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A Novel Integral Transform: W- Transform and its Applications

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Abstract: The main purpose of this work is to introduce a novel transform known as Wisam transform (W - transform), that is defined as:

$$W[h(t)](r,\varphi) = H(r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t)\,dt, \qquad r,\varphi > 0.$$

We explain its basic properties and prove some important results, including the linearity property, the existence theorem, the convolution theorem, and the properties of derivatives. Furthermore, the new transform is applied to the solution of some ordinary differential equations (ODEs). The ability of this proposed integral transform to transform ordinary differential equations into solvable algebraic equations is demonstrated.

Keywords: Laplace transform, Kamal transform, Sumudu transform, Wisam transform, Ordinary differential equations.

1 Introduction

Ordinary and partial differential equations have recently emerged as crucial for simulating a variety of real-world applications in engineering and other sciences, including mathematical biology, fluid dynamics, optics, electrical circuits, and quantum physics [1,2,3,4,5,6,7,8,9]. More recently, the Laplace transform has been widely applied to solve differential equations [10,11]. In addition, researchers have contributed extensions to the original Laplace transform, such as the Fourier transform [12], Sumudu transform [13],Elzaki transform [14,15], Gamar transform [16,17] ,Kamal transform [18], ARA transform [19,20], Jafari transform [21] Melin transform [22] , Sawi transform [23] Mahgoub transform [24]among many others. Integral transformations, including single, double, and triple integrals, are widely recognized as highly effective tools for solving both linear partial and integral differential equations. Numerous researchers have made significant efforts to develop these methods and apply them to deal with a wide range of problems in mathematics. The rest of the paper is organized as follows. Section 2 introduces basic definitions and the existence theorem of the Wissam transform. Section 3 provides basic definitions and theorems about the proposed transform, proving some important results, including the linearity property, the convolution theorem, and the properties of derivatives. Section 4 introduces the Wissam transform for some elementary functions. Section 5 applies the Wissam transform to some types of ordinary differential equations (ODEs). Finally, we conclude with a conclusion.

2 Basic definitions and theorems for W - transform

In this section, we introduce the basic properties of W - transforms.
Definition 2.1. Let $h(t)$ be a continuous function of t specified on the interval $(0,\infty)$ for $t > 0$.Then, W -

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transform of $h(t)$, denoted by $W[h(t)]$, is defined as

$$W[h(t)](r,\varphi) = H(r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t)\,dt, \quad r,\varphi > 0. \tag{1}$$

where r and φ are transform variables for t.

Theorem 2.1. (The sufficient condition for the existence of W – transform). If the function $h(t)$ is a piecewise continuous in every finite interval $0 \le t < \infty$ and it is exponential order ϑ. Then $W[h(t)]$ exists for all $\varphi^{-1} > \vartheta$ and satisfies

$$|h(t)| \le E e^{\vartheta t} \tag{2}$$

where E is positive constant.

Proof of Theorem 2.1. Using the definition of W - transform, we get

$$\begin{aligned}|H(r,\varphi)| &= \left|\varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t)\,dt\right| \\ &\le \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} |h(t)|\,dt \\ &\le E\,\varphi^{r-1}\int_0^\infty e^{-\left(\frac{1}{\varphi^r}-\vartheta\right)t}\,dt \\ &= E\,\varphi^{r-1}\frac{1}{\frac{1}{\varphi^r}-\vartheta} \\ &= \frac{E\,\varphi^{2r-1}}{1-\vartheta\varphi^r}, \qquad \varphi^{-r} > \vartheta.\end{aligned}$$

Thus, W - transform integral converges absolutely for $\varphi^{-1} > \vartheta$. In the following arguments, we present some properties of W - transform.

3 Properties of Wisam Transform:

In this section, we establish some properties of the W - transform, which enable us to calculate further transform of functions in applications.

Property 3.1. (Linearity property) Let $h(t)$ and $g(t)$ be two functions in which W - transform exists , then

$$W\left[p\,h(t)+q\,g(t)\right](r,\varphi) = p\,W[h(t)](r,\varphi)+q\,W[g(t)](r,\varphi), \tag{3}$$

where p and q are nonzero constants.

