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Exponentiated Gamma Exponential Distribution

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Abstract: In T-X distribution, two random variables X, the transformer, and T, the transformed are used to develop a new distribution. A new distribution namely Exponentiated Gamma Exponential Distribution(EGED) is proposed which is generated from family of Exponentiated T-X distributions. Shannon entropy of the resulting distribution is obtained. The hazard function and survival function are provided. Fisher information matrix for the exponentiated gamma exponential distribution is obtained. Parameter estimation of the exponentiated gamma exponential distribution by the maximum likelihood method is also proposed. A real data set is used to illustrate the application of the exponentiated gamma exponential distribution.

Keywords: T-X family, Hazard function, Shannon entropy, Estimation

1 Introduction

The exponential distribution is one of the key distributions in the theory and practice of statistics and are commonly employed in the formation of methods of lifetime distributions and stochastic process in general.[1] referred to the exponential distribution, while discussing the sampling distribution of standard deviation, as Pearson's Type X distribution. Applications of the exponential distribution in actuarial, biological and engineering problems were demonstrated subsequently by [2], [3] and [4]. The extension of the exponential distribution in the form of Weibull distribution were studied by [5]. This family of distributions includes the exponential distribution as a special case when the shape parameter equals one.

Reliability theory and reliability engineering also make extensive use of the exponential distribution. Because of the memoryless property of this distribution, it is well suited to model, the constant hazard rate portion of the bathtub curve used in reliability theory. The basic characterization of the exponential distribution based on lack of memory is simply the logarithmic equivalent of the functional equation $f(x+y) = f(x)f(y)$, which is due to [6], [7] and [8]. A complete solution of the logarithmic equivalent of this functional equation for both continuous and discrete cases was provided by [9] and [10]. [11] characterized exponential through Poisson and renewal process, [12] through order statistics and [13] through range and ratios of order statistics.

Numerous authors have derived various generalizations of the distributions. [14],[15] and [16] introduced the two-parameter generalized exponential distribution can be used in analyzing many life time data. Researchers developed and studied new and more flexible distribution. Parameters estimation of gamma-Pareto distribution was considered by [17]. [18] proposed $T-X$ method for generating families of continuous distributions. The problem of estimation of paremeters of Weibull-Pareto distribution by the method of modified maximum likelihood was considered [19]. [20] studied various structural properties of gamma X family specialised their results on gamma-normal distribution. The gamma and exponential distributions are the most widely used in the reliability and survival studies. In addition, the exponential distribution is one of the members of gamma$-X$ family as well. [21] obtained the structural properties of the T-X family of gamma-exponential distribution. [22] generalizes the method of T-X family by inclusion of an additional parameter c which leads to a new family of exponentiated T-X distribution and studied the properties of exponentiated weibull-exponential distribution. [23] extended the exponentiated weibull-exponential distribution to more general form.

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[22] presented the cumulative distribution function (cdf) of the exponentiated $T-X$ family as follows

$$G(x)=\int_{0}^{-\log(1-F^{c}(x))} r(t)dt \tag{1}$$

Where r(t) is the probability density function (pdf) of random variable T defined over $[0,\infty)$ and $F(x)$ is the cumulative distribution function of random variable X.

The probability density function for the continuous random variable X can be written as

$$g(x)=\frac{cf(x)F^{c-1}(x)}{1-F^{c}(x)}r\left[-\log\left(1-F(x)\right)\right],c>0 \tag{2}$$

If $T\sim\gamma(\alpha,\beta)$, then

$$r(t)=\frac{1}{\beta^{\alpha}\Gamma(\alpha)}t^{\alpha-1}e^{-\frac{t}{\beta}};t\geq 0$$

Using (2), the pdf of gamma$-X$ family is

$$g(x)=\frac{c}{\Gamma(\alpha)\beta^{\alpha}}f(x)F^{c-1}(x)(-\log(1-F^{c}(x)))^{\alpha-1}\times(1-F^{c}(x))^{\frac{1}{\beta}-1} \tag{3}$$

This paper is organised as follows. In section (1.1), we define the exponentiated gamma exponential distribution. Shanon entropy of the EGED is given in section (1.2). Estimation of the model parameters by maximum likelihood is performed in section (1.3). Section (1.4), defines the Fisher's information matrix for EGED. Application to real data set illustrate the performance of the EGED in section (1.5). The paper is concluded in section (1.6).

