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# An Improved Estimators for Finite Population Mean in Sample Surveys

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**Abstract:** Statisticians are always curious to get more precision in estimating the population parameters and this increase in precision is achieved by using auxiliary information in survey sampling and various statisticians have used correlation coefficient, skewness, coefficient of variation etc. as auxiliary information to get more precision, so in this paper we do the same thing by proposing the modified ratio type estimators in SRSWOR by using the linear combination of coefficient of kurtosis and population deciles as auxiliary information of auxiliary variable. The properties associated with the proposed estimators are assessed by mean square error (MSE) and bias. For illustration we also provide empirical study. From empirical study it is confirmed that our proposed estimators are a class of efficient estimators under percent relative efficiency (PRE) criterion.
**Keywords:** Kurtosis, Deciles, Ratio-type estimators, Mean square error, Bias, Efficiency.

## 1 Introduction

The use of auxiliary information has become indispensable for improving the precision of the estimators of population parameters such as mean and variance of a variable under study. A great variety of techniques such as ratio, product and regression methods of estimation are commonly known in this regard and this auxiliary information can be used either at design stage or at estimation stage or at both stages. Keeping this in view, large number of estimators have been suggested in sampling literature by various authors such as Cochran [19] suggested a classical ratio type estimator for estimation of finite population mean using one auxiliary variable under simple random sampling scheme and the product type estimator to estimate population mean or total of study variable by using auxiliary information when correlation coefficient is negative was given by Murthy [13] and the difference type ratio estimator that outperforms conventional ratio and linear regression estimators was introduced by Rao [18]. The modified ratio type estimators using coefficient of variation and coefficient of kurtosis of the auxiliary variate was given by Upadhyaya and Singh [11] and the proposed family of ratio estimators using known values of some parameters in SRSWOR for estimation of population mean of the study variable was given by Singh and Tailor [5] and also Sisodia and Dwivedi [1], Singh *et al* [6] utilized coefficient of variation of auxiliary variable and proposed some modified ratio estimators. Further improvements are achieved by introducing a large number of modified ratio estimators by using the known values of coefficient of variation, kurtosis, skewness, median, correlation coefficient by Subramani and Kumarapandiyan [9], [7], [8]. Some other authors such as Sharma and Singh [14], Sharma *et al.* [17], Sharma and Singh [15] and Sharma and Singh [16] also done the similar work in different sampling schemes.
The objective of the By providing such noteworthy contributions, we also provide some contribution by proposing some ratio type estimators for estimating the population mean in SRSWOR by using the linear combination of coefficient of kurtosis and population deciles as auxiliary information of auxiliary variable in order to get more precision in estimating population parameters than by existing estimators.
Consider a finite population $Z=\{Z_1, Z_2, Z_3, \ldots, Z_N\}$ of $N$ distinct and identifiable units. Let $Y$ be the study variable with value $Y_i$ measured of $Z_i$, $i=1,2,3,\ldots,N$ giving a vector $Y=\{Y_1, Y_2, Y_3, \ldots, Y_N\}$. The objective is to estimate population mean $\bar{Y}=\frac{1}{N}\sum_{i=1}^{N} Y_i$ on the basis of a random sample.

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Before discussing about the proposed estimators, we will mention the estimators in Literature using the notations given in the next section.

### *1.1 Notations*

| | |
|---|---|
| $N$ | Population size |
| $n$ | Sample size |
| $f = n/N$ | Sampling fraction |
| $Y$ | Study variable |
| $X$ | Auxiliary variable |
| $\bar{X}, \bar{Y}$ | Population means |
| $\bar{x}, \bar{y}$ | Sample means |
| $x, y$ | Sample totals |
| $S_x, S_y$ | Population standard deviations |
| $S_{xy}$ | Population covariance between variables |
| $C_x, C_y$ | Population coefficient of variation |
| $\rho$ | Population correlation coefficient |
| $B(.)$ | Bias of the estimator |
| $MSE(.)$ | Mean square error of the estimator |
| $\hat{Y}_i$ | Existing modified ratio estimator of $\bar{Y}$ |
| $\hat{Y}_{pj}$ | Proposed modified ratio estimator of $\bar{Y}$ |
| $\beta_2$ | Population kurtosis |
| $\beta_1$ | Population skewness |
| $M_d$ | Population Median |
| $QD = \frac{Q_3 - Q_1}{2}$ | Quartile Deviation |
| $D_k \ k = 1,2,\ldots,10$ | Deciles |

***Subscript***

$i$ For existing estimator $j$ For proposed estimators

Based on the above mentioned notations, the mean ratio estimator for estimating the population mean $\bar{Y}$ of the study variable $Y$ is given as

$$\hat{\bar{Y}}_r = \frac{\bar{y}}{\bar{x}}\bar{X} = \hat{R}\bar{X},$$

Where $\hat{R} = \frac{\bar{y}}{\bar{x}} = \frac{y}{x}$ is the estimate of $R = \frac{\bar{Y}}{\bar{X}} = \frac{Y}{X}$.

The bias, constant and the mean square error of the mean ratio estimator is given by

$$B(\hat{\bar{Y}}_r) = \frac{(1-f)}{n}\frac{1}{\bar{X}}(RS_x^2 - \rho S_x S_y), \qquad R = \frac{\bar{Y}}{\bar{X}}, \qquad MSE(\hat{\bar{Y}}_r) = \frac{(1-f)}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y).$$

The mean ratio estimator given above is used to improve the precision of the estimate of the population mean in comparison with the sample mean estimator whenever a positive correlation exists between the study variable and the auxiliary variable.

