involving the unknown $u(\xi)$ in R.H.S of (7), one writes

$$u(\xi)=\sum_{n=0}^{\infty}u_n(\xi) \qquad (8)$$

and recasts the nonlinear term into

$$\mathscr{N}[u]](\xi)=\sum_{m=0}^{\infty}\mathscr{A}_m(u_0(\xi),u_1(\xi),\cdots,u_m(\xi)), \qquad (9)$$

where $\mathscr{A}_n(\xi)$, $n\geq 0$ are Adomian polynomials [20,21] for nonlinear term can be obtained by using the formula

$$\mathscr{A}_m(\xi)=\frac{1}{m!}[\frac{d^m}{d\varepsilon^m}\mathscr{N}(\sum_{k=0}^{\infty}u_k\varepsilon^k)]_{\varepsilon=0},\ \ m\geq 0. \qquad (10)$$

The correction to the leading order due to presence of nonlinearities are obtained by executing the following steps

$$u_0(\xi)=c_-\ e^{-\lambda\xi}+\hat{\mathscr{O}}^{-1}[g](\xi) \qquad (11)$$

with

$$u_{n+1}(\xi)=\hat{\mathscr{O}}^{-1}[\mathscr{A}_n](\xi),\ \ n\geq 0. \qquad (12)$$

It is important to note that whenever the domain becomes $(-\infty,0]$, instead of using vanishing boundary condition $u(\infty)=0$, for localized solution of (16) we use $u(-\infty)=0$ in (6) and get

$$u(\xi)=c_+\ e^{+\lambda\ \xi}+\hat{\mathscr{O}}^{-1}[\mathscr{N}[u]](\xi)+\hat{\mathscr{O}}^{-1}[g](\xi),\ \xi\in(-\infty,0]. \qquad (13)$$

similarly following the previous steps, higher order corrections over leading order approximation can be obtained recursively from

$$u_0(\xi)=c_+\ e^{+\lambda\ \xi}+\hat{\mathscr{O}}^{-1}[g](\xi) \qquad (14)$$

and

$$u_{n+1}(\xi)=\hat{\mathscr{O}}^{-1}[\mathscr{A}]_n(\xi),\ \ n\geq 0. \qquad (15)$$

In case of $\lambda<0$, one has to proceed in the same way by retaining the term involving $e^{+\lambda\ \xi}$.

3 Application of the RCAM to chiral NLSE

In [19], M. Eslami studied the chiral NLSE in (1+2)-dimensions

$$\begin{aligned}iq_t+a(q_{xx}+q_{yy})+i\{b_1(qq_x^*-q^*q_x)\\+b_2(qq_y^*-q^*q_y)\}q=0,\end{aligned} \qquad (16)$$

where the first term is the evolution term, while a represents the coefficient of dispersion term. Also, b_1 and b_2 are the coefficients of nonlinear coupling terms. In order to solve Eq.(16)we apply the transformation

$$q(x,y,t)=\{A+u(\xi)\}e^{i\phi(x,y,t)}, \qquad (17)$$

where $u(\xi)$ is the amplitude portion of the solution, while phase portion $\phi(x,y,t)$ and ξ are given below

$$\begin{aligned}&\xi=B_1x+B_2y-2a(k_1B_1+k_2B_2)t,\\&\phi(x,y,t)=k_1x+k_2y+\omega t+\theta.\end{aligned} \qquad (18)$$

Under this transformation NLSE (16) reduces to the travelling wave equation

$$\begin{aligned} &a(B_1^2+B_2^2)u''+\{6A^2(b_1k_1+b_2k_2)-a(k_1^2+k_2^2)-\omega\}u \\ &+6A(b_1k_1+b_2k_2)u^2+2(b_1k_1+b_2k_2)u^3 \\ &+A\{2A^2(b_1k_1+b_2k_2)-a(k_1^2+k_2^2)-\omega\}=0. \end{aligned} \tag{19}$$

For localised solution equating constant term with zero we get

$$A\{2A^2(b_1k_1+b_2k_2)-a(k_1^2+k_2^2)-\omega\}=0. \tag{20}$$

Solving for A we get

$$A=0,\pm\sqrt{-\frac{\lambda^2}{\delta}}, \tag{21}$$

where λ, δ are given by

$$\lambda=\sqrt{\frac{a(k_1^2+k_2^2)+\omega}{a(B_1^2+B_2^2)}},\ \delta=-\frac{2(k_1b_1+k_2b_2)}{a(B_1^2+B_2^2)}. \tag{22}$$