2 Exponentiated Gamma Exponential Distribution

Suppose a random variable X (the transformer) follows the exponential distribution with probability density function as

$$f(x)=\theta e^{-\theta x},x>\theta$$

and the cumulative distribution function is given by

$$F(x)=1-e^{-\theta x}$$

then (3) results in

$$g(x)=\frac{c}{\Gamma(\alpha)\beta^{\alpha}}\theta e^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}\times\left[-\log\left(1-(1-e^{-\theta x}\right)^{c}\right]^{\alpha-1}\times\left[1-\left(1-e^{-\theta x}\right)^{c}\right]^{\frac{1}{\beta}-1} \tag{4}$$

$g(x)$ follows the exponentiated gamma exponential distribution with parameters c, α, β and θ. If c=1 in (4), the exponentiated gamma exponential distribution leads to the gamma-exponential distribution given by [21].

$$g(x)=\frac{1}{\Gamma(\alpha)\beta^{\alpha}}\theta^{\alpha}x^{\alpha-1}\left(e^{-\theta x}\right)^{\frac{1}{\beta}}$$

The cumulative distribution function of exponentiated gamma-exponential results as

$$G(x)=\frac{\gamma\left[\alpha,\frac{-\log\left(1-\left(1-e^{-\theta x}\right)^{c}\right)}{\beta}\right]}{\Gamma(\alpha)} \tag{5}$$

The survival function of the EGED can be obtained as

$$R(x)=1-G(x)=1-\frac{\gamma\left[\alpha,\frac{-\log\left(1-\left(1-e^{-\theta x}\right)^{c}\right)}{\beta}\right]}{\Gamma(\alpha)} \tag{6}$$

and the hazard function is

$$h(x)=\frac{g(x)}{R(x)}$$

$$h(x)=\frac{c\theta e^{-\theta x}(1-e^{-\theta x})^{c-1}\left[1-(1-e^{-\theta x})^{c}\right]^{\frac{1}{\beta}-1}}{\beta^{\alpha}\left[\Gamma(\alpha)-\gamma\left[\alpha,\frac{-\log(1-(1-e^{-\theta x})^{c}}{\beta}\right]\right]}\times\left[-\log\left(1-\left(1-e^{-\theta x}\right)^{c}\right)^{\alpha-1}\right] \tag{7}$$

3 Shannon Entropy

The Shannon entropy for the exponentiated gamma-exponential variable X is given by the following Theorem.

Theorem: If a random variable X follows EGED, then the Shannon entropy of X is given by

$$\eta_{x}=-\log\theta+\left(\frac{1-c}{c}\right)\left[\alpha\beta-\frac{\Gamma(\alpha)}{\gamma(\alpha)}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\frac{1}{\left(n+\frac{1}{\beta}\right)^{2}}\right]+\alpha+\log\left(\frac{\beta\Gamma(\alpha)}{c}\right)+(1-\alpha)\psi(\alpha)$$

where ψ is the digamma function.

Proof: The Shannon entropy of the exponentiated gamma-X family of distribution given by [22]

$$\eta_{x}=-E(\log f(F^{-1}(1-e^{-T})^{\frac{1}{c}}))+\frac{1-c}{c}E(\log(1-e^{-T}))+\alpha(1-\beta)+\log\left(\frac{\beta\Gamma(\alpha)}{c}\right)+(1-\alpha)\psi(\alpha) \tag{8}$$

So, to complete the proof we need to evaluate $-E(\log f(F^{-1}(1-e^{-T})^{\frac{1}{c}}))$, where $f(x)=\theta e^{-\theta x}$. It can be shown that$F^{\frac{-1}{c}}(x)=\log(1-x)^{-\frac{1}{\theta c}}$, thus

$$-E(\log f(F^{-1}(1-e^{-T})^{\frac{1}{c}}))=-\log\theta+\frac{\alpha\beta}{c} \tag{9}$$

$$E(\log(1-e^{-T}))=\frac{1}{\gamma(\alpha)}\int_0^\infty \log(1-e^{-T})t^{\alpha-1}e^{\frac{-t}{\beta}}dt \tag{10}$$