## 2 Estimators in the Literature

The ratio type estimators suggested by Kadilar and Cingi [2] for the population mean in the simple random sampling using some known auxiliary information on coefficient of kurtosis and coefficient of variation. They showed that their suggested estimators are more efficient than traditional ratio estimator in the estimation of the population mean.
The estimators given by Kadilar and Cingi [2] are given below:

$$\hat{Y}_1 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{\bar{x}} \bar{X},\ \hat{Y}_2 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + C_x)}(\bar{X} + C_x),\ \hat{Y}_3 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_2)}(\bar{X} + \beta_2),$$

$$\hat{Y}_4 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + C_x)}(\bar{X}\beta_2 + C_x),\ \hat{Y}_5 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \beta_2)}(\bar{X}C_x + \beta_2),$$

The biases, related constants and the MSE for the Kadilar and Cingi [2] estimators are respectively given as follows:

$$B(\hat{Y}_1) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_1^2, \quad R_1 = \frac{\bar{Y}}{\bar{X}} \quad MSE(\hat{Y}_1) = \frac{(1-f)}{n}(R_1^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_2) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_2^2, \quad R_2 = \frac{\bar{Y}}{(\bar{X} + C_x)} \quad MSE(\hat{Y}_2) = \frac{(1-f)}{n}(R_2^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_3) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_3^2, \quad R_3 = \frac{\bar{Y}}{(\bar{X} + \beta_2)} \quad MSE(\hat{Y}_3) = \frac{(1-f)}{n}(R_3^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_4) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_4^2, \quad R_4 = \frac{\bar{Y}}{(\bar{X}\beta_2 + C_x)} \quad MSE(\hat{Y}_4) = \frac{(1-f)}{n}(R_4^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_5) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_5^2, \quad R_5 = \frac{\bar{Y}}{(\bar{X}C_x + \beta_2)} \quad MSE(\hat{Y}_5) = \frac{(1-f)}{n}(R_5^2 S_x^2 + S_y^2(1-\rho^2)).$$

Some modified ratio estimators given by Kadilar and Cingi [3] using known value of coefficient of correlation, kurtosis and coefficient of variation are as follows:

$$\hat{Y}_6 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \rho)}(\bar{X} + \rho),\ \hat{Y}_7 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}C_x + \rho)}(\bar{X}C_x + \rho),\ \hat{Y}_8 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + C_x)}(\bar{X}\rho + C_x),$$

$$\hat{Y}_9 = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_2 + \rho)}(\bar{X}\beta_2 + \rho),\ \hat{Y}_{10} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\rho + \beta_2)}(\bar{X}\rho + \beta_2).$$

The biases, related constants and the MSE for the Kadilar and Cingi [3] estimators are respectively given as follows:

$$B(\hat{Y}_6) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_6^2, \quad R_6 = \frac{\bar{Y}}{\bar{X} + \rho} \quad MSE(\hat{Y}_6) = \frac{(1-f)}{n}(R_6^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_7) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_7^2, \quad R_7 = \frac{\bar{Y}C_x}{\bar{X}C_x + \rho} \quad MSE(\hat{Y}_7) = \frac{(1-f)}{n}(R_7^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_8) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_8^2, \quad R_8 = \frac{\bar{Y}\rho}{\bar{X}\rho + C_x} \quad MSE(\hat{Y}_8) = \frac{(1-f)}{n}(R_8^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_9) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_9^2, \quad R_9 = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2 + \rho} \quad MSE(\hat{Y}_9) = \frac{(1-f)}{n}(R_9^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$B(\hat{Y}_{10}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}} R_{10}^2, \quad R_{10} = \frac{\bar{Y}\rho}{\bar{X}\rho + \beta_2} \quad MSE(\hat{Y}_{10}) = \frac{(1-f)}{n}(R_{10}^2 S_x^2 + S_y^2(1-\rho^2)).$$

Some modified ratio estimators proposed by Yan and Tian [20] using the known value of coefficient of skewness and kurtosis are as follows:

$$\hat{Y}_{11} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x} + \beta_1)}(\bar{X} + \beta_1),\ \hat{Y}_{12} = \frac{\bar{y} + b(\bar{X} - \bar{x})}{(\bar{x}\beta_1 + \beta_2)}(\bar{X}\beta_1 + \beta_2).$$

The biases, related constants and the MSE for the Yan and Tian [20] estimators are respectively given as follows:

$$B(\hat{Y}_{11}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{11}^2, \quad R_{11} = \frac{\bar{Y}}{\bar{X}+\beta_1} \qquad MSE(\hat{Y}_{11}) = \frac{(1-f)}{n}(R_{11}^2 S_x^2 + S_y^2(1-\rho^2)).$$

$$B(\hat{Y}_{12}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_{12}^2, \quad R_{12} = \frac{\bar{Y}\beta_1}{\bar{X}\beta_1+\beta_2} \qquad MSE(\hat{Y}_{12}) = \frac{(1-f)}{n}(R_{12}^2 S_x^2 + S_y^2(1-\rho^2)).$$

The estimators proposed by Subramani and Kumarapandiyan [10] by utilizing the auxiliary information of population deciles in the simple random sampling for the estimation of the population mean and the estimators are given below

$$\hat{Y}_i = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}+D_k)}(\bar{X}+D_k),$$

The biases, related constant and the MSE for the Subramani and Kumarapandiyan [10] estimators are respectively given as follows:

$$B(\hat{Y}_i) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_i^2, \quad R_i = \frac{\bar{Y}}{\bar{X}+D_k} \qquad MSE(\hat{Y}_i) = \frac{(1-f)}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2)),$$

Where $i = 13,12,\ldots,22$ and $k = 1,2,3,\ldots,10.$

Estimators proposed by Abid *et al.* [11] by utilizing the auxiliary information of correlation coefficient, coefficient of variation and population deciles and their linear combinations in simple random sampling and the estimators are given as

$$\hat{Y}_i = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\rho+D_k)}(\bar{X}\rho+D_k),$$

The biases, related constant and the MSE for the Abid *et al.* [11] estimators are respectively given as follows:

$$B(\hat{Y}_i) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_i^2, \quad R_i = \frac{\bar{Y}\rho}{\bar{X}\rho+D_k} \qquad MSE(\hat{Y}_i) = \frac{(1-f)}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2)),$$