4 Case-I (When $A=0$)

Using $A=0$ in Eq (19) we get the following differential equation

$$u''(\xi)-\lambda^2u(\xi)=\delta\,u(\xi)^3. \tag{23}$$

Application of RCAM, to (19)gives following iteration formulae

$$\begin{aligned} &u_0(\xi)=c_\pm\,e^{\pm\lambda\xi}, \\ &u_{n+1}(\xi)=\hat{\mathscr{O}}^{-1}[\mathscr{A}_n](\xi),\ \ n\geq 0, \end{aligned} \tag{24}$$

where $\mathscr{A}_n(\xi)$, $n\geq 0$ are calculated using formulae (10). Hence from (25) the successive terms are given by

$$\begin{aligned} &u_0(\xi)=c_\pm\,e^{\pm\lambda\xi}, \\ &u_1(\xi)=\frac{\delta\left(c_\pm e^{\pm\lambda\xi}\right)^3}{8\lambda^2}, \\ &u_2(\xi)=\frac{\delta^2\left(c_\pm e^{\pm\lambda\xi}\right)^5}{64\lambda^4}, \\ &\ldots\ldots \\ &u_n(\xi)=c_\pm 8^{-n}e^{\pm\lambda\xi}\left(\frac{\delta c_\pm^2 e^{\pm 2\lambda\xi}}{\lambda^2}\right)^n, \\ &\ldots\ldots \end{aligned} \tag{25}$$

The closed form of series solution of (23) is given by:

$$\begin{aligned} u(\xi)&=\sum_{n=0}^{\infty}u_n(\xi) \\ &=\frac{8c_\pm\lambda^2e^{\mp\lambda\xi}}{8\lambda^2e^{\mp 2\lambda\xi}-\delta c_\pm^2}. \end{aligned} \tag{26}$$

Now using (26) with (21) in (17) we can find solution of NLSE (16) in the following form

$$q=\frac{8c_\pm\lambda^2e^{\mp\lambda\xi}}{8\lambda^2e^{\mp 2\lambda\xi}-\delta c_\pm^2}e^{i\phi(x,y,t)}, \tag{27}$$

where $\phi(x,y,t)$, $\xi(x,y,t)$, λ and δ are given in (18) in (22) respectively. Depending on values of λ and δ this solution can be recast in the following forms

$$\begin{aligned} q_1(\xi)&=\sqrt{-\frac{2\lambda^2}{\delta}} \\ &\times sech\left[\pm\lambda\xi+\frac{1}{2}log\left(-\frac{\delta c_\pm^2}{8\lambda^2}\right)\right]e^{i\phi(x,y,t)}, \end{aligned} \tag{28}$$

$$\begin{aligned} q_2(\xi)&=\sqrt{\frac{2\lambda^2}{\delta}} \\ &\times csch\left[\pm\lambda\xi+\frac{1}{2}log\left(\frac{\delta c_\pm^2}{8\lambda^2}\right)\right]e^{i\phi(x,y,t)}, \end{aligned} \tag{29}$$

these solutions exists when λ is real and $2\lambda^2\delta>0$.

$$\begin{aligned} q_3(\xi)&=\sqrt{\frac{2\lambda_1^2}{\delta}} \\ &\times sec\left[\pm\lambda_1\xi-\frac{i}{2}log\left(\frac{\delta c_\pm^2}{8\lambda_1^2}\right)\right]e^{i\phi(x,y,t)}, \end{aligned} \tag{30}$$

$$\begin{aligned} q_4(\xi)&=\sqrt{\frac{2\lambda_1^2}{\delta}} \\ &\times csc\left[\pm\lambda_1\xi-\frac{i}{2}log\left(-\frac{\delta c_\pm^2}{8\lambda_1^2}\right)\right]e^{i\phi(x,y,t)}, \end{aligned} \tag{31}$$

these solutions exists when λ is pure imaginary (ie, $\lambda=i\,\lambda_1$), $2\lambda^2\delta>0$ and values of $c_\pm$ are so chosen that trigonometric functions becomes real.

5 Case-II (When $A=\pm\sqrt{-\frac{\lambda^2}{\delta}}$)

For $A=\pm\sqrt{-\frac{\lambda^2}{\delta}}$ Eq (19) reduces to the following differential equation

$$u''(\xi)+2\lambda^2u(\xi)=\mp 3\sqrt{-\lambda^2\delta}u(\xi)^2+\delta\,u(\xi)^3. \tag{32}$$

Application of RCAM, to (19)gives following iteration formulae

$$\begin{aligned} &u_0(\xi)=c_\pm\,e^{\pm\lambda\xi}, \\ &u_{n+1}(\xi)=\hat{\mathscr{O}}^{-1}[\mathscr{A}_n](\xi),\ \ n\geq 0, \end{aligned} \tag{33}$$

where $\mathscr{A}_n(\xi)$, $n\geq 0$ are calculated using formula (10). Hence from (33) the successive terms are given by

$$\begin{aligned} &u_0(\xi)=c_\pm e^{\pm\sqrt{2}\sqrt{-\lambda^2}\xi}, \\ &u_1(\xi)=-\frac{c_\pm^2e^{\pm 3\sqrt{2}\sqrt{-\lambda^2}\xi}\left(c_\pm\delta\mp 8\sqrt{-\delta\lambda^2}e^{\pm\sqrt{2}\sqrt{-\lambda^2}\xi}\right)}{16\lambda^2}, \\ &\ldots\ldots \end{aligned} \tag{34}$$