Using the following series expansion of natural logarithm

$$\log(1-x)=-\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \qquad -1<x\leq 1$$

So, we write,

$$\log(1-e^{-t})=-\sum_{n=1}^{\infty}(-1)^{n+1}\frac{e^{-nt}}{n} \qquad -1<e^{-t}\leq 1$$

$$-\frac{1}{\gamma(\alpha)}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\int_0^\infty dt t^{\alpha-1}e^{-t(n+\frac{1}{\beta})}$$

By using the series representation of the logarithm function, Equation (10) can be expressed as

$$\frac{1}{\gamma(\alpha)}\int_0^\infty dt\log(1-e^{-t})t^{\alpha-1}e^{-\frac{t}{\beta}}$$
$$=-\frac{\Gamma(\alpha)}{\gamma(\alpha)}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\frac{1}{\left(n+\frac{1}{\beta}\right)^{\alpha}}$$

Therefore, Equation (8) reduces to

$$\eta_x=-\log\theta$$
$$+\left(\frac{1-c}{c}\right)\left[\alpha\beta-\frac{\Gamma(\alpha)}{\gamma(\alpha)}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\frac{1}{\left(n+\frac{1}{\beta}\right)^{2}}\right]$$
$$+\alpha+\log\left(\frac{\beta\Gamma(\alpha)}{c}\right)+(1-\alpha)\psi(\alpha)$$

4 Parameter Estimation

Suppose $X_1,X_2,...,X_n$ constitute a random sample from a exponentiated gamma exponential distribution as defined in (4), The likelihood function is given by

$$L=g(x)=\left(\frac{c}{\Gamma(\alpha)\beta^{\alpha}}\right)^n\theta^n e^{-\theta\sum_{i=1}^n x_i}(1-e^{-\theta x})^{n(c-1)}$$
$$\times\left[-\log(1-(1-e^{-\theta x})^c\right]^{n(\alpha-1)} \tag{11}$$
$$\times\left[1-(1-e^{-\theta x})^c\right]^{n\left(\frac{1}{\beta}-1\right)}$$

From (11), the log-likelihood function is

$$\log L=n\log c-n\log\Gamma(\alpha)-n\alpha\log\beta+n\log\theta$$
$$-\theta\sum_{i=1}^n x_i+n(c-1)\log(1-e^{-\theta x})$$
$$+n(\alpha-1)\log[-\log(1-(1-e^{-\theta x})^c)]$$
$$+n\left(\frac{1}{\beta}-1\right)\log[1-(1-e^{-\theta x})^c)] \tag{12}$$

The derivatives of (12) with respect to α, c, β and θ are given by

$$\frac{\partial L}{\partial\alpha}=-n\psi(\alpha)-\log\beta$$
$$+n\log\left[-\log(1-(1-e^{-\theta x})^c)\right] \tag{13}$$

where $\psi(\alpha)=\frac{\gamma(\alpha)'}{\gamma(\alpha)}$

$$\frac{\partial L}{\partial\beta}=-\frac{n}{\beta}\left(\alpha+\frac{1}{\beta}\right)\log\left[1-(1-e^{-\theta x})^c\right] \tag{14}$$

$$\frac{\partial L}{\partial\theta}=\frac{n}{\theta}-\sum_{i=1}^n x_i+\frac{n(c-1)xe^{-\theta x}}{1-e^{-\theta x}}-$$
$$\frac{nxc(\alpha-1)e^{-\theta x}(1-e^{-\theta x})^{c-1}}{(1-(1-e^{-\theta x})^c)\log(1-(1-e^{-\theta x})^c)}-$$
$$\frac{nxc\left(\frac{1}{\beta}-1\right)e^{-\theta x}(1-e^{-\theta x})^{c-1}}{(1-(1-e^{-\theta x})^c)} \tag{15}$$

$$\frac{\partial L}{\partial c}=\frac{n}{c}+n\log(1-e^{-\theta x}) \tag{16}$$

The MLE of $\hat{\alpha}$, $\hat{c}$, $\hat{\beta}$ and $\hat{\theta}$ can be obtained by setting (13-16) to zero and solving the resulting equations using Newton Raphson numerical procedure. The Fisher's information matrix for EGED is given in next section.