Where $i = 23,24,\ldots,32$ and $k = 1,2,3,\ldots,10.$

$$\hat{Y}_i = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}C_x+D_k)}(\bar{X}C_x+D_k),$$

$$B(\hat{Y}_i) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_i^2, \quad R_i = \frac{\bar{Y}C_x}{\bar{X}C_x+D_k} \qquad MSE(\hat{Y}_i) = \frac{(1-f)}{n}(R_i^2 S_x^2 + S_y^2(1-\rho^2)),$$

Where $i = 33,34,\ldots,42$ and $k = 1,2,3,\ldots,10.$

## 3 Improved Estimators

Motivated by the mentioned estimators in Section 2, we propose a new class of efficient ratio type estimators using the linear combination of coefficient of kurtosis and population deciles. As deciles divides the series into ten equal parts and every part represents 1/10th of the sample or population and deciles are not affected by extreme values present in data and these proposed estimators perform better than the existing estimators in the literature even if the presence of extreme values in data. The proposed estimators are given below:

$$\hat{Y}_{pj} = \frac{\bar{y}+b(\bar{X}-\bar{x})}{(\bar{x}\beta_2+D_k)}(\bar{X}\beta_2+D_k). \text{ Where } k = 1,2,\ldots,10.$$

The bias, related constant and the MSE for proposed estimator can be obtained as follows:

$$B(\hat{Y}_{pj}) = \frac{(1-f)}{n}\frac{s_x^2}{\bar{Y}}R_j^2, \qquad R_j = \frac{\bar{Y}\beta_2}{\bar{X}\beta_2+D_k} \qquad MSE(\hat{Y}_{pj}) = \frac{(1-f)}{n}(R_j^2 S_x^2 + S_y^2(1-\rho^2)).$$

Where $j = 1,2,\ldots,10$ and $k = 1,2,\ldots,10.$

## 4 Efficiency Comparison

The efficiency conditions for the proposed ratio estimators have been derived algebraically according to usual ratio estimator and existing ratio estimators in literature. The proposed ratio estimators are more efficient than that of the usual ratio estimator if

$$MSE(\hat{Y}_{pj}) \leq MSE(\hat{Y}_r),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2) \leq \frac{(1-f)}{n}(S_y^2 + R^2 S_x^2 - 2R\rho S_x S_y),$$

$$R_{pj}^2 S_x^2 - \rho^2 S_y^2 - R^2 S_x^2 + 2R\rho S_x S_y \leq 0,$$

$$(\rho S_y - RS_x)^2 - R_{pj}^2 S_x^2 \geq 0,$$

$$(\rho S_y - RS_x + R_{pj}^2)(\rho S_y - RS_x - R_{pj} S_x) \geq 0.$$

Condition I: $(\rho S_y - RS_x + R_{pj} S_x) \leq 0$ and $(\rho S_y - RS_x - R_{pj} S_x) \leq 0$

After solving the condition I, we get

$$\left(\frac{RS_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right).$$

Hence,

$$MSE(\hat{Y}_{pj}) \leq MSE(\hat{Y}_r),$$

$$\left(\frac{\rho S_y - RS_x}{S_x}\right) \leq R_{pj} \leq \left(\frac{RS_x - \rho S_y}{S_x}\right),$$

Or

$$\left(\frac{RS_x - \rho S_y}{S_x}\right) \leq R_{pj} \leq \left(\frac{\rho S_y - RS_x}{S_x}\right). \qquad \text{Where } j = 1,2,....,10.$$

From the expressions of the mean square error (MSE) of the proposed estimators and the existing estimators, we have derived the conditions for which the proposed estimators are more efficient than existing modified ratio estimators is as follows:

$$MSE(\hat{Y}_{pj}) \leq MSE(\hat{Y}_i),$$

$$\frac{(1-f)}{n}(R_{pj}^2 S_x^2 + S_y^2(1-\rho^2) \leq \frac{(1-f)}{n}(R_i{}^2 S_x^2 + S_y^2(1-\rho^2)),$$

$$R_{pj}^2 S_x^2 \leq R_i^2 S_x^2,$$

$$R_{pj} \leq R_i,$$

Where $j = 1,2,....,10$ and $i = 1,2,....,42.$

## 5 Applications

The performances of the suggested modified ratio estimators and the existing modified ratio estimators are evaluated by using two populations. For population 1 we use the data of Singh and Chaudhary [4] page 177, and for population 2 we use the data of Murthy [13] page 228. The characteristics of these three populations are given below in table 1, whereas the constants, biases and mean square errors of the usual, existing and suggested estimators are given in table 2.
The percentage relative efficiency (PREs) of the proposed estimators (p), with respective to the existing estimators (e), are computed as

$$PRE = \frac{MSE\, of\ Existing\ Estimator}{MSE\, of\ propoesd\, estimator} \times 100$$

The percentage relative efficiencies of the proposed modified ratio estimators with the usual ratio and existing ratio estimators for population 1 and 2 are given in table 3-8.
The information contained in Table 2 discloses that constants, biases and MSEs for the proposed estimators are much lower as compared to the usual ratio estimator and the existing ratio estimators. Moreover, these values even decrease with increase in the decile orders. From tables 3- 8 it becomes evident that the PREs of the proposed estimators with

regards to usual and the existing estimators are much higher, which indicates that they are more efficient.