Proof of Property 3.1. Using the definition of W - transform, we get

$$W[p\,h(t)+q\,g(t)](r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\left[p\,h(t)+q\,g(t)\right]dt. \tag{4}$$

Using the linearity of improper integral, we have

$$\begin{aligned}W\left[p\,h(t)+q\,g(t)\right](r,\varphi) &= p\,\varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t)\,dt \\ &\quad + q\,\varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} g(t)\,dt \\ &= p\,W[h(t)](r,\varphi) + q\,W[g(t)](r,\varphi).\end{aligned}$$

Property 3.2. (Change of scale property) Let $W[h(t)] = H(r,\varphi)$, then

$$W[h(\xi t)](r,\varphi) = \xi^{-\frac{2r-1}{r}} H\left(r,\, \xi^{1/r}\varphi\right), \qquad \xi \in \mathbb{R}. \tag{5}$$

Proof of Property 3.2. Using the definition of W - transform for $h(\xi t)$, we get

$$W[h(\xi t)](r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(\xi t)\,dt. \tag{6}$$

Substitution of $\mu = \xi t$ in Eq. (6) yields

$$W[h(\xi t)](r,\varphi) = \frac{\varphi^{r-1}}{\xi}\int_0^\infty e^{-\mu/(\xi\varphi^r)} h(\mu)\,d\mu. \tag{7}$$

And,

$$\begin{aligned}W[h(\xi t)](r,\varphi) = \frac{\varphi^{r-1}}{\xi}\left(\xi^{1/r}\varphi\right)^{1-r}\left(\xi^{1/r}\varphi\right)^{r-1} \\ \times \int_0^\infty e^{-\mu/(\xi^{1/r}\varphi)^r} h(\mu)\,d\mu.\end{aligned}$$

Therefore,

$$\begin{aligned}W[h(\xi t)](r,\varphi) &= \xi^{-\frac{2r-1}{r}}\left(\xi^{1/r}\varphi\right)^{r-1}\int_0^\infty e^{-\mu/(\xi\varphi^r)} h(\mu)\,d\mu \\ &= \xi^{-\frac{2r-1}{r}} H\left(r,\, \xi^{1/r}\varphi\right).\end{aligned}$$

Definition 3.1. The convolution of $h(t)$ and $g(t)$is denoted by $(h*g)(t)$ and defined by

$$(h*g)(t) = \int_0^t h(t-\sigma)\,g(\sigma)\,d\sigma. \tag{8}$$

Theorem 3.1. Let $W[h(t)] = H(r,\varphi)$. Then,

$$W\left[h(t-\sigma)H(t-\sigma)\right] = e^{-\sigma/\varphi^r} H(r,\varphi). \tag{9}$$

where $H(t)$ denotes the Heaviside function defined by

$$H(t-\sigma) = \begin{cases} 1, & t > \sigma, \\ 0, & \text{otherwise.}\end{cases}$$

Proof of Theorem 3.1. From the definition of W - transform, we have

$$\begin{aligned}W\left[h(t-\sigma)H(t-\sigma)\right] &= \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t-\sigma)H(t-\sigma)\,dt \\ &= \varphi^{r-1}\int_\sigma^\infty e^{-t/\varphi^r} h(t-\sigma)\,dt.\end{aligned} \tag{10}$$

Putting $t-\sigma = \lambda$ in Eq. (10), we obtain

$$W[h(t-\sigma)H(t-\sigma)] = \varphi^{r-1}\int_0^\infty e^{-(\lambda+\sigma)/\varphi^r} h(\lambda)\,d\lambda. \tag{11}$$

Thus, Eq. (11) can be simplified into

$$W\left[h(t-\sigma)H(t-\sigma)\right] = \varphi^{r-1}\int_0^\infty e^{-(\lambda+\sigma)/\varphi^r} h(\lambda)\,d\lambda$$
$$= \varphi^{r-1} e^{-\sigma/\varphi^r}\int_0^\infty e^{-\lambda/\varphi^r} h(\lambda)\,d\lambda$$
$$= e^{-\sigma/\varphi^r} H(r,\varphi).$$