5 Fisher information matrix

Appling log on both sides in (1), we have

$$L=\log g(x)=\log c-\log\Gamma(\alpha)-\alpha\log\beta+\log\theta$$
$$-\theta x+(c-1)\log\left(1-e^{-\theta x}\right)$$
$$+(\alpha-1)\log\left[-\log\left(1-\left(1-e^{-\theta x}\right)^c\right)\right]$$
$$+\left(\frac{1}{\beta}-1\right)\log\left[1-\left(1-e^{-\theta x}\right)^c\right] \tag{17}$$

Differentiating (17), with respect to α,β,c and θ we get

$$\frac{\partial L}{\partial\alpha}=-\log\beta-\psi(\alpha)+\log\left[-\log\left(1-\left(1-e^{-\theta x}\right)^c\right)\right]$$

where $\psi(\alpha)=\frac{\acute{\Gamma}(\alpha)}{\Gamma(\alpha)}$ is the digamma function

$$\frac{\partial^2 L}{\partial\alpha^2}=(\psi(\alpha))^2-\psi(\acute{\alpha});\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\alpha}\right)=0$$

$$\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial\alpha}\right)=-\frac{cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left(1-\left(1-e^{-\theta x}\right)^c\right)\log\left(1-\left(1-e^{-\theta x}\right)^c\right)}$$

$$\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial\alpha}\right)=-\frac{1}{\beta};\frac{\partial L}{\partial\beta}=-\frac{\alpha}{\beta}-\frac{\log\left[1-\left(1-e^{-\theta x}\right)^c\right]}{\beta^2}$$

$$\frac{\partial^2 L}{\partial\beta^2}=\frac{\alpha}{\beta^2}+\frac{2\log\left[1-\left(1-e^{-\theta x}\right)^c\right]}{\beta^3}$$

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial\beta}\right)=-\frac{1}{\beta};\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\beta}\right)=0$$

$$\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial\beta}\right)=\frac{cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\beta^2\left(1-(1-e^{-\theta x})^c\right)}$$

$$\frac{\partial L}{\partial c}=\frac{1}{c}+\log\left(1-e^{-\theta x}\right);\frac{\partial^2 L}{\partial c^2}=-\frac{1}{c^2}$$

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial c}\right)=0;\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial c}\right)=0$$

$$\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial c}\right)=\frac{xe^{-\theta x}}{1-(1-e^{-\theta x})}$$

$$\begin{aligned}\frac{\partial L}{\partial\theta}=&\frac{1}{\theta}-x+\frac{x(c-1)e^{-\theta x}}{1-e^{-\theta x}}\\&-\frac{cx(\alpha-1)e^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left(1-(1-e^{-\theta x})^c\right)\log\left(1-(1-e^{-\theta x})^c\right)}\\&\left(\frac{1}{\beta}-1\right)\frac{cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left(1-(1-e^{-\theta x})^c\right)}\end{aligned}$$

$$\begin{aligned}\frac{\partial^2 L}{\partial\theta^2}=&-\frac{1}{\theta^2}-\frac{x^2(c-1)e^{-\theta x}}{(1-e^{-\theta x})^2}-\\&\frac{(\alpha-1)x^2c^2e^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left[\left(1-(1-e^{-\theta x})^c\right)\log\left(1-(1-e^{-\theta x})^c\right)\right]^2}\\&\times\left[e^{-\theta x}-e^{-\theta x}\left(1-e^{-\theta x}\right)^c\right.\\&\qquad\times\log\left(1-\left(1-e^{-\theta x}\right)^c\right)\\&\qquad\left.+e^{-\theta x}\left(1-e^{-\theta x}\right)^c\right]\\&-xe^{-\theta x}\left(1-e^{-\theta x}\right)^c\\&-\frac{\left(\frac{1}{\beta}-1\right)cx^2e^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left[1-(1-e^{-\theta x})^c\right]^2}\\&\times\left[(1-\left(1-e^{-\theta x}\right)^c)\right.\\&\qquad\times\left\{(c-1)e^{-\theta x}\left(1-e^{-\theta x}\right)^{-1}-1\right\}\\&\qquad\left.+ce^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}\right]\end{aligned}$$