**Table 1.** Characteristics of the populations.

| Parameters | Population 1 | Population 2 |
|---|---|---|
| $N$ | 34 | 80 |
| $n$ | 20 | 40 |
| $\bar{Y}$ | 856.4117 | 5182.637 |
| $\bar{X}$ | 208.8823 | 1126.463 |
| $\rho$ | 0.4491 | 0.9413 |
| $S_y$ | 733.1407 | 1854.659 |
| $C_y$ | 0.8561 | 0.354193 |
| $S_x$ | 150.5059 | 845.6097 |
| $C_x$ | 0.7205 | 0.7506772 |
| $\beta_2$ | 0.0978 | -0.063386 |
| $\beta_1$ | 0.9782 | 1.050002 |
| $D_1$ | 70.3 | 369.7 |
| $D_2$ | 76.8 | 460.4 |
| $D_3$ | 108.2 | 597 |
| $D_4$ | 129.4 | 676.8 |
| $D_5$ | 150.0 | 757.5 |
| $D_6$ | 227.2 | 850.2 |
| $D_7$ | 250.4 | 1484.5 |
| $D_8$ | 335.6 | 1810 |
| $D_9$ | 436.1 | 2500 |
| $D_{10}$ | 564.0 | 3480 |

**Table 2.** MSE, bias and constant of the usual ratio estimator, existing estimators and proposed estimators

| Estimators | Constant | | Bias | | MSE | |
|---|---|---|---|---|---|---|
| | Population | | | | | |
| | 1 | 2 | 1 | 2 | 1 | 2 |
| $\hat{Y}_r$ | 4.100 | 4.601 | 4.2704 | 60.8770 | 10539.3 | 189775.1 |
| $\hat{Y}_1$ | 4.100 | 4.601 | 9.1539 | 36.5063 | 16673.5 | 193998.1 |
| $\hat{Y}_2$ | 4.086 | 4.598 | 9.0911 | 36.4577 | 16619.6 | 193746.2 |
| $\hat{Y}_3$ | 4.098 | 4.601 | 9.1454 | 36.5104 | 16666.1 | 194019.4 |
| $\hat{Y}_4$ | 3.960 | 4.650 | 8.5387 | 37.2861 | 16146.6 | 198039.9 |
| $\hat{Y}_5$ | 4.097 | 4.601 | 9.1420 | 36.5117 | 16663.3 | 194026.4 |
| $\hat{Y}_6$ | 4.091 | 4.597 | 9.1147 | 36.4453 | 16639.9 | 193682.3 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\hat{Y}_{7}$ | 4.088 | 4.596 | 9.0995 | 36.4251 | 16626.9 | 193577.6 |
| $\hat{Y}_{8}$ | 4.069 | 4.598 | 9.0149 | 36.4546 | 16554.4 | 193730.5 |
| $\hat{Y}_{9}$ | 4.011 | 4.662 | 8.7630 | 37.4882 | 16338.7 | 199087.0 |
| $\hat{Y}_{10}$ | 4.096 | 4.601 | 9.1349 | 36.5106 | 16654.2 | 194020.7 |
| $\hat{Y}_{11}$ | 4.081 | 4.597 | 9.0688 | 36.4383 | 16600.5 | 193645.9 |
| $\hat{Y}_{12}$ | 4.098 | 4.601 | 9.1452 | 36.5102 | 16666.0 | 194018.4 |
| $\hat{Y}_{13}$ | 3.068 | 3.464 | 5.1243 | 20.6939 | 13222.5 | 112048.6 |
| $\hat{Y}_{14}$ | 2.998 | 3.266 | 4.8974 | 18.3960 | 13025.0 | 100138.9 |
| $\hat{Y}_{15}$ | 2.701 | 3.077 | 3.9725 | 15.5954 | 12236.1 | 85624.8 |
| $\hat{Y}_{16}$ | 2.532 | 2.874 | 3.4902 | 14.2457 | 11823.0 | 78629.5 |
| $\hat{Y}_{17}$ | 2.386 | 2.751 | 3.1010 | 13.0514 | 11489.7 | 72439.9 |
| $\hat{Y}_{18}$ | 1.964 | 2.622 | 2.1003 | 11.8559 | 10902.1 | 66244.4 |
| $\hat{Y}_{19}$ | 1.865 | 1.985 | 1.8934 | 6.7952 | 10455.5 | 40016.2 |
| $\hat{Y}_{20}$ | 1.573 | 1.765 | 1.3472 | 5.3722 | 9987.7 | 32641.5 |
| $\hat{Y}_{21}$ | 1.328 | 1.429 | 0.9601 | 3.5224 | 9656.2 | 23054.5 |
| $\hat{Y}_{22}$ | 1.108 | 1.124 | 0.6686 | 2.1783 | 9406.6 | 16088.9 |
| $\hat{Y}_{23}$ | 2.344 | 3.411 | 2.9912 | 20.0707 | 11395.7 | 108818.5 |
| $\hat{Y}_{24}$ | 2.254 | 3.208 | 2.7676 | 17.7480 | 11204.2 | 96780.7 |
| $\hat{Y}_{25}$ | 1.904 | 2.944 | 1.9741 | 14.9429 | 10524.6 | 82243.2 |
| $\hat{Y}_{26}$ | 1.723 | 2.808 | 1.6169 | 13.6016 | 10218.7 | 75291.5 |
| $\hat{Y}_{27}$ | 1.578 | 2.684 | 1.3553 | 12.4208 | 9994.6 | 69171.7 |
| $\hat{Y}_{28}$ | 1.198 | 2.553 | 0.7818 | 11.2447 | 9503.5 | 63076.7 |
| $\hat{Y}_{29}$ | 1.117 | 1.917 | 0.6800 | 6.3378 | 9416.3 | 37646.0 |
| $\hat{Y}_{30}$ | 0.896 | 1.700 | 0.4369 | 4.9819 | 9208.1 | 30618.7 |
| $\hat{Y}_{31}$ | 0.726 | 1.370 | 0.2869 | 3.2380 | 9079.7 | 21580.8 |
| $\hat{Y}_{32}$ | 0.585 | 1.073 | 0.1862 | 1.9867 | 8993.4 | 15095.6 |
| $\hat{Y}_{33}$ | 2.795 | 3.201 | 4.2530 | 17.6739 | 12476.8 | 96397.0 |
| $\hat{Y}_{34}$ | 2.715 | 2.979 | 4.0132 | 15.3043 | 12270.9 | 84116.2 |
| $\hat{Y}_{35}$ | 2.385 | 2.697 | 3.0981 | 12.5432 | 11487.2 | 69806.4 |
| $\hat{Y}_{36}$ | 2.205 | 2.555 | 2.6466 | 11.2627 | 11100.5 | 63170.1 |