$$u_n(\xi) = \frac{c_\pm 2^{-5n-1}\lambda^{-2n} e^{\mp\sqrt{2}\sqrt{-\lambda^2}(-2n-1)\xi}}{\sqrt{c_\pm^3 \delta \left(c_\pm \delta \mp 16\sqrt{-\delta\lambda^2} e^{\mp\sqrt{2}\sqrt{-\lambda^2}\xi}\right)}}$$
$$\times \left\{ \left(\pm 8c_\pm \sqrt{-\delta\lambda^2} e^{\mp\sqrt{2}\sqrt{-\lambda^2}\xi} + \sqrt{c_\pm^3 \delta \left(c_\pm \delta \mp 16\sqrt{-\delta\lambda^2} e^{\mp\sqrt{2}\sqrt{-\lambda^2}\xi}\right)} + c_\pm^2(-\delta) \right)^{n+1} \right. \tag{35}$$
$$\left. - \left(\pm 8c_\pm \sqrt{-\delta\lambda^2} e^{\mp\sqrt{2}\sqrt{-\lambda^2}\xi} - \sqrt{c_\pm^3 \delta \left(c_\pm \delta \mp 16\sqrt{-\delta\lambda^2} e^{\mp\sqrt{2}\sqrt{-\lambda^2}\xi}\right)} + c_\pm^2(-\delta) \right)^{n+1} \right\},$$
$$\ldots\ldots$$

The closed form of series solution of (32) is given by:

$$u(\xi) = \sum_{n=0}^{\infty} u_n(\xi) = \frac{2c_\pm \lambda^2}{2\lambda^2 e^{\pm\sqrt{2}\sqrt{-\lambda^2}\xi} \mp c_\pm \sqrt{-\delta\lambda^2}}. \tag{36}$$

Now using (36) with (21) in (17) we can find solution of NLSE (16) in the following form

$$q = \left[\frac{2c_\pm \lambda^2}{2\lambda^2 e^{\pm\sqrt{2}\sqrt{-\lambda^2}\xi} \mp c_\pm \sqrt{-\delta\lambda^2}} \pm \sqrt{-\frac{\lambda^2}{\delta}} \right] e^{i\phi(x,y,t)}, \tag{37}$$

where $\phi(x,y,t)$, $\xi(x,y,t)$, λ and δ are given in (18) in (22) respectively. Again depending on the values of λ and δ this solution can be recast in the following forms

$$q_5(\xi) = \mp\sqrt{\frac{-\lambda^2}{\delta}} \times tanh \left[\pm\sqrt{-\frac{\lambda^2}{2}}\xi \mp \frac{1}{2} log \left(\pm\frac{c_\pm}{2}\sqrt{-\frac{\delta}{\lambda^2}} \right) \right] \times e^{i\phi(x,y,t)}, \tag{38}$$

$$q_6(\xi) = \pm\sqrt{\frac{-\lambda^2}{\delta}} \times coth \left[\pm\sqrt{-\frac{\lambda^2}{2}}\xi \pm \frac{1}{2} log \left(\pm\frac{c_\pm}{2}\sqrt{-\frac{\delta}{\lambda^2}} \right) \right] \times e^{i\phi(x,y,t)}, \tag{39}$$

these solutions exists when λ is pure imaginary and $-\lambda^2\delta > 0$.

$$q_7(\xi) = \sqrt{\frac{\lambda^2}{\delta}} \times tan \left[\pm\sqrt{\frac{\lambda^2}{2}}\xi + \frac{i}{2} log \left(\frac{c_\pm}{2}\sqrt{-\frac{\delta}{\lambda^2}} \right) \right] \times e^{i\phi(x,y,t)}, \tag{40}$$

$$q_8(\xi) = -\sqrt{\frac{\lambda^2}{\delta}} \times cot \left[\pm\sqrt{\frac{\lambda^2}{2}}\xi + \frac{i}{2} log \left(-\frac{c_\pm}{2}\sqrt{-\frac{\delta}{\lambda^2}} \right) \right] \times e^{i\phi(x,y,t)}, \tag{41}$$

these solutions exists when λ^2 is real, $\lambda^2\delta > 0$ and value of $c_\pm$ is so chosen that trigonometric functions becomes real.

6 Conclusion

In summary, applying RCAM, we have found closed form solution of chiral NLSE, from which eight types solutions constructed. The main strength of this method is that it does not need any guessing of forms of solutions as other methods like tanh, Jacobi elliptic function method, ansatz method etc. requires. It should be noted here that the RCAM not only can generate regular solutions, but also singular ones involving csch, coth functions. The obtained solutions establish that method is an effective and powerful tool for solving nonlinear differential equations arising in mathematical physics. It remains an open problem as how to generalize this method in order to deal with nonlinear differential equations with variable coefficients.

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