Theorem 3.2. (Convolution Theorem)
If $W[h(t)] = H(r,\varphi)$ and $W[g(t)] = G(r,\varphi)$, then

$$W[(h*g)(t)] = \varphi^{r+1} H(r,\varphi)\,G(r,\varphi). \tag{12}$$

Proof of Theorem 3.2. From the definition of W - transform, we have

$$W[(h*g)(t)] = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\left[\int_0^t h(t-\sigma)\,g(\sigma)\,d\sigma\right]dt. \tag{13}$$

The definition of Heaviside function, Eq. (13) can be written as

$$W[(h*g)(t)] = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\left[\int_0^\infty h(t-\sigma)\,H(t-\sigma)\,g(\sigma)\,d\sigma\right]dt \tag{14}$$

Thus, Eq.(14) can be written as

$$W[(h*g)(t)] = \int_0^\infty g(\sigma)\,d\sigma\,\varphi^{r-1}\int_0^\infty e^{-t/\varphi^r} h(t-\sigma)\,H(t-\sigma)\,dt. \tag{15}$$

Using Theorem 3.1, we have

$$W[(h*g)(t)] = H(r,\varphi)\int_0^\infty e^{-\sigma/\varphi^r} g(\sigma)\,d\sigma$$
$$= \varphi^{r+1} H(r,\varphi)\,G(r,\varphi).$$

4 W - Transform for Some Elementary Functions:

In this section, we introduce the W - transform of some elementary functions.
i.Let $h(t) = 1$.Then

$$W[1](r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\cdot[1]\,dt \quad = \varphi^{r-1}\left[-\varphi^r e^{-t/\varphi^r}\right]_0^\infty$$
$$= \varphi^{r-1}\cdot\varphi^r \;= \varphi^{2r-1}. \tag{16}$$

ii.Let $h(t) = t$, $t > 0$.Then

$$W[t](r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\,[t]\,dt. \tag{17}$$

Using integrating by parts, we obtain:

$$\varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\,t\,dt$$
$$= \varphi^{r-1}\left\{t\left(-\varphi^r e^{-t/\varphi^r}\right)\Big|_0^\infty + \varphi^r\int_0^\infty e^{-t/\varphi^r}\,dt\right\} = \varphi^r\, W[1]. \tag{18}$$

From use the property (i), we get

$$W[t](r,\varphi) = \varphi^{3r-1}. \tag{19}$$

iii.Let $h(t) = t^2$, $t > 0$ .Then

$$W[t^2](r,\varphi) = \varphi^{r-1}\int_0^\infty [t^2]e^{-t/\varphi^r}\,dt. \tag{20}$$

Using integrating by parts, we obtain:

$$\varphi^{r-1}\int_0^\infty t^2 e^{-t/\varphi^r}\,dt =$$
$$\varphi^{r-1}\left\{t^2\left(-\varphi^r e^{-t/\varphi^r}\right)\Big|_0^\infty + 2\varphi^r\int_0^\infty t\,e^{-t/\varphi^r}\,dt\right\} = 2\varphi^r\, W[t]. \tag{21}$$

From use the property (ii), we get

$$W[t^2](r,\varphi) = 2\,\varphi^{4r-1}. \tag{22}$$

Similarly, we can prove that:

$$W[t^n](r,\varphi) = n!\,\varphi^{(n+2)r-1}, \qquad \forall n\in\mathbb{N},\ n\geq 0. \tag{23}$$

iv.Let $h(t) = e^{ct}$,$t > 0$ and c is nonzero constant. Then

$$W[e^{ct}](r,\varphi) = \varphi^{r-1}\int_0^\infty e^{-t/\varphi^r}\,[e^{ct}]\,dt \quad = \varphi^{r-1}\int_0^\infty e^{-(\frac{1}{\varphi^r}-c)t}\,dt. \tag{24}$$

From using simple calculations, we have

$$W[e^{ct}](r,\varphi) = \frac{\varphi^{2r-1}}{1-c\varphi^r}, \qquad \forall c < \varphi^{-r}. \tag{25}$$

Similarly,

$$W\left[e^{ict}\right](r,\varphi) = \frac{\varphi^{2r-1}}{1-ic\varphi^r}.$$

Thus, one can obtain

$$W\left[e^{ict}\right](r,\varphi) = \frac{\varphi^{2r-1}}{1+c^2\varphi^{2r}} + i\,\frac{c\,\varphi^{3r-1}}{1+c^2\varphi^{2r}}.$$