| | | | | | | |
|---|---|---|---|---|---|---|
| $\hat{Y}_{37}$ | 2.053 | 2.427 | 2.2962 | 10.1574 | 10800.4 | 57441.3 |
| $\hat{Y}_{38}$ | 1.634 | 2.294 | 1.4535 | 9.0772 | 10078.7 | 51843.3 |
| $\hat{Y}_{39}$ | 1.539 | 1.670 | 1.2901 | 4.8079 | 9938.8 | 29716.9 |
| $\hat{Y}_{30}$ | 1.269 | 1.465 | 0.8775 | 3.7015 | 9585.5 | 23983.0 |
| $\hat{Y}_{41}$ | 1.052 | 1.163 | 0.6026 | 2.3322 | 9350.0 | 16886.1 |
| $\hat{Y}_{42}$ | 0.864 | 0.898 | 0.4062 | 1.3919 | 9181.8 | 12013.1 |
| $\hat{Y}_{p1}$ | 0.923 | 0.745 | 0.4641 | 2.8697 | 9231.6 | 29343.2 |
| $\hat{Y}_{p2}$ | 0.861 | 0.618 | 0.4041 | 1.9743 | 9180.2 | 24702.8 |
| $\hat{Y}_{p3}$ | 0.651 | 0.491 | 0.2309 | 1.2498 | 9031.9 | 20947.96 |
| $\hat{Y}_{p4}$ | 0.559 | 0.439 | 0.1702 | 0.9974 | 8979.9 | 19639.9 |
| $\hat{Y}_{p5}$ | 0.491 | 0.396 | 0.1315 | 0.8126 | 8946.8 | 18682.5 |
| $\hat{Y}_{p6}$ | 0.338 | 0.356 | 0.0623 | 0.6574 | 8887.5 | 17877.8 |
| $\hat{Y}_{p7}$ | 0.309 | 0.211 | 0.0521 | 0.2306 | 8878.8 | 15666.2 |
| $\hat{Y}_{p8}$ | 0.235 | 0.174 | 0.0301 | 0.1577 | 8859.9 | 15288.3 |
| $\hat{Y}_{p9}$ | 0.183 | 0.128 | 0.0183 | 0.0844 | 8849.8 | 14908.5 |
| $\hat{Y}_{p10}$ | 0.143 | 0.093 | 0.0111 | 0.0443 | 8843.7 | 14700.2 |

**Table 3.** Percent relative efficiency of the proposed estimators with the usual estimator given by Cochran [19] for population I and II respectively.

| Existing estimator | Proposed estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{Y}_{p1}$ | $\hat{Y}_{p2}$ | $\hat{Y}_{p3}$ | $\hat{Y}_{p4}$ | $\hat{Y}_{p5}$ | $\hat{Y}_{p6}$ | $\hat{Y}_{p7}$ | $\hat{Y}_{p8}$ | $\hat{Y}_{p9}$ | $\hat{Y}_{p10}$ |
| $\hat{Y}_r$ | 114.165 | 114.805 | 116.701 | 117.365 | 117.799 | 118.585 | 118.701 | 118.955 | 119.090 | 119.182 |
| $\hat{Y}_r$ | 646.743 | 768.233 | 905.936 | 966.273 | 1015.79 | 1061.51 | 1211.37 | 1241.31 | 1272.93 | 1290.97 |

**Table 4.** Percent relative efficiency of the proposed estimators with the existing modified ratio estimators by Kadilar and Cingi [2] for the population I and II respectively.

| Existing | Proposed estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{Y}_{p1}$ | $\hat{Y}_{p2}$ | $\hat{Y}_{p3}$ | $\hat{Y}_{p4}$ | $\hat{Y}_{p5}$ | $\hat{Y}_{p6}$ | $\hat{Y}_{p7}$ | $\hat{Y}_{p8}$ | $\hat{Y}_{p9}$ | $\hat{Y}_{p10}$ |
| $\hat{Y}_1$ | 180.613 | 181.624 | 184.625 | 185.675 | 186.362 | 187.606 | 187.790 | 188.190 | 188.405 | 188.550 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{Y}_2$ | 180.029 | 181.037 | 184.028 | 185.075 | 185.760 | 186.999 | 187.183 | 187.582 | 187.796 | 187.940 |
| $\hat{Y}_3$ | 180.533 | 181.544 | 184.543 | 185.593 | 186.280 | 187.522 | 187.706 | 188.107 | 188.321 | 188.466 |
| $\hat{Y}_4$ | 174.905 | 175.885 | 178.790 | 179.808 | 180.473 | 181.677 | 181.855 | 182.243 | 182.451 | 182.591 |
| $\hat{Y}_5$ | 180.502 | 181.513 | 184.512 | 185.562 | 186.248 | 187.491 | 187.675 | 188.075 | 188.290 | 188.434 |
| $\hat{Y}_1$ | 661.135 | 785.328 | 926.095 | 987.775 | 1038.39 | 1085.13 | 1238.32 | 1268.93 | 1301.26 | 1319.70 |
| $\hat{Y}_2$ | 660.276 | 784.309 | 924.893 | 986.493 | 1037.05 | 1083.73 | 1236.71 | 1267.28 | 1299.57 | 1317.98 |
| $\hat{Y}_3$ | 661.207 | 785.415 | 926.197 | 987.884 | 1038.51 | 1085.25 | 1238.46 | 1269.07 | 1301.40 | 1319.84 |
| $\hat{Y}_4$ | 674.909 | 801.690 | 945.39 | 1008.35 | 1060.03 | 1107.74 | 1264.12 | 1295.37 | 1328.37 | 1347.19 |
| $\hat{Y}_5$ | 661.231 | 785.443 | 926.231 | 987.919 | 1038.55 | 1085.29 | 1238.50 | 1269.12 | 1301.45 | 1319.89 |