$$\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial\theta}\right)=-\frac{cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\left(1-(1-e^{-\theta x})^c\right)\log\left(1-(1-e^{-\theta x})^c\right)}$$

$$\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial\theta}\right)=\frac{cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}}{\beta^2\left(1-(1-e^{-\theta x})^c\right)}$$

$$\begin{aligned}\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\theta}\right)=&\frac{xe^{-\theta x}}{(1-e^{-\theta x})}\\&-\frac{(\alpha-1)cxe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}\left(1-\left(1-e^{-\theta x}\right)^c\right)}{\left[\left(1-(1-e^{-\theta x})^c\right)\log\left(1-(1-e^{-\theta x})^c\right)\right]^2}\\&\times\log\left(1-\left(1-e^{-\theta x}\right)^c\right)-\\&\frac{\left(\frac{1}{\beta}-1\right)xe^{-\theta x}\left(1-e^{-\theta x}\right)^{c-1}\left(1-\left(1-e^{-\theta x}\right)^c\right)}{\left[1-(1-e^{-\theta x})^c\right]^2}\end{aligned}$$

Taking expectations on both sides of the equations, we get

$$I(1,1)=-E\left[\frac{\partial^2 L}{\partial\alpha^2}\right]=\xi_{11};I(1,2)=-E\left[\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\alpha}\right)\right]=\xi_{12}$$

$$I(1,3)=-E\left[\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial\alpha}\right)\right]=\xi_{13}$$

$$I(1,4)=-E\left[\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial\alpha}\right)\right]=\xi_{14}$$

$$I(2,1)=-E\left[\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial\beta}\right)\right]=\xi_{21};I(2,2)=-E\left[\frac{\partial^2 L}{\partial\beta^2}\right]=\xi_{22}$$

$$I(2,3)=-E\left[\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\beta}\right)\right]=\xi_{23}$$

$$I(2,4)=-E\left[\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial\beta}\right)\right]=\xi_{24}$$

$$I(3,1)=-E\left[\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial c}\right)\right]=\xi_{31}$$

$$I(3,2)=-E\left[\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial c}\right)\right]=\xi_{32}$$

$$I(3,3)=-E\left[\frac{\partial^2 L}{\partial c^2}\right]=\xi_{33};I(3,4)=-E\left[\frac{\partial}{\partial\theta}\left(\frac{\partial L}{\partial c}\right)\right]=\xi_{34}$$

$$I(4,1)=-E\left[\frac{\partial}{\partial\alpha}\left(\frac{\partial L}{\partial\theta}\right)\right]=\xi_{41}$$

$$I(4,2)=-E\left[\frac{\partial}{\partial\beta}\left(\frac{\partial L}{\partial\theta}\right)\right]=\xi_{42}$$

$$I(4,3)=-E\left[\frac{\partial}{\partial c}\left(\frac{\partial L}{\partial\theta}\right)\right]=\xi_{43};I(4,4)=-E\left[\frac{\partial^2 L}{\partial\theta^2}\right]=\xi_{44}$$

The Fisher information matrix of exponentiated gamma exponential distribution is given by

$$I(\alpha,\beta,c,\theta)=\begin{bmatrix}\xi_{11} & \xi_{12} & \xi_{13} & \xi_{14}\\ \xi_{21} & \xi_{22} & \xi_{23} & \xi_{24}\\ \xi_{31} & \xi_{32} & \xi_{33} & \xi_{34}\\ \xi_{41} & \xi_{42} & \xi_{43} & \xi_{44}\end{bmatrix}$$