Using Euler's formula: $e^{ict} = \cos(ct) + i\sin(ct)$. .
And the formul $\sinh x = \frac{e^x - e^{-x}}{2}$, $\quad \cosh x = \frac{e^x + e^{-x}}{2}$.
Then, we find the W - transform of the following functions:

$$W[\cos(ct)](r,\varphi) = \frac{\varphi^{2r-1}}{1+c^2\varphi^{2r}},$$
$$W[\sin(ct)](r,\varphi) = \frac{c\,\varphi^{3r-1}}{1+c^2\varphi^{2r}},$$
$$W[\cosh(ct)](r,\varphi) = \frac{\varphi^{2r-1}}{1-c^2\varphi^{2r}}, \qquad (c^2\varphi^{2r} < 1),$$
$$W[\sinh(ct)](r,\varphi) = \frac{c\,\varphi^{3r-1}}{1-c^2\varphi^{2r}}, \qquad (c^2\varphi^{2r} < 1).$$

The following theorem presents W - transform for the derivatives.

Theorem 4.1. (Derivative properties).If $W[h(t)] = H(r,\varphi)$, then

i. $W[h'(t)](r,\varphi) = \frac{1}{\varphi^r} H(r,\varphi) - \varphi^{r-1} h(0),$

ii. $W[h''(t)](r,\varphi) = \frac{1}{\varphi^{2r}} H(r,\varphi) - \varphi^{r-1} h'(0) - \frac{1}{\varphi} h(0),$

iii. $W\left[h^{(n)}(t)\right](r,\varphi) = \frac{1}{\varphi^{nr}} H(r,\varphi) - \varphi^{r-1} \sum_{k=0}^{n-1} \frac{h^{(k)}(0)}{\varphi^{(n-k-1)r}}.$

Proof of Theorem 4.1. From the definition of W - transform, we have

i. $W[h'(t)] = \varphi^{r-1} \int_0^\infty e^{-t/\varphi^r} h'(t)\, dt.$

Using integrating by parts, we obtain:

$$\varphi^{r-1} \int_0^\infty e^{-t/\varphi^r} h'(t)\, dt = \varphi^{r-1} \left(-h(0) + \frac{1}{\varphi^r} \int_0^\infty e^{-t/\varphi^r} h(t)\, dt \right).$$

Thus,

$$W[h'(t)](r,\varphi) = \frac{1}{\varphi^r} H(r,\varphi) - \varphi^{r-1} h(0). \tag{26}$$

ii. $W[h''(t)] = \varphi^{r-1} \int_0^\infty e^{-t/\varphi^r} h''(t)\, dt.$

Using integrating by parts, we obtain:

$$\varphi^{r-1} \int_0^\infty e^{-t/\varphi^r} h''(t)\, dt = \varphi^{r-1} \left(-h'(0) + \frac{1}{\varphi^r} \int_0^\infty e^{-t/\varphi^r} h'(t)\, dt \right). \tag{27}$$

Using Eq. (26), we get

$$W[h''(t)] = \frac{1}{\varphi^{2r}} H(r,\varphi) - \varphi^{r-1} h'(0) - \frac{1}{\varphi} h(0). \tag{28}$$

Similarly, we can prove (iii)

Now, we present the basic properties and relations of W - transform in the following table:

Table 1: Basic properties of the W-transform

h(t)	**W[h(t)]**
1	φ^{2r-1}
$t^n,\ n \geq 0$	$n!\,\varphi^{(n+2)r-1}$
e^{ct}	$\frac{\varphi^{2r-1}}{1-c\varphi^r}$
$\sin(ct)$	$\frac{c\varphi^{3r-1}}{1+c^2\varphi^{2r}}$
$\cos(ct)$	$\frac{\varphi^{2r-1}}{1+c^2\varphi^{2r}}$
$\sinh(ct)$	$\frac{c\varphi^{3r-1}}{1-c^2\varphi^{2r}}$
$\cosh(ct)$	$\frac{\varphi^{2r-1}}{1-c^2\varphi^{2r}}$
$h'(t)$	$\frac{1}{\varphi^r} H(r,\varphi) - \varphi^{r-1} h(0)$
$h''(t)$	$\frac{1}{\varphi^{2r}} H(r,\varphi) - \varphi^{r-1} h'(0) - \frac{1}{\varphi} h(0)$
$h^{(n)}(t)$	$\frac{1}{\varphi^{nr}} H(r,\varphi) - \varphi^{r-1} \sum_{k=0}^{n-1} \frac{h^{(k)}(0)}{\varphi^{(n-k-1)r}}$