**Table 5.** Percent relative efficiency of the proposed estimators with the existing modified ratio estimators by Kadilar and Cingi [3] for the population I and II respectively.

| Existing estimators | Proposed estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{Y}_{p1}$ | $\hat{Y}_{p2}$ | $\hat{Y}_{p3}$ | $\hat{Y}_{p4}$ | $\hat{Y}_{p5}$ | $\hat{Y}_{p6}$ | $\hat{Y}_{p7}$ | $\hat{Y}_{p8}$ | $\hat{Y}_{p9}$ | $\hat{Y}_{p10}$ |
| $\hat{Y}_6$ | 180.249 | 181.258 | 184.234 | 185.301 | 185.987 | 187.228 | 187.411 | 187.811 | 188.025 | 188.155 |
| $\hat{Y}_7$ | 180.108 | 181.117 | 184.090 | 185.156 | 185.841 | 187.081 | 187.265 | 187.664 | 187.878 | 188.008 |
| $\hat{Y}_8$ | 179.323 | 180.327 | 183.288 | 184.349 | 185.031 | 186.266 | 186.448 | 186.864 | 187.059 | 187.189 |
| $\hat{Y}_9$ | 176.986 | 177.977 | 180.899 | 181.947 | 182.620 | 183.839 | 184.019 | 184.411 | 184.622 | 184.750 |
| $\hat{Y}_{10}$ | 180.404 | 181.414 | 184.393 | 185.460 | 186.147 | 187.389 | 187.572 | 187.972 | 188.187 | 188.317 |
| $\hat{Y}_6$ | 660.059 | 784.050 | 924.588 | 986.167 | 1036.70 | 1083.37 | 1236.31 | 1266.87 | 1299.14 | 1317.55 |
| $\hat{Y}_7$ | 659.702 | 783.626 | 924.088 | 985.634 | 1036.14 | 1082.78 | 1235.64 | 1266.18 | 1298.44 | 1316.84 |
| $\hat{Y}_8$ | 660.223 | 784.245 | 924.818 | 986.413 | 1036.96 | 1083.64 | 1236.61 | 1267.18 | 1299.46 | 1317.88 |
| $\hat{Y}_9$ | 678.477 | 805.929 | 950.388 | 1013.69 | 1065.63 | 1113.60 | 1270.81 | 1302.22 | 1335.39 | 1354.31 |
| $\hat{Y}_{10}$ | 661.212 | 785.42 | 926.203 | 987.890 | 1038.52 | 1085.26 | 1238.47 | 1269.08 | 1301.41 | 1319.85 |

**Table 6.** Percent relative efficiency of the proposed estimators with existing modified ratio estimators by Yan and Tian [20] for the population I and II respectively.

| Existing estimators | Proposed estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{Y}_{p1}$ | $\hat{Y}_{p2}$ | $\hat{Y}_{p3}$ | $\hat{Y}_{p4}$ | $\hat{Y}_{p5}$ | $\hat{Y}_{p6}$ | $\hat{Y}_{p7}$ | $\hat{Y}_{p8}$ | $\hat{Y}_{p9}$ | $\hat{Y}_{p10}$ |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\bar{Y}}_{11}$ | 179.822 | 180.829 | 183.798 | 184.862 | 185.546 | 186.784 | 186.967 | 187.366 | 187.581 | 187.710 |
| $\hat{\bar{Y}}_{12}$ | 180.532 | 181.542 | 184.523 | 185.592 | 186.278 | 187.521 | 187.705 | 188.106 | 188.321 | 188.451 |
| $\hat{\bar{Y}}_{11}$ | 659.934 | 783.903 | 924.414 | 985.982 | 1036.51 | 1083.16 | 1236.07 | 1266.63 | 1298.90 | 1317.30 |
| $\hat{\bar{Y}}_{12}$ | 661.204 | 785.411 | 926.192 | 987.879 | 1038.5 | 1085.25 | 1238.45 | 1269.06 | 1301.39 | 1319.84 |

**Table 7.** Percent relative efficiency of the proposed estimators with existing modified ratio estimators by Subramani and Kumarapandiyan [10] for the population I and II respectively.