6 Application

The procedure can be best explained by using some real data set. For the purpose we use data of Badar and Priest (1982). The estimates of parameters for the proposed model were estimated using the maximum likelihood method through R-software. It may be noted that Raqab, Madi and Kundu (2008) fitted the 3-parameter generalized exponential distribution to the same data set. Single carbon fibres were tested at gauge lengths of 1, 10, 20 and 50 mm. Impregnated tows of 1000 fibres were tested at gauge lengths of 20, 50, 150 and 300 mm. The data Set (of size 63) corresponding to single fibre with 20mm of gauge length is reproduced as under:

1.901 2.132 2.203 2.228 2.257 2.350 2.361 2.396 2.397 2.445
2.454 2.474 2.518 2.522 2.525 2.532 2.575 2.614 2.616 2.618
2.624 2.659 2.675 2.738 2.740 2.856 2.917 2.928 2.937 2.937
2.977 2.996 3.030 3.125 3.139 3.145 3.220 3.223 3.235 3.243
3.264 3.272 3.294 3.332 3.346 3.377 3.408 3.435 3.493 3.501
3.537 3.554 3.562 3.628 3.852 3.871 3.886 3.971 4.024 4.027
4.225 4.395 5.020

Table 1: Parameter estimates for the strength data

Distribution	Weibull	GPW	KGPW	EWED
Estimates	$\hat{\alpha}$=5.049 $\hat{\lambda}$=3.315	$\hat{\alpha}$=3.151 $\hat{\lambda}$=19.824 $\hat{\theta}$=179.363	$\hat{a}$=40.071 $\hat{b}$=1.407 $\hat{\alpha}$=2.215 $\hat{\lambda}$=0.67 $\hat{\theta}$=0.467	$\hat{\alpha}$=8.562 $\hat{\beta}$=0.375 $\hat{\theta}$=1.842 $\hat{c}$=11.637
Log-Likelihood	61.957	69.271	56.346	56.32843
AIC	127.914	144.542	122.692	120.6569
BIC	132.2003	150.9714	133.4077	129.2294
AICC	128.114	144.9488	123.7446	121.3465

The results in Table 1 indicate that the exponentiated gamma exponential distribution, kumaraswamy generalized power Weibull distribution, generalized power Weibull distribution and Weibull distribution fits the strength data reasonably well. However, EGED produces the lowest values of AIC, BIC, AICC and Log-Likelihood and therefore fitted better than KGPW, GPW and Weibull distribution. The Figure 1 also confirms a good fit of the EGED for the data set.

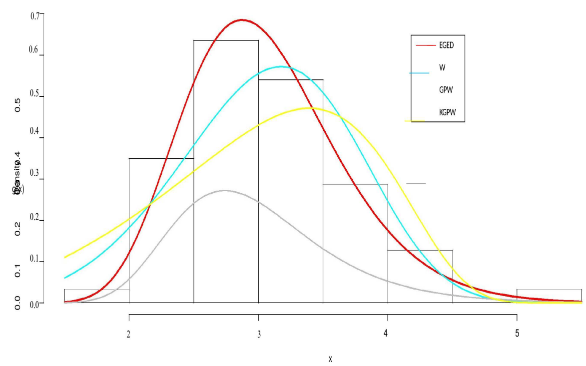

Fig. 1: Estimated pdfs of the EGED and its sub-models for the strength data

7 Conclusion

In this paper, a special case of exponentiated gamma-X family, the exponentiated gamma exponential distribution is derived. The exponentiated gamma exponential distribution is a generalization of exponential distribution. In general, the exponentiated gamma exponential distribution is a generalization of the X distribution. Some mathematical properties are derived and plots of the pdf is presented to show the flexibility of the new distribution. A real data set is fitted to the exponentiated gamma exponential distribution and compared with other known distributions. The results show that the exponentiated gamma exponential distribution provides a good fit to data set. It is hoped that the findings of the paper will be useful for the practitioners in various fields of theoretical and applied sciences.

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