5 Applications

In this section, we illustrate the basic idea of the W - method for solving a first and second order linear ordinary differential equation.

Example 5.1. Let's consider the first order differential equation:

$$h'(t) + h(t) = 0, \qquad h(0) = 1. \tag{29}$$

Applying W - transform on both sides of Eq. (29), we have

$$W[h'(t) + h(t)] = 0. \tag{30}$$

By linearity and derivative properties of W - transform, we get

$$\frac{1}{\varphi^r} H(r,\varphi) - \varphi^{r-1} h(0) + H(r,\varphi) = 0. \tag{31}$$

Substituting the given conditions and rearranging the terms, we have

$$H(r,\varphi) = \frac{\varphi^{2r}}{(1+\varphi)^r}. \tag{32}$$

Taking the inverse W - transform, we get

$$h(t) = W^{-1}\left[\frac{\varphi^{2r}}{(1+\varphi)^r}\right] = e^{-t}. \tag{33}$$

Example 5.2. Let's consider the first order differential equation:

$$h'(t) + 2h(t) = t, \qquad h(0) = 1. \tag{34}$$

Applying W - transform on both sides of Eq. (34), we have

$$W[h'(t) + 2h(t)] = W[t]. \tag{35}$$

By linearity and derivative properties of W - transform, we get

$$\frac{1}{\varphi^r} H(r,\varphi) - \varphi^{r-1} h(0) + 2H(r,\varphi) = \varphi^{3r-1}. \tag{36}$$

Substituting the given conditions and rearranging the terms, we have

$$H(r,\varphi) = \frac{\varphi^r\left(\varphi^{3r-1} + \varphi^{r-1}\right)}{(1+2\varphi)^r} = \frac{\varphi^r\left(4\varphi^{3r-1} + 5\varphi^{r-1} - \varphi^{r-1}\right)}{4(1+2\varphi)^r}, \tag{37}$$

And ,

$$H(r,\varphi)=\frac{1}{2}\varphi^{3r-1}+\frac{5}{4}\frac{\varphi^{2r-1}}{(1+2\varphi)^{r}}-\frac{1}{4}\varphi^{2r-1}. \quad (38)$$

Taking the inverse transform W^{-1} for Eq. (38), we get the solution of Eq. (34)

$$h(t)=\frac{1}{2}t+\frac{5}{4}e^{-2t}-\frac{1}{4}. \quad (39)$$

Example 5.3. Let's consider the second order differential equation:

$$h''(t)+h(t)=0, \qquad h'(0)=h(0)=1. \quad (40)$$

Applying W - transform on both sides of Eq. (40), we have

$$W[h''(t)+h(t)]=0. \quad (41)$$

By linearity and derivative properties of W - transform, we get

$$\frac{1}{\varphi^{2r}}H(r,\varphi)-\varphi^{r-1}h'(0)-\frac{1}{\varphi}h(0)+H(r,\varphi)=0. \quad (42)$$

Substituting the given conditions and rearranging the terms, we have

$$H(r,\varphi)=\frac{\varphi^{2r}\left(\varphi^{r-1}+\varphi^{-1}\right)}{(1+\varphi)^{2r}}=\frac{\varphi^{3r-1}}{(1+\varphi)^{2r}}+\frac{\varphi^{2r-1}}{(1+\varphi)^{2r}}. \quad (43)$$

Taking the inverse transform W^{-1} for Eq. (43), we get the solution of Eq. (40)

$$h(t)=W^{-1}\left[\frac{\varphi^{3r-1}}{(1+\varphi)^{2r}}+\frac{\varphi^{2r-1}}{(1+\varphi)^{2r}}\right]=\sin t+\cos t. \quad (44)$$