| Existing estimator | Proposed estimators | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\bar{Y}}_{p1}$ | $\hat{\bar{Y}}_{p2}$ | $\hat{\bar{Y}}_{p3}$ | $\hat{\bar{Y}}_{p4}$ | $\hat{\bar{Y}}_{p5}$ | $\hat{\bar{Y}}_{p6}$ | $\hat{\bar{Y}}_{p7}$ | $\hat{\bar{Y}}_{p8}$ | $\hat{\bar{Y}}_{p9}$ | $\hat{\bar{Y}}_{p10}$ |
| $\hat{\bar{Y}}_{13}$ | 143.230 | 144.032 | 146.397 | 147.245 | 147.790 | 148.776 | 148.922 | 149.239 | 149.410 | 149.513 |
| $\hat{\bar{Y}}_{14}$ | 141.091 | 141.881 | 144.211 | 145.046 | 145.582 | 146.554 | 146.697 | 147.010 | 147.178 | 147.280 |
| $\hat{\bar{Y}}_{15}$ | 132.545 | 133.287 | 135.476 | 136.261 | 136.765 | 137.677 | 137.812 | 138.106 | 138.264 | 138.360 |
| $\hat{\bar{Y}}_{16}$ | 128.071 | 128.788 | 130.902 | 131.660 | 132.147 | 133.029 | 133.159 | 133.443 | 133.596 | 133.688 |
| $\hat{\bar{Y}}_{17}$ | 124.460 | 125.157 | 127.212 | 127.949 | 128.422 | 129.279 | 129.406 | 129.682 | 129.830 | 129.920 |
| $\hat{\bar{Y}}_{18}$ | 118.095 | 118.756 | 120.706 | 121.405 | 121.854 | 122.667 | 122.788 | 123.049 | 123.190 | 123.275 |
| $\hat{\bar{Y}}_{19}$ | 113.257 | 113.891 | 115.761 | 116.432 | 116.863 | 117.642 | 117.758 | 118.009 | 118.144 | 118.225 |
| $\hat{\bar{Y}}_{20}$ | 108.190 | 108.796 | 110.582 | 111.222 | 111.634 | 112.379 | 112.493 | 112.729 | 112.858 | 112.936 |
| $\hat{\bar{Y}}_{21}$ | 104.599 | 105.185 | 106.912 | 107.531 | 107.929 | 108.649 | 108.755 | 108.987 | 109.112 | 109.187 |
| $\hat{\bar{Y}}_{22}$ | 101.895 | 102.466 | 104.148 | 104.751 | 105.139 | 105.840 | 105.944 | 106.170 | 106.292 | 106.365 |
| $\hat{\bar{Y}}_{13}$ | 381.855 | 453.587 | 534.890 | 570.515 | 599.752 | 626.747 | 715.225 | 732.904 | 751.575 | 762.225 |
| $\hat{\bar{Y}}_{14}$ | 341.268 | 405.375 | 478.037 | 509.875 | 536.004 | 560.130 | 639.204 | 655.003 | 671.690 | 681.208 |
| $\hat{\bar{Y}}_{15}$ | 291.805 | 346.620 | 408.750 | 435.974 | 458.316 | 478.945 | 546.558 | 560.068 | 574.335 | 582.474 |
| $\hat{\bar{Y}}_{16}$ | 267.965 | 318.302 | 375.356 | 400.356 | 420.872 | 439.816 | 501.905 | 514.312 | 527.414 | 534.887 |
| $\hat{\bar{Y}}_{17}$ | 246.871 | 293.246 | 345.809 | 368.840 | 387.742 | 405.195 | 462.396 | 473.826 | 485.897 | 492.782 |
| $\hat{\bar{Y}}_{18}$ | 225.757 | 268.166 | 316.233 | 337.295 | 354.580 | 370.540 | 422.849 | 433.301 | 444.340 | 450.636 |
| $\hat{\bar{Y}}_{19}$ | 136.373 | 161.991 | 191.027 | 203.750 | 214.191 | 223.832 | 255.430 | 261.744 | 268.412 | 272.215 |
| $\hat{\bar{Y}}_{20}$ | 111.240 | 132.137 | 155.822 | 166.200 | 174.717 | 182.581 | 208.356 | 213.506 | 218.946 | 222.048 |
| $\hat{\bar{Y}}_{21}$ | 78.5685 | 93.3275 | 110.056 | 117.386 | 123.402 | 128.956 | 147.161 | 150.798 | 154.640 | 156.831 |
| $\hat{\bar{Y}}_{22}$ | 54.8301 | 65.1299 | 76.8041 | 81.9195 | 86.1175 | 89.9937 | 102.698 | 105.237 | 107.918 | 109.447 |

**Table 8.** Percent relative efficiency of the proposed estimator existing modified ratio estimators by Abid *et al.* [11] for the population I and II respectively.

| Existing | **Proposed estimators** |
|---|---|

| | $\overset{J}{Y}_{p1}$ | $\overset{J}{Y}_{p2}$ | $\overset{J}{Y}_{p3}$ | $\overset{J}{Y}_{p4}$ | $\overset{J}{Y}_{p5}$ | $\overset{J}{Y}_{p6}$ | $\overset{J}{Y}_{p7}$ | $\overset{J}{Y}_{p8}$ | $\overset{J}{Y}_{p9}$ | $\overset{J}{Y}_{p10}$ |
|---|---|---|---|---|---|---|---|---|---|---|
| $\overset{J}{Y}_{23}$ | 123.442 | 124.133 | 126.171 | 126.902 | 127.371 | 128.221 | 128.347 | 128.621 | 128.768 | 128.857 |
| $\overset{J}{Y}_{24}$ | 121.367 | 122.047 | 124.051 | 124.769 | 125.231 | 126.066 | 126.190 | 126.459 | 126.604 | 126.691 |
| $\overset{J}{Y}_{25}$ | 114.006 | 114.644 | 116.527 | 117.201 | 117.635 | 118.420 | 118.536 | 118.789 | 118.925 | 119.007 |
| $\overset{J}{Y}_{26}$ | 110.692 | 111.312 | 113.140 | 113.795 | 114.216 | 114.978 | 115.091 | 115.336 | 115.468 | 115.548 |
| $\overset{J}{Y}_{27}$ | 108.265 | 108.871 | 110.658 | 111.299 | 111.711 | 112.456 | 112.567 | 112.807 | 112.936 | 113.014 |
| $\overset{J}{Y}_{28}$ | 102.945 | 103.521 | 105.221 | 105.830 | 106.222 | 106.931 | 107.035 | 107.264 | 107.387 | 107.461 |
| $\overset{J}{Y}_{29}$ | 102.000 | 102.571 | 104.256 | 104.859 | 105.247 | 105.949 | 106.053 | 106.28 | 106.401 | 106.475 |
| $\overset{J}{Y}_{30}$ | 99.7454 | 100.303 | 101.950 | 102.541 | 102.920 | 103.607 | 103.708 | 103.930 | 104.049 | 104.120 |
| $\overset{J}{Y}_{31}$ | 98.3545 | 98.9052 | 100.529 | 101.111 | 101.485 | 102.162 | 102.262 | 102.480 | 102.598 | 102.669 |
| $\overset{J}{Y}_{32}$ | 97.4197 | 97.9651 | 99.5737 | 100.150 | 100.520 | 101.191 | 101.290 | 101.506 | 101.623 | 101.693 |
| $\overset{J}{Y}_{33}$ | 135.153 | 135.909 | 138.141 | 138.941 | 139.455 | 140.385 | 140.523 | 140.823 | 140.984 | 141.081 |
| $\overset{J}{Y}_{34}$ | 132.922 | 133.667 | 135.861 | 136.648 | 137.154 | 138.069 | 138.204 | 138.499 | 138.657 | 138.753 |
| $\overset{J}{Y}_{35}$ | 124.433 | 125.130 | 127.184 | 127.921 | 128.394 | 129.251 | 129.377 | 129.653 | 129.802 | 129.891 |
| $\overset{J}{Y}_{36}$ | 120.244 | 120.917 | 122.903 | 123.615 | 124.072 | 124.900 | 125.022 | 125.292 | 125.432 | 125.519 |
| $\overset{J}{Y}_{37}$ | 116.993 | 117.648 | 119.580 | 120.273 | 120.718 | 121.523 | 121.642 | 121.902 | 122.041 | 122.125 |
| $\overset{J}{Y}_{38}$ | 109.176 | 109.787 | 111.590 | 112.236 | 112.651 | 113.403 | 113.514 | 113.756 | 113.886 | 113.965 |
| $\overset{J}{Y}_{39}$ | 107.660 | 108.263 | 110.041 | 110.678 | 111.087 | 111.829 | 111.938 | 112.177 | 112.305 | 112.383 |
| $\overset{J}{Y}_{30}$ | 103.833 | 104.414 | 106.129 | 106.744 | 107.138 | 107.853 | 107.959 | 108.189 | 108.313 | 108.388 |
| $\overset{J}{Y}_{41}$ | 101.282 | 101.849 | 103.522 | 104.121 | 104.506 | 105.203 | 105.307 | 105.531 | 105.652 | 105.725 |
| $\overset{J}{Y}_{42}$ | 99.4605 | 100.017 | 101.659 | 102.248 | 102.626 | 103.311 | 103.412 | 103.633 | 103.751 | 103.823 |
| $\overset{J}{Y}_{23}$ | 370.847 | 440.511 | 519.471 | 554.069 | 582.462 | 608.679 | 694.607 | 711.776 | 729.909 | 740.252 |
| $\overset{J}{Y}_{24}$ | 329.823 | 391.78 | 462.005 | 492.776 | 518.029 | 541.346 | 617.768 | 633.038 | 649.165 | 658.363 |
| $\overset{J}{Y}_{25}$ | 280.280 | 332.931 | 392.607 | 418.756 | 440.215 | 460.03 | 524.972 | 537.949 | 551.653 | 559.470 |
| $\overset{J}{Y}_{26}$ | 256.589 | 304.789 | 359.422 | 383.36 | 403.005 | 421.145 | 480.598 | 492.478 | 505.024 | 512.180 |
| $\overset{J}{Y}_{27}$ | 235.733 | 280.016 | 330.207 | 352.200 | 370.249 | 386.914 | 441.535 | 452.449 | 463.975 | 470.549 |
| $\overset{J}{Y}_{28}$ | 214.962 | 255.342 | 301.111 | 321.166 | 337.625 | 352.821 | 402.629 | 412.582 | 423.092 | 429.087 |
| $\overset{J}{Y}_{29}$ | 128.295 | 152.396 | 179.712 | 191.681 | 201.504 | 210.574 | 240.301 | 246.241 | 252.514 | 256.092 |
| $\overset{J}{Y}_{30}$ | 104.347 | 123.948 | 146.166 | 155.900 | 163.89 | 171.267 | 195.444 | 200.275 | 205.377 | 208.288 |
| $\overset{J}{Y}_{31}$ | 73.5462 | 87.3618 | 103.021 | 109.882 | 115.513 | 120.713 | 137.754 | 141.159 | 144.755 | 146.806 |

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{Y}_{32}$ | 51.4450 | 61.1089 | 72.0624 | 76.8619 | 80.8007 | 84.4377 | 96.3578 | 98.7396 | 101.255 | 102.690 |
| $\hat{Y}_{33}$ | 328.516 | 390.227 | 460.174 | 490.822 | 515.975 | 539.199 | 615.318 | 630.528 | 646.591 | 655.753 |
| $\hat{Y}_{34}$ | 286.663 | 340.513 | 401.548 | 428.292 | 450.241 | 470.506 | 536.928 | 550.200 | 564.216 | 572.211 |
| $\hat{Y}_{35}$ | 237.896 | 282.585 | 333.237 | 355.432 | 373.646 | 390.464 | 445.586 | 456.600 | 468.232 | 474.867 |
| $\hat{Y}_{36}$ | 215.280 | 255.720 | 301.557 | 321.642 | 338.124 | 353.344 | 403.225 | 413.192 | 423.719 | 429.723 |
| $\hat{Y}_{37}$ | 195.757 | 232.530 | 274.210 | 292.472 | 307.460 | 321.300 | 366.658 | 375.721 | 385.292 | 390.752 |
| $\hat{Y}_{38}$ | 176.679 | 209.868 | 247.486 | 263.969 | 277.497 | 289.987 | 330.925 | 339.104 | 347.743 | 352.671 |
| $\hat{Y}_{39}$ | 101.274 | 120.298 | 141.861 | 151.309 | 159.063 | 166.222 | 189.688 | 194.377 | 199.329 | 202.153 |
| $\hat{Y}_{30}$ | 81.7320 | 97.0862 | 114.488 | 122.114 | 128.371 | 134.150 | 153.088 | 156.872 | 160.868 | 163.147 |
| $\hat{Y}_{41}$ | 57.5460 | 68.3570 | 80.6098 | 85.9785 | 90.3846 | 94.4529 | 107.787 | 110.451 | 113.265 | 114.870 |
| $\hat{Y}_{42}$ | 40.9400 | 48.6305 | 57.3474 | 61.1668 | 64.3014 | 67.1956 | 76.6816 | 78.5771 | 80.5789 | 81.7207 |

## 6 Conclusions

Thus from the above study we conclude that our proposed estimators for estimating population mean in simple random sampling without replacement are more efficient than the classical and existing estimators as their MSE and bias is lower than the classical and existing estimators and also by PRE criterion we also conclude that they are more efficient than the classical and existing estimators, hence we strongly recommend that our suggested estimators preferred over the classical and existing estimators for use in practical applications.

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