Example 5.4. Let's consider the following differential equation :

$$h''(t)-3h'(t)+2h(t)=0, \qquad h'(0)=4, h(0)=1. \quad (45)$$

Applying W - transform on both sides of Eq. (45), we have

$$W[h''(t)-3h'(t)+2h(t)]=0. \quad (46)$$

By linearity and derivative properties of W - transform, we get

$$\frac{1}{\varphi^{2r}}H(r,\varphi)-\varphi^{r-1}h'(0)-\frac{1}{\varphi}h(0)-3\left(\frac{1}{\varphi^{r}}H(r,\varphi)-\varphi^{r-1}h(0)\right)+2H(r,\varphi)=0. \quad (47)$$

Substituting the given conditions and rearranging the terms, we have

$$H(r,\varphi)=\frac{\varphi^{r-1}+\varphi^{-1}}{\varphi^{-2r}-3\varphi^{-r}+2}=\varphi^{2r-1}\left(\frac{3}{1-2\varphi^{r}}-\frac{2}{1-\varphi^{r}}\right). \quad (48)$$

Taking the inverse transform W^{-1} for Eq. (48), we get the solution of Eq. (45)

$$h(t)=W^{-1}\left[\frac{3\varphi^{2r-1}}{1-2\varphi^{r}}-\frac{2\varphi^{2r-1}}{1-\varphi^{r}}\right]=-2e^{t}+3e^{2t}. \quad (49)$$

Example 5.5. Let's consider the following second - order differential equation:

$$h''+9h=\cos(2t), \qquad h(0)=1, \qquad h\left(\frac{\pi}{2}\right)=-1. \quad (50)$$

Since $h'(0)$ is not known, let $h'(0)=c$. Applying W - transform on both sides of Eq. (50), we have

$$W[h''+9h]=W[\cos(2t)]. \quad (51)$$

By linearity and derivative properties of W - transform, we get

$$H(r,\varphi)=\frac{\dfrac{\varphi^{2r-1}}{1+4\varphi^{2r}}+\varphi^{r-1}c+\varphi^{-1}}{\dfrac{1+9\varphi^{2r}}{\varphi^{2r}}}. \quad (52)$$

Eq. (52) can be written as

$$H(r,\varphi)=\frac{\varphi^{2r-1}\,\varphi^{2r}}{(1+4\varphi^{2r})(1+9\varphi^{2r})}+\frac{c\,\varphi^{3r-1}}{1+9\varphi^{2r}}+\frac{\varphi^{2r-1}}{1+9\varphi^{2r}}. \quad (53)$$

And ,

$$H(r,\varphi)=\frac{\varphi^{2r-1}}{5\left(1+4\varphi^{2r}\right)}-\frac{4\varphi^{2r-1}}{5\left(1+9\varphi^{2r}\right)}+\frac{c\,\varphi^{3r-1}}{1+9\varphi^{2r}}. \quad (54)$$

Taking the inverse transform W^{-1} for Eq. (54), we get

$$h(t)=\frac{1}{5}\cos(2t)+\frac{4}{5}\cos(3t)+\frac{c}{3}\sin(3t). \quad (55)$$

Since $h\left(\frac{\pi}{2}\right)=-1$, we can determine the value of c , which is equal to 12/5 .Therefore, the solution of Eq. (50) is

$$h(t)=\frac{1}{5}\cos(2t)+\frac{4}{5}\cos(3t)+\frac{4}{5}\sin(3t). \quad (56)$$

6 Conclusion

In this paper, we demonstrate the flexibility of the new Wisam integral transform for some fundamental functions. We discuss and prove the definition of the new transform and its basic properties, such as linearity, existence, convolution, and ordinary derivatives. The obtained results are also applied to solve various types of ordinary differential equations. The results demonstrate the feasibility and applicability of the new transformation. We look forward to soon solving differential equations using the mechanical vibration equation as a practical example of this method, which plays an important role in engineering and physics. This is done by choosing a specific value for the variable r to suit the required problem. We also have many future studies based on this research manuscript for any real-life problems involving mathematical systems.

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